Functional Form Specification In The Quarterly Demand For Red Meats In Canada

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Most demand studies, particularly those for food products, choose somewhat arbitrarily either a linear or a log-linear functional form. Since choice of functional form has implications for elasticities and their properties over time, either form may be considered restrictive. An empirical test of functional form in the short-run (quarterly) demand for four red meats is presented in this paper. For beef demand, neither a linear nor a log-linear form is appropriate. Similarly for pork and lamb demand, a linear functional form is not appropriate, but the log-linear one is. For veal demand, although data supported both functional forms, a linear form is preferred when seasonal influences are accounted for.

Most studies of demand for food products have used two functional forms — a linear (additive) formulation, where product demand is hypothesized as a linear combination of specified independent variables; and, a log-linear form where the logarithm of quantity demanded is linearly related to the logarithms of specified independent variables. If one hypothesizes that the demand (QD) for a food product (k) is determined by its own price (Pk), by prices of close substitutes or complements (Pk′) and by consumer disposable income (DI), then the deterministic relationships[1] are written as:

$$(1) \quad QDk_t = \beta_0 + \beta_1 Pk_t + \beta_2 Pk'_t + \beta_3 DI_t, \text{ and}$$

$$(2) \quad QDk_t = \beta_0 (Pk_t)^{\beta}1 (Pk'_t)^{\beta}2 (DI_t)^{\beta}3$$

Based on economic theory, there is no *a priori* reason for selecting either form. Works by Eisner, Chow, and Zarembka point out such indecision between the two forms in studies of monetary theory. Choice of functional form in agricultural product demand studies analysis is a difficult area that has not received adequate attention in the past.[2] Either a linear specification or a log-linear specification have been commonly used. Studies[3] by Kulshreshtha and Wilson, Brandow, Kulshreshtha and Reimer, Hassan and Katz, Tryfos and Tryphonoponlous, Hayenga and Hacklander, and Fuller and Ladd have assumed a linear functional form. Briemyer, Fox, and Working used a log-linear specification.

In empirical market demand studies, choice of functional form is an important decision since it has implications for price and income elasticities. Both of the formulations in equations (1) and (2) may be restrictive. The log-linear specification implies that the price and income elasticity coefficients are constant over the entire range of the specified variable. Such an implication is re-

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[1]A deterministic relation refers to the deletion of the stochastic error term.

[2]Difficulties arise primarily because there is no *a priori* rationale for the choice of one or the other form.

[3]This list of studies is restricted to those on the demand for red meats, or individual products comprising red meats.

strictive if a sample contains large variation in incomes or price or, over time, similar variations occur. On the other hand, a linear functional form implies that the income elasticity, if it is less than one, is rising and tends toward unity as consumer incomes increase over time (Chang, 1977A, p. 355). Similarly, if the income elasticity is greater than one, it would tend towards unity as income increases. Restrictiveness of the linear form assumption can be argued on the basis of growth in consumption as family income increases. With lower income, certain foods such as meats are a luxury, and their consumption (on a per capita basis) should increase with rising income. After consumption reaches a certain level, associated with the growth in income, the product may become a necessity. Any further growth in its consumption will slow down. The income elasticity in some cases, therefore, would decrease or even become negative as incomes increase.

The Box-Cox parametric transformation allows the estimation procedure to select the functional form that satisfies a pre-selected objective function.[4] The selection is determined by the data and associated analytical results and not by the researcher's restrictions. Since such restrictions generally are specified arbitrarily, a test of functional form should be useful to many applied economists working in the demand area, and especially those working on demand for red meats. A test of the functional form has been reported by Chang for red meats demand in the United States. Similar results for Canada are not available. Furthermore, the work by Chang (1977A) utilized annual data. Whether his results would equally apply to short-run demand functions remains to be tested.[5]

The primary objective of this paper is to estimate a more general demand function for individual red meats in Canada using quarterly data. Demand relationships for four individual red meats — beef, pork, lamb, and veal — are examined using quarterly data for the 1958 (I Quarter) to 1977 (IV Quarter) period.

A General Functional Form

The Box-Cox parametric transformation was applied first to monetary theory analyses by Zarembka, and several other studies have extended his original model and specification. The Box-Cox transformation procedure allows the estimation procedure to select a functional form which maximizes the likelihood function of the sample. The functional form is selected on the basis of the value the parameter of transformation for which the likelihood has attained a maximum. Since there are no restrictions on the value of this parameter value, it can be called a general functional form. For a particular red meat the general function form model can be represented by equation (3). Deleting the time subscript,

$$(3) \quad QDk^{(\lambda)} = \beta_0 + \beta_1 Pk^{(\lambda)} + \beta_2 Pk'^{(\lambda)} + \beta_3 DI^{(\lambda)}$$

where, QDk, Pk, Pk′, and DI are as defined above, and λ is the parameter of transformation, such that,

$$(4.1) \quad QDK^{(\lambda)} = [\,(QDK)^{\lambda} - 1]/\lambda$$

$$(4.2) \quad Pk^{(\lambda)} = [\,(Pk)^{\lambda} - 1]/\lambda, \text{ and so on}$$

$$\text{For } \lambda \neq 0$$

As $\lambda \to 1$ equation (3) becomes equation (1), and as $\lambda \to 0$, equation (3) approaches equation (2)[6]. Thus, both linear and log-linear functional forms are special sub-cases of the more general functional form.

[4]This objective function is described in the next subsection.

[5]A short-run framework is used here for a time frame shorter than a year.

Equation (3) like equation (1) allows elasticities to vary with changes in income levels or prices, such that,

$$(5.1) \qquad \epsilon_{DI} = \beta_3 \cdot \frac{DI}{QDk} \quad \lambda$$

and

$$(5.2) \qquad \epsilon_P = \beta_1 \cdot \frac{Pk}{QDk} \quad \lambda$$

The signs of the elasticities are still determined by the regression coefficient, but the elasticities can decrease or increase over time depending upon the sign of estimated λ.

It is assumed in the estimation of parameters for equation (3) that the disturbance term e_t can be brought in additively. Furthermore, the errors are assumed to be normally and independently distributed with mean zero and a constant variance. The normality assumption for e_t is, however, approximate, as suggested by Spitzer (1977, p. 119). For this assumption to be valid, the error term should extend from $-\infty$ to $+\infty$, and consequently, the dependent variable, as shown by equation (4.1), should also extend over this range. For certain values of λ's, this range may not be possible.

Parameters in equation (3) are estimated with the maximum likelihood approach.[7] The likelihood in relation to the original observation QDk is expressed as

$$(6) \qquad \frac{1}{(2\pi)^{\frac{1}{2}n} \sigma^n} J(\lambda, QDk) \exp.$$

$$- \frac{\Sigma(QDk^{(\lambda)} - \beta_0 - \beta_1 Pk^{(\lambda)} - \beta_2 Pk'^{(\lambda)} - \beta_3 DI^{(\lambda)})^2}{2\,\sigma^2}$$

$$\text{where} \qquad J(\lambda, QDk) = \prod_{i=1}^{n} \frac{dY_i^{(\lambda)}}{dY_i}$$

[6]The proof of equivalence between equation (3) and (1) as $\lambda \to 1$, is straightforward. As $\lambda \to 0$, equations (4.1) and (4.2) become $\ell n\,(QDk_t)$ and $\ell n\,(Pk_t)$, respectively. Proof of this equivalence is shown in Zarembka.

[7]For more details, see Box and Cox (1964, p. 215-216).

The maximum likelihood criteria, according to Box and Cox (1964, p. 215) leads directly to point estimates of the parameters and to approximate tests and confidence intervals based on the Chi-square distribution.

The estimation procedure is iterative in nature. A range of trial values of λ is selected, and parameters estimated. A criterion called likelihood is calculated for each trial value of λ, such that

$$(7) \qquad L_{max}(\hat{\lambda}) = -\frac{n}{2} \log \hat{\sigma}^2$$

$$(\hat{\lambda}) + (\hat{\lambda} - 1) \sum_{t=1}^{n} \log QDk_t$$

where $\hat{\sigma}^2(\hat{\lambda})$ is the maximum likelihood estimate of variance for the given value of $\hat{\lambda}$. The value of $\hat{\lambda}$ which maximizes the criterion in equation (7) is then chosen to be the parameter of functional form for a given behavioral equation.

Model Specification and Data Needs

The demand function for individual red meats was specified according to equation (3). Specifically, demand for a red meat was specified in the following manner:

$$(8) \quad \text{Beef: } PCB_t = f\,[PB_t, PK_t, PCH_t, DI_t]$$

$$(9) \quad \text{Pork: } PCP_t = f\,[PB_t, PK_t, PCH_t, DI_t]$$

$$(10) \quad \text{Lamb: } PCL_t = f\,[PB_t, PK_t, PL_t, PV_t, PCH_t, DI_t]$$

$$(11) \quad \text{Veal: } PCV_t = f\,[PB_t, PK_t, PV_t, PCH_t, DI_t]$$

where: PCB_t, PCP_t, PCL_t, PCV_t = Per capita domestic disappearance of Beef, Pork, Lamb, and Veal, respectively, during the time period t; PB_t, PK_t, PL_t, and PV_t,

PCH_t = Retail Price Index for Beef, Pork, Lamb, Veal, and Chicken, respectively, during the time period t; and DI_t = Personal Disposable Income per capita, in 1961 dollars.

This specification was selected on the basis of several criteria, such as results of previous studies, *a priori* expectations with respect to sign of the elasticity, and statistical significance of estimated coefficients. For example, Reimer and Kulshreshtha (p. 63) have suggested that veal prices show virtually no influence on beef consumption, and vice-versa. This was also supported by results of Tryfos and Tryphonoponlous (p. 649).

Quarterly time series data for the period extending from the first quarter of 1958 to the fourth quarter of 1977 were used to estimate equations (8) through (11). Choice of this period was based on availability of quality and comparable data.[8]

Domestic disappearance of a meat was based only on production under Federal inspection; no attempt was made to account for farm slaughter.[9] The retail price of beef was measured as an average price in cents per pound of three retail cuts: sirloin steak, prime rib roast, and hamburger. For the period 1975-1977 (inclusive) these prices were reported in terms of an index based on 1971. The price series were extended in cents per pound using these indexes. These data were obtained from Statistics Canada (62-002). The retail price of pork was a simple average price in cents per pound of three retail cuts: shoulder roast, rib chops and ham. Retail price of lamb was measured in cents per pound for lamb leg roast.[10] Veal prices were obtained for veal loin chops. Chicken prices were also in cents per pound for grade A eviscerated. Personal disposable income was measured in dollars in adjusted annual levels.[11] These data were collected from Statistics Canada (11-003).

Short-run variations in quantity demanded are often associated with seasonal factors that recur about every 12 months. Logan and Boles have reported the presence of such influences for U.S.A.; such effects likely are present in the Canadian data too.

Seasonal influences are accounted for by using three binary variables, BY1, BY2, BY3. Variable BYI was set equal to 1 if an observation belonged to the first quarter, and equal to 0 otherwise. Variables BY2 and BY3 are defined similarly.

Two different models were employed to test for the general functional form. Model 1 contains equations 8 to 11 with prices measured in constant dollars, thereby removing secular changes in the general price level. Model 2 adds the seasonal Binary Variables to Model 1. These demand functions were estimated using a single equation method, applying the method of maximum likelihood. A computer program as developed by Chang (1977B) was used.

Empirical Results

Results for the value of λ that maximizes equation (7) are presented in Tables 1 and 2 for the two models. Results were satisfactory on the basis of R^2 and d-W statistics. The R^2 values are reasonably high for most equations, except for pork. The hypothesis for autocorrelation in the error term is either inconclusive at $\alpha = 0.01$ or rejected.

All signs are consistent with *a priori* expectations. Consumer incomes show a positive influence on beef consumption, but a negative

[8]All data correspond to Canada as a whole, and are either totals or average for the quarter. Data were obtained from secondary sources such as Statistics Canada publications.

[9]These data series were obtained from Agriculture Canada.

[10]For 1977, this price was combined with other meat prices. The correlation between the latter series and price of lamb leg roast for the period 1974 to 1977 was used to estimate the series for 1977.

[11]Statistics Canada reports national income on a quarterly basis in two forms: i) incomes received during that quarter, and ii) incomes received during the quarter converted to a 12-month period. The latter series was used to create the data on personal disposable income.

influence on consumption of pork, lamb, and veal. With minor exceptions these coefficients are significant statistically.

Results for the model 2, as shown in Table 2, are significant only for pork and veal demand. The coefficients for seasonal binary variables in the demand for beef and lamb are not significant.[12] These results are not presented since model 2 is equivalent to model 1.

Test of Hypothesis About Functional Form

In Tables 1 and 2, functions were chosen at a value of $\hat{\lambda}$ where the $L_{max}(\hat{\lambda})$, as expressed by equation (7), was the highest. To test whether the approximate 95 percent confidence interval around $\hat{\lambda}$ would still include zero or one, that is, the linear or logarithmic functional form, the following criterion was used:

$$(12) \quad L_{max}(\hat{\lambda}) - L_{max}(\lambda) < \tfrac{1}{2}\chi_1^2(\alpha = 0.05)$$

The plot of values of the maximum likelihood criteria was of a fairly flat surface for veal and pork.[13] This is also reflected in a wider 95 percent confidence region for these products as compared to beef.

In Table 3, the functional form for demand for red meats is tested. For beef demand, the hypothesis of a linear or logarithmic form is rejected. This conclusion is based on the fact that the 95 percent confidence interval does not contain either $\lambda = 1$ or $\lambda = 0$. For pork and lamb, the functional form is not linear, and for veal the demand function could be linear or log-linear. Based on Model 2, no change in the conclusion reached with respect to the above was made for pork demand. However, the hypothesis that veal demand function is log-linear is rejected.

Elasticity Estimates

The estimated coefficients in Tables 1 and 2 were converted into elasticities and compared with those of the linear and log-linear functional forms. Results are presented in Table 4.

For all products, the relative magnitudes of mean price and income elasticities obtained by using general, linear and log-linear forms were not different. For beef and pork, the response to a change in price or in income is estimated to be inelastic in nature. Results were different for the other two products. For example, direct price elasticities were greater than one for lamb and veal demand. Quantities demanded of these products therefore fluctuate more than beef and pork for a given change in price. The lowest price elasticity was for beef which suggests that beef has become a staple food in the diets of Canadians, and does not undergo large variations due to price changes in the short-run.

The general functional form parameters also indicate that the income elasticity for beef decreased from 0.5547 in 1958 (II) to 0.190 in 1977 (IV). Similarly for veal, using model II, the change in the income elasticity decreased from −.373 to −1.452 for the same period.

Conclusions

This paper has tested the functional form for demand for individual red meats. Results indicate that: 1) The demand for beef cannot be expressed in either a log or linear functional form; 2) For pork, and lamb the hypothesis of a linear demand function is not accepted; and 3) For veal the hypothesis that the demand function is linear or log-linear is accepted.

These results have implications for applied econometric research for other agricultural

[12]A F-test was carried out to test for the marginal contribution made by the seasonal binaries. For beef a F value of 1.18 and for lamb a value of 0.87 were obtained. Compared to a critical F value at $\alpha = 0.5$ and $\nu_1 = 3$, $\nu_2 = 65$ of 2.75, these values were insignificant, suggesting that model 1 and model 2 are statistically equivalent for these two products.

[13]These plots are not presented here for space considerations. They simply show the value of the maximum likelihood plotted against value of $\hat{\lambda}$.

TABLE 1. Results for Model 1 for General Functional Form

Demand for	β_0 (t)	Regression Coefficient and t-values						R^2 (F)	d-W	$\hat{\lambda}$
		PB	PK	PL	PV	PCH	DI			
Beef	−502.93	−1.309	2.433	--	--	−.591	825.27	.831	1.23	−1.66
	(−16.97)	(−2.50)	(3.58)			(−2.72)	(16.87)	(98.00)		
Pork	4.342	.741	−1.009	--	--	−.443	−.461	.659	1.465	−.37
	(9.01)	(4.83)	(−7.76)			(−5.35)	(−2.83)	(39.25)		
Lamb	−2.04	.530	1.5788	−1.992	2.30	−.542	−.868	.783	1.823	−.03
	(−1.17)	(.70)	(4.6)	(−6.52)	(4.38)	(−1.25)	(−3.87)	(48.4)		
Veal	2.469	.108	.051	--	−.158	−.028	−.0096	.769	1.624	+.57
	(4.57)	(2.28)	(2.30)		(−5.74)	(−1.23)	(−5.64)	(53.57)		

TABLE 2. Results for Model 2 for General Functional Form

Demand For	β_0 (t)	Regression Coefficient and t-values									R^2 (F)	d-W	$\hat{\lambda}$
		PB	PK	PL	PV	PCH	DI	BY1	BY2	BY3			
Pork	2.342	.395	−.560	--	--	−.266	−.068	−.0002	−.0006	−.0006	.732	1.563	−.64
	(13.02)	(5.58)	(−9.07)			(−5.59)	(−3.04)	(−1.42)	(3.99)	(−4.23)	(31.91)		
Veal	1.253	.054	.027	--	−.064	−.011	−.0022	−.137	−.020	−.200	.827	1.850	+.79
	(3.09)	(2.88)	(3.18)		(−6.44)	(−1.13)	(−6.73)	(−3.15)	(−.46)	(−4.73)	(48.33)		

[a]For Beef and Lamb Model 2 did not have any significant coefficients for the seasonal binaries. Therefore, the results are equivalent to those for Model 1.

TABLE 3. Optimal Value and 95 Percent Confidence Interval for $\hat{\lambda}$

Demand For	Optimum Value for $\hat{\lambda}$	95% Confidence Interval: Lower Limit	95% Confidence Interval: Upper Limit	Hypothesis For: Linear Form	Hypothesis For: Log Form
		Model 1			
Beef	−1.66	−1.93	−1.39	Rejected	Rejected
Pork	− .37	−1.56	+ .82	Rejected	Accepted
Lamb	− .03	− .35	+ .29	Rejected	Accepted
Veal	+ .56	− .09	+1.21	Accepted	Accepted
		Model 2			
Pork	− .64	−1.83	+ .55	Rejected	Accepted
Veal	+ .79	+ .32	+1.26	Accepted	Rejected

products. An initial assumption of linearity or log-linearity may not always be supported by the data. A violation of the assumption implies specification error, which could result in biased estimates of parameters. Furthermore, if certain functions are not linear, they may also affect the econometric simulations for models where some functions may be linear while others are not. Simulation of mixed models are more complex, and quite frequently linear approximations for nonlinear variables are used.

To summarize, this paper has demonstrated that an appropriate functional form for the demand for a red meat may depart from either the linear or the logarithmic formulations. In view of these findings, empirical studies on demand for other food products should be cognizant of this property. Furthermore, since evidence indicates that elas-

TABLE 4. Quarterly Demand Elasticities for Red Meats, By Functional Form[a]

Demand For	Func. Form	Income Elasticity: I[b]	Income Elasticity: II	Direct Price Elasticity: I	Direct Price Elasticity: II	Cross-Price Elasticity: I	Cross-Price Elasticity: II	Cross-Price Elasticity: Product
Beef	General	.488	--	− .144	--	.290	--	Pork
	Log	.500	--	− .198	--	.345	--	Pork
	Linear	.495	--	− .171	--	.406	--	Pork
Pork	General	−.076	−.030	− .543	− .394	.392	.231	Beef
	Log	−.072	−.068	− .545	− .566	.380	.395	Beef
	Linear	−.061	−.062	− .543	− .573	.344	.365	Beef
Lamb	General	−.694	--	−1.751	--	1.389	--	Pork
	Log	−.677	--	−1.749	--	1.382	--	Pork
	Linear	−.267	--	−1.752	--	1.299	--	Pork
Veal	General	−.576	−.641	−1.801	−1.866	.495	.630	Pork
	Log	−.625	−.598	−1.916	−1.888	.564	.652	Pork
	Linear	−.540	−.570	−1.737	−1.763	.455	.580	Pork

[a]All elasticities are calculated at the mean.
[b]I and II refer to Model I and Model II, respectively.

ticities for some products are changing over time, estimation of parameters using the time-varying parameters method as advocated by Belsley and Kuh may be more appropriate. However, as Wallace has suggested, statistical technique is a complement and not a substitute for the logical foundation in a field of application. These results hopefully are useful toward this purpose.

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