Tax Reform and Beef Cow Replacement Strategy

Robert Innes and Hoy Carman

This paper models optimal beef cow replacement strategy in a stochastic environment under U.S. income tax rules effective before and after the Tax Reform Act of 1986. Under each tax regime, the producer's buy versus raise decision and optimal culling age choice are analyzed. Per-cow profit levels are also calculated. Results of the numerical analysis indicate that tax law changes, particularly the loss of the capital gains exclusion and restrictions on preproduction expensing, will have significant effects on both optimal decisions and profitability of beef cow operations. When provisions of the Tax Reform Act of 1986 are fully effective in 1988, the optimum age for culling beef cows will increase, as will the after-tax costs of beef cow operations.

Key words: asset replacement, beef cow management, culling, risk, tax reform.

Beef cattle producers must decide when to replace aging cows and whether to raise or buy the replacements. A variety of income tax provisions influences both choices, including availability of tax credits, special treatment of capital gains income, preproductive expense limitations, depreciation schedules, and marginal tax rates. This paper examines optimal replacement strategies and associated per-cow profit levels under U.S. tax rules effective before and after the Tax Reform Act of 1986 (TRA).

Conceptually, the analysis draws on asset replacement theory developed by Burt, Perrin, and Chisholm (1966). As in Burt's model, stochastic disturbances are permitted by incorporating probabilities of a cow dying, of a live cow not producing a calf, and of early culling because of illness or poor productivity.

Several other authors have also investigated the application of replacement theory to beef cow operations, but in settings and with objectives that are quite different than here. The previous research which is closest in spirit to this study was done by Kay and Rister. Using a nonstochastic model, they derived optimal culling ages and buy versus raise strategies in the presence of tax provisions prevailing in the mid-1970s. Another article by Rister and Kay analyzed the impact of capital gains provisions on optimal replacement age but not the buy versus raise choice and, again, in a nonstochastic environment. Other studies have included stochastics but not taxes or the buy versus raise choice (Bentley, Waters, and Shumway; Bentley and Shumway; Trapp).

The importance of income tax rules to beef cow management practices can hardly be exaggerated. Investment tax credits (ITCs) give a major bonus to buyers; by subsidizing replacement heifer purchase costs, ITCs also encourage early replacement. Favorable tax treatment of capital gains income permits raisers to sell cows and exclude 60% of the proceeds from taxable income; they thereby favor a "raise" strategy and also shorter cow lives. Preproductive expense provisions and depreciation schedules affect the timing of tax deductions, with different effects depending on whether a "buy" or a "raise" strategy is pursued. As the expensing provisions become more liberal and the depreciation regimes more rapid, tax benefits of replacement increase, tending to favor earlier culling. Moreover, the for-

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mer provisions have a greater effect on raisers, who incur greater preproductive costs. In contrast, the latter schedules have a greater effect on buyers who must capitalize the purchase cost of a replacement. Finally, tax rates affect the relative magnitudes of tax-generated and nontax-generated cash flows as well as their temporal distribution; thus, changes in cow replacement strategy will likely result from revised tax rate schedules.

The TRA changes all of these tax provisions. Expensing of preproductive costs, ITCs and capital gains exclusions have been eliminated. In addition, tax depreciation has been accelerated and the number of tax brackets, as well as the relevant marginal tax rates, have been reduced. A priori, the combined effects of all these changes on optimal beef cow replacement strategy are ambiguous.

The following analysis is intended to sort out these tax effects and their implications for producer profits in a realistic model of the cow-calf producer's choice problem. The analytical model is presented in the next section, followed by a description of the data inputs. The final section presents results of the analysis.

The Model

Theoretical Development

Because risk of cow death implies a risk of the cow's unplanned and early replacement, the starting point for our analysis is Burt's model of asset replacement under risk. Formally, let T be planned replacement age and ϕ a management policy parameter which affects cash flows. Then the present value of cash flows from a single animal (asset) without replacement is

$$(1)\quad PV(T, \phi) = \left\{R(0, \phi) + \sum_{t=1}^{T} \beta^t R(t, \phi)\cdot Q(t-1)\right\} + Q(T)M(T, \phi)\beta^T,$$

where β is the discount factor; $R(t, \phi)$ is the net expected revenues from an animal in year t, assuming it lives to year $(t - 1)$; $Q(t)$ is the probability that the animal survives to age t; and $M(T, \phi)$ is the animal's salvage value in year T. In order to convert the present value in (1) into a value with replacement, the operator is assumed to replace the animal whenever it dies and then to repeat the policy regime of (T, ϕ). Thus, if $L(T)$ is the lifespan of the animal (a random variable with a distribution which depends on T), the value of the repeated replacement policy is

$$(2)\quad V(T, \phi) = PV(T, \phi) + E\{\beta^{L(T)^r} V(T, \phi)\},$$

where the expectation operator is over $L(T)$.[1] Solving for $V(T, \phi)$,[2]

$$(3)\quad V(T, \phi) = \{1 - E(\beta^{L(T)})\}^{-1}\cdot PV(T, \phi).$$

The optimal T and ϕ will maximize $V(T, \phi)$ in (3).

Structure of the Beef Cow Replacement Problem

This analysis adapts and applies the foregoing model to the beef cow producer's choice problem by positing (*a*) stationary prices, technology, and opportunity costs; (*b*) stochastic birth rates; and (*c*) stochastic cow illness and death. Each operator maximizes the present value of the infinite stream of risky cash flows by choice of planned culling age, T, and a replacement strategy of either raising his own replacements or purchasing bred yearlings.

The adapted optimization problem has several important features:

Timing. In practice, heifer and steer calves (in the winter cycle) are weaned at the beginning of November. A heifer is bred in the subsequent spring to produce a weaned offspring two years after its own weaning. Producers pursuing a buy replacement strategy purchase a bred yearling heifer approximately one year after it is weaned. These timing attributes of the cow replacement problem are incorporated in the analysis below.

Large numbers. The producer will be assumed to operate a large ranch so that the law of large number applies. Hence, if pursuing a raise strategy, the producer will anticipate death loss and will plan replacements accordingly.[3]

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[1] $L(T)$ takes on a value of $t < T$ with probability $Q(t - 1) - Q(t)$ and a value of T with probability $Q(T - 1)$.

[2] The nonstochastic analog to (3) involves no risk of unplanned replacement; thus, without death risk, $L(T)$ would equal T with a probability of one and the familiar annuity operator, $(1 - \beta^T)^{-1}$, would emerge (as in Perrin and Chisholm 1974).

[3] The large ranch assumption also implies that IRC Sec. 179 expensing is irrelevant to the present choice problem. This tax provision permits the expensing of up to \$10,000 of capital acquisitions for operators with capital investments of less than \$200,000 for the year; before 1987, the allowance was \$5,000. For an operation which is large (but not too large to be disqualified from Sec. 179 treatment), the \$10,000 expense allowance will be

Cow death. When a cow dies, the producer leaves the remains to a rendering plant at no cost or benefit to himself.

Cost/revenue realization. Costs of maintaining a heifer or cow are assumed to be incurred at the beginning of each period, while revenues are realized at the end of each period.

Bayesian updating. In a crude way, this analysis recognizes that when a cow does not give birth to a live offspring, the producer will update his subjective probability assessments of the cow's productivity. This updating will often lead to early culling. Specifically, this analysis assumes that a fixed proportion of unproductive cows, γ, are culled.[4]

Fixed capacity. Implicit in the infinite horizon construction is a fixed capacity for animals. In other words, the problem of interest here is not the choice of optimal capacity but rather the choice of management strategy which maximizes the value of each capacity unit.[5] Analytically, this per-unit criterion must be recognized explicitly and value measures adjusted accordingly. Specifically, a raise decision requires use of extra capacity in the raising process, capacity which is not required with a buy strategy. Hence, the value of cash flows must be adjusted by a factor which reflects the number of animal slots required for each productive cow.

Extra replacements. For every replacement heifer raised on a ranch, there are, in general, other heifers kept beyond calving age. The actual replacements are chosen among the possible replacements according to timing of cycling and weight. The University of California Cooperative Extension Service, for example, recommends that two heifers be kept for every replacement heifer desired. In practice, however, ranchers retain only 10% to 20% more heifers than they will need for replacement.[6] Some of these extra heifers are retained to deal with death risk for planned replacements. And most are not kept for a full year after weaning but rather for only seven to eight months. Adjusting the percentage so as to reflect full-year-equivalent retentions and not to reflect an allowance for death of replacements (which is incorporated elsewhere in the analysis), extra retention of 10% is considered realistic. In the analysis, an extra retention parameter, η, is varied around 10% to determine the sensitivity of predicted outcomes.

Taxable income. In order to avoid carryback/carryforward complications, we assume that the cow-calf operator has sufficient income to use all tax deductions and tax credits during the period in which they arise.

Problem Formalization

The following notation is now introduced:

Prices:

P_c	heifer calf price
P_s	steer calf price
P_y	bred yearling heifer price
P_h	cull cow price, age greater than two
P_{h2}	slaughter heifer price

Weights:

$\bar{W}_{ct}$	mean weight of a heifer calf born to a cow of age t
$\bar{W}_{st}$	mean weight of a steer calf born to a cow of age t
$\bar{W}_{ht}$	mean weight of age t cow
$\bar{W}_c$	mean weight of a heifer calf retained for replacement
$\bar{W}_y$	mean weight of a yearling heifer kept for replacement
OC_t	cost of maintaining a cow of age t for one year
θ_t	probability that a cow of age t produces live offspring
q_t	probability that a cow of age (t − 1) will live to age t
v_t	proportion of productive age t cows which are culled due to illness
γ	proportion of nonproductive cows which are culled
η	extra retention for replacement heifers
β	$(1 + i)^{-1}$, where i is the opportunity cost of funds
T	planned culling age

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used regardless of replacement practices. However, for the small operator, some of this allowance may go unused when a raise approach to replacement is taken. In this case, the allowance gives an additional incentive to buy some of the replacements, an incentive which is not considered here.

[4] Other studies (Bentley and Shumway; Bentley, Waters, and Shumway; Trapp) assume that all unproductive heifers are culled (i.e., $\gamma = 1$).

[5] If annual capacity rents were incorporated in cash flows, the cow-calf operator would maximize $PV(T, \phi)$ in (1) and choose a capacity level (herd size) such that the maximized value of $PV()$ for the marginal cow was zero. For discussion and analysis of such herd size choices, see Trapp.

[6] We are indebted to the University of California Livestock Farm Advisor, Daniel Drake, for these observations.

Table 1. Cash Flows with Raise Strategy Under Pre-TRA Tax Law

Time	Cow Age	Expected Cash Flow
-2	1	$R_{-2} = -(P_c\bar{W}_c + OC_2)(1 - \tau)(1 + \eta)$
-1	2	$R_{-1} = -OC_3(1 - \tau) + \eta\bar{W}_{h2}P_{h2}(1 - \tau(1 - CGE))$
$0, \ldots, (T - 4)$	$t = \{3, \ldots, (T - 1)\}$	$R_{t-3} = \{½(P_c\bar{W}_{ct} + P_s\bar{W}_{st})\theta_t - OC_{t+1}(\theta_t(1 - v_t) + (1 - \theta_t)(1 - \gamma))\}(1 - \tau) + [\theta_t v_t + (1 - \theta_t)\gamma]P_h\bar{W}_{h2}(1 - \tau(1 - CGE))$
$T - 3$	T	$R_{T-3} = \{½(P_c\bar{W}_{ct} + P_s\bar{W}_{sT})\theta_t\}(1 - \tau) + P_h\bar{W}_{hT}(1 - \tau(1 - CGE))$

Note: $\theta_t v_t + (1 - \theta_t)\gamma$ is the probability that an age $t(<T)$ heifer is culled. Likewise, $\theta_t(1 - v_t) + (1 - \theta_t)(1 - \gamma)$ is the probability that an age t cow is not culled.

Tax parameters:

τ	marginal tax rate of the producer
D_t	depreciation percentage on a capital asset, five-year tax life, in year t of the tax life
CGE	capital gains exclusion percentage
ITC	investment tax credit percentage.[7]

The producer's maximization problem is solved by first computing the present value of cash flows (with repeated replacements) for each of the planned cull age and buy versus raise strategies. The optimal replacement policy is then found in stages. First, the optimal planned culling age, T, and present value, V, are found for each of the raise and buy approaches. Second, the two maximal values are compared in order to determine which replacement strategy, buy or raise, is best. By carrying out this two-stage maximization under different tax regimes, the effects of the tax law changes on optimal decisions are revealed.

Present Values with a Raise Replacement Strategy

Consider first the case of pre-TRA tax law. To calculate the raise strategy present value for this case, we must first identify expected cash flows, $\{R_t\}$, for each period in the life of a single cow, assuming the cow lives to that period (see table 1). Note that time 0 is selected to correspond with the first year in which the cow is productive. Thus, when a cow is replaced in time t, it is as if a replacement calf was set aside for breeding two years before. This construction permits comparison with the purchase strategy, in which such forethought is unnecessary.

The next step is to identify the probabilities associated with each of the R_t cash flows given in table 1, as well as the probabilities of required replacement. To this end, define $Q_{s,t}$ as the probability that a cow will survive to age t (considering death risk as well as early culling due to illness or lack of productivity), and $Q_{R,t}$ as the probability that a cow will be replaced at age t. These variables are expressed in table 2.

Based on tables 1 and 2 and the logic of part A, the present value of the infinite stream of risky cash flows can be written as

$$(4) \quad V(T) = \left(\frac{1}{1 - \sum_{t=0}^{T-2} \beta^t Q_{R,t}}\right) \cdot \left(\sum_{t=-2}^{T-3} \beta^t R_t Q_{s,t}\right).$$

As noted earlier, this value measure relates to more than a single unit of productive cow capacity. In order to maintain a productive cow every period, some capacity must be reserved for the raising process. To convert $V(T)$ to a per-productive-unit basis, two expectations must be calculated: first, the expected number of productive periods with a single animal $E(N \mid T)$, and second, the expected total number of periods in which a single animal will absorb a unit of capacity. Based on table 2 (where q_t^* is defined),

$$(5) \quad E(N \mid T) = \left\{\sum_{t=1}^{T-2} (t - 1) \cdot \left(\prod_{n=1}^{t} q_n^*\right) \cdot (1 - q_{t+1}^*)\right\} + (T - 2) \cdot \left(\prod_{n=1}^{T-1} q_n^*\right),$$

[7] Under pre-TRA tax law, a "buyer" of a replacement heifer could take a 10% investment tax credit, in which case he was obligated to reduce the depreciable basis of the heifer by one-half of the ITC. However, he could alternately have elected to take an 8% ITC with no adjustment to basis. In our analysis, we tested for the optimal ITC election, finding that it always paid buyers to take the full 10% ITC.

and the second expectation will be $E(N|T) + (1 + \eta)$. Note that a productive period is defined as one in which a cow is held and can produce, even though the cow may die during the period. This treatment is symmetric with the buy strategy.

Using (5), $V(T)$ must be adjusted to a per-productive-unit value, $V^*(T)$, as follows:

$$(6) \qquad V^*(T) = \left(\frac{E(N|T)}{E(N|T) + (1 + \eta)}\right) \cdot V(T).$$

The optimal planned culling age will maximize $V^*(T)$ by choice of T; denoting this optimal culling age by T^*, the associated per-capacity-unit value is $V^*(T^*)$.

With post-TRA tax law, table 1 changes to reflect capitalization and depreciation of preproductive expenses, and loss of the capital gains exclusion (see table 3A).[8] In addition, there is a tax write-off for the undepreciated capital value of a cow in the event of death. This write-off yields the tax savings shown in table 3B, wherein χ_t is tax savings in year t if the cow dies in that year, $Q_{D,t}$ is the probability that the cow survives to year $t - 1$ and dies (of natural causes) in year t.[9]

The discounted expected value of the latter cash flows, $\sum_{t=-1}^{T-3} \beta^t \chi_t Q_{D,t}$, must be added to the second term in (4) to calculate $V(T)$. Thus,

$$(7) \qquad V^*(T) = \left(\frac{E(N|T)}{E(N|T) + (1 + \eta)}\right) \cdot \left\{\frac{1}{1 - \sum_{t=0}^{T-2} \beta^t Q_{R,t}}\right\} \cdot \left\{\sum_{t=-2}^{T-3} \beta^t (R_t Q_{s,t} + \chi_t Q_{D,t})\right\},$$

where $\chi_{-2} = 0$.

Present Values with a Buy Replacement Strategy

Under pre-TRA tax law, the major differences between cash flows under the buy strategy and those presented in table 1 are due to the investment tax credit and depreciation on the capital asset, the purchased heifer (see table 4A). The associated probabilities are identical to those in table 2, with one exception: q_2 is replaced by 1. (Buyers need not be concerned with prepurchase death risk.)

Because a buy strategy produces a depreciable asset base, the owner will receive some tax benefits in the event of death loss before the animal is fully depreciated (see table 4B).[10] Using the same logic as before, the present value per unit of productive cow capacity is as follows:

$$(8) \qquad V^*(T) = \left(\frac{1}{1 - \sum_{t=1}^{T-2} \beta^t Q^*_{R,t}}\right) \cdot \left(\sum_{t=-1}^{T-3} \beta^t (R^*_t Q^*_{s,t} + \chi^*_t Q^*_{D,t})\right),$$

where $Q^*_{s,t}$ and $Q^*_{R,t}$ are the adjusted table 2 probabilities.

Under the post-TRA tax law, this formulation changes in only the following respects: (*a*) the ITC is now zero; (*b*) the depreciation rates, D_n, have changed; and (*c*) the depreciable basis is $CV = -R^*_{-1} = P_y \bar{W} y + OC_3$, reflecting capitalization of the preproductive expense, OC_3.

Steady State Profits

The foregoing model yields optimal cow-calf management strategies which do not depend on a modeler's essentially arbitrary choice among alternative cash flow sequences (e.g., whether costs of replacement are born at the outset or at the time of culling). However, the dollar value measures, $V^*(\,)$, are sensitive to this ordering, and thus do not provide reliable measures of profits.[11] To obtain such a mea-

[8] The TRA still permits expensing of preproductive costs provided the operator also takes straight-line depreciation on his capital assets. Though we assume here that the raising operator has elected to capitalize in order to retain accelerated depreciation allowances, we also perform the analysis with post-1987 expensing of preproductive costs. This procedure permits us to determine the sensitivity of outcomes to the tax treatment of preproductive costs, as well as the value to cow-calf operators of the expensing alternative. However, our model does not permit us to address the choice between expensing and capitalizing since we do not incorporate measures of nonanimal assets.

[9] $Q_{D,t}$ does not include the extra probability of early culling in year t since the tax savings from early culling are included in R_t (table 3A).

[10] These tax benefits are net of required ITC recapture on animals which die or are replaced before living five productive years.

[11] Changes in $V^*(\,)$ do, however, measure changes in the present value of profits.

Table 2. Event Probabilities with a Raise Strategy

Time	Cow Age	Probability that Animal Survives to Age t	Probability of Replacement with Time t Producing Cow
-2	1	$Q_{s,-2} = 1$	0
-1	2	$Q_{s,-1} = q_2$	0
0	3	$Q_{s,0} = q_2 q_3$	$Q_{R,0} = (1 - q_2)$
1	4	$Q_{s,1} = q_2 q_3^* q_4$	$Q_{R,1} = q_2(1 - q_3^*)$
2	5	$Q_{s,2} = q_2 q_3^* q_4^* q_5$	$Q_{R,2} = q_2 q_3^*(1 - q_4^*)$
.	.	.	.
.	.	.	.
.	.	.	.
$T-4$	$T-1$	$Q_{s,T-4} = q_2 q_3^* \cdots q_{T-2}^* q_{T-1}$	$Q_{R,T-4} = q_2 q_3^* \cdots q_{T-3}^*(1 - q_{T-2}^*)$
$T-3$	T	$Q_{s,T-3} = q_2 q_3^* \cdots q_{T-1}^* q_T$	$Q_{R,T-3} = q_2 q_3^* \cdots q_{T-2}^*(1 - q_{T-1}^*)$
$T-2$		0	$Q_{R,T-2} = q_2 q_3^* \cdots q_{T-2}^* q_{T-1}^*$

Note: $q_t^* \equiv q_t(1 - \theta_t v_t - (1 - \theta_t)\gamma)$ = probability that the cow lives and is retained beyond age t. For $t < 3$, $q_1^* = 1$ and $q_2^* = q_2$.

sure, we construct the steady state distribution of cow ages which results from the optimal replacement policy. Associated with this steady state distribution is an annual expected profit level (per unit of capacity). The latter profit measure represents the long-run annual expected cash flow per cow with the optimal policy, regardless of the initial starting point. Based on table 5 (where N_R and N_B are defined), these steady state profit (SSP) measures are

(9) $$SSP_R = \frac{\sum_{t=-2}^{T-3} (R_t Q_{s,t} + \chi_t Q_{D,t})}{N_R}, \text{ and}$$

$$SSP_B = \frac{\sum_{t=-1}^{T-3} (R_t^* Q_{s,t}^* + \chi_t^* Q_{D,t}^*)}{N_B},$$

with R and B indexing raise and buy strategies.

Data

Physical data utilized for the analysis are shown in table 6. The two sets of birth rates are from Rogers (also see Bentley, Waters, and Shumway) and Patterson et al. The analysis was run for both cases. Rates of death and culling due to illness (v_t) are extrapolated from Greer,

Table 3. Cash Flows with Raise Strategy under Post-TRA Tax Law

(A) Survival Flows

Time	Cow Age	Expected Cash Flow
-2	1	$R_{-2} = -(P_c\bar{W}_c(1 - \tau) + OC_2)(1 + \eta)$
-1	2	$R_{-1} = -OC_3 + \eta(P_{h2}\bar{W}_{h2}(1 - \tau) + \tau OC_2)$
$0, \ldots, (T-4)$	$t = \{3, \ldots, (T-1)\}$	$R_{t-3} = \{½((P_c\bar{W}_{ct} + P_s\bar{W}_{st})\theta_t$
		$- OC_{t+1}(\theta_t(1 - v_t) + (1 - \theta_t)(1 - \gamma))\}(1 - \tau)$
		$+ \tau D_{t-2}(OC_2 + OC_3) + [\theta_t v_t + (1 - \theta_t)\gamma][P_h\bar{W}_{ht}(1 - \tau)$
$T-3$	T	$R_{T-3} = + \tau(OC_2 + OC_3)(1 - \sum_{n=1}^{t-2} D_n)]$
		$\{½(P_c\bar{W}_{cT} + P_s\bar{W}_{sT})\theta_t\}(1 - \tau)$
		$+ P_h\bar{W}_{hT}(1 - \tau) + \tau(OC_2 + OC_3)(1 - \sum_{n=1}^{t-3} D_n)$

(B) Cash Flows from Death Loss

Time	Age t	Tax ,Savings	Probability
-1	2	$\chi_{-1} = \tau OC_2(1 + \eta)$	$Q_{D,-1} = (1 - q_2)$
0	3	$\chi_0 = \tau(OC_2 + OC_3)$	$Q_{D,0} = q_2(1 - {}_1q_3)$
$t-3$	$t > 3$	$\chi_{t-3} = \tau(OC_2 + OC_3)(1 - \sum_{n=1}^{t-3} D_n)$	$Q_{D,t-3} = q_2 q_3^*\ ;{}_1\text{pd}\cdot\cdot\ q_{T-1}^*(1 - q_t)$

Table 4. Cash Flows with Buy Strategy under Pre-TRA Tax Law

(A) Survival Flows

Time	Cow Age	Expected Cash Flow
-1	2	$R^*_{-1} = -P_y\bar{W}_y(1 - ITC) + OC_3(1 + \tau)$
$0, \ldots, (T-4)$	$t = \{3, \ldots, (T-1)\}$	$R^*_{t-3} = \{\frac{1}{2}(P_c\bar{W}_{ct} + P_s\bar{W}_{st})\theta_t - OC_{t+1}(\theta_t(1 - v_t) + (1 - \theta_t)(1 - \gamma))\}(1 - \tau) + \tau D_{t-2}CV \cdot [\theta_t v_t + (1 - \theta_t)\gamma]\,\{P_h\bar{W}_{ht} - \tau(P_h\bar{W}_{ht} - CV(1 - \sum_{n=1}^{t-2} D_n)) - ITC\cdot P_y\bar{W}_y\max(\frac{7-t}{5}, 0)(1 - \frac{1}{2}\tau)\}$
$T-3$	T	$R_{T-3} = \{\frac{1}{2}(P_c\bar{X}_{cT} + P_s\bar{W}_{sT})\theta_t\}(1 - \tau) + P_h\bar{W}_{hT} - \tau(P_h\bar{W}_{hT} - CV(1 - \sum_{n=1}^{t-3} D_n)) - ITC\cdot P_y\bar{W}_y\max(\frac{7-t}{5}, 0)(1 - \frac{1}{2}\tau)$

(B) Cash Flows from Death Loss

Time	Age t	Tax Savings	Probability
0	3	$\chi^*_0 = \tau\cdot CV - ITC\cdot P_y\bar{W}_y\max(\frac{7-t}{5}, 0)(1 - \frac{1}{2}\tau)$	$Q^*_{D,0} = (1 - q_3)$
$1, \ldots$	$t = 4, \ldots$	$\chi^*_{t-3} = \tau\cdot CV(1 - \sum_{n=1}^{t-3} D_n) - ITC\cdot P_y\bar{W}_y\max(\frac{7-t}{5}, 0)(1 - \frac{1}{2}\tau)$	$Q^*_{D,t-3} = (1 - q_t)\prod_{n=3}^{t-1} q^*_n$

Notes: $CV \equiv P_y\bar{W}_y(1 - \frac{1}{2}ITC)$ is the depreciable basis. The cash flow expressions in table 4 assume that $(P_h\bar{W}_{ht} - CV) < 0$, implying that the capital gains exclusion does not come into play. For the data used here, this inequality holds. The last term in the R^*_{t-3} expression ($t \geq 3$) reflects ITC recapture.

Table 5. Steady State Age and Cash Flow Distribution

Cow Age	Steady State Age Frequency	Survival Flow	Probability	Death Flow	Probability
		Cash Flows for Each Age Group			
		(A) With Raise Strategy			
1	$(1 + \eta)/N_R$ [a]	$R_{-2}/(1 + \eta)$	1	0	0
2	$1/N_R$	R_{-1}	q_2	χ_{-1}	$(1 - q_2)$
3	q_2/N_R	R_0	q_3	χ_0	$(1 - q_3)$
4	$q_2q^*_3/N_R$	R_1	q_4	χ_1	$(1 - q_4)$
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T	$q_2q^*_3 \ldots q_{T-1}/N_R$	R_{T-3}	q_T	χ_{T-3}	$(1 - q_T)$
		(B) With Buy Strategy			
2	$1/N_B$ [b]	R^*_{-1}	1		0
3	$1/N_B$	R^*_0	q_3	0	$(1 - q_3)$
4	q^*_3/N_B	R^*_1	q_4	χ^*_0	$(1 - q_4)$
5	$q^*_3q^*_4/N_B$	R^*_2	q_5	χ^*_1	$(1 - q_5)$
.				χ^*_2	
.					
.					
T	$q^*_3q^*_4 \ldots q^*_{T-1}/N_B$	R^*_T	q_T	χ^*_T	$(1 - q_T)$

[a] $N_R = (1 + \eta) + 1 + q_2 + q_2q^*_3 + \ldots + (q_2q^*_3 \ldots q^*_{T-1})$.
[b] $N_B = 2 + q^*_3 + q^*_3q^*_4 + \ldots + (q^*_3q^*_4 \ldots q^*_{T-1})$.

Table 6. Data

(A) Physical Data Utilized for the Analysis

Cow Age	Birth Rates: Source (1)[a]	Birth Rates: Source (2)[b]	Death Rates[c]	Culling due to Illness[c]	Weights: Cow	Weights: Calf Heifer	Weights: Calf Steer
2			.01	.008	850		
3	.890	.90	.0135	.0276	1,000	444	474
4	.927	.93	.0113	.0234	1,100	464	496
5	.945	.95	.0073	.0367	1,100	486	520
6	.943	.94	.0146	.0403	1,100	486	520
7	.930	.93	.0160	.063	1,100	486	520
8	.908	.93	.0160	.0736	1,100	486	520
9	.870	.92	.0166	.0922	1,100	486	520
10	.820	.92	.0166	.1070	1,100	486	520
11	.766	.90	.0184	.122	1,100	464	496
12	.700	.90	.0192	.137	1,075	464	496
13	.636	.87	.0200	.152	1,050	464	496
14	.562	.82	.0208	.167	1,025	464	496
15	.450	.77	.0216	.182	1,000	464	496

(B) Depreciation Schedule on Five-Year Asset

Year of Asset Life	Calendar Year Rates: Pre-1987 D_n	Calendar Year Rates: Post-1987 D_n	Planning Year Rates[d]: Pre-1987 D_n	Planning Year Rates[d]: Post-1987 D_n
1	.15	.2000	.3150	.4400
2	.22	.3200	.2125	.2240
3	.21	.1920	.2100	.1344
4	.21	.1152	.2100	.1152
5	.21	.1152	.0525	.0720
6	.00	.0576	.0000	.0144

(C) Price Data

Price	Low	Intermediate	High
		($/customer)	
P_c	74.14	83.00	91.86
P_s	80.00	88.50	97.00
P_h	35.38	42.50	49.62
P_{h2}	56.79	69.00	81.21

[a] See Rogers.
[b] These figures are our approximations based on the 1987 study by Patterson et al.
[c] Greer, Whitman, and Woodward present statistics only through age 10. Subsequent percentages represent judgmental extrapolations. Some of the earlier percentages are also adjusted slightly to preserve an increasing rate structure.
[d] The planning year is assumed to be October 1 to October 1.

Whitman, and Woodward. The weights are taken from Kay and Rister, though updated to reflect technological improvements according to the advice of Daniel Drake, University of California Livestock Farm Advisor. The alterations include (*a*) higher cow weights at ages three through five and (*b*) an extra 15 pounds on all weaning weights. The mean weights of retained heifer calves and replacement yearlings are set at 500 and 900 pounds, respectively; these weights are higher than average because of the selection of heavier heifers for replacement.

Operating costs, which do not include capital (i.e., interest) or replacement expenditures, are estimated from the livestock budgets of the University of California Cooperative Extension Service. A producing cow is estimated to cost between $275 and $325 per year, while the cost of raising a yearling from a weaned heifer calf is estimated at $220 to $260. In the analysis, three levels of costs are considered: high ($325 and $260), intermediate ($300 and $240) and low ($275 and $220). Intermediate cost levels are considered normal.

Pre- and post-1987 depreciation schedules are shown in table 6B.[12] Since the winter cycle gives us roughly a planning year of October to October, depreciation schedules must be adjusted to reflect the allowances for this interval, rather than for the calendar year. This adjustment implies that the first planning year depreciation percentage is the first calendar year depreciation, plus three-quarters of the second calendar year depreciation amount. Likewise, the second planning year depreciation is one-quarter of the second and three-quarters of the third calendar year amounts, and so on.

Of the thirteen pre-1987 marginal income tax rates, four are considered here: 15%, 26%, 35%, and 50%. Post-1987 tax rates of 15% and 28% are examined with the TRA tax regime. Rate parameters are varied as follows, with the normal case indicated by an asterisk: γ (culling rate for non producers) $\in \{.5, 1^*\}$, η (extra retention) $\in \{0, .1^*, .2\}$, i (interest rate) $\in \{.07, .1^*, .13\}$.

Finally, prices are set at three levels: high, intermediate, and low. The intermediate price level is an estimate of recent California prices taken from *California Livestock Market News.* High and low prices are selected to be two standard deviations above and below current prices, with the adjustment based on a historical measure of the prices' covariance matrix, Σ. Specifically, Σ is estimated using three years of monthly data (1984–86) taken from California Department of Food and Agriculture statistics. Decomposing Σ so that $\Sigma = ZZ'$, letting $\bar{P}$ (a vector) denote current prices, and letting ι denote a vector of ones, $P_{high} = \bar{P} + 2Z\iota$, and $P_{low} = \bar{P} - 2Z\iota$. The resulting prices are shown in table 6C.

The quoted prices used for P_c, P_s, P_y, P_h, and P_{h2} are as follows: P_c is feeder heifer, 400–500 pounds, Stockton; P_s and P_y, feeder steer, 400–500 pounds, Stockton; P_h, cutter cows, Stockton; and P_{h2} and P_y, heavy feeder heifer, Shasta.

Setting $P_y = P_{h2}$ is consistent with expert advice received in constructing the model. To determine the predicted outcomes' sensitivity to this specification, we also set $P_y = .5(P_{h2} + P_s)$.

Results

Table 7 presents selected outcomes of the numerical analysis. In highlighting these and other results, we will first focus on the TRA's impact on replacement strategy and then discuss the effects on profitability.

Replacement Strategy

Buy versus raise. In many circumstances, a rancher's optimal buy versus raise choice changes under the new tax law. For example, with calving rates from source (2) (see table 6A) and a low relative yearling price, formerly high tax bracket ranchers switch from raise to buy under the new tax act. With a high relative yearling price, a rancher who stays in the 15% tax bracket often shifts from buy to raise.

To explain these effects, note that the value of a capital gains exclusion (with raising) increases with the rancher's tax rate, while the value of an ITC (with buying) stays constant. Thus, for a low tax bracket operator, the loss of the capital gains exclusion (CGE) under the TRA does not lead to a significant additional cost of raising replacements, though the loss of the ITC eliminates a significant inducement to buy replacements. With a high relative yearling price discouraging a buy strategy, the presence or absence of the ITC will make the difference between the relative profitability of buy and raise strategies for the low tax bracket operators. In contrast, the loss of the CGE is more important for formerly high tax bracket ranchers. With a low relative yearling price favoring a buy strategy, the loss of the CGE eliminates the key inducement for the high tax bracket ranchers to raise replacements.

This explanation attributes policy changes to new tax rates and elimination of the CGE and ITC, rather than to new limits on expensing of preproductive costs or to changes in depreciation regimes. To confirm (or refute) this reasoning, we performed the analysis without the latter two changes in the tax law; the resulting replacement strategies are identical

[12] Under post-1987 tax law, the calendar year depreciation schedule depends on whether 40% or more of new capital assets are acquired within the last quarter of the year; if so, the mid-quarter convention applies; if not, the midyear convention applies. Here we assume that the midyear convention is relevant, essentially implying that buyers purchase the bulk of their replacements (and/or other capital assets) by the end of September rather than in October or November. However, we test for sensitivity of outcomes to application of the mid-quarter convention for buyers, finding that replacement policies are unaffected and annual equivalent values change by very small amounts (relative to steady state profits).

Table 7. Optimum Replacement Age (*T*), Buy (*B*) Versus Raise (*R*) Decision, and Steady State Profit (*SSP*) Under Alternative Income Tax Rate and Price Regimes, Pre-TRA and Post-TRA

θ Series Source[a]	Relative Yearling Price[b]	Tax Rate[c]	Base Case[d]			High Price[e]			Low Price[e]		
			T	*B/R*	*SSP*	*T*	*B/R*	*SSP*	*T*	*B/R*	*SSP*
1	Low	.15*A*	10	*B*	40	10	*B*	64	9	*B*	16
		.26*A*	10	*B*	35	9	*R*	56	10	*B*	13
		.35*A*	9	*R*	31	8	*R*	56	10	*B*	12
		.50*A*	6	*R*	35	5	*R*	57	7	*R*	14
		.15*X*	10	*B*	32	10	*B*	55	10	*B*	8
		.28*X*	10	*B*	27	10	*R*	47	10	*B*	7
2	Low	.15*A*	11	*B*	45	12	*B*	70	11	*B*	19
		.26*A*	12	*B*	39	11	*R*	61	12	*B*	17
		.35*A*	10	*R*	35	10	*R*	58	12	*B*	15
		.50*A*	6	*R*	36	5	*R*	58	10	*R*	15
		.15*X*	13	*B*	37	13	*B*	61	13	*B*	13
		.28*X*	13	*B*	31	13	*B*	52	13	*B*	11
1	High	.15*A*	10	*R*	26	10	*R*	56	10	*B*	1
		.26*A*	9	*R*	29	9	*R*	56	10	*R*	2
		.35*A*	9	*R*	31	8	*R*	56	9	*R*	6
		.50*A*	6	*R*	35	5	*R*	57	7	*R*	14
		.15*X*	10	*R*	20	10	*R*	48	10	*B*	−7
		.28*X*	10	*R*	17	10	*R*	41	10	*R*	−7
2	High	.15*A*	13	*B*	34	12	*R*	61	13	*B*	6
		.26*A*	11	*R*	33	11	*R*	59	11	*R*	6
		.35*A*	10	*R*	34	10	*R*	58	11	*R*	9
		.50*A*	6	*R*	36	5	*R*	58	10	*R*	15
		.15*X*	13	*R*	26	13	*R*	54	14	*B*	0
		.28*X*	13	*R*	22	13	*R*	46	13	*R*	−2

[a] The source numbers correspond to those in table 6A.
[b] The low yearling price is P_{h2} and the high yearling price is $.5(P_{h2} + P_s)$. Note that these yearling prices are relative in that they follow changes in P_{h2} and P_s (see e).
[c] *A* signifies pre-TRA and *X* signifies post-TRA.
[d] The base case sets prices, costs, the interest rate, the extra retention parameter (η), and the culling rate for nonproducing cows at their normal values as indicated in the text.
[e] High price and low price indicate that P_c, P_s, P_h and P_{h2} are set two standard deviations above and below their current (Dec.1987) values, as described in the text.

to those that emerge in the presence of these TRA changes.[13]

Planned culling age. Optimal culling ages increase under the TRA because of elimination of the ITC and the capital gains exclusion. Though the new ban on expensing of preproductive costs should also favor higher cull ages, we find that this tax change has no effect on replacement policy.[14,15]

The increase in cull ages under the TRA is greatest for the formerly high tax bracket ranchers. This is because, under pre-TRA tax law, the optimal raise strategy culling age decreases as the tax rate rises. With a CGE, the relative value of the capital gain from culling (relative to other after-tax cash flows) increases with the tax rate; thus, before the TRA, high tax rate ranchers had a greater incentive to cull early.

Variation in parameters. Perhaps the most remarkable outcome of varying economic parameters is the insensitivity of optimal management practices to these changes. For example, a two standard deviation change in beef prices leads to only occasional and minor

[13] Post-TRA policies are also found to be insensitive to expensing of preproductive costs without preservation of the old depreciation schedule.

[14] These results are consistent with Rister and Kay, who pointed out that the capital gains exclusion will lead to earlier culling. However, as noted, the results here also reflect the impact of other tax provisions not examined by the latter authors.

[15] In interpreting planned culling ages, note that they do not represent average culling ages. Rather, they represent the age at which a cow will be culled if not culled earlier for illness or non-production. For example, with planned culling ages of 6, 10, and 12, the average lifespan of the cow is only 5.3, 7.3, and 7.9, respectively [using θ series (2)].

Table 8. Tax Brackets/Relative Yearling Price [Low (*L*), High (*H*)]

	.15*A*/*L*	.26*A*/*L*	.35*A*/*L*	.50*A*/*L*	.15*A*/*H*	.26*A*/*H*	.35*A*/*H*	.50*A*/*H*
Two standard deviation reduction in prices ($)	−25.50	−21.64	−19.54	−20.74	−27.08	−26.58	−24.45	−20.74
Increase in costs to high cost case	−18.91	−16.36	−14.38	−11.94	−18.71	−16.18	−14.26	−11.94
50% reduction in cow illness/death rates	7.43	6.32	5.53	3.79	7.88	6.73	5.82	3.79
Decrease in calving rates [to θ series (1)]	−5.43	−4.04	−3.99	−.83	−7.11	−3.84	−2.98	−.83

changes in replacement strategies (though it leads to dramatic changes in the level of profits). The insensitivity of policies to price changes is consistent with earlier research (see, for example, Kay and Rister; Rister and Kay; Bentley, Waters, and Shumway). This insensitivity is attributable to the following observation: cull age and buy versus raise choices are determined by the relative magnitudes of returns over time and across policies and not by absolute levels of returns.

To the extent that price changes affect optimal policies, the impacts are as follows: Under pre-TRA tax law, higher prices lead to occasional switching from buy to raise and, sometimes, slightly lower culling ages; no changes occur under the TRA. Under both tax regimes, lower prices lead to a few more buy decisions. We attribute these effects to a lower relative importance of raising costs (*vis-à-vis* yearling costs and net revenues) in a high (versus low) price situation.

Changes in other economic parameters [including operating costs, the interest rate, extra retention (η), and culling of unproductive cows (γ)] have no appreciable effect on management practices. However, the more recent calving rate data [i.e., θ series (2)] leads to higher culling ages; this is due to higher calving percentages for older cows, which reduces the incentive to cull. We also tested for the effect of a proportional reduction in rates of death and culling due to illness; this change had no impact on replacement policy, again because relative magnitudes of returns are not appreciably altered by the change.

Profitability

By any standards, the TRA leads to large losses in profit for cow-calf operations. For example, with base case parameters and θ series (2) (table 6A), steady state profits (per cow per year) decline by the following dollar amounts (and percentages) for the indicated tax brackets:

	Tax Brackets			
	.15*A*/.15*X*	.26*A*/.28*X*	.35*A*/.28*X*	.50*A*/.28*X*
Low rel. Yearling price	$8.16 (18.1%)	$7.83 (20.1%)	$3.54 (10.2%)	$4.58 (12.8%)
High rel. Yearling price	$7.69 (22.9%)	$10.76 (32.9%)	$11.82 (35.0%)	$13.87 (38.8%)

(As in table 7, *A* and *X* designate pre- and post-TRA tax rates, respectively.)

To put these statistics into perspective, consider the following changes in steady state profits [from the base case with θ series (2)] when prices, costs, cow death rates, and calving rates are altered (see table 8). These numbers reveal that the TRA has a larger impact on steady state profits than major changes in either calving or death rates. Though large changes in costs or prices lead to larger swings in steady state profits than caused by the TRA, the TRA's impact is substantial even in the presence of such fluctuations.

While steady state profits give a rough indicator of effects on long-run profit levels, they do not capture benefits and costs of different time paths for cash flows. Thus, while the cash flow consequences of eliminating the ITC and CGE are reflected in steady state profits, these profits are invariant to changes in preproductive expense allowances and depreciation regimes. To obtain an indication of these provisions' relative importance to ranchers' profits, we now look at changes in the value of an infinite stream of replacement cows that would result from (*a*) allowing preproductive costs to be expensed under current law, and (*b*) returning to the pre-TRA depreciation regime (also with preproductive expensing).

As noted earlier, neither of these changes alters management practices. Hence, preproductive expensing (which benefits only raisers) is relevant only for the high relative yearling price case in which a raise strategy is chosen. Likewise, reversion to old depreciation rates is relevant only for the low relative yearling price case in which a buy strategy is chosen. The following numbers are the annual equivalent per-cow value changes [with base case parameters and θ series (2)] that the above reforms would induce:

	Relative Yearling Price	Tax Bracket .15*X*	Tax Bracket .28*X*
		($)	
Preproductive expensing	High	3.95	7.38
Pre-TRA depreciation	Low	−.67	−1.26

Because these measures are in the same units as steady state profits, we can conclude that the TRA's elimination of the ITC, CGE and preproductive expense allowances are each very costly to beef-cow producers, while producers' profit gains from the TRA's more rapid depreciation of purchased animals is modest. However, this last statement should be qualified by noting that our analysis does not incorporate capital investments other than cows themselves. Particularly if these other investments tend to be made early in the calendar year, beef producers' total per-cow gain from the depreciation change can be much larger than suggested by the statistics above. For the same reason, these numbers do not imply that ranchers who raise replacements should elect to expense their preproductive costs and use straight-line depreciation on all other capital assets (see footnote 8). However, they do imply that there is good reason for the livestock industry's interest in restoring preproductive expensing to the tax code.

Summary and Conclusion

This paper presents a model of optimal beef cow replacement strategy in a stochastic environment under U.S. tax rules effective before and after the Tax Reform Act of 1986. Both the buy versus raise decision and the optimal culling age choice are analyzed. Results of the numerical analysis indicate that the tax law changes will have significant effects on both optimal decisions and present values of beef cow operations. More specifically, when provisions of the TRA of 1986 are fully effective in 1988, the optimum age for culling beef cows will increase, as will the after-tax costs of beef cow operations.

Two limitations of the analysis are apparent. (*a*) While economic and technical parameters are varied, prices and culling rates for nonproductive cows are considered exogenous. A more general model would endogenize price and specify a bayesian updating procedure to determine optimal cow-specific culling choices. (*b*) Perhaps more important, the model incorporates stationarity assumptions with respect to beef prices, costs, and technology. Hence, stochastic and/or cyclical changes in prices and other parameters are not considered.

Both limitations suggest that further work may be warranted, and this work would be aided by the recent contributions of Bentley and Shumway, Hertzler, and Trapp on cattle price cycles in stochastic models without taxes. The generalizations could be particularly useful in discerning tax law effects on prices, the price cycle, and the change in optimal choices over time, issues which are beyond the scope of our analysis.

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References

Bentley, E., and R. Shumway. "Adaptive Planning over the Cattle Price Cycle." *S. J. Agr. Econ.* 13(July 1981): 139–48.

Bentley, E., J. Waters, and R. Shumway. "Determining Optimal Replacement Age of Beef Cows in the Presence of Stochastic Elements." *S. J. Agr. Econ.* 8(December 1976):13–18.

Burt, O. "Optimal Replacement Under Risk." *J. Farm Econ.* 47(1965):324–45.

California Department of Food and Agriculture. *Livestock and Meat Prices and Receipts,* 1986.

California Livestock Market News, Dec. 1987.

Chisholm, A. "Criteria for Determining the Optimum Replacement Pattern." *Amer. J. Agr. Econ.* 48(1966): 107–12.

———. "Effects of Tax Depreciation Policy and Investment Incentives on Optimal Equipment Replacement Decisions." *Amer. J. Agr. Econ.* 56(1974):776–83.

Greer, R., R. Whitman, and R. Woodward. "Estimation of Probability of Beef Cows Being Culled and Calculation of Expected Herd Life." *J. Animal Sci.* 51(1987):10–19.

Hertzler, G. "Information, Expectations, Market Efficiency and Welfare: Long-Run Equilibrium in the Beef Market." Ph.D. thesis, University of California, Davis, 1983.

Kay, R., and E. Rister. "Income Tax Effects on Beef Cow Replacement Strategy." *S. J. Agr. Econ.* 9(July 1977): 169–72.

Patterson, D., R. Bellows, P. Burfening, and J. Carr. "Occurrence of Neonatal and Postnatal Mortality in Range Beef Cattle." *Theriogenology,* in press.

Perrin, R. "Asset Replacement Principles." *Amer. J. Agr. Econ.* 54(1972):60–67.

Rister, E., and R. Kay. "The Effect of Capital Gain Income on Beef Cattle Replacement Policy and Profit." *J. Amer. Soc. Farm Mgrs. and Rural Appraisers* Vol. 41 No. 2 (1977):62–67.

Rogers, L. *Replacement Decisions for Commercial Beef Herds.* Washington State University Agr. Exp. Sta. Bull. No. 726, 1971.

Trapp, J. "Investment and Disinvestment Principles with Nonconstant Prices and Varying Firm Size Applied to Beef-Breeding Herds." *Amer. J. Agr. Econ.* 68(1986): 691–703.