Managing Risk by Coordinating Investment, Marketing, and Production Strategies: Comment

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The importance of enterprise selection in managing risk has been recognized since the early beginnings of the agricultural economics profession (Taylor). Production and marketing activities represent different but important sources of risk to farmers. Risk management can be critical to continued viability of a farm entity.

Recently, Johnson and Boehlje (JB) conducted a study to identify investment, marketing, and production strategies that minimized total variance, subject to achievement of a minimum ending net worth level. They used a multiyear quadratic programming (QP) model in the analysis. Their study represents an original approach to identifying risk management strategies for farmers. An apparent error in model specification, however, raises questions about the value of their results.

The major criticism of the JB approach lies in the specification of the representative farm’s activities. As stated in the paper, “Activities for investment, financing, production, marketing, input supply (including land rental), and accounting are specified for each period” (p. 159). Uncertain activities that enter the objective function in additive form (e.g., investments) are correctly modeled in a QP model as separate activities. Those variables entering the objective function in multiplicative fashion, however, cannot be treated in the same manner. As an example, gross revenue is calculated as price times output. If these two random variables (price and output) are normally distributed, their product (gross revenue) is a new random variable that is univariate normally distributed. As a result, one activity (gross revenue) would be entered in the QP model to represent the production and marketing activities. Specification of mean and variance for the new univariate distribution is quite different than that specified by JB when treating production and marketing as separate activities.

For the new univariate distribution (NUD)

\[ E[PY] = \mu_p \mu_y + \sigma_{py} \]

where \( \mu_p \) is expected price, \( \mu_y \) expected yield, and \( \sigma_{py} \) covariance between price and yield. Similarly, variance for the new univariate distribution is

\[ V[PY] = \sigma_p^2 \mu_y^2 + 2 \mu_p \mu_y \sigma_{py} + \sigma_y^2 \sigma_p^2 \]

where \( \sigma_p^2 \) is price variance and \( \sigma_y^2 \) yield variance (Goodman). Covariance between the two univariate normal distributions is

\[ C[PY] = \frac{-\sigma_{py}}{\sigma_p \sigma_y} \]

The NUD approach has been used by others in the agricultural economics profession (Tew and Boggess) and has been recommended by Anderson, Dillon, and Hardaker when treating price and output as random variables (chap. 2). Musser, Mapp, and Barry also suggest this approach when making production and marketing linkages, as was done by JB.

The NUD results are in contrast to the relationships used by JB when treating production and marketing as separate activities. Assuming everything produced is marketed, expected return, variance, and covariance expressions are

\[ E[PY]_{JB} = \mu_p \mu_y + \sigma_{py} \]

\[ V[PY]_{JB} = \sigma_p^2 \mu_y^2 + 2 \mu_p \mu_y \sigma_{py} + \sigma_y^2 \sigma_p^2 \]

\[ C[PY]_{JB} = \frac{-\sigma_{py}}{\sigma_p \sigma_y} \]

Equations (4), (5), and (6) all differ from their counterparts derived by Bohrnstedt and Goldberger. The NUD approach has been used by others in the agricultural economics profession (Tew and Boggess) and has been recommended by Anderson, Dillon, and Hardaker when treating price and output as random variables (chap. 2). Musser, Mapp, and Barry also suggest this approach when making production and marketing linkages, as was done by JB.

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terparts among (1), (2), and (3). The differences between the JB and NUD results are

\[
E[PY] - E[PY]^{JB} = \sigma_{pY} \\
V[PY^2] - V[PY^2]^{JB} = \sigma_{pY}^2 + \sigma_{pY}^2(\sigma_1^2 + \sigma_2^2 - 1) + 2\mu_{pY}(\mu_1 - 1) \\
C[Y, Y_1, Y_2] - C[Y, Y_1, Y_2]^{JB} = \mu_{pY}\sigma_{pY}(\mu_1 - 2) + \sigma_{pY}(\sigma_{pY} + \mu_1\mu_2 - 2) + \sigma_{pY}(\sigma_{pY} + \mu_1\mu_2 - 2) - 2\mu_{pY} - \mu_{pY}\sigma_{pY}(\mu_1 - 2)
\]

Based on (7), JB return either over- or understates expected return by \( \sigma_{pY} \). The most important result to note in equations (8) and (9) is the effect of expected price and price variance on the difference between the two approaches. The larger expected price and price variance is for each activity, the greater the error in estimating variance by the JB method. In addition, activities with higher variance because of high price mean and (or) variance will tend to enter the optimal solution at greater levels for the JB approach than will occur for the NUD approach. Finally, the covariance \( \sigma_{pY} \) may not be zero in many situations, making it a serious error to omit or understate the influence of this variable.

Example Problem

An empirical example is useful to contrast the difference between the JB and NUD approaches. Since criticism is being directed at the JB article, it seemed appropriate to use data from the article in the example problem. This was easier said than done, however, because of the difficulty in verifying calculations for some of the data, particularly variance (table 3 of JB) for crop production activities. A reading of the model documentation provided by Johnson suggests that historical yield variance was somewhat combined with cost-of-production variance to form production variance. A clear explanation concerning the procedure used was not given, and several attempts by the author to duplicate the JB calculations were not successful. Use of incorrect data in the example should not be a serious problem, however, as long as consistency in use exists between the two approaches.

The example problem focuses on production and marketing of corn and soybeans. For the JB approach, the model formulation was

\[
\text{Max } D'X - \left(\frac{r}{2}\right)X'QX \\
\text{subject to } \\
C + S \leq 1 \\
110C - MC = 0 \\
40S - MS = 0,
\]

where

\[
D' = [-49.54 -27.68 1.29 3.20] \\
X' = [C S MC MS] \\
Q = \begin{bmatrix}
317.31 & 182.86 & -4.81 & -9.21 \\
182.86 & 116.05 & -2.50 & -5.17 \\
-4.81 & -2.50 & .11 & .20 \\
-9.21 & -5.17 & .20 & .42
\end{bmatrix},
\]

\( C, S, MC, \) and \( MS \) are, respectively, corn production, soybean production, corn marketing, and soybean marketing activities. The first constraint causes production activity levels to be in percentage terms. The second and third constraints require all production to be marketed. All data were obtained from tables 2 and 3 of the JB article with the exception of expected yields for corn and soybeans (110 bu/acre and 40 bu/acre, respectively), which were obtained from Johnson.

The model formulation for the NUD approach was much the same as for the JB approach, although matrix and vector sizes and contents were different. Values used in the NUD approach were

\[
D' = [87.55 91.11] \\
X' = [CC CS] \\
Q = \begin{bmatrix}
552.0 & 339.2 \\
339.2 & 612.3
\end{bmatrix},
\]

where \( CC \) and \( CS \) are the combined corn and soybean production and marketing activities, respec-
approach would tend to recommend strategies that have a larger percentage of production in a crop with higher marketing mean price and variance.

Implications

Both theoretical and empirical results presented here suggest the JB study has serious methodological problems. Their approach underestimates total variance and tends to generate recommendations biased toward activities with high price means and variances. The example problem indicated how large an error can occur both in $E$–$SD$ frontier calculations and crop mix recommendations where the wrong approach is used. Given the error in their approach, the conclusions and recommendations made by JB are very much open to question. Researchers in general would do well to follow the NUD approach when analyzing the combined production-marketing problem in a QP framework.

References


