Stochastic Dynamic Optimization and Rangeland Investment Decisions

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One of the most uncertain resources for a western beef cattle ranch is the availability of reliable spring forage. The impact on ranch equity position and income variability of investments in crested wheatgrass seedings designed to stabilize spring forage supplies is examined. Expected ending net worth under stochastic forage production and cattle prices is maximized subject to secondary safety-first objectives. Seedings increase expected ending net worth and increase annual net ranch incomes after plant establishment.

Key words: crested wheatgrass, investment analysis, range economics, risk, stochastic programming.

Parameter uncertainty affects production and marketing decisions and, consequently, financial performance for most farms and ranches. The literature is replete with analytical and numerical techniques for incorporating parameter uncertainty in agricultural decision models.

Much of this literature concerns the impact of uncertainty on production and/or marketing decisions. These studies often model short-run decisions, ignoring long-term consequences of decisions made under uncertainty on whole-farm financial performance. Although appropriate for many decisions, short-run perspectives may be inappropriate for budgeting decisions such as capital investment and optimal beef cattle herd size.

This article considers the long-term consequences of a rangeland investment designed to increase spring forage supplies, a traditional source of risk to the western cattle producer. Optimal investment level and herd size over time under stochastic prices and forage production levels are determined using a stochastic nonlinear programming model. The primary ranch objective is assumed to be maximization of the discounted value of ending net worth. However, secondary objectives relating to net ranch income goals are incorporated using Atwood, Watts, and Helmers' lower partial moment procedure of incorporating safety-first considerations in a dynamic model. Variations on chance constraints (Charnes and Cooper) provide probabilistic estimates of forage availabilities under different rainfall patterns.

Range Investment Decisions under Uncertainty

Risk-reducing production alternatives available to livestock producers in semiarid portions of the western United States are limited. Climatic severity, poor soils, lack of inexpensive water supplies, and distance to demand centers limit the set of cropping or other management alternatives available to these agricultural producers.

Although the potential for enterprise diversification is limited for the livestock producer, fluctuations in annual net ranch income might be dampened by selective management practices designed to reduce environmental perturbations affecting net returns. Benefits from such practices might include stabilizing forage supplies, as well as permitting a larger number of animals on the ranch.

A limiting resource on many western ranches is the availability of high nutrient quality
forage in the early spring. Spring calving cows have high nutrient requirements. However, native range growth often is delayed due to low soil temperature and moisture level. Calf gains thus are retarded unless alternative sources of high quality feed are available.

One investment alternative available to the rancher to improve the quantity and quality of spring forage is to replace native range species with an earlier maturing seeded grass. Crested wheatgrass (Agropyron desertum) is a common replacement for the native range, having been established on more than 12 million acres of western rangelands (Dewey and Asay). Crested wheatgrass reaches its highest nutritional content in early spring (Rauzi) and develops two to three weeks earlier than native grasses (Frischknecht, Harris, and Woodward). Production increases resulting from establishment of crested wheatgrass for a spring forage supply are well documented (Jeffries et al.; Hart et al.).

Investment projects designed to increase range forage production share many characteristics common to other agricultural investment problems. For example, weather uncertainty affects expected future production levels, and uncertainty regarding input and output prices renders projections of future economic returns difficult. As Bernardo and Conner recently have written, the same physical and biological factors that make range analysis unique also contribute to the difficulty and expense associated with acquiring adequate information upon which to base decisions. Specifically, range economists have long been frustrated by either the lack of data necessary for the application of a particular analytical method or by the intertemporal and spatial variation in observed forage and animal growth relationships.

**Modeling Resource Uncertainty and Safety-First Financial Constraints**

The model used in the study can be represented:

\[ \begin{align*}
\text{(1a)} & \quad \text{Max} \ e^{-rT}E(NW_T), \\
\text{(1b)} & \quad A_1 X \leq b, \\
\text{(1c)} & \quad A_2 X \leq \beta, \\
\text{(1d)} & \quad YX - F = 0, \\
\text{(1e)} & \quad Y_t X - t + d \geq 0, \\
\text{(1f)} & \quad p d - Q = 0, \text{ and} \\
\text{(1g)} & \quad t - L Q \geq g.
\end{align*} \]

Parameter uncertainty enters the model both exogenously and endogenously. The objective function is the maximization of discounted expected ending net worth, where \( e^{-rT} \) discounts net worth, \( E(NW_T) \), in year \( T \) to present value. A discount factor of 6% was used for the present value calculations.

Constraints (1b) are deterministic constraints relating resource usage to known supplies, as well as transfer functions characteristic of a dynamic model. Forage production levels are uncertain, arising from stochastic annual precipitation levels. Annual forage supplies enter the model as chance constraints (Charnes and Cooper), based on alternative cumulative probability values of a hyperbolic tangent distribution function (Taylor) describing forage yield. Constraints (1c) constrain annual forage supplies on both native range and crested wheatgrass by \( A_2 \), a random forage production level based on annual precipitation. For a given precipitation level, forage production will exceed \( \beta (1 - \alpha) \) percent of the time, where \( \alpha \) is the probability that resource supplies will be less than or equal to \( \beta \).

Financial performance indicators such as net ranch income, cash surplus, and annual changes in net worth result from values of the choice variables and random prices. \( Y \) is a matrix of financial coefficients relating choice variables \( X \) to cash flow, profitability, and solvency measures. Standard farm management definitions are used to determine financial performance measures (Hawkins et al.). Table 1 illustrates the components used in the calculation of the measures.

Constraints (1e) to (1g) incorporate Atwood, Watts, and Helmers' safety-first criteria. \( Y_i \) is a submatrix of \( Y \) relating \( X \) to annual net ranch income (NRI) under each state of nature. An endogenously selected reference level of income, \( t \), is used to compute the lower partial moment of annual NRI. \( d \) is a vector of negative deviations from target levels of annual income. \( p \) is the probability associated with the state of nature corresponding to each element of \( d \). \( Q \) is the average value of the negative deviations over all states. The final constraint, (1g), relates the reference level of income, \( t \), to target level, \( g \). This constraint
ensures that the safety-first relationship, \( \Pr(NRI < g) < 1/L^* \), is satisfied.

Three ten-year rainfall series were generated to allow calculation of available forage supplies (the generation procedures for rainfall and prices are discussed below). In addition, three correlated price series for each class of livestock considered were generated. Thus, nine states of nature were possible for each of the ten years included in the model. Accounting rows tracked net cash income, net ranch income, end-of-year cash surplus or deficit, and net worth at the end of each year. Cash deficits induced additional non-real estate borrowing at interest rates assumed for the year of the deficit.¹

**Behavioral Assumptions of the Model**

Results described here result from maximizing expected ending net worth, without explicit consideration of the distribution of this random variable. However, employing safety-first constraints on annual distributions of \( NRI \) makes the model consistent with the class of mean-risk models shown earlier by Fishburn, and later by Atwood, to result in solutions that will be stochastically efficient. Using the linear negative deviation from target measures [constraint (1e)] assures the results will be second-degree stochastically efficient.

**Data**

Initial herd size and forage resources in this study were based on Cooperative Extension budgets for Elko County, Nevada (Myer and Hackett). The first-year breeding herd consists of 507 mature cows, 76 replacement heifers, and 25 bulls. All factor prices are in 1985 dollars. Animal costs, excluding feed and fixed costs, are $94.44 per cow [U.S. Department of Agriculture (USDA)]. Variable costs of removing sagebrush/native range and planting crested wheatgrass are $23.16 per acre (Soneman et al.). Variable feed costs include hay production ($45.01 per acre), hay purchase ($65 per ton), and BLM grazing fees [$1.35 per animal unit month (AUM)]. Beginning balance sheet data for the ranch are in table 2.

The representative ranch utilizes approxi-

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¹ Although the model allows interest rates to vary, real estate interest was assumed to be 11.5% and non-real estate rates were assumed constant at 12.5% for each year in the model.
Figure 1. Annual rainfall under states of nature 1-3

mately 78,000 acres of public rangeland. The ranch's forage and feed requirements and supplies are assumed to be balanced initially. Specifically, range forage production is adequate to supply initial herd requirements in years of average rainfall. In years of adequate rainfall the cattle are on range for 215 days (15 April–15 November). In low rainfall years, when range forage production is less than herd requirements, animals are removed from the public range early and fed hay. Hay is priced at market rates.

Annual hay production is a choice variable, yet cow numbers are limited by the amount of hay produced for a four-and-a-half-month winter feeding period. Harvested hayfields are grazed prior to the winter feeding period.

Rainfall Data

Annual rainfall data were available from the Elko, Nevada, weather reporting station for the period January 1930 through December 1984. Sample autocorrelation and partial autocorrelation functions were calculated for the 55 annual observations. No significant year-to-year correlation structure was found in the series.

Goodness-of-fit tests failed to reject the hypothesis that the annual data fit the gamma distribution. Maximum likelihood estimators were calculated for the distribution's two parameters ($A = 11.28$ and $B = .86$). Annual rainfall values for each year of the model were generated using the random number generating routine for the gamma distribution (GGAMR) from the IMSL library of statistical subroutines. Annual rainfall levels generated for the three states of nature included in the model are illustrated in figure 1.

Range Forage Production

Sneva and Hyder report annual precipitation and forage production observations as percentages of normal for various range sites representative of the Intermountain West. These 95 paired observations were used to estimate an empirical distribution of forage and precipitation using procedures reported in Taylor.

OLS estimates of coefficients served as starting points for maximum-likelihood estimators. OLS regression related

\[ u = .5 \ln \left( \frac{F(Y, R)}{1 - F(Y, R)} \right) = P(Y, R) \]

\[ = b_0 + b_1Y + b_2Y^2 + b_3R, \]

Lambert and Harris
where \( F(Y, R) \) is the cumulative distribution for the sorted observations of forage yield \((Y)\) and rainfall \((R)\). \( P(Y, R) \) are fitted values of the polynomial regression where the dependent variables are observed probability densities. Observed densities are derived by grouping similar values of yield and calculating densities by ranging over discrete values of observed \( R \) (see Taylor for additional details).

First-order conditions characterizing the maximum value of the (natural log of the) maximum-likelihood function then were solved simultaneously using GAMS/MINOS (Brooke, Kendrick, and Meeraus). The resulting empirical distribution was

\[
F(Y, R) = .5 + .5 \tanh \left[ P(Y, R) \right],
\]

where \( \tanh \) is the hyperbolic tangent function. \(^2\)

Forage yield in the model thus was stochastic, represented by the distribution function (3). Precipitation levels generated by the IMSL subroutine were used to represent state-specific values of \( R \) in \( P(Y, R) \).

Different values of \( F(Y, R) \) of each year’s distribution of yields were used to provide chance-constrained right-hand-side values for forage production. Values were derived by inverting (3) and solving for \( Y \) for different values of \( \alpha \):

\[
\alpha = F(Y, R) = .5 + .5 \tanh \left[ P(Y, R) \right] = .5 + .5 \frac{(e^\alpha - e^{-\alpha})}{(e^\alpha + e^{-\alpha})}
\]

yields, by inverting,

\[
u = .5[\ln(\alpha) - \ln(1 - \alpha)].
\]

Since \( u \) is quadratic, \(^3\) \( Y \), expressed as a percentage of average forage production, can be solved for any specified values of \( \alpha \) and \( R \):

\[
Y = \frac{-b_1 \pm [b_1^2 - 4b_2(b_0 - \alpha + b_3R)]^{.5}}{2b_2},
\]

where \( Y \) will be the two roots of the quadratic expression. No difficulty was encountered in choosing the “more reasonable” root in the empirical application.

Stochastic rainfall and, consequently, forage production were assumed to determine number of days of grazing available on the range. Animal growth on crested wheat (native range) was fixed as 2.31 and 1.91 (1.45 and 1.21) pounds per day for steer and heifer calves, respectively (Williams).

Several assumptions were made concerning any crested wheat investment entering the solution. To reduce model size, any crested wheat investment occurred in year one. Two years of rest followed seeding. Investment costs exceeding surplus cash available after satisfying net ranch income target levels came from increased non-real estate borrowing.

**Cattle Price Generator**

November cattle prices were collected for the period 1950–84. The choice of November prices was based on observed sales dates for most western cow/calf ranches (Gilliam). Kansas City prices were used for light steers and heifers and Omaha prices were used for bulls and utility cows. Kansas City and Omaha prices were used by default: no complete price series exists for Nevada. Prices were deflated to 1985 dollars using the Consumer Price Index.

Two considerations motivated the choice of procedures used to generate prices for the simulation model. First, prices display a cyclical pattern over time. Autocorrelation and partial autocorrelation functions supported the existence of this time trend for all four price series. Second, prices for steer calves, heifer calves, cull cows, and bulls are highly correlated.

Generation of prices for the model thus relied on time-series techniques to capture the trend component and on correlation techniques to preserve the relationship among the four price sets.

Generation procedures concentrated on the steer price series since income from steer sales constitutes the major component in a cattle ranch’s gross receipts (steer receipts generally were slightly over 50% of the total receipts for the representative ranch used in the study). An ARIMA \((4,0,1)\) model was deemed best among those estimated:

\[
(1 - .7532B - .2403B^2)p_t = (1 + .3240B)a(t)
\]

\[
(1 - .0354)(.0476)(.1425)
\]

\[
(7) \quad \text{Residual Mean Square} = 437.571 \text{ (dollars per cwt.)},
\]

where \( p_t \) is deflated November steer price, \( a(t) \) are model error terms, \( B \) is the backspace operator of the ARIMA process, and standard errors are in parentheses. Price series of ten

\(^2\) The hyperbolic tangent of \( u \) is \( \tanh u = (e^u - e^{-u})/(e^u + e^{-u}) \).

\(^3\) Coefficient values resulting from the reduced gradient procedures of MINOS were \( b_0 = -.871, b_1 = .068, b_2 = -.0001094, \) and \( b_3 = -.047 \).
years each were generated using the ARIMA structure for steers. Random deviates were generated from the GGNML subroutine of the statistical package IMSL from a normal distribution with the same first two moments of the errors from the ARIMA estimation. These generated values were added to each year's ARIMA forecast to simulate randomness in the series. The resulting series retained the trend characteristics of the historical steer price data.

The generated steer price series next was normalized, and procedures described in Richardson and Condra were used to produce correlated and time-trended series for heifer calves, cull cows, and bulls. Steer prices under the three price states of nature considered are shown in figure 2.

Experimental Procedures

Nine price and weather states were incorporated for each of the ten years in the model. Sensitivity of the model to the probability level associated with the cumulative distribution of forage ($\alpha$) and to target levels of $NRI$, as well as probability limits $1/L^*$ of the safety-first constraints, was tested by successive runs under different parameter values.

The model contained 37 and 24 blocks of equations and variables, respectively. Total model size was 1,313 equations and 1,293 variables. Approximately 14% of the 5,880 nonzero elements in the model were nonlinear. The National Science Foundation's supercomputing facility at Cornell University was used for the production runs.

Model Results

Initial model runs were conducted with crested wheatgrass acreage fixed at zero. This provided a basis for comparison between the situations with and without the seeding investment. Results are reported in tables 3 and 5.

Native range forage production was adequate to meet the nutritional needs of the initial 507-cow herd during years of average or greater rainfall. With a forage cumulative distribution (cdf) $\alpha$ value of .50 and mean annual rainfall levels of 9.74 inches, 42 pounds per acre of consumable forage is produced. This provides sufficient forage for 581 animal units

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4 Twenty acres/AUM is typical of unimproved range production in Nevada at 60% utilization. Improvements to three acres/AUM are possible with a crested wheatgrass seeding.
Table 3. Solution Characteristics when Crested Wheat Acreage Is Fixed at 0

<table>
<thead>
<tr>
<th>Target NRI ($)</th>
<th>Probability Constraint 1/L*</th>
<th>α Value</th>
<th>Ending Net Worth ($)</th>
<th>Number of States</th>
<th>Table 5 Case Number</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>.50</td>
<td>.33</td>
<td>660,763</td>
<td>13</td>
<td>I</td>
</tr>
<tr>
<td>20,000</td>
<td>.50</td>
<td>.50</td>
<td>673,565</td>
<td>13</td>
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</tr>
<tr>
<td>50,000</td>
<td>.50</td>
<td>.33</td>
<td>658,456</td>
<td>13</td>
<td>III</td>
</tr>
</tbody>
</table>

Benefits with respect to both profitability and ending net worth result from converting unimproved range to crested wheatgrass (table 4). For example, with target NRI equal to $0 and both the safety-first probability value and α equal to .5, 6,193 acres of range are seeded. Under average rainfall, total forage production on the crested acres plus the remaining unimproved acreage provides grazing for 842 AUs, a large increase from the 581 AUs calculated above under average conditions.

Benefits of the investment result from several sources. First, greater forage supplies permit herd expansion (table 5). Benefits from herd expansion derive from greater annual net ranch incomes and from increased net worth at the end of the planning horizon. Second, increased forage reduces the number of states of nature under which animals must be removed from the range due to inadequate forage production in states of below-average rainfall. Cost savings result from reducing the number of states in which hay must be purchased (or fed rather than sold).

The substitute between annual net ranch income and ending net worth observed above for the unimproved range situation also is evident when seeded acreage is a choice variable. As the safety-first constraints become more binding, under either higher specified targets or more stringent probability limits, ending net worth is reduced to satisfy annual income requirements.

Ranch cash flows also are affected by the seeding investment (figure 3). A slight drop in cash surplus occurs in year one resulting from loss of native range grazing on the approximately 6,000 acres being seeded. The effect is

Table 4. Solution Characteristics when Crested Wheat Acreage Is Endogenous

<table>
<thead>
<tr>
<th>Target NRI ($)</th>
<th>Probability Constraint 1/L*</th>
<th>α Value</th>
<th>Ending Net Worth ($)</th>
<th>Acres</th>
<th>Number of States Hay Purchased</th>
<th>Table 5 Case Number</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>.25</td>
<td>.50</td>
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<td>6,193</td>
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<tr>
<td>20,000</td>
<td>.25</td>
<td>.50</td>
<td>796,117</td>
<td>6,378</td>
<td>71,446</td>
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<tr>
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<td>.50</td>
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<td>5,994</td>
<td>71,830</td>
<td>9</td>
</tr>
</tbody>
</table>

Note: CW = crested wheatgrass; Nat. = native grasses.
magnified in year two when additional non-real estate payments resulting from increased borrowing to finance the investment begin. However, beginning in year 4 and continuing over the remainder of the period, cash surplus exceeds the solution obtained when no investment is allowed.

**Summary and Conclusions**

A dynamic nonlinear optimization model was developed to determine optimal rangeland investment levels and herd sizes for a western beef cattle ranch. Expected discounted value of ending net worth was maximized, subject to probabilistic safety-first constraints on annual net ranch incomes under various stochastic rainfall and output price values. The procedure was shown to be useful in determining optimal capital investments when planning horizons are long, production is uncertain, and annual financial return requirements are present.

The benefits of an investment designed to provide a more stable supply of high quality spring forage were reflected in improved financial performance indicators, as well as increasing expected end-of-period net worth above the “no investment” alternative. Although the analysis results are dependent upon the parameters assumed for the representative ranch, the analysis does suggest the possibility of substantial benefits for the rancher considering a similar investment. The investment analysis also will depend upon the ranch’s initial resource base. Although the operation was assumed initially to be balanced with respect to forage needs and supplies, benefits of the investment will differ depending upon initial stocking levels. An excess of forage supplies over requirements will reduce the advantages of the investment. Conversely, the seeding may be more attractive for a ranch initially operating at or over the capacity of the forage/feed resources.

The model is a large-scale application of the dynamic safety-first model presented by Atwood, Watts, and Helmers. An additional feature of the model addresses the problem of inadequate data availability often encountered in range economic research. Utilizing range data on rainfall and subsequent forage production, an empirical cumulative distribution function of forage production was calculated. The procedure allows the analyst’s confidence in data quality to be directly incorporated by altering the probability level, $\alpha$, associated with the forage constraints. If one is doubtful of the quality of available data on forage availabilities, a lower value of $\alpha$ can be used. This variation on chance constraints directly incorporates an estimation of the empirical distribution function developed by Taylor rather than, for example, requiring the assumption that uncertain resource supplies are normally distributed.

One caution regarding model results stems from the high values of net ranch income and increases in net worth occurring over the plan-
Average Cash Surplus with and without the investment horizon. Variable cash costs reported in USDA are very low compared to the values of ranch production used in this study. Decreasing net ranch incomes either through increased costs or lower output prices certainly will influence optimal herd size and investment acreage. Use of the model in ranch planning thus will require careful consideration of actual input costs and output characteristics of the individual ranch.

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References


