Green Payment Programs for Nonpoint Source Pollution Control: How Important Is Targeting for Cost-Effectiveness?

Kenneth A. Baerenklau

Mechanism design theory is used to examine the case of a cost-minimizing regulator who uses input-reduction subsidies to meet an exogenously imposed ambient standard for nonpoint source pollution. A general result claimed for a welfare-maximizing regulator is clarified to show that an optimal contract scheme may involve a pooling equilibrium. Numerical results suggest the ability to directly target contracts reduces costs significantly for the regulator. But in the absence of this ability, indirect targeting reduces costs only slightly.

Key words: ambient standard, cost minimization, input-reduction subsidy, mechanism design, nonpoint source pollution, targeting

Introduction and Background

Point source pollution of surface and groundwater resources has received the majority of regulatory attention in the past few decades. This emphasis on point source pollution is largely because nonpoint source (NPS) pollution is much more difficult to control. The sources of observed ambient NPS pollution are diffuse, and therefore difficult to identify and monitor. Further, the mechanism by which NPS pollution migrates from its sources into the environment is site-specific and stochastic. It is not surprising, then, that the U.S. Environmental Protection Agency has labeled NPS pollution as the "greatest source of water quality problems in the United States today" (p. 52).

Considerable academic research on NPS pollution control has focused on market-based pollution incentives. Work on ambient taxes and subsidies dates back to Tietenberg's reply to Baumol and Oates' 1971 theorem regarding the efficiency of a uniform pollution tax. In 1988, Segerson contributed a seminal paper which incorporates both taxes and subsidies with a flat penalty, and Xepapadeas (1991, 1992, 1995) has examined various stochastic-dynamic elements of subsidy and penalty schemes. Marketable permits and point-nonpoint trading also continue to receive a significant amount of attention in the literature, with some of the more often-cited works including analyses by Shortle (1987, 1990); Letson; and Malik, Letson, and Crutchfield.
Each of these market-based approaches unfortunately has serious practical problems that limit applicability to NPS pollution control in the field—including significant information costs, the use of Draconian penalties, scarcity of permit traders, and the issue of establishing an appropriate trading ratio. In light of these difficulties, some researchers instead have considered voluntary “green payment” programs which are much more common in practice. Essentially, these programs offer subsidies to polluters who voluntarily choose to adopt less-polluting (but often more expensive, less productive, or higher variability) production practices. Research in this area has focused on either input-reduction subsidies or cost-sharing programs for the installation and maintenance of less-polluting technologies (often called “best management practices”).

Malik and Shoemaker were first to examine the problem of designing an economically efficient cost-sharing program to control agricultural NPS pollution. Their model assumes each profit-maximizing agent operates a farm with heterogeneous land quality on which two different technologies may be employed: a “cleaner” technology that is more productive on lower quality land, and a “dirtier” technology that is more productive on higher quality land. The regulator’s goal is to choose the ranges of land quality over which each technology may be employed as well as the technology subsidies which maximize net social benefits subject to an ambient pollution standard.

In a different approach to a similar problem, Wu and Babcock use mechanism design theory to examine the case of profit-maximizing agents operating heterogeneous farms with homogeneous land quality (i.e., there is a distribution of land quality across farms but not within a single farm). The regulator’s problem is to specify a set of incentive-compatible and individually rational contracts \((x_i, s_i)\) that maximizes net social welfare from agricultural production and pollution. The term \(x_i\) represents a vector of per acre inputs and \(s_i\) is the associated per acre subsidy.

The incentive compatibility requirement is a result of the information asymmetry inherent in the NPS pollution control problem. Specifically, while each farmer knows his own resource endowment, the regulator effectively does not. Therefore, because incentives may exist for farmers to misrepresent their resource endowments to take advantage of government subsidies, the regulator must specify self-selecting contracts if she wishes to separate farmers according to their resource endowments. If such a separation is not desired, these constraints are unnecessary and the regulator simply bunches the agents together and treats them as a single group. When the regulator chooses the former contract scheme, the solution is said to be “separating,” and when she chooses the latter, the solution is said to be “pooling.”

In their analysis, Wu and Babcock state, “Chambers (1992) and Guesnerie and Seade (1982) have shown that with only two groups bunching is not optimal if the government’s objective depends on the payment level, \(s_i\)” (p. 319). The present study seeks to clarify this result by showing how a pooling contract can be optimal, and by assessing the relative cost-effectiveness of different contract mechanisms.

---

1 Smith and Tomasi (1995, 1999) also have applied principles of mechanism design to the problem of nonpoint source pollution control.

2 This may be because it is prohibitively costly for the regulator to obtain such information, or because the regulator must appear to be “fair” by offering all agents the same contract menu, thus effectively eliminating the ability to directly target contracts based on farm-specific information even if it is known. This latter justification is cited by Chambers. It also is acknowledged by regulators in Wisconsin's Priority Watershed Program, a state-level cost-sharing program tasked with controlling NPS pollution.
Because regulators typically are not explicitly faced with welfare maximization, but rather with the dual problem of cost minimization subject to an ambient constraint, this analysis adopts the latter framework. Numerical results suggest (a) if the regulator is able to target contracts directly at specific agents, and thus achieve the first-best separating solution, total subsidy costs are significantly lower than for the optimal second-best solution; and (b) if instead the regulator is able to target contracts only indirectly, and thus achieve the second-best separating solution, total subsidy costs are only slightly lower than for the optimal pooling solution.

Problem Framework and Notation

Consider the case where agents possess the same preferences and objectives but different resource endowments. Specifically, assume that each agent is a price-taking profit-maximizer endowed with a fixed quantity of homogeneous land, and that land types differ across agents along two dimensions: productivity ($i$) and pollution potential ($j$), as shown in the illustration below.

<table>
<thead>
<tr>
<th>Productivity</th>
</tr>
</thead>
<tbody>
<tr>
<td>Low</td>
</tr>
<tr>
<td>High</td>
</tr>
<tr>
<td><strong>Pollution Potential</strong></td>
</tr>
<tr>
<td>Low</td>
</tr>
<tr>
<td>High</td>
</tr>
</tbody>
</table>

For any given level of inputs, it is assumed type $2j$ land produces more output, generates larger profits, and has larger marginal productivities relative to type $1j$ land. Type $i2$ land produces more emissions and has a larger marginal impact on ambient pollution relative to type $i1$ land. Also assume all agents employ the same types of variable inputs to generate the same type of output.

Letting $x$ denote a vector of per acre inputs, $w$ the vector of input prices, and $p$ the output price, each agent’s private per acre profit-maximization problem (without government intervention) may be stated as follows:

\[ \pi_i = \max_x \left[ pf_i(x) - w'x \right], \quad i = 1, 2, \]

where $f_i(\cdot)$ is the twice differentiable, increasing and strictly concave production function for type $ij$ land. Because agents disregard pollution potential, $j$ does not appear in the optimization problem. Also note that the productivity assumptions imply:

\[ f_2(x) > f_1(x), \quad \forall x > 0 \]

and

\[ \frac{\partial f_2(x)}{\partial x_k} > \frac{\partial f_1(x)}{\partial x_k}, \quad \forall x \text{ and } \forall k. \]

\[ \text{(1)} \]

\[ \text{(2)} \]

\[ \text{(3)} \]

\[ \text{This duality is not presented here formally, but is easily derived using straightforward algebraic manipulations of either problem statement.} \]

\[ \text{Throughout, this analysis relies heavily on the notation used in Wu and Babcock.} \]

\[ \text{The analysis may be extended to include additional land types and heterogeneous endowments for each agent, but the simplifications employed here are convenient for the analysis that follows.} \]
The regulator’s problem is to present the agents with a menu of per acre input-reduction subsidy contracts \((x_i, s_i; i = 1, 2)\) which minimizes the regulator’s cost of achieving the ambient standard.\(^6\) Note that these contracts are indexed by land productivity but not by pollution potential because pollution potential does not enter the agent’s objective function, whereas productivity enters through \(f_i(\cdot)\). Letting \(n_{ij}\) denote the number of acres of type \(ij\) land, the regulator’s problem may be stated as:

\[
\min_{x_i, s_i} \left[ \sum_i \sum_j n_{ij} s_i \right] \quad \text{s.t.: } h(n_{ij}, z_j(x_i)) \leq A.
\]

Here, \(A\) is the ambient standard and \(h(\cdot)\) is the ambient pollution function assumed to depend on the number of acres of each type of land, \(n_{ij}\), and the pollution produced by each type of land on a per acre basis, \(z_j(x_i)\).\(^7\) Assume both \(h(\cdot)\) and each \(z_j(x_i)\) are twice differentiable and increasing functions. Also note that the pollution potential assumptions imply:

\[z_2(x) > z_1(x), \quad \forall x > 0\]

and

\[
\frac{\partial z_2(x)}{\partial x_k} > \frac{\partial z_1(x)}{\partial x_k}, \quad \forall x \text{ and } \forall k \text{ that pollute.}
\]

Assuming the regulator knows \(n_{ij}\), but either cannot identify the type of any individual parcel of land or cannot directly target contracts at specific agents for reasons such as those mentioned earlier (see footnote 2), additional constraints must be added to ensure contracts intended for type \(ij\) land are accepted by agents who actually operate type \(ij\) land. In principal-agent theory, these constraints are referred to as “individual rationality” and “incentive compatibility” constraints. The individual rationality constraints guarantee that agents operating type \(ij\) land prefer accepting the contract intended for type \(ij\) land as opposed to rejecting it. These constraints may be written as:

\[
\pi_1(x_1) + s_1 \geq \pi_1(x_1^0),
\]

\[
\pi_2(x_2) + s_2 \geq \pi_2(x_2^0),
\]

where \(x_i^0\) denotes the solution to the agent’s optimization problem without government intervention. The incentive compatibility constraints ensure agents operating type \(ij\) land prefer contracts intended for type \(ij\) land as opposed to contracts intended for any other type of land. These constraints may be written as:

\[
\pi_1(x_1) + s_1 \geq \pi_1(x_2) + s_2,
\]

\[
\pi_2(x_2) + s_2 \geq \pi_2(x_1) + s_1.
\]

Again, because agents disregard pollution potential, \(j\) does not appear in any of these constraints.

---

\(^6\) Such standards are ubiquitous in environmental regulation (e.g., Clean Water Act and Clean Air Act standards in the United States) and provide motivation for using this more realistic constrained optimization problem framework.

\(^7\) This representation allows for each of the four land types to contribute a different amount of emissions to the ambient concentration.
Optimality with Perfect Information
and No Targeting Constraints

With perfect information and no targeting constraints, the regulator can identify the type of each parcel of land and can assign the appropriate contract to each agent. Therefore a pooling solution cannot be optimal. Furthermore, there is no need for the incentive compatibility (IC) constraints, although the individual rationality (IR) constraints remain important. The Lagrangian for the regulator’s problem with perfect information may be stated as:

\begin{equation}
L = -n_1s_1 - n_2s_2 + \lambda \left[ A - h\left(n_{ij}, z_j(x_i)\right) \right] + \mu_1\left[ \pi_1(x_1) + s_1 - \pi_1(x_1^0) \right] + \mu_2\left[ \pi_2(x_2) + s_2 - \pi_2(x_2^0) \right],
\end{equation}

where \( n_i = \sum_j n_{ij} \). Furthermore, assume the regulator has the ability to sign contracts only for a single production input, \( x_k \). Letting \( x'_i = x'_i(p, w, x_{ik}) \) denote each agent’s optimal input choices given the regulator’s choice of \( x_{ik} \), algebraic manipulations of (11) give the following set of necessary conditions for an interior solution \( (x_{ik}^*, s_i^*; i = 1, 2) \):

\begin{align}
&\frac{n_1 * (\frac{\partial \pi_1}{\partial x_{1k}})}{\frac{\partial h}{\partial x_{1k}}} = \frac{n_2 * (\frac{\partial \pi_2}{\partial x_{2k}})}{\frac{\partial h}{\partial x_{2k}}}, \\
&h\left(n_{ij}, z_j(x'_i)\right) = A, \\
&s_1^* = \pi(x_1^0) - \pi_1(x_1^*), \\
&s_2^* = \pi(x_2^0) - \pi_2(x_2^*).
\end{align}

Conditions (12) and (13) define two loci of points \( (x_{1k}^*, x_{2k}^*) \). The locus defined by (12) gives the cost-minimizing allocation of inputs for all possible ambient standards. Notice that (12) is the optimality condition requiring the regulator’s marginal cost of reducing ambient pollution to be equalized across land types. The locus defined by (13) gives all possible allocations of inputs meeting a given ambient standard. Therefore, the intersection of these two loci gives the regulator’s optimal input levels. Optimal payments are then given by conditions (14) and (15), the IR constraints: \( s_i^* = \pi_i(x_i^0) - \pi_i(x_i^*), i = 1, 2 \).

Optimality with Imperfect Information
or Targeting Constraints

Assuming the regulator knows \( n_{ij} \), but either has imperfect information and therefore cannot identify the type of any individual parcel of land or, equivalently, cannot assign contracts to specific agents (see footnote 2), she must rely on both the IC and IR constraints to implement a cost-effective second-best mechanism. Wu and Babcock (p. 319) show that in any feasible mechanism under imperfect information, the IC constraints and productivity assumptions imply \( x_{2k}^* \geq x_{1k}^* \). When both IC constraints bind, \( x_{2k}^* = x_{1k}^* \) and the mechanism is therefore pooling (i.e., only one contract is offered); when no more than one IC constraint binds, \( x_{2k}^* > x_{1k}^* \) and the mechanism is therefore separating (i.e., two contracts are offered).
As mentioned previously, for the constrained cost-minimizing regulator with perfect information and no targeting constraints, a pooling solution cannot be optimal. However, this result does not hold for the constrained cost-minimizing regulator with imperfect information or targeting constraints. Under certain conditions, the optimal separating solution in this case collapses to an optimal pooling solution.

**Optimal Separating Mechanism**

In a separating mechanism, the regulator offers two contracts, and therefore both IR and IC constraints must be included. The Lagrangian for this case may be stated as follows:

\[
L = -n_1 s_1 - n_2 s_2 + \lambda \left[ A - h(n_{ij}, z_j(x_i)) \right] + \mu_1 \left[ \pi_1(x_1) + s_1 - \pi_1(x_1^0) \right] \\
+ \mu_2 \left[ \pi_2(x_2) + s_2 - \pi_2(x_2^0) \right] + \mu_3 \left[ \pi_1(x_1) + s_1 - \pi_1(x_2) - s_2 \right] \\
+ \mu_4 \left[ \pi_2(x_2) + s_2 - \pi_2(x_1) - s_1 \right].
\]

One possible solution to this problem framework already has been addressed in the perfect information case. When this first-best solution is not feasible, a second-best separating solution instead may exist. Given the productivity assumptions presented earlier and some additional regularity conditions (that are satisfied, for example, if \( f_i(\cdot) \) is homothetic),\(^8\) equation (16) can be manipulated to give the following necessary conditions for the second-best separating solution, where \( n = \sum_i \sum_j n_{ij} \) is the total number of acres:

\[
\frac{n_1 * (\partial \pi_1 / \partial x_{1k})}{\partial h / \partial x_{1k}} \bigg|_{x_i^*} = \frac{n * (\partial \pi_2 / \partial x_{2k}) - n_1 * (\partial \pi_1 / \partial x_{2k})}{\partial h / \partial x_{2k}} \bigg|_{x_2^*},
\]

\[
h(n_{ij}, z_j(x_i^*)) = A,
\]

\[
s_2^* = \pi_2(x_2^0) - \pi_2(x_2^*),
\]

\[
s_1^* = \pi_1(x_1^0) - \pi_1(x_1^*) + s_2^*.
\]

Condition (17) is again an optimality condition requiring the equalization of marginal abatement costs across land types, taking into account the premium being paid to agents with lower productivity land in order to make the contract menu incentive compatible. Condition (18) is the ambient constraint, and condition (19) is the IR constraint for agents with higher productivity land. Condition (20) is the IC constraint for agents with lower productivity land, and requires the premium they receive be minimized.

---

\(^8\) Homotheticity of \( f_i(\cdot) \) is sufficient but not necessary for the results that follow.
Optimal Pooling Mechanism

In a pooling mechanism, the regulator offers only one contract, and hence the IC constraints may be omitted. Thus, the Lagrangian is similar to that under perfect information, but now there are only two choice variables, \( x_k \) and \( s \):

\[
L = -ns + \lambda \left[ A - h(n_{ij}, z_j(x)) \right] + \mu_1 \left[ \pi_1(x) + s - \pi_1(x_1^0) \right] + \mu_2 \left[ \pi_2(x) + s - \pi_2(x_2^0) \right].
\]

Algebraic manipulations of (21) give the following set of necessary conditions for an interior solution \((x_k^*, s^*)\):

\[
h(n_{ij}, z_j(x^*)) = A, \quad s^* = \pi_2(x_2^0) - \pi_2(x^*). \quad (23)
\]

Condition (22) is the ambient constraint, and condition (23) is the IR constraint for the higher productivity land. This is a relatively simple problem for the regulator—first, determine the maximum common per acre input level that will achieve the ambient standard, and then determine the minimum common payment that will induce both types of agents to sign contracts.

Optimality of Separating versus Pooling Solutions

If the first-best separating solution (S1) is feasible, then it must be optimal because it corresponds to the perfect information case with no targeting constraints. The additional separating (S2) and pooling solutions are second-best mechanisms and correspond to the case of imperfect information and/or targeting constraints. Consequently, in theory, the S2 and pooling solutions are less desirable from the regulator's perspective. The pooling mechanism is simpler because it consists of a single contract offer and therefore involves no targeting. The S2 mechanism is more complicated because it consists of a menu of contracts and thus involves "indirect" targeting—i.e., all agents select from the same menu, but the menu is designed such that all agents of the same type choose the same contract.

When the S1 solution is not feasible, it is possible to derive conditions under which the S2 mechanism collapses to an optimal pooling solution. To do this, it is convenient to examine the pooling solution and determine whether there exists a feasible S2 mechanism in a neighborhood of that solution. When no such mechanism exists, an additional convexity condition is sufficient to establish global optimality of the pooling solution.
The algebraic condition under which the S2 mechanism collapses to an optimal pooling solution is derived in appendix A and may be stated as follows:

\[
\frac{\partial h}{\partial x_{2k}} \bigg|_{x^*_k} > \frac{n \ast (\partial \pi_2 / \partial x_{2k}) - n_1 \ast (\partial \pi_1 / \partial x_{1k})}{n_1 \ast (\partial \pi_1 / \partial x_{1k})} ,
\]

where \(x^*_k\) is the optimal input level in the pooling solution, and \(n\) is the total number of acres. From this condition, three factors are shown to influence the relative optimality of pooling versus S2 solutions: (a) relative pollution potential, (b) relative productivity, and (c) number of acres of each land type.

For a given productivity differential and distribution of land types, the left-hand side of condition (24) becomes large as the ambient concentration becomes more sensitive to emissions from high-productivity land. In such a case, even a small positive increase in pollution from high-productivity land must be offset by a large decrease in pollution from low-productivity land that is expensive for the regulator to subsidize. Thus, pooling tends to be optimal. In the limit, the left-hand side of (24) approaches infinity, in which case pooling always is optimal. Conversely, as the ambient concentration becomes less sensitive to emissions from high-productivity land, the left-hand size becomes small and separating tends to be optimal. In the limit, the left-hand side approaches zero and pooling never is optimal because the right-hand side must be nonnegative.

For a given pollution potential differential and distribution of land types, the right-hand side of condition (24) becomes small when the productivity differential is small. When there is no productivity differential (i.e., when there is only one land productivity type), the right-hand side equals zero and pooling always is optimal. Conversely, when high-productivity land is much more productive than low-productivity land, the right-hand side of (24) becomes large and separating tends to be optimal. In such a case, even a small increase in emissions from high-productivity land is valued greatly by these land operators, and therefore confers on the regulator significant savings. Thus, even a large decrease in emissions from low-productivity land can be subsidized while still lowering total costs.

For given productivity and pollution potential differentials, the distribution of land types has competing effects. As the amount of high-productivity land increases, both numerators become larger while both denominators remain fixed, and thus both sides increase. As the amount of low-productivity land increases, the left-hand denominator becomes larger while the numerator remains fixed and the entire right-hand side becomes smaller; thus both sides decrease. The net effect of changes in the distribution of land types therefore is ambiguous for the general case.

Discussion

The preceding results can be summarized as follows. If a feasible S1 mechanism exists, it must be optimal because it is equivalent to the perfect information case with no targeting constraints. When no such mechanism exists, then if a feasible S2 mechanism

\[12\] Reversing the inequality results in an existence condition for a feasible S2 mechanism. An analogous condition for a feasible S1 mechanism is not addressed here, but would be a useful extension.
exists, it must be optimal because the pooling solution is simply a special case of an S2 mechanism (i.e., the S2 solution must be at least as desirable as the pooling solution). When no feasible S2 mechanism exists, then the pooling solution is optimal.\footnote{Note, although the pooling solution is a unique case of an S2 mechanism, it is not optimal merely at a single point, but rather over an infinite range defined by condition (24).}

These results provide practical usefulness. By knowing what types of solutions may be optimal in this framework, some interesting empirical questions may be addressed. For example, how do the costs of a typical green payment program compare with what is achievable in this framework? Specifically, how much “room for improvement” is there in a typical green payment program? And how would changes in the ambient standard for a particular watershed affect total program costs for that watershed? Answers to these questions, of course, depend on the specific characteristics of the watershed of concern, and require data regarding production functions, the ambient pollution function, and the distribution of land types for that watershed.

However, it is possible to examine without detailed empirical information how the second-best solutions in this framework compare with the first-best solution—in other words, to examine the magnitude of the loss generated by information asymmetry and/or the inability to target contracts directly for political reasons.

Ideally, it would be desirable to place theoretical bounds on the additional cost incurred under a second-best solution versus a first-best solution. This would measure either the value of acquiring additional information necessary for a first-best solution, or the cost imposed on the regulator (and thus on other social programs from which funding is redirected) by political barriers which prohibit direct targeting. But finding such theoretical bounds is a fairly daunting task because it involves comparing the solutions to systems of implicit functions defined by the first-order conditions corresponding to different problems. While it would be somewhat easier to assume specific and relatively simple functional forms, here numerical examples are used to cast some light on the relative magnitude of the loss.

Details of two numerical examples are given in appendix B, but both imply the increased cost imposed on the regulator due to the inability to target contracts directly is relatively large. In the first example, budgetary outlays for the optimal second-best solution are about 125% larger than for the first-best solution ($39.62 versus $17.57). In the second example, the increase is about 80% ($6.96 versus $3.86). These results, although derived from stylized examples, suggest efforts by regulators to acquire detailed information regarding resource endowments and to resist political pressures to treat farmers “fairly” may be well justified.

Based on the numerical results when direct targeting is not an option, indirect targeting, when possible, can produce only modest cost reductions. In the first example, where indirect targeting is possible, the regulator realizes a cost reduction of only about 1% by moving from the optimal pooling solution ($39.95) to the optimal feasible S2 solution ($39.62). To the extent that a menu of contracts is more costly to develop and administer in practice, these results reveal that pooling contracts may be preferred in even more cases than would be indicated by the theoretical results presented earlier.
Summary and Conclusion

This analysis derives the necessary conditions describing the candidate solutions to a class of nonpoint source pollution control problems. Under certain conditions, the optimal contract mechanism for a constrained cost-minimizing regulator will involve a pooling equilibrium. This result illustrates the possibility that, when direct targeting of green payment programs is not possible, simpler arrangements offering fewer choices may be able to achieve an ambient standard at the lowest possible cost. Though not conclusive, the numerical results presented here lend support to efforts to overcome barriers to first-best mechanisms because direct targeting appears to produce significant cost savings for regulators. The same results also suggest second-best separating solutions may provide only slight cost savings over simpler pooling solutions, implying the returns to indirect targeting efforts may be small.

[Received March 2002; final revision received August 2002.]

References


Appendix A:
Derivation and Discussion of the Optimality Condition for a Pooling Solution

Start with the ambient pollution function: \( h(n_i, z_j(x_i)) = A \). Take the total derivative in \((x_{1k}, x_{2k})\) space:
\[
\frac{\partial h}{\partial x_{1k}} * dx_{1k} + \frac{\partial h}{\partial x_{2k}} * dx_{2k} = 0.
\]
Rearranging gives the slope of the constraint:
\[
\frac{dx_{2k}}{dx_{1k}} = -\frac{\partial h/\partial x_{1k}}{\partial h/\partial x_{2k}}.
\]

Recalling that the IR constraint for high-productivity land must bind, the incremental subsidy required to move high-productivity land operators from the pooling solution to a nearby feasible S2 mechanism is given by:
\[
s_2^* = \pi_2(x_2^*) - \pi_2(x_2^* - s_p^*) - s_p^* - \pi_1(x_1^*)
\]
\[
= s_p^* + \pi_2(x_2^*) - \pi_2(x_2^* - s_p^*)
\]
\[
= \Delta s_2 = \pi_2(x_2^*) - \pi_2(x_2^* - s_p^*).
\]

Taking the first-order approximation gives: \( \Delta s_2 = -\partial \pi_2(x_2^*)/\partial x_{2k} * dx_{2k} \).

Recalling that the IC constraint for low-productivity land operators must bind, the incremental subsidy required to move low-productivity land operators from the pooling solution to a nearby feasible S2 mechanism is given by:
\[
s_1^* = \pi_1(x_2^*) + s_2^* - s_1^* - \pi_1(x_1^*)
\]
\[
= \pi_1(x_2^*) - \pi_1(x_1^*) + \Delta s_1
\]
\[
= [\pi_1(x_2^*) - \pi_1(x_1^*)] - [\pi_1(x_1^*) - \pi_1(x_1^*)] + \Delta s_2.
\]

Again taking the first-order approximation, and noting that \( dx_{1k} < 0 \), gives:
\[
\Delta s_1 = -\frac{\partial \pi_1(x_2^*)}{\partial x_{1k}} * dx_{2k} - \frac{\partial \pi_1(x_2^*)}{\partial x_{1k}} * dx_{1k} + \Delta s_2
\]
\[
= -\frac{\partial \pi_1(x_2^*)}{\partial x_{1k}} * (dx_{1k} + dx_{2k}) - \frac{\partial \pi_2(x_2^*)}{\partial x_{2k}} * dx_{2k}.
\]

Because the pooling solution is a degenerate case of an S2 mechanism, it is optimal if and only if any attempt to move to a nearby feasible S2 mechanism results in higher costs to the regulator (i.e., if and only if \( n_1 * \Delta s_1 + n_2 * \Delta s_2 > 0 \)). Substituting the preceding results into this inequality gives:
\[
n_1 \left[ -\frac{\partial \pi_1(x_2^*)}{\partial x_{1k}} * (dx_{1k} + dx_{2k}) - \frac{\partial \pi_2(x_2^*)}{\partial x_{1k}} * dx_{2k} \right] + n_2 \left[ \frac{\partial \pi_2(x_2^*)}{\partial x_{2k}} * dx_{2k} \right] > 0.
\]
Rearranging gives:
\[
-n_1 \left[ -\frac{\partial \pi_1(x_2^*)}{\partial x_{1k}} * dx_{1k} \right] > (n_1 + n_2) \left[ \frac{\partial \pi_2(x_2^*)}{\partial x_{1k}} * dx_{2k} \right] - n_1 \left[ \frac{\partial \pi_1(x_2^*)}{\partial x_{1k}} * dx_{2k} \right].
\]
Noting that $\frac{\partial \pi_i(x_p^*)}{\partial x_k} > 0$, and $dx_{2k} > 0$, rearranging further gives:

$$- \frac{dx_{2k}}{dx_{1k}} > \frac{n \cdot \frac{\partial \pi_2(x_p^*)}{\partial x_k} - n_1 \cdot \frac{\partial \pi_1(x_p^*)}{\partial x_k}}{n_1 \cdot \frac{\partial \pi_1(x_p^*)}{\partial x_k}} \geq 0.$$ 

Substituting

$$\frac{dx_{2k}}{dx_{1k}} = -\frac{\partial h/\partial x_{1k}}{\partial h/\partial x_{2k}}$$

yields:

$$\frac{\partial h/\partial x_{2k}}{\partial h/\partial x_{1k}} > \frac{n \cdot \frac{\partial \pi_2(x_p^*)}{\partial x_k} - n_1 \cdot \frac{\partial \pi_1(x_p^*)}{\partial x_k}}{n_1 \cdot \frac{\partial \pi_1(x_p^*)}{\partial x_k}} \geq 0.$$

Condition (A1) is both necessary and sufficient for local optimality of the pooling solution. An additional condition sufficient to establish global optimality of a pooling solution that satisfies (A1) may be derived as follows. Consider again the regulator’s problem, and assume the implicit function theorem holds so that the regulator’s total cost function may be written as:

$$TC = \min_{s_2} \left[ n_1 s_1(s_2 | p, w, A, n_{ij}, \theta) + n_2 s_2 \right],$$

where $\theta$ is a vector of parameters describing the production, per acre pollution, and ambient pollution functions. This exposition makes clear that the regulator’s problem essentially involves choosing only a single variable, $s_2$. Selection of $s_2$ determines $x_2$ through the IR constraint for type 2j land; $x_2$ then determines $x_1$ through the ambient constraint; and $x_1$ then determines $s_1$ through the IC constraint for type 1j land. Suppressing some notation and taking the derivative of $TC$ with respect to $s_2$ gives the first-order necessary condition for cost minimization, as well as the slope of $s_1(s_2 | \cdot)$ at a candidate solution:

$$\left. \frac{\partial TC}{\partial s_2} \right|_{s_2^*} = n_1 \frac{\partial s_1}{\partial s_2} \bigg|_{s_2^*} + n_2 = 0 \Rightarrow \frac{\partial s_1}{\partial s_2} \bigg|_{s_2^*} = -\frac{n_2}{n_1} < 0.$$ 

Notice this expression implies $s_1(s_2 | \cdot)$ is downward sloping at a candidate solution, which makes sense. A smaller payment to one agent type means that type will make a smaller input reduction. The other agent type must then make a larger input reduction to meet the ambient constraint, and this larger reduction requires a larger payment.

Taking the second derivative gives the second-order sufficient condition establishing $s_2^*$ as a global minimum:

$$\frac{\partial^2 TC}{\partial s_2^2} = n_1 \frac{\partial^2 s_1}{\partial s_2^2} > 0 \Rightarrow \frac{\partial^2 s_1}{\partial s_2^2} > 0.$$ 

In other words, $s_1(s_2 | \cdot)$ must be globally convex. Although it may be difficult to establish this property algebraically, an intuitive argument confirms it holds if the production functions are strictly concave and the ambient constraint is concave in $(x_1, x_2)$ space. First, recall that a decrease in $s_2$ implies a necessary increase in $s_1$ because of the ambient constraint. Moreover, a strictly concave production function implies inputs become increasingly valuable to producers as their usage levels decline. This means input reduction becomes increasingly expensive to subsidize as input levels decrease. Combining these two results demonstrates that as $s_2$ decreases, $s_1$ must increase at an increasing rate to meet the ambient constraint provided this constraint is concave (a convex constraint would correspond to the unlikely scenario where an input has a larger marginal impact on ambient pollution when it is used sparingly than when it is used copiously). Therefore, $s_1(s_2 | \cdot)$ is globally convex under these conditions.
Appendix B:
Numerical Examples

Consider the following numerical example involving only two types of land and a single production input. The single input case greatly simplifies the analysis and renders unnecessary the assumptions regarding the marginal input productivities and the homotheticity of \( f_i(\cdot) \). The breakdown of land types is shown in the illustration below.

<table>
<thead>
<tr>
<th>POLLUTION</th>
<th>LOW</th>
<th>HIGH</th>
</tr>
</thead>
<tbody>
<tr>
<td>POTENTIAL</td>
<td>Low</td>
<td>0</td>
</tr>
<tr>
<td></td>
<td>High</td>
<td>( n_2 )</td>
</tr>
</tbody>
</table>

Production functions, per acre pollution functions, and the ambient pollution function are shown in the left-hand column of the table below. Parameter values are shown in the right-hand column. All regularity conditions mentioned previously are satisfied here. Also notice the production functions are strictly concave and the ambient constraint is concave (see discussion in appendix A).

<table>
<thead>
<tr>
<th>PRODUCTION FUNCTIONS:</th>
<th>PARAMETER VALUES:</th>
</tr>
</thead>
<tbody>
<tr>
<td>( f_1 = \ln(x_1 + 1) )</td>
<td>( p = 10 )</td>
</tr>
<tr>
<td>( f_2 = \gamma \ln(x_2 + 1) )</td>
<td>( w = 1 )</td>
</tr>
<tr>
<td>PER ACRE POLLUTION FUNCTIONS:</td>
<td></td>
</tr>
<tr>
<td>( z_1(x_1) = \frac{1}{2} x_1 )</td>
<td>( n_1 = 3 )</td>
</tr>
<tr>
<td>( z_2(x_2) = 2x_2 )</td>
<td>( n_2 = 2 )</td>
</tr>
<tr>
<td>AMBIENT POLLUTION FUNCTION:</td>
<td>( A = 30 )</td>
</tr>
<tr>
<td>( h(\cdot) = n_1 z_1 + n_2 z_2 )</td>
<td>( \gamma = 2, 1.2 )</td>
</tr>
</tbody>
</table>

**CASE 1**

Consider the case where \( \gamma = 2 \), namely, a large productivity differential. To solve this problem, it is first necessary to determine whether the S1 solution is feasible. If it is, then it must be optimal. If not, then the second-best solutions may be investigated to find the minimum-cost contract scheme. The initial operating positions for each type of land are as follows:

\[
\begin{align*}
\pi_1 &= p \ln(x_1 + 1) - wx_1 \\
\pi_2 &= p \ln(x_2 + 1) - wx_2 \\
\pi_1' &= \frac{p}{x_1 + 1} - w = \frac{10}{x_1 + 1} - 1 \\
\pi_2' &= \frac{2p}{x_2 + 1} - w = \frac{20}{x_2 + 1} - 1 \\
\Rightarrow x_1^0 &= 9 \\
\Rightarrow \pi_1^0 &= 14.03 \\
\Rightarrow x_2^0 &= 19 \\
\Rightarrow \pi_2^0 &= 40.91
\end{align*}
\]

The necessary conditions for an S1 mechanism are given by:

\[
\frac{n_1 \cdot (\partial \pi_1 / \partial x_1)}{\partial h / \partial x_1} \bigg|_{x_1^0} = \frac{n_2 \cdot (\partial \pi_2 / \partial x_2)}{\partial h / \partial x_2} \bigg|_{x_2^0},
\]

\[
h(n_{ij}, z_j(x_i)) = A,
\]
\[ S_1 = \pi(x_1^0) - \pi_1(x_1^*), \]
\[ S_2 = \pi(x_2^0) - \pi_2(x_2^*). \]

Solving this set of equations gives:
\[ x_1^* = 6.580, \quad x_2^* = 5.855, \quad s_1^* = 0.35, \quad s_2^* = 8.26 \Rightarrow \text{Total Cost} = 17.57. \]

While IC_2 holds, it is straightforward to verify that IC_1 is violated (i.e., \( x_{2k}^* > x_{1k}^* \) does not hold). Therefore, the S1 solution is not feasible.

Next, consider the necessary conditions for a pooling mechanism:
\[ h(n_{ij}, z_j(x^*)) = A, \]
\[ s^* = \pi_2(x_2^0) - \pi_2(x_2^*). \]

Solving this set of equations gives:
\[ x^* = 6.000, \quad s^* = 7.99 \Rightarrow \text{Total Cost} = 39.95. \]

Here it is easy to check that IR_1 is satisfied, so this is a candidate solution.

Last, consider the necessary conditions for an S2 mechanism:
\[ \frac{n_i * (\partial \pi_1 / \partial x_{1k})}{\partial h / \partial x_{1k}}|_{x_1^*} = \frac{n_i * (\partial \pi_2 / \partial x_{2k}) - n_i * (\partial \pi_1 / \partial x_{2k})}{\partial h / \partial x_{2k}}|_{x_2^*}, \]
\[ h(n_{ij}, z_j(x^*)) = A, \]
\[ s_2^* = \pi_2(x_2^0) - \pi_2(x_2^*), \]
\[ s_1^* = \pi_1(x_2^*) - \pi_1(x_1^*) + s_2^*. \]

Solving this set of equations yields:
\[ x_1^* = 5.272, \quad x_2^* = 6.182, \quad s_1^* = 8.10, \quad s_2^* = 7.66 \Rightarrow \text{Total Cost} = 39.62. \]

Both IR_1 and IC_2 are satisfied, so this is also a candidate solution. Furthermore, the total cost for the S2 solution is less than the total cost for the pooling solution (as expected), so the S2 solution is optimal for \( \gamma = 2 \).

**CASE 2**

It can be shown that the algebraic condition under which the pooling solution is optimal for this problem framework simplifies to \( \gamma < 1.6 \). So consider the case in which \( \gamma = 1.2 \) (small productivity differential). Similar calculations to those presented above yield the following results:

- **S1 mechanism**: \( x_1^* = 7.784, \quad x_2^* = 5.554, \quad s_1^* = 0.08, \quad s_2^* = 1.81 \Rightarrow \text{Total Cost} = 3.86; \)
- **Pooling mechanism**: \( x^* = 6.000, \quad s^* = 1.47, \Rightarrow \text{Total Cost} = 7.35; \)
- **S2 mechanism**: \( x_1^* = 6.984, \quad x_2^* = 5.763, \quad s_1^* = 1.22, \quad s_2^* = 1.65 \Rightarrow \text{Total Cost} = 6.96. \)

However, IC_1 is violated for the S1 solution, so it is not feasible. And IC_2 is violated for the S2 solution, so it also is not feasible. This leaves the pooling solution as the only feasible mechanism, and therefore it is optimal for \( \gamma = 1.2 \).