Journal of Agricultural and Resource Economics 26(1):291–308

Feeder Cattle Price Slides

B. Wade Brorsen, Nouhoun Coulibaly, Francisca G. C. Richter, and DeeVon Bailey

A theoretical model is developed to explain the economics of determining price slides for feeder cattle. The contract is viewed as a dynamic game with continuous strategies where the buyer and seller are the players. The model provides a solution for the price slide that guarantees an unbiased estimate of cattle weight. An empirical model using Superior Livestock Auction (SLA) data shows price slides used are smaller than those needed to cause the producer to give unbiased estimates of weight. Consistent with the model's predictions, producers slightly underestimate cattle weights.

Key words: asymmetric information, feeder cattle, game theory, price slide

Introduction

Feeder cattle prices per pound normally decrease as cattle weights increase. Also, a given buyer only wants cattle that are within a specific weight range. Thus, feeder cattle weight is critical in determining price. Estimating the weight of cattle can be difficult for both buyers and sellers, especially when cattle are sold for future delivery. In many private-treaty sales, the buyer never sees the cattle before purchase, although an order buyer might. In video auctions, the buyer only sees the cattle on a television screen. Thus, the seller is often better able to estimate weights than the buyer. Since sellers and buyers have asymmetric information about cattle weights, contracts need to be structured to provide sellers with an incentive to accurately estimate average delivery weights.

The usual approach to dealing with uncertain weight is to adjust the original contract price by a "price slide." The price slide (sometimes called a one-way slide) specifies the rate at which the contract price will be reduced when the average delivered weight is greater than the weight established in the contract plus a specified tolerance. With a one-way slide, no adjustment is made to the contract price if delivered cattle weigh less than the specified limit.

Suppose, for example, a producer estimates average delivered weight at 500 lbs. The producer could sell cattle at $70/cwt with a price slide of 10 cents per cwt for each pound of actual average weight over 520 lbs. If cattle average 530 lbs. at delivery, then $1/cwt (10 cents/cwt/lb. × 10 lbs.) is deducted from the $70/cwt contract price. If, however, actual

---

B. Wade Brorsen is a regents professor and Jean and Patsy Neustadt chair in the Department of Agricultural Economics, Oklahoma State University; Nouhoun Coulibaly is an associate with a research development firm in Côte d'Ivoire; Francisca G. C. Richter is a research associate in the Department of Agricultural Economics, Oklahoma State University; and DeeVon Bailey is a professor in the Department of Agricultural Economics, Utah State University. Senior authorship is shared by the first three authors.

Review coordinated by J. Scott Shonkwiler and Russell Tronstad; publication decision made by Russell Tronstad.

average weight is 515 lbs., no adjustment is made from the contract price.[1] The one-way price slide is an implicit option, and therefore the value of the option should be reflected in the price.

Superior Livestock Auction (SLA) currently sells over a million head of feeder cattle a year—more than any other auction in the United States. Feeder cattle sold through SLA are sold with a price slide and, in general, buyers only see the cattle through a television screen (Bailey, Peterson, and Brorsen). Most private-treaty sales also use a price slide, though price slides are not used in traditional auctions. The interest of producers in the topic is demonstrated by two extension articles (Bailey and Holmgren; and Prevatt) on price slides. However, a review of the literature reveals no research has yet been conducted in support of extension efforts.

A contract has four essential variables: the contract price (base price), the price slide, the allowable weight difference (weight tolerance), and the estimated cattle average delivery weight (base weight). Bailey and Holmgren argued that sellers may obtain higher contract price offers if they select small allowable weight differences (or weight tolerances) and large price slides. Other important elements of the contract are time to delivery and cattle weight variability. Characteristics such as breed, sex, lot size, condition, location, and frame size are also likely to be considered when setting the contract price.

Bailey, Brorsen, and Fawson found the surprising result that time to delivery has a positive effect on prices at Superior Livestock Auction, while other empirical studies on cash-forward contracting have consistently found a decrease in forward-contract prices as time to delivery increases (e.g., Brorsen, Coombs, and Anderson; Elam). The positive relationship between time to delivery and the contract price could be due to the implicit option created by the price slide.

In this study, a theoretical model is developed to explain the economics of determining price slides for feeder cattle that will encourage sellers to accurately estimate cattle weight. The contract between buyer and seller is viewed as a dynamic game with continuous strategies where buyer and seller are the players. If, as in reality, the seller is to set the value of the price slide, necessary conditions for subgame perfect equilibrium can be obtained. It is also possible to determine the value of the slide as an exogenous variable so that equilibrium is reached when the seller gives an unbiased estimate of cattle weight. In other words, optimal values of the slide are obtained so that it is in the seller's best interest to give an accurate estimate of the cattle's weight. Based on our research, price slides should be set higher than Prevatt suggests, or the specified weight tolerance should be lower. The model's predictions are compared to actual SLA observations. The slides used in SLA are smaller than those needed to give the producer an incentive not to underestimate weight. Consistent with the model's predictions, producers slightly underestimate cattle weights.

Analytical Model

Consider a feeder cattle buyer who contracts with a seller for future delivery of cattle. To simplify the model, perfect competition is assumed so that neither buyer nor seller are able to make profits. The seller estimates the average weight of the cattle to be sold,

[1] Prevatt refers to the compensation scheme described as a one-way slide. If the buyer and seller had symmetric information, a two-way slide could be used where premiums are paid if cattle are lighter than expected, but one-way slides are more commonly used.

called the base weight (y_0), and sets the price slide ($\gamma > 0$). The buyer responds by offering a contract price (p_0) per cwt which maintains her expected utility at zero. The seller then decides either to accept or reject the contract. The price slide modifies the contract price in the following way:

$$(1) \qquad p(y; y_0, p_0, \gamma, \delta) = \begin{cases} p_0 - \gamma(y - y_0 - \delta) & \text{if } y \geq y_0 + \delta, \\ p_0 & \text{if } y < y_0 + \delta. \end{cases}$$

The tolerance in feeder cattle weight estimation error is known as weight tolerance and is represented by $\delta > 0$. We assume it is preestablished, and thus neither buyer nor seller can decide upon its value.[2] The delivery weight is given by y, and $p(y)$ is the price actually paid per cwt at the time of delivery, when the average weight is finally revealed to buyer and seller. The payment $p(y)$ is a compensation scheme which penalizes the seller if delivered weights are greater than $y_0 + \delta$. Compensation schemes of this type are used in many real-world contractual relationships where asymmetric information exists (Phlips; Harris and Raviv).

Let r_s and s_b be the seller's payoff and the buyer's share, respectively, of the cattle's value in dollars per head:

$$(2) \qquad r_s = \begin{cases} (p_0 - \gamma(y - y_0 - \delta))y & \text{if } y \geq y_0 + \delta, \\ p_0 y & \text{if } y < y_0 + \delta, \end{cases}$$

and

$$(3) \qquad s_b = \begin{cases} (v(y, \mathbf{z}) - p_0 + \gamma(y - y_0 - \delta))y & \text{if } y \geq y_0 + \delta, \\ (v(y, \mathbf{z}) - p_0)y & \text{if } y < y_0 + \delta, \end{cases}$$

where $v(y, \mathbf{z})$ is the value per cwt of the cattle when weight is known, and $\mathbf{z}$ is a vector of other relevant variables. So, if weight were known with certainty, the buyer would be paying the real market value for cattle and her share would be zero. Because heavier cattle are normally worth less per pound than lighter cattle, we assume $v(y, \mathbf{z})$ is a monotonically decreasing function of y. The utility of buyer and seller will depend on s_b and r_s, respectively.

The contract can be viewed as a two-person dynamic game with continuous strategies to be approached using "backward induction" (Gibbons; Fundenberg and Tirole). According to SLA's procedures, the stages of the game are as follows:

1. The seller offers an estimate of the weight (y_0) and the price slide (γ).
2. The buyer offers a price per cwt, the contract price (p_0).
3. The seller either accepts or rejects the offer.

Assume the seller accepts the contract at stage 3. This scenario implies the buyer offered a contract price which, given y_0 and γ, maximizes the seller's utility while keeping the buyer's utility at its reservation level, which is assumed to be zero. Consequently, the seller knows the problem with which the buyer is confronted. If the seller could solve

[2] In practice, the weight tolerance is also a choice variable of the seller. For a time, the SLA did fix the weight tolerance, but quit because it was unpopular. To keep the model as simple as possible, weight tolerance is assumed fixed. Weight tolerances observed in the SLA vary little for a given weight range, so the assumption is reasonable.

the buyer's problem—i.e., obtain the buyer's best response function $p_0^*(y_0, \gamma)$ that guarantees the buyer her reservation utility—the seller would be able to select the optimum values for weight and price slide (y_0^* and γ^*), so that $p_0^*(y_0^*, \gamma^*)$ maximized the seller's own utility. The existence of $p_0^*(y_0, \gamma)$, y_0^*, and γ^* would guarantee subgame perfection, and the problem would be solved.

Unfortunately, although the seller knows the general problem faced by the buyer, he is not able to "rationally guess" the buyer's subjective probability distribution of cattle weights (note that the distribution of weights is crucial in calculating expected profit or utility). The buyer will most likely choose a probability distribution of weights based on the information provided by the seller (y_0 and γ). Given this probability distribution, the buyer obtains her best response function $p_0^*(y_0, \gamma)$.

A unique closed-form solution for the optimal slide problem cannot be obtained unless we impose conditions on the probability distributions and their parameters. Assuming the distribution is normal, we obtain a necessary condition for optimally selecting the price slide. This condition suggests the price slide should be bigger than the market's weight discount and not equal to it, as has been proposed by Prevatt. Still, with further assumptions on the probability distribution of weights used by buyer and seller, values for the slide are found *as if determined exogenously*, whereby the seller has no incentive to give an erroneous estimate of the cattle's weight. In the empirical section, however, we use regression analysis to estimate the buyer's subjective mean and variance of cattle weights, to verify whether the model corresponds to reality.

A Lower Bound for the Price Slide

The buyer's problem is to find p_0 that makes her expected utility from the transaction equal to her reservation utility, i.e.:

$$\int_{y_{\min}}^{y_{\max}} u_b\left[s_b(p_0; y_0, \gamma)\right] f_b(y; y_0, \gamma)\, dy = 0, \tag{4}$$

where y_0 and γ are taken as constants, u_b is the buyer's utility function, and f_b is the buyer's subjective density function of cattle weights. Note how the buyer will base her distribution of weights on the information provided by the seller. Assume the solution to the buyer's problem is the best response function $p_0^*(y_0, \gamma)$. The seller's problem is then to find y_0 and γ that satisfy

$$\max_{y_0, \gamma} \int_{y_{\min}}^{y_{\max}} u_s\left[r_s(p_0^*, y_0, \gamma)\right] f_s(y)\, dy, \tag{5}$$

where u_s is the seller's utility as a function of his payoff, and f_s is the density function of cattle weights according to the seller's knowledge. Unlike f_b, f_s does not depend on the base weight or slide.

Assuming risk neutrality, we can directly substitute (3) into (4) and express the buyer's problem as:

$$\int_{y_{\min}}^{y_0+\delta} \left[v(y, \mathbf{z}) - p_0\right] y f_b(y)\, dy$$
$$+ \int_{y_0+\delta}^{y_{\max}} \left[v(y, \mathbf{z}) - p_0 + \gamma(y - y_0 - \delta)\right] y f_b(y)\, dy = 0,$$

or

$$\int_{y_{\min}}^{y_{\max}} v(y, \mathbf{z}) y f_b(y)\, dy + \int_{y_0+\delta}^{y_{\max}} \gamma(y - y_0 - \delta) y f_b(y)\, dy$$
$$= p_0 \int_{y_{\min}}^{y_{\max}} y f_b(y)\, dy.$$

Rearranging, the contract price should satisfy

$$p_0^* = \frac{E_b[v(y, \mathbf{z})y] + \gamma \int_{y_0+\delta}^{y_{\max}} (y - y_0 - \delta) y f_b(y)\, dy}{E_b(y)}, \tag{6}$$

where E_b denotes the expectation with respect to the buyer's density function of cattle weights.

From this result, the contract price is the buyer's expected value of cattle per cwt plus the discount per cwt the buyer expects due to the slide. In other words, the buyer includes the expected discount in the contract price.

To analyze the seller's problem, substitute equation (2) into (5):

$$\max_{y_0, \gamma} E(r_s) = p_0^*(y_0, \gamma)\mu - \gamma \int_{y_0+\delta}^{y_{\max}} (y - y_0 - \delta) y f_s(y)\, dy. \tag{7}$$

The first-order conditions are $\partial E(r_s)/\partial \gamma = 0$ and $\partial E(r_s)/\partial y_0 = 0$, represented by:

$$\partial E(r_s)/\partial \gamma = \mu \partial p_0^*/\partial \gamma - \int_{y_0^*+\delta}^{y_{\max}} (y - y_0^* - \delta) y f_s(y)\, dy = 0 \tag{7a}$$

and

$$\partial E(r_s)/\partial y_0 = \mu \partial p_0^*/\partial y_0 + \gamma^* \int_{y_0^*+\delta}^{y_{\max}} y f_s(y)\, dy = 0. \tag{7b}$$

From (7a), the contract price is an increasing function of the slide, a result which can also be derived from (6). Note that the integrand in condition (7b) is the one designated for the expected value of y, although the integral is computed over only part of the range. Making use of the following inequality,

$$\int_{y_0^*+\delta}^{y_{\max}} y f_s(y)\, dy < \int_{y_{\min}}^{y_{\max}} y f_s(y)\, dy = \mu,$$

(7b) can be rearranged:

$$-\frac{\partial p_0^*/\partial y_0}{\gamma^*} = \frac{\int_{y_0^*+\delta}^{y_{\max}} y f_s(y)\, dy}{\mu} < 1.$$

Therefore, a necessary but not sufficient condition for optimality is:

$$\gamma^* > -(\partial p_0^*/\partial y_0). \tag{8}$$

Equation (8) states the slide should be set above the absolute value of the slope of the buyer's best response function. The buyer's best response for the contract price is a decreasing function of base weight, as is the true value of cattle, $v(y)$. In fact, it is reasonable to assume the buyer will discount cattle according to the market discount. Thus, equation (8) also suggests the slide should be greater than the market's weight discount, and not equal to it (as extension articles have suggested).

A Price Slide to Provide Incentives for Unbiased Estimates of Cattle Weight

Now let us assume the slide could be determined by a "supervising entity" in order to promote fair contracts. Rather than letting the seller set the price slide value to his own convenience, we prefer to set the value of the slide such that equilibrium is reached when the seller gives an unbiased estimate of the weight.

To solve this problem, we specify probability distributions of weights for buyer and seller. Let the seller's probability distribution of weights be normal with mean and variance parameters μ and σ^2, respectively. The seller's revenue in equation (7) is still maximized, but now γ is set exogenously. Thus the seller has only the base weight as a choice variable, and only one first-order condition holds: $\partial E(r_s)/\partial y_0 = 0$. This condition gives base weight y_0 as a function of the price slide. If we want the base weight to be the seller's best estimate of the true mean weight, then impose $y_0 \equiv \mu$, and solve for γ^{**}. The price slide γ^{**} is the value of the slide that makes the seller want to *accurately state* average cattle weight. Because the buyer knows the price slide is no longer a variable for the seller, she will take the seller's estimate of weight as the mean of her own subjective probability distribution of weights, i.e., $\mu_b \equiv y_0$. The variance of weights is assumed to be equal for buyer and seller, $\sigma_b^2 = \sigma^2$.

With γ^{**}, the seller optimizes revenue by letting his estimate of the mean weight (μ_b) be the base weight (y_0). The value of the slide derived by proceeding in this manner is specified as:

$$(9) \quad \gamma^{**} = \frac{\int_{y_{\min}}^{y_{\max}} v(y)y\left(\frac{1}{\mu} - \frac{y-\mu}{\sigma^2}\right)\exp\left(-\frac{1}{2}\left(\frac{y-\mu}{\sigma}\right)^2\right)dy}{\int_{\mu+\delta}^{y_{\max}} (y-\mu-\delta)y\left(\frac{1}{\mu} + \frac{y-\mu}{\sigma^2}\right)\exp\left(-\frac{1}{2}\left(\frac{y-\mu}{\sigma}\right)^2\right)dy + (\mu+\delta)\exp\left(-\frac{\delta^2}{2\sigma^2}\right)},$$

where $y_{\min}$ and $y_{\max}$ are interpreted as realistic lower and upper bounds for the mean weight of cattle. The probability of y values occurring beyond those limits can be considered negligible. The derivation of equation (9) is given in the appendix. By plotting this slide against δ, it was verified that the slide is an increasing function of the weight tolerance, so the smaller the weight tolerance, the smaller the slide needed to guarantee unbiased estimates of weight. Equation (9) above is used to interpret results in the following section.

Empirical Models

In this section, we take an empirical approach to better understand feeder cattle contracts and check the theoretical findings from the previous section. First, we test whether base weights are unbiased predictors of actual weights. Then, using regression analysis, we obtain estimated mean and variance equations of cattle weight at delivery based on market characteristics, delivery time, and the information the buyer can access: base weight, price slide, and weight tolerance. We assume the buyer may use these equations to obtain her subjective distribution of cattle weight, and thus the expected discount applied to the price per cwt of cattle as well as the base price.

Because the model suggests the contract price be a function of the discount due to the slide as well as market and cattle characteristics [equation (6)], a second regression is performed to regress the contract price on the expected discount and other characteristics. This price regression allows us to observe how the slide, through the expected discount, affects the relationship between price and time to delivery. Finally, the slide required for sellers to provide an unbiased estimate of the weight, γ^{**}, is obtained for the data and compared to slides actually used.

The data used in this section are actual Superior Livestock Auction data for the 1987–1989 period (3,370 observations) and the 1993–1994 period (2,299 observations). The data contain information on lot characteristics, contract prices, base weights, and other relevant variables needed to estimate the models for cattle weighing not more than 900 pounds. The weight tolerance for the 1993–1994 data was set at 10 lbs., and thus does not enter as a variable in the regressions. In what follows, the 1987–1989 period is referred to as period 1, and the 1993–1994 period as period 2.

Test for Unbiasedness

If the base weight proposed by the seller reflects his estimate of average delivered weights, and if his estimate is accurate, the mean of the difference between actual and estimated delivery weights should be zero. This hypothesis is tested using a paired differences t-test.

The t-ratio of the paired t-test is 8.15 for period 1, and 7.24 for period 2. Their corresponding one-tailed p-values are less than 0.0001. These values indicate the actual and estimated weights are significantly different at the 5% level, so sellers *understate* average weights. Table 1 shows the bias was small (5.88 and 5.58 lbs. for periods 1 and 2, respectively). Because the raw data deviate from normality, we also test the bias nonparametrically with the sign test and the sign ranked test, and confirm the bias is statistically different from zero at least at the 5% level. P-values for both tests are smaller than 0.0001.

Weight Bias and Weight Variability

Recall the buyer is to propose a contract price. According to equation (6), the contract price should be a function of the expected discount and the expected market value of the cattle. To construct these expectations, the buyer needs to assume a probability distribution of weights. If she assumes normality, it is necessary to estimate the mean and variance of weight based on the information received from the seller and some other information accessible to the buyer.[3] Although we did not derive or assume values for μ_b and σ^2 in the theory section, it is possible to obtain them empirically with the data available.

The following equations are used to estimate the mean and variance of weight:

[3] Other variables, such as dummies for quarter and regions in the mean equation, a time by weight interaction in the variance equation, or a variable signaling economic conditions after signing the contract and before delivery, could have been included. However, we keep the model simple because we assume the buyer cannot access all this information. Weather, for instance, needs to be predicted in order to include seasonality in the model. When seasonal dummies were included, parameters changed substantially across periods, most likely due to insufficient variation in the data to obtain accurate estimates.

Table 1. Summary Statistics for Selected Variables: Superior Livestock Video Cattle Auction Data, Period 1 (1987–1989) and Period 2 (1993–1994)

		Period 1, 1987–1989				Period 2, 1993–1994			
Variable	Units	Mean	Min.	Max.	Std. Dev.	Mean	Min.	Max.	Std. Dev.
Base Weight (y_0)	lbs.	627.87	270.0	890.0	132.6	608.44	225.0	895.0	126.3
Actual Weight (y)	lbs.	633.75	304.0	960.4	132.9	614.03	283.5	933.3	127.6
Contract Price (p_0)	$/cwt	82.61	57.0	130.0	10.4	87.42	55.0	134.5	10.8
Weight Difference [a]	lbs.	5.88	-190.3	178.3	39.1	5.58	-147.3	193.0	35.4
Price Slide	(cents/cwt)/lb.	5.29	3.0	10.0	2.8	7.03	1.0	15.0	2.7
Weight Tolerance (*W_Tol*)	lbs.	14.97	0.1	35.0	7.3	10.00	10.0	10.0	0.0
Head		129.04	10.0	2,000.0	115.4	117.02	25.0	1,000.0	83.4
Time to Delivery	days	30.46	0.0	89.0	23.0	26.11	2.0	89.5	22.0
Discount	$/cwt	0.61	0.0	15.7	1.3	0.79	0.0	11.0	1.4
Estimated Discount (*Ep*)	$/cwt	0.68	0.2	3.8	0.5	0.90	0.2	6.1	0.5
Estimated Discount – Discount		0.07	-13.32	2.3	1.5	0.11	-9.9	4.1	1.3

Note: Number of observations for period 1 (1987–1989) = 3,370; for period 2 (1993–1994) = 2,299.

[a] Weight difference is mean weight at delivery minus base weight.

$$y - y_0 = \alpha_0 + \alpha_1 y_0 + \alpha_2 y_0^2 + \alpha_3 W\_Tol + \alpha_4 Slide + \nu, \tag{10}$$

where the ν's are independent and normally distributed with mean zero and variance

$$\begin{aligned}\sigma_v^2 = \exp(&\beta_0 + \beta_1 y_0 + \beta_2 y_0^2 + \beta_3 Slide + \beta_4 W\_Tol + \beta_5 Head^{-1} \\ &+ \beta_6 Steers + \beta_7 Time + \beta_8 Time^2 + \beta_9 MidWest + \beta_{10} West \\ &+ \beta_{11} South + \beta_{12} Upper + \beta_{13} WCoast + \beta_{14} LSW).\end{aligned} \tag{11}$$

The variable y is actual weight, y_0 is base weight, *Steers* is a dummy variable for steer versus heifer, *Time* denotes days to delivery, *Slide* is the price slide (cents/cwt), and *W_Tol* is the weight tolerance (lbs./head). *MidWest*, *West*, *South*, *Upper*, *WCoast*, and *LSW* are dummy variables representing the regions where the cattle are located.[4] The reciprocal of the number of head in the lot, $Head^{-1}$, is included to capture the reduced variability from averaging over a large number of animals. Recall that for n random variables with equal variance v,

$$\mathrm{var}(\bar{x}) = \mathrm{var}\left(\sum_{i=1,\ldots,n} x_i/n\right) = vn^{-1}.$$

Equation (11) specifies a deterministic variance that imposes multiplicative heteroskedasticity in the model (Greene). The model is estimated in SAS using the Mixed procedure with the local=exp( ) option (SAS Institute, Inc.), obtaining MLE estimates of the parameters.

These equations are used to define the buyer's expectations about mean and variance of weight difference. With them and the base weight given by the seller, the buyer estimates cattle weight distribution at delivery and proposes a contract price.

The parameter estimates of equations (10) and (11) are reported in table 2. The assumption that weight variability increases with time to delivery is also tested using these same equations. The parameter estimates of time to delivery in the cattle weight variance equation (table 2) indicate time to delivery has a positive effect on the variance of base weight. The parameter for the price slide is not consistently significant in the mean equation, but it is negative and significant in the variance equation for both periods, suggesting sellers do use slightly larger slides when more certain about weights.

Our variables are better able to explain the variability in the bias than the bias itself. The parameter estimate for weight tolerance in the first period suggests that reducing the weight tolerance decreases the bias. The bias varies greatly with weight. With low base weights the bias tends to decrease, while high base weights tend to increase the bias. The variance is also heavily influenced by weight, with sellers being much less accurate at estimating weight of light-weight cattle. The inverse of number of head per lot in the variance equation has a positive coefficient as expected, revealing more error variance with small lot sizes.

Although our general model [equations (4)–(8)] does not rely on assumptions over the probability distribution of cattle weight, we check the standardized residuals from the weight difference equation for normality. The normality assumption is imposed in

[4] There are seven regions: *MidWest* (Nebraska, Kansas, Colorado, Missouri, Illinois, and Iowa); *West* (Montana, Wyoming, Idaho, Utah, and Nevada); *South* (Mississippi, Florida, Louisiana, Alabama, Arkansas, North Carolina, Georgia, Tennessee, and Kentucky); *Upper* (South Dakota, North Dakota, Minnesota, and Wisconsin); *WCoast* (California, Arizona, Oregon, and Washington); *LSW* (Texas, Oklahoma, and New Mexico); and *East* (states east of Illinois and north of Kentucky).

Table 2. Parameter Estimates of Cattle Weight Difference (Actual Weight minus Base Weight) Mean and Variance Equations: Feeder Cattle Auction Data, Period 1 (1987–1989) and Period 2 (1993–1994)

	Period 1, 1987–1989		Period 2, 1993–1994	
Variable[a]	Parameter Estimate	Standard Error	Parameter Estimate	Standard Error
Mean Equation:				
Intercept	56.24***	13.73	156.48***	16.80
Slide	-0.24	0.35	-2.61***	0.50
Weight Tolerance (W_Tol)	0.46***	0.11	—	—
Base Weight (y_0)	-13.89***	4.22	-36.92***	5.04
Base Weight^2 (y_0^2)	0.75**	0.34	2.39***	0.41
Variance Equation:				
Exp(Intercept)	776.50*	475.87	8,904.09	7,343.99
Slide	-0.09***	0.01	-0.11***	0.02
Weight Tolerance (W_Tol)	-6.7E-3*	3.9E-3	—	—
Base Weight (y_0)	0.12	0.18	-0.22	0.23
Base Weight^2 (y_0^2)	-5.4E-3	1.5E-2	0.01	0.02
Steers	9.0E-2*	0.05	-0.12*	0.07
Time	1.4E-2***	3.9E-3	1.8E-2***	4.7E-3
Time^2	-9.5E-3*	5.0E-3	-1.3E-2**	6.1E-3
Head^{-1}	0.17***	0.03	0.17**	0.08
MidWest	-5.1E-2	5.9E-2	-0.58**	0.25
West	6.5E-2	7.4E-2	-0.73***	0.26
South	-6.7E-2	9.8E-2	-0.68***	0.26
Upper	0.24	0.34	-0.56*	0.30
WCoast	0.21	0.15	-0.31	0.28
LSW	—	—	-0.73***	0.25

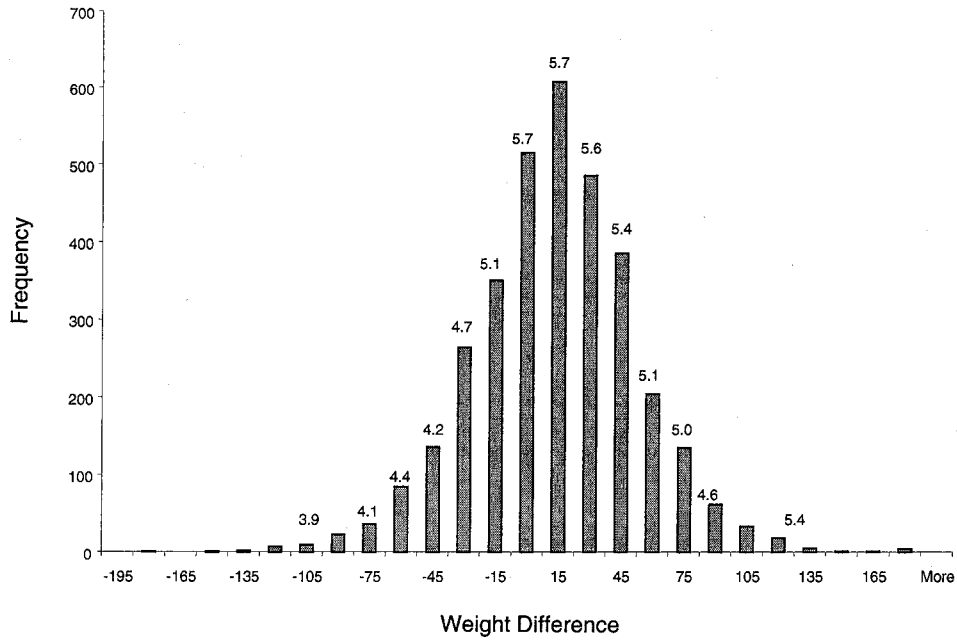
Note: Single, double, and triple asterisks (*) denote significance at the 10%, 5%, and 1% levels, respectively.
[a] *Base Weight* in the regression is in cwt, and *Base Weight*^{2} is in cwt^2; *Head* is in hundreds, and *Time* is in days; the dependent variable, *Cattle Weight Difference*, is measured in pounds; *Weight Tolerance* is fixed at 10 pounds for the 1993–1994 period.

equation (9) as well as the empirical models. The Kolmogorov-Smirnov test rejects normality, in part, due to a large sample. Normal probability plots, histograms, and skewness and kurtosis measures reveal the data are somewhat leptokurtic, but not far from normal. Histograms for the weight difference variable are presented in figures 1 and 2 for period 1 and period 2, respectively.

Contract Price

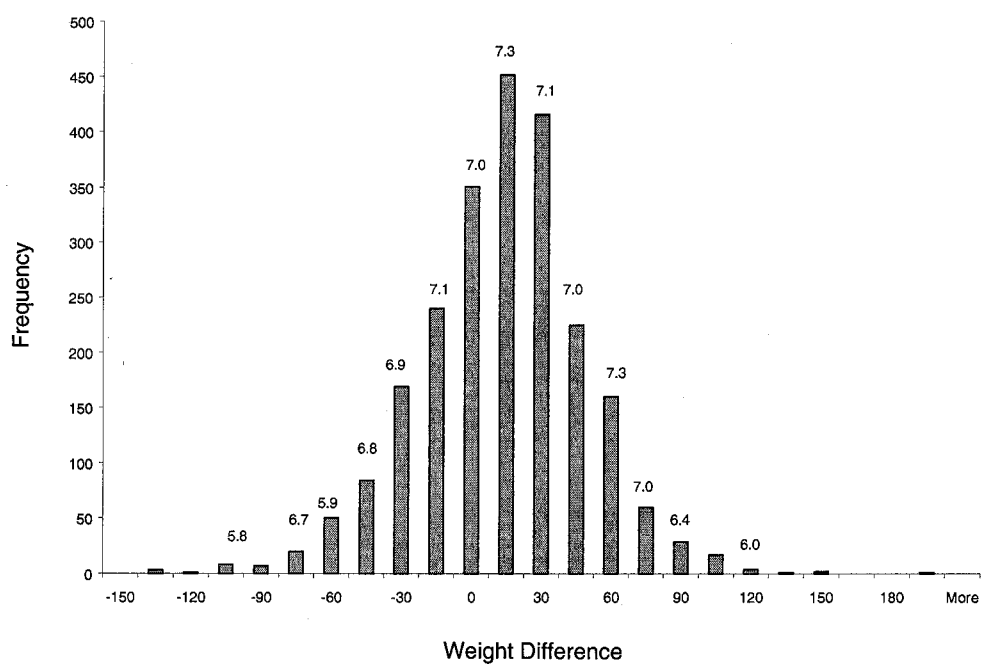
With estimates for the mean and variance of weight obtained from equations (10) and (11), then, according to equation (6), the buyer should be able to construct the expected discount per cwt, which is given by:

$$Ep = \frac{\gamma \int_{y_0+\delta}^{y_{\max}} (y - y_0 - \delta) y f_b(y)\, dy}{E_b(y)}.$$



Note: Numbers on top of each bar are the average price slide for that category, except the end values which are averages for the tails.

Figure 1. Histogram for weight difference (average cattle weight at delivery minus base weight), period 1, 1987–1989



Note: Numbers on top of each bar are the average price slide for that category, except the end values which are averages for the tails.

Figure 2. Histogram for weight difference (average cattle weight at delivery minus base weight), period 2, 1993–1994

Assuming normality, these two parameters completely specify a distribution of weights at delivery time, allowing for estimation of the expected discount. The contract price is explained as follows:

$$(12)\quad p_0 = \alpha_0 + \alpha_1 y_0 + \alpha_2 y_0^3 + \alpha_3 Head + \alpha_4 Head^2 + \alpha_5 Steers + \alpha_6 Time + \alpha_7 Time^2 + \alpha_8 Futures + \alpha_9 MidWest + \alpha_{10} West + \alpha_{11} South + \alpha_{12} Upper + \alpha_{13} WCoast + \alpha_{14} LSW + \alpha_{15} Ep + \sum_{i=16}^{32} \alpha_i OC_i + u + \varepsilon,$$

where ε has mean zero and variance σ_ε^2, whereas u is an error component associated with the day of the sale having zero mean and variance σ_u^2.

The variable p_0 is the contract price, *Ep* is the estimated expected discount per cwt,[5] *Futures* is the current price of the futures contract that will be the nearby futures at the time of delivery. The OC_i's are discrete variables measuring other market and lot characteristics such as breed, flesh, and frame.[6] *Head* is the number of animals, and all other variables are as defined previously. This random-effects model is estimated using the Mixed procedure in SAS. The estimated mean equation is used to plot the contract price against base weight and time to delivery.[7]

The difference between the average expected discount and the average of actual discounts is small, although statistically significant (table 1). Thus, the model used for the distribution of weights is imperfect. This could be due to some minor misspecification such as incorrect functional form or nonnormality.

Regression estimates are reported in table 3. The base weight was included in equation (12) in quadratic form. As seen in table 3, the parameter estimate of the base weight is negative while that of the square of the base weight is positive. Figure 3 illustrates the effect of the base weight on the contract price for periods 1 and 2. As expected, the contract price decreases as base weight is increased.

Figures 4 and 5 show, for periods 1 and 2, respectively, the contract price as a function of time to delivery resulting from estimating equation (12), i.e., when the estimated discount is entered as an explanatory variable. In each graphic, these relationships are compared with the relationship between contract price and time to delivery when the estimated discount variable is not in the model. For the first period (1987–1989, figure 4), including the expected discount widens the range in which time to delivery has a negative effect on the contract price. Price increases with small values of time to delivery and decreases otherwise. In the data set, however, most of the values of time to delivery are within the range where the contract price slightly increases. An explanation for this could be that buyers in this market pay a premium to reduce their input risk, or perhaps

[5] The use of the variable *Ep* creates a generated regressor problem. As Hoffman demonstrates, parameter estimates are still consistent with a generated regressor, but estimates of standard errors are biased. Monte Carlo studies by Hoffman show this bias is small (< 10%) except when using a lagged dependent variable. No correction is made here since the estimated coefficient is several times its standard error.

[6] The *OC* variables are *Miles*, which measures miles from location to delivery point, and the following binary variables: *Truck* = 1 if there are at least 40,000 lbs. per truckload, *Horn* = 1 if cattle are not horned, and *Mixed* = 1 if cattle are unmixed by sex. There are also five binary variables for breed (*English-Exotic Cross*, *English Cross*, *Exotic Cross*, *Angus*, and *Dairy*), four for flesh (*Heavy*, *Medium Heavy*, *Medium Flesh*, and *Light-Medium Flesh*), and three for frame (*Large Frame*, *Medium-Large Frame*, and *Medium Frame*).

[7] Corn prices were relatively steady all through the second period and most of the first. As shown by figure 3, the slope of the price-weight equation remains fairly constant, suggesting corn prices did not significantly influence the weight coefficient in the base price equation.

Table 3. Parameter Estimates of Contract Price Equation: Superior Livestock Video Cattle Auction Data, Period 1 (1987–1989) and Period 2 (1993–1994)

	Period 1, 1987–1989		Period 2, 1993–1994	
Variable[a]	Parameter Estimate	Standard Error	Parameter Estimate	Standard Error
Intercept	71.41***	4.83	58.07***	9.85
Futures Price	0.83***	0.05	0.79***	0.09
Steers	-6.90***	0.13	-5.31***	0.19
Head	0.56***	0.10	0.88***	0.30
Head^2	-3.6E-2***	0.01	-9.0E-2*	5.2E-2
Base Weight (y_0)	-12.78***	0.56	-9.59***	1.63
Base Weight^2 (y_0^2)	0.67***	0.04	0.40***	0.12
Expected Discount (*Ep*)	4.50***	0.36	3.83***	0.70
Time	1.5E-2	9.9E-3	2.3E-2	1.6E-2
Time^2	-1.7E-2	1.2E-2	-3.7E-2**	1.9E-2
English-Exotic Cross	-0.95**	0.40	8.1E-2	0.67
English Cross	-0.88**	0.41	1.24*	0.72
Exotic Cross	-1.39***	0.45	0.78	0.76
Angus	-0.31	0.72	5.16***	1.11
Dairy	-8.98***	0.58	-14.48***	0.95
Heavy	-2.68**	1.12	—	—
Medium Heavy	-2.61***	0.47	2.20	2.03
Medium Flesh	-2.32***	0.43	2.13	2.00
Light-Medium Flesh	-2.07***	0.46	2.40	2.00
Large Frame	6.77***	1.08	4.05***	0.89
Medium-Large Frame	5.97***	1.08	2.70***	0.64
Medium Frame	4.45***	1.16	2.02***	0.63
Horn	-1.77***	0.54	-1.27***	0.37
MidWest	1.00***	0.15	5.82***	0.76
West	1.58***	0.19	6.14***	0.81
South	-1.61***	0.24	0.16	0.78
Upper	2.05**	0.87	6.19***	0.90
WCoast	-0.81**	0.37	1.98**	0.80
LSW	—	—	2.97***	0.78
Truck	-0.71***	0.26	—	—
Mixed	1.85***	0.21	1.99***	0.25
Miles	-0.10***	0.02	—	—
Sale (random effect)	4.31***	1.19	2.03**	0.85

Note: Single, double, and triple asterisks (*) denote significance at the 10%, 5%, and 1% levels, respectively.
[a] *Base Weight* in the regression is in cwt, and *Base Weight*^{2} is in cwt^2; *Head* and *Miles* are in hundred units.

sellers wanting to sell cattle immediately must pay a liquidity cost because most buyers are demanding cattle for future delivery.

In contrast, in the second period (1993–1994, figure 5), including the expected penalty does make the price negatively related to delivery time in the range of 0 to 40 days. So once the discount generated by the price slide is accounted for, the positive effect of time to delivery on price is reversed for a good part of the relevant range. Thus, results in

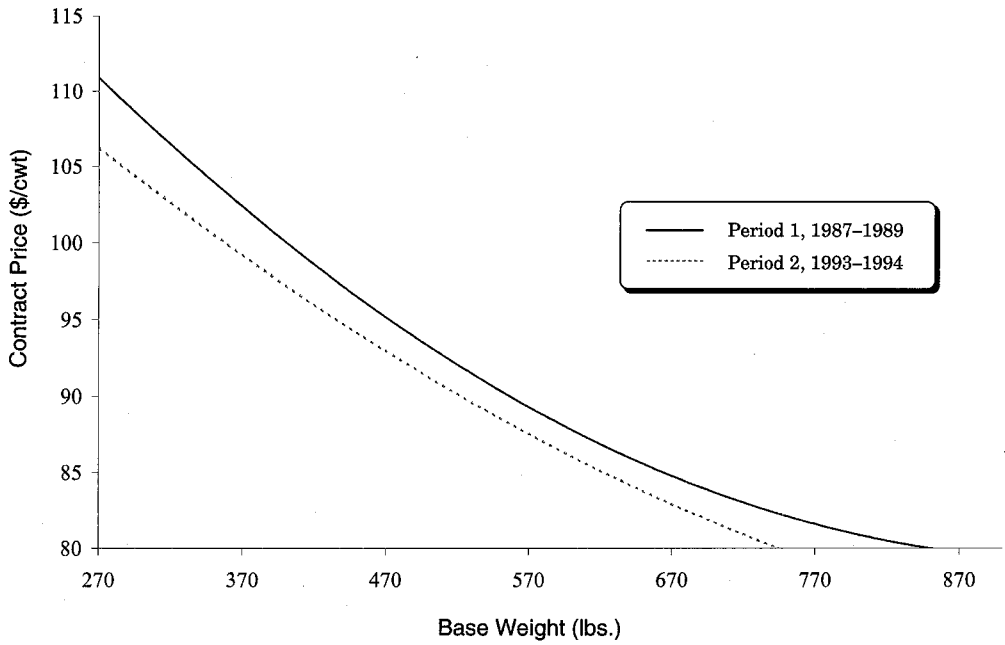


Figure 3. Effect of cattle weight on contract price: Superior Livestock Auction data, periods 1 and 2

period 2 support our hypothesis that the positive relationship of price and delivery time may be due to the implicit option created by the slide.

The parameter for expected discount was significant and positive for both data sets (4.50 and 3.83, table 3). Yet, according to equation (6), a value close to one was expected. The coefficient estimate was also sensitive to changes in the specification of the weight-difference mean equation used to obtain the expected penalty. Although the magnitude of the coefficient was sensitive, it always remained positive and significant, implying the option-like value of the slide is indeed recognized by buyers. The fact that the value of the parameter is above one indicates sellers could use larger price slides or lower weight tolerances. However, recall our model assumes risk-neutral agents. In a risk-averse setting, a lower weight tolerance might be preferred over a higher price slide.

Comparing the Actual Price Slide with the Model's Predictions

To determine if the price slide is set at a value which encourages the seller to provide an unbiased estimate of cattle weight (according to the analytical model), an estimate of the market weight discount is needed. This is taken to be the change in the contract price due to a unit increase in weight. In other words, the market weight discount is estimated, at each observation, as the derivative of the price equation with respect to weight.[8] The average over all observations is called the estimated mean weight discount.

[8] We use $\partial P_0/\partial y_0$ as an estimate of $\partial v/\partial y$.

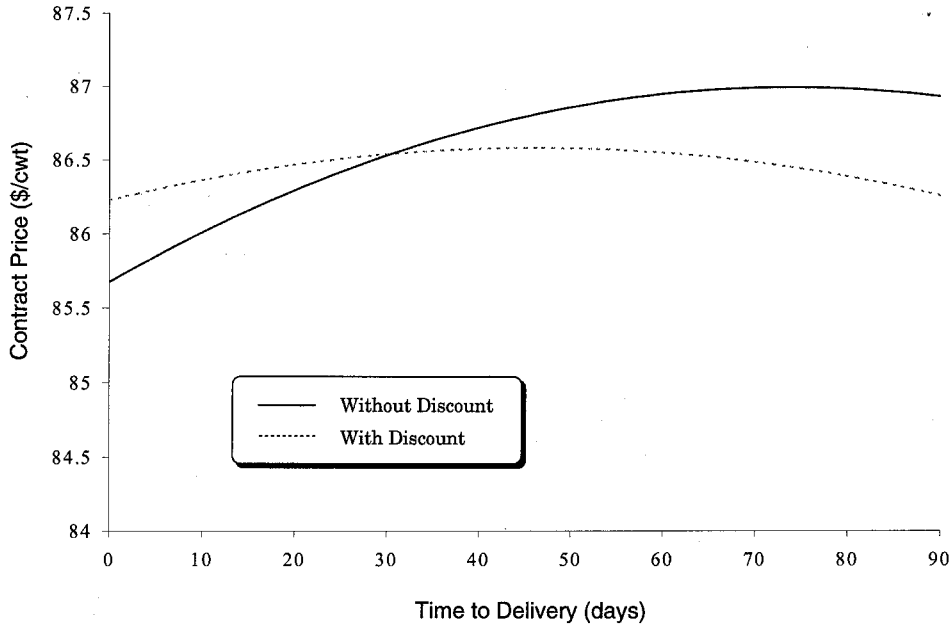


Figure 4. Effect of delivery time on contract price, accounting and not accounting for discount due to price slide: Superior Livestock Auction data, period 1 (1987–1989)

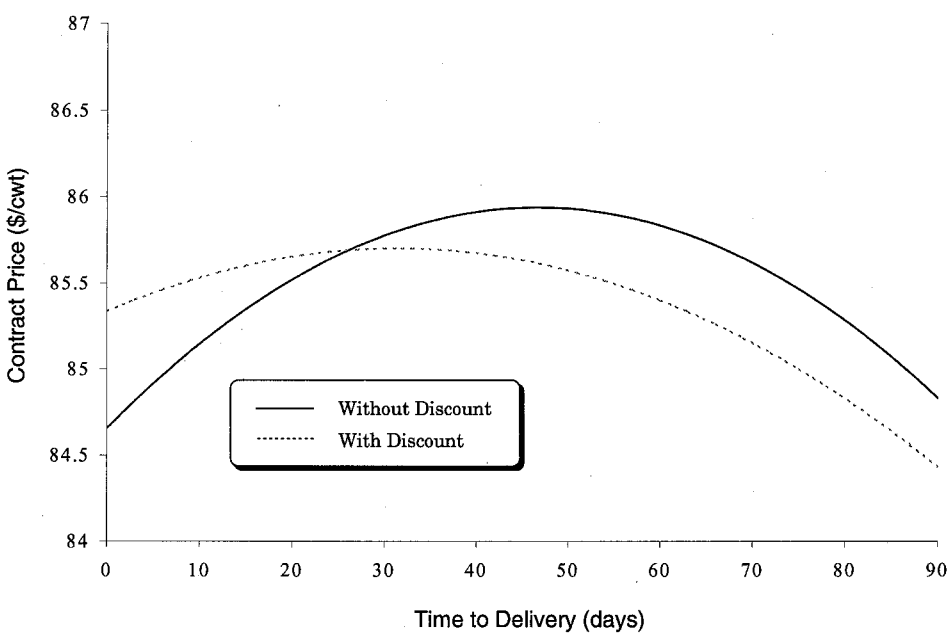


Figure 5. Effect of delivery time on contract price, accounting and not accounting for discount due to price slide: Superior Livestock Auction data, period 2 (1993–1994)

For periods 1 and 2, estimated mean weight discounts are 4.42 and 4.71 (cents/cwt)/lb., respectively. On average, the slide is around 1.25 times the estimated mean weight discount for the first period and increases to 1.45 times the weight discount in the second period.[9] This finding suggests there has been a tendency to increase the slide above the market weight discount over time. But when calculating the slide which, according to the model, should give unbiased estimates of weight [equation (9)], we obtain 11.07 (cents/cwt)/lb. for period 1 as the optimal slide for the mean weight, which is about 2.5 times the estimated mean weight discount (11.07/4.42). For period 2, a similar situation occurs. The optimal slide for the average base weight, according to equation (9), is 10.80 (cents/cwt)/lb., about 2.3 times the corresponding estimated mean weight discount (10.8/4.71).

Thus in both cases, equation (9) indicates that either a bigger price slide or a smaller weight tolerance is needed to guarantee unbiased estimates of weight. If the slide is to be kept at about the market weight discount, an alternative to increasing the price slide is to reduce the weight slide. This strategy is possible because the optimal slide (γ^{**}) is increasing in the weight slide. Thus, a smaller weight slide in equation (9) lowers the optimal slide. Note the optimal slide is smaller in the second period, perhaps due in part to the fact that the average weight slide is already smaller in the second period (10 lbs.) as compared to the first period (15 lbs.).

Because the model predicts that the price slides used at SLA are not big enough to avoid unbiased estimates of cattle weight, we would expect to see a difference between actual and base weights in the data. In fact, as seen in table 1, actual weights are slightly larger than base weights.

Conclusions

Feeder cattle sold through video auctions and by private treaty are often for future delivery. Because delivery weights are not known when cattle are contracted, sellers must estimate them. Sellers and buyers have asymmetric information about cattle weights. Consequently, contracts need to be structured to provide sellers with an incentive to accurately represent their estimates of average delivery weights.

The usual approach to dealing with weight uncertainty is to adjust the contract price by a price slide. The analytical model provides the solution as to how cattle should be valued in the presence of a price slide.

Comparative statics results show that the price slides used are not sufficient to impose unbiased predictions of cattle weights. Furthermore, empirical results confirm the price slides used are too small to impose unbiasedness, and sellers tend to understate weights.

Sellers often ask extension economists for advice on how to pick slides. Theoretical results suggest choosing price slides of double the market's discount for weight if large weight tolerances are used, or selecting small weight tolerances if a price slide close to the market's weight discount is to be used. Our research data reveal there was, in fact, a tendency from the 1980s to the 1990s toward higher slide-weight discount ratios.

[9] We estimate the weight discount and obtain the slide-weight discount ratio for each observation, and then average over all observations.

Based on our empirical results, sellers receive more than the expected discount in higher contract prices. Thus, our findings suggest that larger price slides or lower weight tolerances should be encouraged.

[*Received June 1999; final revision received April 2001.*]

References

Bailey, D., B. W. Brorsen, and C. Fawson. "Buyer Concentration at Feeder Cattle Auctions." *Rev. Agr. Econ.* 15(January 1993):103–19.

Bailey, D., and L. N. Holmgren. "Price Slides for Feeder Cattle." Work. pap., Dept. of Agr. Econ., Utah State University, 1996.

Bailey, D., M. C. Peterson, and B. W. Brorsen. "A Comparison of Video Cattle Auction and Regional Market Prices." *Amer. J. Agr. Econ.* 73(May 1991):465–75.

Brorsen, B. W., J. Coombs, and K. Anderson. "The Cost of Forward Contracting Wheat." *Agribus.: An Internat. J.* 11(1995):349–54.

Elam, E. "Cash Forward Contracting versus Hedging of Fed Cattle, and the Impact of Cash Contracting on Cash Prices." *J. Agr. and Resour. Econ.* 17(1992):205–17.

Fundenberg, D., and J. Tirole. *Game Theory.* Cambridge MA: The MIT Press, 1992.

Gibbons, R. *Game Theory for Applied Economists.* Princeton NJ: Princeton University Press, 1992.

Greene, W. *Econometric Analysis*, 3rd ed. Englewood Cliffs NJ: Prentice-Hall, 1997.

Harris, M., and A. Raviv. "Optimal Incentive Contracts with Imperfect Information." *J. Econ. Theory* 20(April 1979):231–59.

Hoffman, D. L. "Two-Step Generalized Least Squares Estimators in Multi-Equation Generated Regressor Models." *Rev. Econ. and Statis.* 69(1987):336–64.

Phlips, L. *The Economics of Imperfect Information.* Cambridge, England: Cambridge University Press, 1988.

Prevatt, J. W. "Marketing Feeder Cattle with a Price Slide." Circular No. ANR-800, Alabama Coop. Ext. Ser., Dept. of Agr. Econ., Auburn University AL, 1992.

SAS Institute, Inc. *SAS/ETS User's Guide*, Version 6.12. Cary NC: SAS Institute, Inc., 1996.

Superior Livestock Auction. Unpublished data. SLA, Greeley CO, 1987–89, 1993–94.

Appendix: Derivation of Text Equation (9)

The seller's problem is represented by:

$$\text{(A1)} \qquad \max_{y_0} E(r_s) = p_0^*(y_0, \gamma^{**})\mu - \frac{\gamma^{**}}{\sigma\sqrt{2\pi}} \int_{y_0+\delta}^{y_{\max}} (y - y_0 - \delta) y \exp\left[-\frac{(y-\mu)^2}{2\sigma^2}\right] dy,$$

where the slide is assumed fixed at γ^{**} by the supervising entity.

The first-order condition is given as:

$$\text{(A2)} \qquad \partial E(r_s)/\partial y_0 = \mu \partial p_0^*/\partial y_0 + \gamma^{**} \int_{y_0+\delta}^{y_{\max}} \frac{1}{\sigma\sqrt{2\pi}} y \exp\left[-\frac{(y-\mu)^2}{2\sigma^2}\right] dy = 0.$$

The value of $\partial p_0^*/\partial y_0$ remains to be determined. If the slide is given such that it is in the seller's best interest to accurately estimate cattle weight, then the buyer can trust the seller's estimate y_0, and take it as the mean of weight distribution. Thus, $E_b(y) = y_0$. Also, we assume both buyer and seller take the variance of weights to be σ^2.

With these assumptions, we can obtain the derivative of the buyer's best response function with respect to the base weight:

(A3) $\partial p_0^*/\partial y_0$

$$= \frac{\partial}{\partial y_0}\left(\frac{1}{\sigma y_0\sqrt{2\pi}}\left[\int_{-\infty}^{\infty} v(y)y\exp\left[-\frac{(y-y_0)^2}{2\sigma^2}\right]dy + \gamma\int_{y_0+\delta}^{\infty}(y-y_0-\delta)y\exp\left[-\frac{(y-y_0)^2}{2\sigma^2}\right]dy\right]\right)$$

$$= \frac{1}{\sigma y_0\sqrt{2\pi}}\left[\int_{-\infty}^{\infty} v(y)y\frac{(y-y_0)}{\sigma^2}\exp\left[-\frac{(y-y_0)^2}{2\sigma^2}\right]dy - \gamma\int_{y_0+\delta}^{\infty} y\exp\left[-\frac{(y-y_0)^2}{2\sigma^2}\right]dy\right.$$

$$+ \gamma\int_{y_0+\delta}^{\infty}(y-y_0-\delta)y\frac{(y-y_0)}{\sigma^2}\exp\left[-\frac{(y-y_0)^2}{2\sigma^2}\right]dy + \gamma(y_0+\delta)\exp\left(-\frac{\delta^2}{2\sigma^2}\right)$$

$$\left. - \frac{1}{y_0}\int_{-\infty}^{\infty} v(y)y\exp\left[-\frac{(y-y_0)^2}{2\sigma^2}\right]dy + \frac{\gamma}{y_0}\int_{y_0+\delta}^{\infty}(y-y_0-\delta)y\exp\left[-\frac{(y-y_0)^2}{2\sigma^2}\right]dy\right].$$

Replacing (A3) in (A2) gives the base weight as a function of the slide. Therefore, if we want the value of the slide that makes the base weight equal to the real average cattle weight, μ, we need only to replace y_0 with μ, and solve for the slide, γ^{**}:

(A4) $\partial E(r_s)/\partial y_0|_{y=\mu}$

$$= \frac{1}{\sigma\sqrt{2\pi}}\left[\frac{\gamma}{\mu}\int_{\mu+\delta}^{y_{\max}}(y-\mu-\delta)y\exp\left[-\frac{(y-\mu)^2}{2\sigma^2}\right]dy\right.$$

$$- \frac{1}{\mu}\int_{-\infty}^{y_{\max}} v(y)y\exp\left[-\frac{(y-\mu)^2}{2\sigma^2}\right]dy + \int_{-\infty}^{\infty} v(y)y\frac{(y-\mu)}{\sigma^2}\exp\left[-\frac{(y-\mu)^2}{2\sigma^2}\right]dy$$

$$\left. + \gamma^{**}\int_{\mu+\delta}^{\infty}(y-\mu-\delta)y\frac{(y-\mu)}{\sigma^2}\exp\left[-\frac{(y-\mu)^2}{2\sigma^2}\right]dy + \gamma^{**}(\mu+\delta)\exp\left(-\frac{\delta^2}{2\sigma^2}\right)\right] = 0.$$

Thus, solving for the slide yields text equation (9). □