Application of the Economic Threshold for Interseasonal Pest Control

Darwin C. Hall and L. Joe Moffitt

We show how an interseasonal pest control problem can be simplified to enable an intraseasonal model to be empirically applied, extending the range of application of the intraseasonal model. Three alternative economic thresholds are compared. The optimal solution requires repeated computations by the farmer to compute the profit maximizing dose, with a corresponding threshold, for each pest infestation. Two alternative decision rules require a single computation by the farmer for the threshold and dosage rate. An empirical illustration shows that, relative to the optimal solution which is computationally burdensome to the farmer, little net revenue is lost by using one of the thresholds based upon a simpler decision rule.

Pesticides are an important input in modern agricultural production. Negative externalities from pesticide use most notably include the risk to humans of chronic toxic effects, such as carcinogenesis, mutagenesis, teratogenesis, as well as acute toxic effects. Farmers apply pesticides to control damage and prophylactically to reduce uncertainty of the subjective probability of damage. Estimates of the economic threshold and optimal dosage rates can reduce prophylactic applications.

Hall and Moffitt developed an intraseasonal model of pest control for an individual farmer, based upon the subjective probability of the level of infestation. Moffit et al. apply that model to corn nematodes. Here we extend the range of application of that model to a class of interseasonal pest control problems, with an empirical illustration. With our empirical illustration, we compare three alternative economic thresholds for interseasonal control of the Egyptian Alfalfa Weevil in California.

Economic Thresholds

Headley was the first economist to rigorously define the economic threshold. In Headley's model, the control cost is solely a function of the level to which the pest population is reduced, and doesn't depend upon the pre-application infestation. Moreover, there is no fixed (application) cost, only variable (material) cost. Due to the assumed cost function, if the pest population exceeds Headley's threshold by any amount, the optimal dose is that which reduces the pest population back to the threshold. His model allows for a single application during the growing season where the timing of the application is predetermined. If the threshold is exceeded, then the threshold is the post-application
pest population. Hall and Norgaard [1973, 1974] generalized the concept of the economic threshold to include the timing of application. In their model, the economic threshold is defined as the pre-application pest population, the level at which it is economical to apply pesticides, and does not equal the level to which the pest population is reduced. Hall and Norgaard show that the solution to the threshold depends upon the initial infestation.

In the economics literature, the first empirical estimation of an economic threshold was by Talpaz and Borosh. They estimated the optimal dose and optimal frequency of applications. The calculation of the Talpaz–Borosh threshold is based upon an assumed constant pre-application infestation level. Consequently, the economic threshold they estimate depends upon the level of the infestation, but for a farmer that value varies from season to season. Moreover, the Talpaz–Borosh threshold is the post-application pest population. That is, the farmer would have to apply the dosage rate which resulted in a particular reduction. Finally, they assume that the optimal post-application pest population is the same for every application, rather than developing a model which is truly dynamic in that it allows for other than a steady-state solution.

Three economic thresholds can be derived from a three equation system comprised of a profit equation, yield function (including pest damage), and population (growth/kill) function, given respectively by equations (1), (2), and (3):

\[ \pi = r_Y - \delta f - \nu P \]  
\[ Y = Y_o - \alpha B + u \]  
\[ B = B_{\text{ex}} \exp(-\beta P) + v \]

where

- \( \pi \) = net return after pest management cost,  
- \( r_Y \) = price per unit of yield,  
- \( Y \) = yield net of pest damage,  
- \( Y_o \) = parameter which equals yield with no pest damage,  
- \( f \) = fixed pesticide application cost,  
- \( \delta = 1 \), if an application is made and 0 otherwise,  
- \( \nu \) = price per unit of pesticide,  
- \( P \) = rate of pesticide application,  
- \( B \) = post-application pest population,  
- \( B_{\text{ex}} \) = pre-application pest population,  
- \( \alpha \) = damage coefficient,  
- \( \beta \) = efficacy coefficient,  

and where \( u \) and \( v \) are random disturbances reflecting the sampling procedure which generates our data, with \( E[u] = E[v] = 0; E[u^2] = \sigma_u^2, E[v^2] = \sigma_v^2, E[uv] = \sigma_{uv} \) and are assumed to be bivariate normal.

The first threshold starts with a fixed dosage rate, \( P^N \), which could equal the maximum legal rate specified on the label of the pesticide container. The farmer applies \( P^N \) if the pest population exceeds the Fixed Dosage Rate Threshold:

\[ B^N = \frac{(f + \nu P^N)}{(\alpha r_Y \delta - \exp(-\beta P^N))} \]

The second threshold allows the dosage rate to vary. Equations (1)–(3) generalize Headley’s model to allow the threshold to depend upon the initial infestation. The Generalized Headley Threshold and dosage rate are given by

\[ B^H = \frac{\nu}{r_Y \beta \alpha} \]  
\[ P^H = \frac{[\ln(r_Y \beta \alpha B_{\text{ex}}/\nu)]}{\beta} \]

The third economic threshold will be referred to here as the Stochastic Threshold, derived by the authors from a stochastic model:

\[ \nu \]  
\[ P^H = \frac{[\ln(r_Y \beta \alpha B_{\text{ex}}/\nu)]}{\beta} \]
\[ B^* = \frac{(f + v_p P^*)}{(\alpha r (1 - \exp(-f P^*)))} \] (7)

and the optimal dosage rate from the stochastic model is the unique positive root of

\[ \sigma \beta \alpha_r (2\pi)^{-\frac{1}{2}} \exp\left(-\frac{\beta^2}{2}\right) \left( \frac{f + v_p P^* - \mu \alpha_r (1 - \exp(-\beta P^*))}{\alpha_r (1 - \exp(-\beta P^*))} \right) - \left(1 - f\left[ f + v_p P^* - \mu \alpha_r (1 - \exp(-\beta P^*)) \right] / \sigma \alpha_r (1 - \exp(-\beta P^*)) \right) = 0 \] (8)

where \( \phi(\cdot) \) is the standard normal distribution. Derivations of equations (4) through (8) can be found in either Hall and Moffitt [1982] or Moffitt et al. [1984].

**Interseasonal Pest Control**

We motivate the model which follows with the example of the Egyptian Alfalfa Weevil, a pest which has continued to attract attention [Regev et al., 1983].

The Egyptian Alfalfa Weevil (EAW) is a major pest of alfalfa in California and is capable of inflicting significant damage to the crop if an outbreak occurs and is uncontrolled. Current control practice requires a series of pesticide applications to control developing weevils during late winter and spring. Aside from costly multiple pesticide applications for the EAW, current practice is also plagued by secondary outbreaks of lepidopterous, mite and aphid pests, some of which migrate to other crops. Additional pesticide applications then become necessary on alfalfa and other crops.

Economists and entomologists [see Regev et al. and references cited there] have developed an experimental EAW control program as an alternative to current practice. The experimental program consists of a single pesticide application in late fall. A further advantage of the experimental program is that the timing of the single application avoids secondary pest outbreaks.

Current and experimental control programs are depicted in the context of EAW alfalfa development in Figure 1. Adult weevils typically emerge from aestivation outside the field during mid fall and migrate to the alfalfa plants over a period of several weeks to feed, mate, and lay eggs during late fall and early winter. The eggs hatch in 5–10 days and the larvae move to the terminal of the stems to feed in the shoot apex. The larvae develop over the next few weeks and pupate prior to the first cutting of the alfalfa stand. After a couple of weeks adults emerge from the pupae in early spring, feeding on the alfalfa until early summer when they migrate out of the field for summer aestivation. The experimental program exploits the EAW lifecycle by reducing the population level prior to oviposition. Implementation of the program requires that the population be estimated during late fall and that a control decision be made then.

Let \( A_e \) be the number of adults emerging after summer aestivation. Let \( A_p \) be the post-application population. Then the kill function, \( k(\cdot) \) is given by:

\[ A_p = k(A_e, P) \] (9)

The adults, \( A_e \), lay eggs which pupate and turn into larvae. According to Gutierrez et al., the larvae population, \( B_n \), grows as a function of degree days, \( t \), reaches a maximum and then diminishes due to death and because the larvae become adults and leave the field.

So the larvae growth function, \( b(\cdot) \), can be written:

\[ B_t = b(A_p, t) \] (10)

Figure 2 is based upon Gutierrez et al., and shows the larvae population growing to a maximum, \( B_{max} \) at time \( t^* \) in degree days.

If no pesticides are applied, then \( B_{max} \) becomes

\[ B_{max} = b(A_e, t^*) \] (11)

and if pesticides are applied,
\[ B_{\text{max}} = b(A, t^*) \] (12)

Substitute (9) into (12):

\[ B_{\text{max}} = b(k(A, P), t^*) \] (13)

For pests, such as the EAW which have a single maximum, \( B_{\text{max}} \), as shown in Figure 2, (11) is a one-to-one correspondence. Suppressing the constant \( t^* \) and taking the inverse of (11):

\[ A, = b^{-1}(B_{\text{max}}) \] (14)

Substituting (14) into (13),

\[ B_{\text{max}} = b(k(b^{-1}(B_{\text{max}}), P), t^*) \] (15)

Again, suppressing the constant \( t^* \) and simplify notation with \( B(\cdot) = b(k(b^{-1}(\cdot))) \) so that

\[ B_{\text{max}} = B(P, B_{\text{max}}) \] (16)

Equation (16) states that the maximum pest population depends upon both what the maximum would have been in the absence of pesticide application and the amount of pesticide applied.

Damage to the crop is a function of the time path of the pest population. Given the growth function, \( b(\cdot) \), shown in Figure 2, a one-to-one correspondence exists between the time path of the larvae to the maximum larvae population, \( B_{\text{max}} \). Therefore, we simply let yield be a function of \( B_{\text{max}} \):

\[ Y = y(B_{\text{max}}) \] (17)

Equation (17) corresponds to Headley's equations (1) and (4), and lends a justification for his damage function, which he does not provide.

At this point, we need specific functional forms for (16) and (17). We can use the same functional forms given by equations (2) and (3) with appropriate changes in notation:

\[ Y = y_\alpha - \alpha B_{\text{max}} + u \] (18)

Equation (18) corresponds to Headley's equations (1) and (4), and lends a justification for his damage function, which he does not provide.

At this point, we need specific functional forms for (16) and (17). We can use the same functional forms given by equations (2) and (3) with appropriate changes in notation:

\[ Y = Y_\alpha - \alpha B_{\text{max}} + u \] (18)

A number of alternative functional forms were analyzed by Hall. The authors also compared estimation between an additive and a multiplicative error term for the kill equation (19). We also compared FIML with nonlinear 3SLS.
Parameters from equation (1) are \( r_v = \$100/\text{ton}; f = \$4.35/\text{acre}; \) \( P^N = .25 \text{ gallons/acre}; \) and \( v_p = \$30/\text{gallon}. \) Equations (16) and (17) were estimated by full information maximum likelihood. The estimated form is

\[
Y = 5.83 - 0.012 B_{\text{max}}; \quad R^2 = 0.88 \quad (20)
\]

\[
B_{\text{max}} = 159.23 \exp(-12.03P); \quad R^2 = 0.94 \quad (21)
\]

where numbers in parentheses are estimated asymptotic standard errors.

Observations on the uncontrolled pest population were used to estimate the mean and standard deviation of \( B_{\text{max}}. \) Normality was also assumed; thus \( B_0 \), the probability distribution for \( B_{\text{max}} \), is

\[
g_{B_0}(x) = \frac{1}{\sqrt{2\pi}a^2} \exp\left[-\frac{1}{2a^2}(x - \mu)^2\right] \quad (22)
\]

where \( \mu = 155.7 \text{ insects/sq. ft.} \) and \( a = 19.4 \text{ insects/sq. ft.} \) were estimated from the control test plots on which no pesticide was applied.

One purpose of this empirical illustration is to compare the three economic thresholds given by equations (4), (5) and (7). These three thresholds are not, however, directly comparable. Recall that the Generalized Headley Threshold (equation 5) requires the farmer to compute both the threshold and the dosage rate, which depends upon \( B_{\text{max}} \) and must therefore be computed each growing season by the farmer. The Fixed Dosage Rate Threshold given by equation (4) only requires that the farmer apply the maximum rate on the label if the threshold is exceeded: No computation is required beyond the initial estimation of the threshold. The Stochastic Threshold also only requires that the farmer applies the optimal dose if the threshold is exceeded: Again, only an initial computation is required. In order to compare these three thresholds, we calculated expected net revenue and expected insecticide use, given in Table 1.5

\[5 \] The formulas for expected net revenue and expected insecticide use are given in the Appendix.
As expected, the largest expected net revenue is achieved with the optimal application rate associated with the Generalized Headley Threshold. In this numerical illustration, the efficiency loss of switching to the Fixed Dosage Rate Threshold is $3.50 per acre. However, only $0.03 per acre in expected net revenue is lost by replacing the Generalized Headley Threshold by the Stochastic Threshold.

Several limitations of the data prohibit direct application of our results to EAW control in California. First, the pesticide material is not registered for use to control the EAW and is more persistent and efficacious than those materials currently registered. Second, the data were collected from a single location and consequently may not reflect conditions for farms in other parts of the region. Third, the data are old and the EAW may have subsequently developed resistance to the insecticide. Fourth, only one material and only one time of application are permitted by the model.

Computation costs in pest control decision making may be avoided with little concomitant loss in net revenue, at least in the empirical illustration provided here.

References


Expressions for expected profit and expected insecticide use for pest control decisions based on the three thresholds given in equations (4), (5) and (7) are provided below. Notation is the same as in the text. Values reported in Table 1 of the text were calculated using these formulas.

Expected profit and expected insecticide use based on equation (4) are respectively

\[
\begin{align*}
E_{n}[\text{Profit}](B^*, P^*) &= r_Y \phi((B^* - \mu)/\sigma) \\
&+ \alpha r_Y (\phi(1 - \phi((B^* - \mu)/\sigma)) \\
&+ \mu([1 - \phi((B^* - \mu)/\sigma)]) \\
&+ [r_Y \phi((B^* - \mu)/\sigma)]
\end{align*}
\]

where the subscript \(E_n\) denotes that the expectation is taken with respect to \(B_{n}^{\text{max}}\).

Expected profit and expected insecticide use based on the generalized Headley threshold (equation (5)) are respectively

\[
\begin{align*}
E_{n}[\text{Profit}](B^*, P^*) &= r_Y \phi((B^* - \mu)/\sigma) \\
&+ \alpha r_Y (\phi(1 - \phi((B^* - \mu)/\sigma)) \\
&+ \mu([1 - \phi((B^* - \mu)/\sigma)]) \\
&+ [r_Y \phi((B^* - \mu)/\sigma)]
\end{align*}
\]

Expected profit and expected insecticide use based on equation (7) utilize the same formulas (A.1 and A.2) with \(B^*\) replacing \(B^\text{max}\) and \(P^*\) replacing \(P^\text{max}\).