Farmland Preservation and Differential Taxation: Evaluating Optimal Policy Under Conditions of Uncertainty

Robert J. Johnston

Differential property tax policy for farmland is often set under conditions of uncertainty and limited information regarding landowners' objective functions. This study examines optimal differential tax policy for a parcel of agricultural land facing uncertain development, identifying instances in which common farmland taxation policies may be non-optimal. Optimal tax rates are characterized given three possible causes of uncertain development: exogenous offers from developers, tax-related reductions in landowner wealth, and a combination of these factors. Model results indicate that underlying causes of uncertain development are critical when seeking to assess the optimality of differential taxation policies, and the use of a single, time-invariant differential tax levy is rarely optimal given uncertain development.

Key Words: differential taxation, farmland preservation, land conversion, optimal control, optimal policy, property tax, uncertainty

Differential taxation is a common tool employed to prevent conversion of farmland to higher-density residential or commercial uses. Most theoretical analyses of differential taxation and land conversion are based on models of land development and timing. These approaches model development from the perspective of the landowner; decisions regarding potential enrollment of land in differential tax programs and conversions from agricultural to non-agricultural use are based on a comparison of the discounted value of land in various uses (Anderson, 1986, 1993; Bentick, 1979, 1997; Bentick and Pogue, 1988; England and Mohr, 2003; Capozza and Helsley, 1989; Hennessy, 1999; Rose, 1973; Shoup, 1970; Skouras, 1978).

Where uncertainty or limited information is addressed by existing work, it is typically introduced in terms of landowner uncertainty regarding income, land rents, and other factors (e.g., Parks and Quimio, 1996; Anderson, 1993). In contrast, local and state officials often choose tax policies without detailed knowledge of landowner objective functions, and under conditions of considerable uncertainty associated with the timing and ultimate causes of farmland conversion. Findings of the empirical literature do not allay this uncertainty. For example, while many empirical studies demonstrate little or no impact of differential taxation programs (Ferguson, 1988; Parks and Quimio, 1996; Wunderlich, 1997), others suggest differential taxation may indeed influence the extent and rate of development (Lopez, Shah, and Altobello, 1994).

Optimal farmland tax policy under conditions of uncertainty will not in general coincide with analogous policy in a deterministic context. Moreover, the potential causes of uncertain development may influence optimal policy. This study models optimal tax policy under three scenarios, each characterizing a different cause of uncertain development.

The first scenario assumes development is triggered solely by an exogenous (i.e., not tax related) event—in this case, an offer from developers that

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occurs at an uncertain future date. The second scenario assumes development is triggered solely by taxation which reduces the wealth of the landowner to some uncertain threshold. Under the third scenario, it is assumed that development is triggered by a combination of these uncertain factors. All models address optimal tax policy from the limited information perspective of community policy makers, in which detailed knowledge of landowner objective functions is unavailable.

To determine optimal tax policy, an optimal control model is developed for a parcel of agricultural land subject to uncertain and irreversible development. As noted above, development may be triggered by an exogenous offer from developers at an uncertain future date, by excessive taxation that reduces the wealth of the landowner to some uncertain threshold, or by a combination of these factors. Optimal policy is compared for each case and contrasted with the case in which the risk of development is not present.

The primary purpose of this presentation is to offer new insights into how communities may improve tax policies given that development is uncertain. Although the model is purposefully stylized and abstracts from many issues addressed elsewhere in the literature, it offers a perspective largely absent from existing work. In particular, the model demonstrates that the cause of uncertain development has significant implications for optimal farmland taxation, and that use of a single, time-invariant tax levy is rarely optimal given uncertain development.

General Theoretical Framework

The literature provides numerous models addressing resource allocation under uncertainty. These include, among others, models of nonrenewable resource depletion given uncertain technological change (Dasgupta and Heal, 1974; and Dasgupta and Stiglitz, 1981), an examination of optimal forest rotation given uncertain risk of forest fire (Reed, 1984), analyses of consumption given the risk of pollution-induced catastrophe (Cropper, 1976; and Clarke and Reed, 1994), an analysis of fishery harvest policy given uncertain stock collapse (Reed, 1988), and a model of fishery harvest policy given uncertain biomass shift (Johnston and Sutinen, 1996). Although these models focus on consumption and resource depletion choices, similar perspectives may be used to address the choice of tax policy given uncertain land conversion.

We begin with a simple model in which development of a particular agricultural parcel of interest is independent of property tax policy, as suggested by the empirical studies of Ferguson (1988), and Parks and Quimio (1996). While optimal control models of resource policy under uncertain conditions are often rich in potential solutions and policy implications (Cropper, 1976), they generally maintain a relatively high degree of abstraction to preserve generality and simplify analysis. The following model continues in this tradition; the basic model is kept simple to maintain a focus on the fundamental questions of interest.

The model addresses optimal taxation and assessment choices pertaining to a single parcel of agricultural land. Although communities may be precluded from establishing tax rates for individual parcels, differential assessment levels for individual parcels may be set on a case-by-case basis, allowing policy makers some flexibility regarding the total property tax levied on individual parcels.[1] Even if constraints prevent community officials from moving toward more optimal policies, it may nonetheless be useful to characterize optimal policy under various scenarios and to assess implications of non-optimal paths.

Tax policy is assumed to be determined by a central authority seeking to maximize net community benefits resulting from the collection of property taxes, $q(t)$, from a parcel of interest, and from the amenity benefits generated by the same agricultural parcel. The central authority would most likely influence $q(t)$ through changes in the assessed value of the parcel, assuming the land is subject to a fixed community-wide tax rate per dollar of assessed value. Regardless of the method used to control property taxes, the ultimate annual tax paid by the landowner is assumed constrained by total wealth and income, such that

(1) $$\dot{x} = h(x) - q(t),$$

where $x > 0$ represents the accumulated wealth or capital stock of the landowner (not including the market value of the agricultural land), $h(x)$ represents the annual increment to landowner wealth resulting from agricultural use of the parcel, and the

[1] For example, Rhode Island provides only recommended differential assessment levels for individual parcels enrolled in the Farm, Forest, and Open Space Act [Rhode Island Department of Environmental Management (RIDEM), 2002]. Individual community tax assessors are given considerable flexibility in determining assessments on enrolled parcels, although in practice relatively standardized assessments are most often used.

dot $(\cdot)$ represents a partial derivative with respect to time.

We specify $h(x) = i(x, \bar{\mathbf{k}}, \bar{\mathbf{g}}) - e(i(\cdot), x, \bar{\mathbf{p}})$, where $i(x, \bar{\mathbf{k}}, \bar{\mathbf{g}})$ represents income from the agricultural use of the parcel, $\bar{\mathbf{k}}$ represents characteristics of the land parcel influencing its productivity as farmland (assumed fixed), and $\bar{\mathbf{g}}$ represents characteristics of the landowner (also assumed fixed). Landowner consumption expenditures are denoted $e(i(\cdot), x, \bar{\mathbf{p}})$ and are assumed to depend on income, accumulated wealth, and a vector of fixed prices $(\bar{\mathbf{p}})$. As $\bar{\mathbf{k}}$, $\bar{\mathbf{g}}$, and $\bar{\mathbf{p}}$ are assumed fixed, and $i(\cdot)$ is a function of x, $\bar{\mathbf{k}}$, and $\bar{\mathbf{g}}$, we suppress $\bar{\mathbf{k}}$, $\bar{\mathbf{g}}$, $\bar{\mathbf{p}}$, and $i(\cdot)$, and denote the growth function $h(x)$ as given above. Although this is a stylized specification, it allows a focus on the more fundamental questions of interest (i.e., the influence of uncertainty on optimal taxation).

Existing wealth at time t is assumed to have a diminishing but always positive marginal impact on the increment to wealth at t, i.e., $h'(x) > 0$ and $h''(x) < 0$, where the prime $(')$ denotes a partial derivative. The rationale for $h''(x) < 0$ is assumed diminishing returns to fixed factors of farm production such as land acreage. It is further assumed that the *entire* agricultural parcel is either maintained in its existing agricultural use or sold for development, and land holdings remain constant until the land is sold.

Social benefits are derived from tax collections, $q(t)$, as taxes support community services. Because the community is assumed to place value on the welfare of farm owners, social benefits also depend on aggregate landowner wealth (x). Hence, the first element of the planner's objective function is denoted $U(q, x)$. In the range of tax levies imposed on farmland, we assume the following:

$$\frac{\partial U(\cdot)}{\partial q} = U_q > 0, \quad \frac{\partial U(\cdot)}{\partial x} = U_x > 0;$$

and

$$\frac{\partial^2 U(\cdot)}{\partial q^2} = U_{qq} < 0, \quad \frac{\partial^2 U(\cdot)}{\partial x^2} = U_{xx} < 0,$$
$$\frac{\partial^2 U(\cdot)}{\partial q \, \partial x} = U_{qx} > 0.$$

That is, the marginal social benefits of taxes collected on the parcel are positive, and the social benefits of the tax levy increase with the stock of landowner wealth. The community benefits of the tax levy increase at a decreasing rate, as either the levy or the stock of wealth increases. Finally, the impact of an increased stock of wealth on the marginal benefits of additional taxes is positive.

In addition to benefits associated with taxation of the parcel, social benefits are provided by amenity benefits of the agricultural parcel, given by $V(\mathbf{k})$, where $\mathbf{k}$ again denotes the fixed characteristics of the parcel. To minimize unnecessary notation and complication, we abstract from other exogenous factors that might affect aggregate net social benefits.

Optimal Taxation Given Exogenous Development Pressure

We first consider optimal taxation when there is a risk of development due solely to exogenous developer behavior (i.e., an offer to purchase the land parcel), independent of property tax levels. Assume an offer sufficient to induce sale of the land occurs at time T, and results in the conversion of the parcel to residential or commercial use. At this point, the parcel ceases to generate amenity benefits of farmland $V(\mathbf{k})$ and the benefits associated with taxation $U(q, x)$ of the farmland parcel, and instead generates annual net benefits associated with developed land—i.e., at conversion time T, $U(q, x) = V(\mathbf{k}) = 0$.

The net benefits (or costs) of the developed parcel, realized subsequent to time T, are given by $W(\mathbf{k}^*)$, where $\mathbf{k}^*$ represents the attributes of the developed property, assumed exogenous and fixed. Specifically, where $\mathbf{k}$ describes the characteristics of the parcel as farmland, $\mathbf{k}^*$ represents its attributes as a developed property (e.g., the number of houses, size of subdivided lots, extent of sewer and drainage infrastructure, etc.).

Implicit in the assumption of a fixed optimal $W(\mathbf{k}^*)$ is the assumption that once developed, the property will retain a constant generation of tax revenues (based on standard residential or commercial tax rates) and community costs. This assumption is made for simplification purposes only; relaxing it does not change fundamental model results. It is hypothesized that $W(\mathbf{k}^*)$ may be positive or negative, depending on the relationship between the tax revenues and community costs generated by the developed parcel.

The function $W(\mathbf{k}^*)$ does not include the wealth of the agricultural landowner as an argument, implying the individual's wealth is of no concern to the community once the land has been sold. Although made for convenience, this assumption corresponds to the notion that communities place particular value on the welfare of active farmers. Thus, once the landowner converts his or her property to nonfarm use, the community ceases to consider the landowner's wealth in its objective function.

To incorporate a random date of an offer sufficient to induce land development, we follow Johnston and Sutinen (1996) and assume T is a random number with a probability density function $f(t)$, where $f(t) > 0$ and

$$\int_0^\infty f(t)\,dt = 1.$$

Given this specification, the probability that an offer sufficient to cause development has not yet occurred at t is given by:

$$F(t) = \int_t^\infty f(\tau)\,d\tau,$$

where

$$F(0) = 1,\ F(\infty) = 0,\ \text{and}$$
$$\dot{F} = -f(t) = \frac{\partial F(t)}{\partial t} < 0.$$

These expressions indicate the land has not yet been developed at time zero, will definitely be developed at some point in the future, and the probability that the land will continue in its undeveloped state diminishes as one looks further into the future.

To allow for a steady-state result, we follow Dasgupta and Heal (1974) and assume a constant hazard function, given by

$$\Delta = \frac{f(t)}{F(t)} = \frac{-\dot{F}(t)}{F(t)} > 0.$$

Here, the risk of development is assumed to be independent of time, or the conditional probability of development at t (given development has not yet occurred) is assumed constant.

Assume the community government maximizes expected social tax and amenity benefits generated by the land parcel of interest. Following Dasgupta and Heal (1974, p. 20),

$$\text{(2)}\quad J = \int_0^\infty f(t)\left\{\int_0^T e^{-rt}\left[U(q,x) + V(\bar{\mathbf{k}})\right]dt + \int_T^\infty e^{-r(t-T)}\left[W(\mathbf{k}^*)\right]dt\right\}dT.$$

Integration by parts yields

$$\text{(3)}\quad J = \int_0^\infty e^{-rt}F(t)\left[U(q,x) + V(\bar{\mathbf{k}})\right] + e^{-rt}\left[1 - F(t)\right]W(\mathbf{k}^*)\,dt.$$

The authority maximizes (3) subject to (1). The Hamiltonian follows directly from (1) and (3).

Necessary conditions defining the optimal path of property taxes and savings stock include:

$$\text{(4)}\quad F(t)U_q = \lambda$$

and

$$\text{(5)}\quad \dot{\lambda} = \lambda\left[r - h'(x)\right] - F(t)U_x,$$

where λ represents the present-value costate variable.

The optimal steady-state tax rate (q^*) and stock of savings (x^*) are found by solving (1), (4), and (5) when $\dot{q} = \dot{\lambda} = 0$. This steady state is characterized by

$$\text{(6)}\quad h'(x) = r + \Delta - \frac{U_x}{U_q},$$

where $\Delta = -\dot{F}(t)/F(t)$ as before.

In the absence of development risk, (6) is replaced with $h'(x) = r - U_x/U_q$. The addition of the positive hazard rate, Δ, given the functional characteristics of $h'(x)$ and U_x/U_q, implies steady-state wealth (x^*) is lower than in the no-development case, and hence will support a lower optimal steady-state tax levy (q^*).

Optimal tax and wealth trajectories leading to the steady state may be assessed through the phase diagram characterized by (1), (4), and (5). From (1), the $\dot{x} = 0$ isocline is characterized by $q(t) = h(x)$. From (4) and (5), it may be shown that

$$\text{(7)}\quad \dot{q} = \frac{U_q\left(r + \Delta - h'(x)\right) - U_x}{U_{qq}},$$

whereby the $\dot{q} = 0$ isocline is characterized by $U_x = U_q(r + \Delta - h'(x))$. Although ambiguities regarding the relative magnitudes and functional forms of the arguments in (1) and (7) prevent exact and unambiguous specification of the $\dot{x} = 0$ and $\dot{q} = 0$ isoclines, common properties of the phase diagram are illustrated by figure 1.[2]

Figure 1 shows the $\dot{x} = 0$ isocline, together with the $\dot{q} = 0$ isoclines given both $\Delta > 0$ and $\Delta = 0$—i.e., under both positive and zero exogenous development risk, respectively. The corresponding steady states are located at the intersections of the isoclines, and are saddlepoints. Approach paths depicted by the phase diagram show that for any initial level of wealth x_0 greater than both steady states, the optimal tax levy q will begin higher and decline more quickly when there is an exogenous risk of development ($\Delta > 0$, figure 1). As a result of higher taxes

[2] The phase diagram in figure 1 is characterized based on the general functional forms $U(q,x) = Aq^{\alpha}x^{\beta}$ ($A > 0$; $0 < \alpha < 1$; $0 < \beta < 1$; $\alpha + \beta \leq 1$) and $h(x) = M - Ne^{-\gamma x}$ ($M, N > 0$; $0 < \gamma < 1$), where e is the exponential operator. Similar phase diagrams result from other common functional forms.

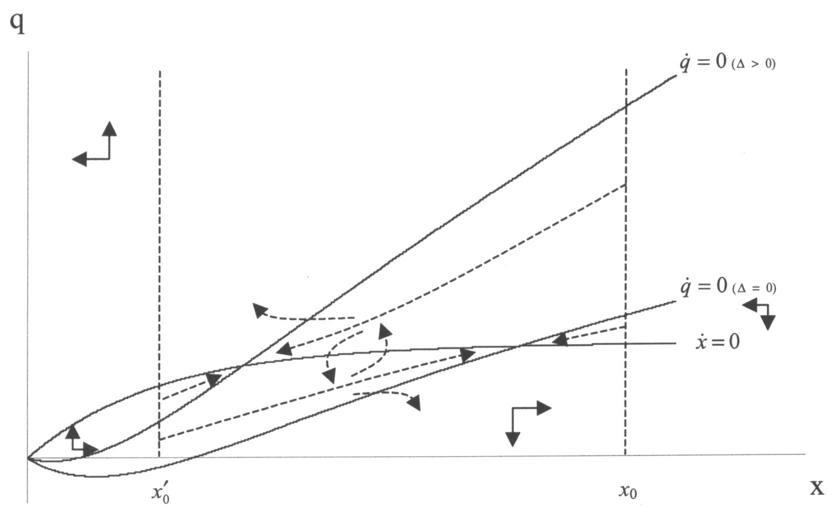

Figure 1. Phase plane analysis and comparative dynamics of the exogenous development case

in early periods, wealth declines more rapidly in the $\Delta > 0$ case compared to the Δ ' 0 case, leading to lower steady-state wealth. This is illustrated by the two dashed approach trajectories to the steady states from initial landowner wealth x_0.

Analogous patterns apply for an initial stock x_0' less than the steady states; initial tax levies are higher when the risk of development is positive, leading to lower steady-state wealth. For initial wealth between the two steady states, the optimal approach path for Δ ' 0 (no risk of development) involves lower initial taxes such that wealth increases to a higher steady state. For $\Delta > 0$ (positive risk of development), the optimal path involves higher initial tax levies whereby wealth decreases to its lower steady state.

In all cases, once the steady state is reached, steady-state taxes are lower in the $\Delta > 0$ case, corresponding to the lower optimal steady-state wealth. However, these lower steady-state taxes are only optimal once landowner wealth has been diminished through the higher initial taxation that is optimal given an exogenous risk of development.

The rationale behind such patterns is intuitive. If a community knows that a farmer will sell land to a developer at some point in the future, and that development is caused entirely by exogenous factors (i.e., is unrelated to tax policy), then optimal policy will extract greater tax revenues from the parcel while it remains in agricultural use. From a neoclassical perspective, the risk of development reduces the expected future value to society of an additional dollar of landowner wealth. Once the land is developed, the stock of landowner wealth generates no additional benefits for the community, as the original landowner no longer owns the parcel and no longer pays property taxes. Accordingly, optimal taxation is characterized by increased tax levies in early periods, leading to reduced landowner wealth in the steady state.

An additional feature of model results here is that neither the value of developed land nor the amenity benefits of farmland has any effect on optimal pre-development tax policy. That is, neither the annual post-development flow of benefits, $W(\mathbf{k}^*)$, nor the external amenity benefits of farmland, $V(\mathbf{k})$, appear in the equations specifying the steady state. This result applies because tax policy in this case has no effect on the date of development. Accordingly, the best the tax authority can do is to maximize expected tax benefits prior to the uncertain development date, and then follow the new optimal tax policy on the property once developed.

The above model applies to cases such as those discussed by Parks and Quimio (1996), Ferguson (1988), and Wunderlich (1997), in which differential taxation has no discernable impact on development decisions. In such cases, the results here suggest common tax policies may be non-optimal. For example, communities may react to increased likelihood of farmland conversion by reducing taxes on farmland at risk of development. However, in the case where the risk of development is exogenous, the optimal response to an increase in exogenous development pressure is an increase in short-term tax levies, leading to lower landowner wealth than would be optimal in the no-development case. This finding illustrates the potentially counter-intuitive result that increased development risk can increase short-run optimal tax levies on farmland.

Optimal Taxation Given Uncertain Wealth-Related Development

We now turn to the case in which the uncertain date of development is related solely to the accumulated wealth of the landowner. This scenario reflects the "farmland as retirement policy" case, in which the farmer will sell his/her land to finance retirement unless sufficient wealth is available.

Assume there is a standing offer from developers to purchase the property, and the landowner would prefer to maintain the land in agricultural use. However, if property taxes drive wealth down to a critical threshold, S, then the landowner will sell the land. S is assumed to be known to the landowner but unknown to the central authority. It is further assumed that wealth at time zero (x_0) is greater than S, and x_0 is of sufficient size to prevent the "extinction" strategy associated with sub-marginal resources (Clark, 1971).

Under this scenario, community officials do not know with certainty the critical wealth threshold below which the landowner will sell the parcel. To model the uncertain wealth stock at which conversion will occur, we apply a probability specification similar to that used to model uncertain pollution catastrophe (e.g., Cropper, 1976; Clarke and Reed, 1994) and uncertain biomass shift (Johnston and Sutinen, 1996).

From the perspective of the central authority, S is a random variable, distributed over the interval $[0, x_0]$, with a probability density function $f(S)$. The likelihood that sale of the property will not occur at wealth stock x is given by the probability that $S < x$, or

$$(8) \qquad F(x) = \int_0^x f(S)\, dS,$$

where

$$F_x = \frac{\partial F(x)}{\partial x} > 0, \ \forall x < x_0,$$

and

$$F_x = \frac{\partial F(x)}{\partial x} = 0, \ \forall x \geq x_0.$$

Other aspects of the model remain as specified above. Accordingly, the central authority maximizes net benefits given by

$$(9) \qquad J = \int_0^\infty e^{-rt}\left[F(x)\left[U(q,x) + V(\bar{\mathbf{k}})\right]\right] + e^{-rt}\left[1 - F(x)\right] W(\mathbf{k}^*)\, dt,$$

subject to (1) as before. The necessary conditions for a maximum include (1) and

$$(10) \qquad F(x)U_q = \omega,$$

$$(11) \qquad \dot{\omega} = \omega\left[r - h'(x)\right] - F_x\left[U(q,x) + V(\bar{\mathbf{k}})\right] + F_x W(\mathbf{k}^*) - F(x)U_x,$$

where ω is the current-value costate variable. Equations (10) and (11) provide necessary but not sufficient conditions. Given the characteristics of the expected benefit function (9), joint concavity may not be an appropriate general assumption (Cropper, 1976). Without this assumption, one cannot guarantee the usual sufficiency conditions.

However, assuming a steady state exists, and assuming (10) and (11) identify a global maximum, the steady state is characterized by

$$(12) \qquad h'(x) = r - \theta(q, x, \bar{\mathbf{k}}) + \psi(q, x, \mathbf{k}^*) - \frac{U_x}{U_q},$$

where

$$(13) \qquad \theta(q, x, \bar{\mathbf{k}}) = \frac{F_x\left[U(q,x) + V(\bar{\mathbf{k}})\right]}{F(x)U_q} \geq 0$$

and

$$(14) \qquad \psi(q, x, \mathbf{k}^*) = \frac{F_x W(\mathbf{k}^*)}{F(x)U_q},$$

where $\psi(q, x, \mathbf{k}^*)$ may be greater, less than, or equal to zero.

The mathematics of the phase diagram are more complex than those in the exogenous development case. The $\dot{x} = 0$ isocline is characterized by the equation $q(t) = h(x)$. From (10) and (11), it may be shown that

$$(15) \qquad \dot{q} = \frac{U_q}{U_{qq}}\left[r - h'(x) - \frac{F_x[U(\cdot) + V(\bar{\mathbf{k}}) - W(\mathbf{k}^*)]}{F(x)U_q} - \frac{U_x}{U_q} - \frac{F_x[h(x) - q]}{F(x)}\right].$$

While the phase diagram is rich in potential solutions, a characteristic pattern is illustrated by

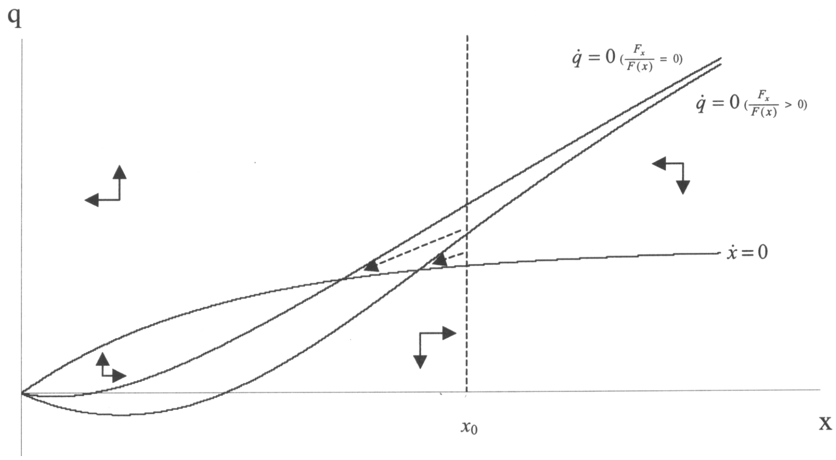

Figure 2. Phase plane analysis and comparative dynamics of the wealth-related development case

figure 2.[3] This pattern presumes the net value to the community of the undeveloped parcel exceeds the net value of the developed parcel, or $U(@)$ % $V(\bar{\mathbf{k}}) > W(\mathbf{k}^*)$. This assumption appears reasonable in the current policy context, given that a rational community would only apply differential taxation in cases where preservation of farmland is beneficial [i.e., $U(@)$ % $V(\bar{\mathbf{k}}) > W(\mathbf{k}^{(})$]. Hence, the discussion emphasizes this case.

If $U(@)$ % $V(\bar{\mathbf{k}}) > W(\mathbf{k}^{(})$, then optimal tax levies in this case are lower in early periods leading to greater steady-state landowner wealth, compared to the case in which there is no wealth-related risk of development. Approach paths to the steady state are illustrated by the dashed lines in figure 2. For any initial x_0 greater than the two steady states, the presence of wealth-related development risk ($F_x/F(x) > 0$, figure 2) leads to a decrease in initial tax levies, leading ultimately to higher steady-state wealth. In the absence of development risk ($F_x/F(x)$ ' 0, figure 2), higher initial tax levies lead to lower steady-state wealth. Simply put, in the case where reduced landowner wealth leads to an increased probability of development, optimal policy maintains higher wealth to discourage land conversion.

These conclusions may be verified through analysis of the different terms in (12). The term $\theta(q, x, \bar{\mathbf{k}})$ reflects the prospect of inducing development with higher taxes and associated wealth reductions. Where $\theta(q, x, \bar{\mathbf{k}}) > 0$, the inclusion of this term in (12) leads to higher steady-state wealth compared to the no-development case.[4]

The term $\psi(q, x, \mathbf{k}^*)$ reflects the net value of the developed property to the existing community. If the annual net benefits of the developed parcel, $W(\mathbf{k}^*)$, are positive, then $\psi(q, x, \mathbf{k}^*)$ will be greater than zero, and its presence in (12) tends to offset that of $\theta(q, x, \bar{\mathbf{k}})$, reducing the optimal steady-state stock of savings.[5] If $\psi(q, x, \mathbf{k}^*)$ is negative, then its inclusion augments the effect of $\theta(q, x, \bar{\mathbf{k}})$, leading to higher steady-state savings. These results are intuitive—if development is dependent on tax policy, then anticipated positive net benefits from the developed parcel make preservation of the agricultural land (and hence landowner wealth) less crucial, and vice versa.

[3] The phase diagram for figure 2 is characterized based on functional forms analogous to those in figure 1. The probability that development will not occur at a given stock is specified using the general functional form $F(x) = (e^{\eta x}$! $1)/(e^{\eta x} + 1)$, $(0 < \eta < 1)$.

[4] The only exception to this general case holds when $\theta(q, x, \bar{\mathbf{k}}) = \psi(q, x, \mathbf{k}^*) = 0$, which occurs if $F_x/F(x) = 0$ (where the marginal effect of stock changes on the risk of development are zero). Given our definitions of F_x and $F(x)$, this can only occur for $x > x_0$, or in cases where the optimal stock of landowner savings in the no-development case is greater than the initial stock x_0. Specifically, since the critical threshold $S < x_0$, the risk of development drops to zero for all $x > x_0$ (if $S > x_0$, development would have already occurred at time zero). Accordingly, if the optimal path in the no-development case leads to a steady-state $x > x_0$, then the risk of development can have no additional impact on the steady state. This result also implies that $\theta(q, x, \bar{\mathbf{k}})$—despite its (generally) positive impact on the optimal stock of steady-state landowner savings—cannot by itself lead to optimal steady-state landowner savings greater than the initial level x_0.

[5] The discussion of this term assumes that $F_x/F(x) > 0$. If this condition does not hold, then $\theta(q, x, \bar{\mathbf{k}}) = \psi(q, x, \mathbf{k}^*) = 0$, as discussed in footnote 3.

The contrast between optimal policy in this case (wealth-related development risk) and that found in the prior model (exogenous development risk) is one of the fundamental results of the analysis. If the probability of development is independent of tax levels, then the optimal approach path unambiguously leads to lower steady-state landowner wealth. If, in contrast, the community can influence the probability of development through tax policy, then the optimal approach path leads to higher steady-state landowner wealth as long as preservation of farmland is beneficial to the community [i.e., $U(\cdot) + V(\bar{\mathbf{k}}) > W(\mathbf{k}^*)$]. Optimal policy under exogenous development risk (i.e., higher initial tax levies leading to lower wealth) is the opposite of optimal policy under wealth-related development risk (i.e., lower initial tax levies leading to higher wealth).

Optimal Policy in the General Case

We now consider the general and more complex case, in which development is influenced both by uncertain developer offers and community tax rates. To model this case, it is assumed that a developer offer of a specific size may cause the landowner to sell (develop) the agricultural property if the landowner's existing stock of wealth is small, but will be rejected if the landowner's wealth is large. We define $f(t, x(t))$ as the joint probability density function of T, where T is the uncertain date of development given a specific savings level x.

Following the development of the previous models, define

$$F(x, t) = \int_t^\infty f\big(\tau, x(\tau)\big)\, d\tau$$

as the probability that development has not yet occurred at time t. Further, assume

$$F_t = \frac{\partial F}{\partial t} < 0 \quad \text{and} \quad F_x = \frac{\partial F}{\partial x} > 0, \qquad \forall\, x < x_0,$$

indicating that a greater stock of wealth diminishes the probability of development due to uncertain developer purchase offers. All other features of the model remain as specified above.

The central authority in this case maximizes net benefits given by

$$(16)\qquad J = \int_0^\infty e^{-rt}\Big[F(t,x)\,[U(q,x) + V(\bar{\mathbf{k}})]\Big] + e^{-rt}\,[1 - F(t,x)]\,W(\mathbf{k}^*)\, dt,$$

subject to (1) as before.

As noted in the prior model, the dynamics of the optimal harvest path and approach to the steady state are complex; a steady-state optimum is not ensured. If an optimal steady-state tax policy does exist, it is characterized by

$$(17)\qquad r + \hat{\Delta}(\cdot) - \hat{\theta}(\cdot) + \hat{\psi}(\cdot) - \frac{U_x}{U_q} = h'(x),$$

where

$$\hat{\Delta}(\cdot) = \frac{-F_t}{F(t,x)}, \quad \hat{\theta}(\cdot) = \frac{F_x\,[U(\cdot) + V(\bar{\mathbf{k}})]}{F(t,x)\,U_q}, \quad \text{and} \quad \hat{\psi}(\cdot) = \frac{F_x\,W(\mathbf{k}^*)}{F(t,x)\,U_q}.$$

In general, a steady state can exist only if it is assumed that

$$\frac{\partial \hat{\Delta}(\cdot)}{\partial t} = 0, \quad \frac{\partial \hat{\theta}(\cdot)}{\partial t} = 0, \quad \text{and} \quad \frac{\partial \hat{\psi}(\cdot)}{\partial t} = 0;$$

i.e., an autonomous solution is assumed.

Despite the complexity of the equations specifying the approach path and steady state, some general policy conclusions may be drawn. Since $\hat{\Delta}(\cdot) > 0$ and $\hat{\theta}(\cdot) > 0$, then the optimal steady-state wealth accumulation ($\hat{x}$) will be greater than that in the case where the probability of development was exogenous (x^*), or independent of differential tax rates. In other words, initial tax collections are lower, such that steady-state wealth accumulates to a higher level. This higher steady-state wealth ultimately supports higher steady-state tax levies, but only after lower tax collections in early periods allow wealth to grow to the higher steady-state level.

Compared to the case in which the probability of development is due solely to tax policy, steady-state savings are lower ($\hat{x} < \tilde{x}$), and steady-state taxes are lower, representing more aggressive taxation approaching the steady state. Whether the steady state will more closely approximate the exogenous development case or the tax-related development case depends on the relative magnitudes of $\hat{\Delta}(\cdot)$ (representing the impact of exogenous development risk) and $\hat{\theta}(\cdot) + \hat{\psi}(\cdot)$ (representing the impact of wealth related development risk).

Comparison to the no-development case is simplified by manipulating (17) to yield

$$(18)\qquad h'(x) = r - \frac{F_t}{F(t,x)} - \frac{F_x}{F(t,x)} \times \left[\frac{[U(q,x) + V(\bar{\mathbf{k}}) - W(\mathbf{k}^*)]}{U_q}\right] - \frac{U_x}{U_q}.$$

The term $!F_t/F(t, x)$ represents the impact of the exogenous portion of the development probability, or the conditional probability of development at t in the absence of any changes in x. As before, this term leads one to maintain lower steady-state landowner savings, based on more aggressive optimal taxation as one approaches the steady state.

The next term,

$$\& \frac{F_x}{F(t,x)}\left[\frac{[U(q,x) \,\%\, V(\bar{\mathbf{k}}) \,\&\, W(\mathbf{k}^{(})]}{U_q}\right],$$

represents the impact of development probability associated with tax-related changes in the stock of landowner savings. Note that the relative impact of this term not only depends on the underlying probability density functions, but also on the social benefits received from taxation ($U(q, x)$), the amenity benefits of farmland ($V(\mathbf{k})$), and the net benefits of the land once developed ($W(\mathbf{k}^*)$). As long as the term in brackets $[U(q, \mathrm{x}) + V(\mathbf{k})\ !\ W(\mathbf{k}^*)]$ is greater than zero (and assuming $F_x/F(t,x) > 0$, as discussed above), the entire term will tend to offset the impact of $!F_t/F(t, x)$. That is, it will lead to a higher optimal stock of steady-state savings.

The impact of $W(\mathbf{k}^*)$ depends on its sign. If the net benefits of the developed property are positive, then larger values for this term will lead to more aggressive taxation approaching the steady state, leading to lower steady-state taxes and wealth. If net benefits of the developed property are negative, then larger (more negative) magnitudes of $W(\mathbf{k}^*)$ will augment the impact of $U(q, x)$ and $V(\mathbf{k})$, leading to higher optimal steady-state savings.

Discussion and Conclusions

This study characterizes optimal differential tax policy from the limited knowledge perspective of community tax authorities, in cases where landowner objective functions are unknown and development prospects are uncertain. The model illustrates that in such cases, theoretical approaches similar to those used to assess optimal natural resource use under uncertainty may be applied to tax policy.

Although the model is purposefully kept simple, it yields findings not evident in the existing literature. Of particular relevance are the conclusions that the underlying causes of uncertain development are critical when seeking to assess the optimality of differential taxation policies, and that the use of a single, fixed differential tax levy is rarely optimal given uncertain development. Model results are summarized by table 1.

For states in which differential taxation of farmland is based on current use value assessment, statewide regulations and guidelines typically determine assessment policy. Yet, states such as Rhode Island provide only nonbinding assessment recommendations, leaving local communities considerable flexibility in determining differential assessments for particular parcels or types of farmland (RIDEM, 2002). While this flexibility may not allow annual changes in tax levies necessary to achieve optimal policies of the type identified here, it nevertheless allows communities some ability to tailor differential taxation to increase net benefits. However, the existing literature provides little guidance—either empirical or theoretical—to assist communities in assessing the optimality of existing tax policies when development is uncertain.

Results here may assist communities in determining whether changes to farmland tax policies would increase net community benefits related to taxation and farmland conservation. Although theoretical results alone fall short of providing a simple quantitative formula that communities might use to determine specific tax levels for farmland parcels, they nonetheless provide insight into optimal tax policies—policies which may in some cases run contrary to common wisdom.

For example, while common wisdom might dictate reducing initial tax levies (to increase farm profits) in response to increased development risk, model results indicate such actions are only optimal if the probability of development is correlated with landowner wealth and the net community benefits of farmland exceed those of developed land (table 1, case IIa). More specifically, in cases where development is influenced solely by tax policy, optimal differential tax rates depend on a comparison of net pre-development social value of farmland and net post-development value. If community benefits of farmland exceed those of developed land, then optimal tax levies will be lower in early periods, leading to greater steady-state landowner wealth.

In contrast, if development is driven solely by exogenous developer behavior (table 1, case I), optimal differential tax levies are higher in early periods, leading to lower landowner wealth in the steady state. In such cases, communities might consider increasing taxes on farmland in the near term, with resulting revenues used to fund alternative conservation policies. Comparison to the

Table 1. Summary of Model Results: Optimal Tax Policy Given Uncertain Development

Case	Cause of Uncertain Development	Initial Taxation (Property Taxes Prior to Steady State)[a]	Steady-State Wealth/Taxes[a]
I	Development caused solely by exogenous developer offer, $\Delta > 0$	Higher	Lower/Lower
IIa	Development caused by reduction of landowner wealth to critical threshold, $U(@) \% V(\bar{\mathbf{k}}) > W(\mathbf{k}^{(})$[b]	Lower	Higher/Higher
IIb	Development caused by reduction of landowner wealth to critical threshold, $U(@) \% V(\bar{\mathbf{k}}) < W(\mathbf{k}^{(})$[c]	Higher	Lower/Lower
III	Development caused by combination of exogenous developer offer and reductions in landowner wealth	May be Higher or Lower	May be Higher or Lower

[a] Compared to the case in which there is no risk of development.
[b] Represents the case emphasized in the text, in which the value to the community of the undeveloped parcel exceeds that of the developed parcel, justifying differential taxation policy.
[c] Represents a case deemphasized in the text, in which the value to the community of the developed parcel exceeds that of the undeveloped parcel. Conservation activity and standard differential taxation is unjustified in this case; hence it represents a less relevant scenario in the current discussion.

wealth-related development case shows that a simple shift in the cause of uncertain development (exogenous versus wealth-related) can generate diametrically opposing policy prescriptions (table 1).

In the general case where development may be related to both exogenous and endogenous factors, optimal policy depends on the balance between uncertain elements underlying development probability. Unlike the prior cases, this more complex model does not generate unambiguous guidance regarding increases or decreases in initial tax levies (table 1). However, even the more complex general case illustrates the type of insights that may be gained though models addressing optimal policy under conditions which more closely simulate situations facing community policy makers. In the case of differential taxation and farmland conservation, this situation includes a lack of access to landowner objective functions and corresponding uncertainty regarding the time at which development may occur.

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