Price-Band Stabilization Programs and Risk: An Application to the U.S. Corn Market

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The impacts of introducing a partial price stabilization scheme in the U.S. corn market are investigated by using a modified version of the bounded price variation model. Specifically, a model is developed and estimated that includes rational expectations of the first three central moments of the (truncated) equilibrium price distribution. The estimated model is used to simulate market equilibrium effects of introducing upper and lower price limits through a tax-subsidy scheme. The results show that corn producers are downside risk averse, and that market feedback effects of price stabilization can, at times, be more important than direct effects.

Key words: downside risk aversion, price stabilization, rational expectations equilibrium.

Introduction

Government programs have played an important role in U.S. agricultural markets for over 50 years (Gardner). While a variety of policy instruments have been used, price support loans, target price-deficiency payment schemes, and acreage set-aside programs have emerged as the cornerstone of U.S. agricultural policies for many major field crops. As a result, much research has focused on determining the effects of government programs on agricultural supply, demand, and price relationships (Lee and Helmberger; Chavas and Holt; Holt 1992).

There also is growing evidence, presumably due to risk aversion, that price, production, and/or income variability are significant factors in aggregate agricultural supply relationships (Pope and Just; Coyle; Holt and Moschini). If, as the empirical evidence suggests, risk is important in agricultural supply decisions, and if these risks cannot be shared fully through existing contingent claims markets, it is desirable to explore ways of controlling or mitigating the effects of uncertainty on agricultural markets (Newbery and Stiglitz). If recent studies have provided insights into the effects of reducing or eliminating agricultural price and income risks in a market equilibrium framework (e.g., Brorsen, Chavas, and Grant; Myers; Innes 1990a, b). While much of this work quantifies benefits associated with eliminating risk, it follows that formal relationships between government policy instruments and risk perceptions of agricultural producers have not been well established. In other words, the mechanism by which agricultural price and income instability can be reduced through government intervention is not well defined.

Developing linkages between government price support activities and the price and risk perceptions of agricultural producers is desirable. This is because complete elimination of risk, either through available contingent claims markets or through government pro-
grams, is likely an unattainable goal. Alternatively, partial price or revenue stabilization may be the most realistic outcome (Miranda and Helmberger).

Considering the above, the objectives of this article are: (a) to develop a framework for examining the effects of government programs on producers’ price and risk expectations, and (b) to determine empirically the potential impacts of partial price stabilization implemented through a tax-subsidy scheme on the U.S. corn market. The conceptual framework developed follows Eeckhoudt and Hansen in that price supports are deemed to effectively truncate producers’ subjective probability distributions of price. The model is closed by assuming producers form expectations rationally.

Essential features of the conceptual model are incorporated into an econometric model of the U.S. corn market by modifying the bounded prices model under rational expectations (Shonkwiler and Maddala; Holt and Johnson) to include risk terms. Unlike prior estimates of risk response in aggregate supply models, the model specified here incorporates, in addition to a price variance term, a measure of the third central moment of price. Inasmuch as price support programs are designed to moderate downside price movements, and to the extent that producers exhibit downside risk aversion, inclusion of a third moment term can be empirically and economically important (Antle 1987; Menezes, Geiss, and Tressler).

The resulting model is used to simulate endogenous market response associated with stabilizing prices partially through a system of minimum and maximum price bands. The results show, for instance, that as band widths are reduced, expected production actually can increase even though expected market and producer prices decline. This result is due exclusively to the reduction in price risk associated with “squeezing” minimum and maximum price limits. Such findings underscore the importance of analyzing government intervention in a market equilibrium setting.

Conceptual Framework

This section develops a market model that includes rational expectations, price uncertainty, and risk-averse producers. The conceptual framework provides a foundation for empirical work reported in following sections.

Consider a competitive industry consisting of \( N \) identical firms, each producing a homogeneous commodity and facing a random output price, \( p \). Although free entry and exit are permitted, the number of firms \( N \) is fixed in the short run. Due to production lags, the output price is not observed at the time production decisions are made. Production technology is represented by the concave production function \( q = f(x) \), where \( x \) is a vector of inputs. To simplify the analysis, production is assumed to be nonstochastic. Random market price \( p \) is characterized by the stochastic inverse demand function,

\[
 p = D(Q, \epsilon), \quad \partial p / \partial Q < 0,
\]

where \( \epsilon \) is a random variable with distribution function \( H(\epsilon) \) and \( Q = Nq \) is industry output. A given value for anticipated output, say \( Q^e \), determines the conditional price distribution, \( F(p \mid Q^e) \). Consequently, expected market price is

\[
 \hat{p}(Q^e) = \int_0^\infty p \, dF(p \mid Q^e) = \int_0^\infty D(Q^e, \epsilon) \, dH(\epsilon),
\]

and second- and higher-order central moments of the price distribution are given by

\[
 \sigma_k(Q^e) = \int_0^\infty (p - \hat{p})^k \, dF(p \mid Q^e) = \int_0^\infty \left[ D(Q, \epsilon) - \bar{D} \right]^k \, dH(\epsilon), \quad k \geq 2.
\]

Government intervention occurs through a system of minimum and maximum prices that are used to support and stabilize prices received by producers. The minimum price is denoted by \( p_m \) and the maximum price is given by \( p_M \). Following Quiggin and Anderson,
this minimum-maximum price system is enforced by a tax-subsidy or buffer-fund scheme. Producers receive a direct per-unit subsidy equal to \((p_m - p)\) if the realized market price falls below the minimum price. Likewise, if the market price exceeds \(p_M\), producers pay a direct per-unit tax equal to \((p - p_M)\). Otherwise, the stabilization agency takes no action.

Minimum and maximum price limits truncate the probability density function of price as perceived by producers from below at \(P_m\) and above at \(PM\) (Eeckhoudt and Hansen). The resulting random price \(v\) is equal to \(P_m\) when \(p < P_m\) and is equal to \(PM\) when \(p > PM\). Hence, the truncated conditional density \(g(v \mid Q)\) takes a zero value in the intervals \([0, P_m]\) and \([PM, \infty)\). At \(p = P_m\), \(g(v \mid Q)\) assumes a value equal to the probability \(F(P_m \mid Q)\); at \(p = PM\), \(g(v \mid Q)\) takes a value equal to the probability \(1 - F(PM \mid Q)\). For values of \(p\) strictly between \(P_m\) and \(PM\), the densities \(f(p \mid Q)\) and \(g(v \mid Q)\) are identical.

Under risk aversion, firms attempt to maximize expected utility of profit. Assuming that each firm possesses a von Neumann–Morgenstem utility function \(u(r)\), which is increasing \((du/dr < 0)\) and concave \((d^2u/dr^2 < 0)\) under risk aversion, the problem is:

\[
\max_{\pi} \mathbb{E} u(\pi \mid Q) = \int_{P_m}^{PM} u[vf(x) - r'x] \, dG(v \mid Q),
\]

where \(\pi = p'f(x) - r'x\) is random profit \((r\) being a vector of known input prices), and \(G(v \mid Q)\) represents the decisionmaker’s subjective beliefs about the price distribution \(G(v \mid Q)\). The first-order conditions associated with (4) are:

\[
\frac{\partial \mathbb{E} u(\pi)}{\partial x} = \int_{P_m}^{PM} u' \cdot [vf(x) - r'] \, dG(v \mid Q) = 0'.
\]

Assuming sufficient second-order conditions are satisfied, (5) can be solved for the optimal choice functions, \(x^*(r, v, \sigma^*_v; Q^*)\) and \(q^*(r, v, \sigma^*_q; Q^*) = f(x^*(r, v, \sigma^*_v; Q^*))\). Here, \(v\) denotes the mean of the density \(g(v \mid Q^*)\) and \(\sigma^*_v\) denotes a vector of second- and (possibly) higher-order central moments associated with \(g(v \mid Q^*)\). The notation used indicates that each firm’s optimal decisions depend on the subjective estimate, \(Q^*\), of industry output.

Industry supply is obtained by summing firm-level supply across all producers and is given by

\[
Q = Nq^*(r, v, \sigma^*_q; Q^*) = Q(r, v, \sigma^*_q; Q^*).
\]

To close the model, it is necessary to relate anticipated output \((Q^*)\) to actual production \((Q)\). This correspondence is obtained by assuming that agents form rational expectations about the truncated price distribution, \(g(v \mid Q)\) (Newbery and Stiglitz). This implies that if industry output \(Q\) in (6) differs from expected output \(Q^*\), agents will revise their output estimates. Hence, a short-run equilibrium is characterized by the condition \(Q^* = Q\), the closing identity in a rational expectations model with price uncertainty and risk-averse agents.

Under a competitive rational expectations equilibrium, industry output represents the fixed point of the mapping from the right-hand side to the left-hand side of (6). The equilibrium of industry output and expected moments of price solves the system of \(m + 1\) equations:

\[
(7a) \quad Q^* = Q(r, v, \sigma^*_v; Q^*); \tag{7a}
\]

\[
(7b) \quad \bar{v} = p_mF(p_m \mid Q^*) + \int_{P_m}^{PM} p \, dF(p \mid Q^*) + p_M[1 - F(p_M \mid Q^*)]; \tag{7b}
\]

\[
(7c) \quad \sigma^*_j = [p_m - \bar{p}]^jF(p_m \mid Q^*) + \int_{P_m}^{PM} [p - \bar{p}]^j \, dF(p \mid Q^*)
\]

\[
+ [p_M - \bar{p}]^j[1 - F(p_M \mid Q^*)], \quad j = 2, \ldots, k. \tag{7c}
\]
Eeckhoudt and Hansen investigated comparative statics associated with a mean-preserving price squeeze implemented through a minimum-maximum price system. While their results show a mean-preserving price squeeze will unambiguously increase output of risk-averse firms, they did not consider market feedback. In the present case, a positive supply response induced by a minimum-maximum price squeeze will result in lower expected market prices. But expected market prices determine the values of the moments for the truncated price distribution in (7). The net result is that changes in minimum and maximum prices will have both a direct (i.e., truncation) and an indirect (i.e., changes in expected price due to changes in output) impact on the moments of the effective producer price distribution, \( g(v \mid Q) \).

As illustrated in subsequent sections, the indirect effect may at times dominate the direct effect. This means the expected producer price, \( \bar{v} \), would fall as the minimum-maximum price band is squeezed. Consequently, the implications of stabilizing prices within a pre-specified band for production, expected producer price, and other variables of interest become an empirical issue when market feedback is incorporated.

**Estimation Framework**

In this section, an empirical framework is developed which maintains key elements of the stylized model of the previous section including rational expectations, exogenous price limits, stochastic demand, and risk-averse producers. Specifically, the bounded prices model under rational expectations considered by Shonkwiler and Maddala, by Holt and Johnson, and by others is extended to include higher-order moments of the (truncated) producer price distribution (Holt 1989).

Consider the following market model with an exogenously set lower price limit, \( \bar{P}_t \):

\[
\begin{align*}
D_t &= \alpha_0 X_{t1} + \alpha_1 P_t + \epsilon_{t1}; \\
S_t &= \beta_0 X_{t2} + \beta_1 P_t^r + \beta_2 \sigma_{t2}^2 + \beta_3 \sigma_{t3}^3 + \epsilon_{t2}; \\
Q_t &= D_t = S_t, \quad \text{if } \bar{P}_t \leq P_t; \\
Q_t &= D_t < S_t, \quad \text{if } \bar{P}_t > P_t,
\end{align*}
\]

where \( D_t \) is quantity demanded, \( S_t \) is quantity supplied, \( Q_t \) is quantity transacted, \( P_t \) is market clearing price, and \( P_t^r \) is the rational expectation of price formed when production decisions are made. Likewise, \( \sigma_{t2}^2 \) is the rational expectation of price variance and \( \sigma_{t3}^3 \) denotes the rational expectation of the third central moment of price. Terms \( X_{t1} \) and \( X_{t2} \) denote vectors of supply and demand shifters, respectively, and \( \epsilon_{t1} \) and \( \epsilon_{t2} \) are joint normally distributed random variables with mean zero and variance-covariance matrix \( \Sigma \), where \( \text{vech}(\Sigma) = (\sigma_1^2, 2, \sigma_3^3) \) denotes unique elements in \( \Sigma \). With observations on \( P_t \) and \( \bar{P}_t \), the data points belonging to equilibrium (\( \Psi_1 \)) and those belonging to excess supply (\( \Psi_2 \)) can be classified.

The model in (8)–(11) represents a market for a commodity where price supports truncate the equilibrium price distribution and where agents form rational expectations. The aggregate supply equation differs in a novel way from many previous specifications in that both second and third central moments of price are included. The third moment is incorporated because price support programs not only have reduced the range over which prices can vary, but they also have modified the “shape” of the price distribution. The distribution’s “shape” can be important if producers exhibit downside risk aversion (Menezes, Geiss, and Tressler). One way of determining empirically the degree of downside risk aversion is to include a measure of skewness in the supply equation, such as the third central moment (Antle 1987).

Expressions for rational expectations of relevant moments of corresponding market and producer price distributions are derived in the following manner. The restricted reduced-form price equation derived from (8)–(10) is:
(12) \[ P_t = (\alpha^*)^{-1}\left(\beta_1'X_{1t} + \beta_2'^*P_t + \beta_3'^*P_t^2 + \beta_4'^*X_{2t} + \alpha'X_{1t} + \epsilon_{t1} - \epsilon_{t2}\right). \]

Taking the expectation of (12) conditional on \( \Omega_{t-1} \), the information set available at the time production decisions are made, gives the rational expectation of market price in the absence of government intervention:

(13) \[ P_t^* = (\alpha^*)^{-1}\left(\beta_1'X_{1t} + \beta_2'^*P_t + \beta_3'^*P_t^2 + \beta_4'^*X_{2t} - \alpha'X_{1t}\right), \]

where \( X_{1t} \) and \( X_{2t} \) denote, respectively, the expectations of (unknown) demand and supply shifters.

Truncation effects of the price support program are incorporated by accounting for the probability that the support price will be effective. Given joint normality of error terms \( \epsilon_{t1} \) and \( \epsilon_{t2} \) and linearity of the structural equations, it follows that the underlying market price (e.g., untruncated) distribution is also normal. Expressions relating the rational expectation of market price in (13) to expectations of the first three central moments of the effective producer price distribution are:

(14) \[ P_t = \tilde{P}_t \Phi(K_t) + \sigma_\theta \phi(K_t) + P_t^*[1 - \Phi(K_t)], \]

(15) \[ \sigma_\theta^2 = \tilde{P}_t \Phi(K_t) + \sigma_\theta^2 K_t \Phi(K_t) + 2P_t^* \sigma_\theta \phi(K_t) + [P_t^* + \sigma_\theta^2][1 - \Phi(K_t)] - P_t^2, \]

(16) \[ \sigma_\mu^2 = \tilde{P}_t^2 (\tilde{P}_t - 3P_t^*) \Phi(K_t) + \sigma_\mu^2 K_t^2 \phi(K_t) \\
+ 3\sigma_\mu^2 (P_t^* - P_t^2) K_t \phi(K_t) \\
+ [2\sigma_\mu^2 + 3P_t^* \sigma_\theta - 6P_t^* \sigma_\theta] \phi(K_t) \\
+ [P_t^* - 3P_t^* + 3\sigma_\mu^2 (P_t^* - P_t^2)][1 - \Phi(K_t)] + 2P_t^2, \]

where

(17) \[ K_t = [\tilde{P}_t - (\alpha^*)^{-1}(\beta_1'X_{1t} + \beta_2'^*P_t + \beta_3'^*P_t^2 + \beta_4'^*X_{2t} - \alpha'X_{1t})]/\sigma_\theta \]

and

(18) \[ \sigma_\theta^2 = (\alpha^*)^{-2}(\beta_1' \psi_1 \beta_1 + \alpha_1' \psi_2 \alpha_1 + \alpha_1' \psi_3 \beta_1 + \sigma_1^2 + \sigma_2^2 - 2\sigma_{12}). \]

Here, \( \Phi(\cdot) \) and \( \phi(\cdot) \) denote, respectively, the distribution and density functions of the standard normal, \( \psi_1 \) is the variance-covariance matrix associated with (unknown) supply shifters, \( \psi_2 \) is similarly defined for demand shifters, and \( \psi_3 \) is the variance-covariance matrix between \( X_{1t} \) and \( X_{2t} \). Likewise, \( 1 - \Phi(K_t) = \text{prob}[P_t \geq \tilde{P}_t] \), the probability the support price is not effective. Expressions for moments of the effective producer price distribution in equations (14)–(16) are nonlinear functions of the support price, the probability of market equilibrium, and the mean and variance of the underlying market price distribution.

The simultaneous solution of equations (13)–(16) yields rational expectations of the first three central moments of the effective producer price distribution. Clearly, an analytical solution cannot be obtained; but given some initial estimates of the structural parameters, it is possible to solve the system numerically. Following Fair and Taylor (1983, 1990), a Gauss–Seidel algorithm is embedded in an iterative maximum likelihood estimation routine to obtain full information maximum likelihood (FIML) estimates of a nonlinear rational expectations model. See Holt and Johnson, or Holt (1992), for further details.

**Empirical Model and Estimation Results**

The above producers are used to estimate a bounded prices model of the U.S. corn market that includes risk terms and rational expectations. The model consists of four structural equations: (a) aggregate corn demand (including stock demand), (b) aggregate corn production, (c) total acres planted to corn, and (d) average corn yield per planted acre. Four
autoregressive models also are specified in the model for predicting values of (unknown) exogenous variables used in the iterative rational expectations simulations.

**Model Specification**

The demand equation is specified as

\[ Q_D = \alpha_0 + \alpha_1 P_C + \alpha_2 E X_R + \alpha_3 I N C + \alpha_4 G C U + \epsilon_{1t}, \]

where \( P_C \) is average price of corn received by farmers, \( E X_R \) denotes the exchange rate weighted by corn exports to foreign markets, \( I N C \) is total disposable income, and \( G C U \) is a grain-consuming animal units index. The exchange rate is included to account for growth in foreign demand for corn during the sample period. Income is included to reflect shifts in derived demand for corn due to increased consumer demand for corn-based products. Likewise, higher livestock inventories as measured by grain-consuming animal units should enhance demand. All prices and income are deflated by the Consumer Price Index (CPI) (1967 = 100). For observations belonging to \( \Psi_1 \), the market is in equilibrium, \( Q_D = Q_S + S T K_{t-1} \), where \( S T K_{t-1} \) represents carryover stocks, and \( P_C \) is freely determined. For observations in \( \Psi_2 \), the market is in disequilibrium, \( Q_D = Q_S + S T K_{t-1} - C C C_t \), where \( C C C_t \) denotes government removals, and \( P_C \) is set equal to the loan rate.

A linear equation is specified for total production, which depends on aggregate corn acres planted and average yields. That is,

\[ Q_S = \beta_0 + \beta_1 A C R_t + \beta_2 Y LD_t + \epsilon_{2t}, \]

where \( A C R \) denotes total acres planted to corn and \( Y LD \) is average corn yield in bushels per planted acre.

The corn planted acreage equation is specified as

\[ A C R_t = \gamma_0 + \gamma_1 P_t + \gamma_2 \sigma^2_t + \gamma_3 \sigma^3_t + \gamma_4 D A_t + \gamma_5 C P_t + \gamma_6 W L T_{t-1} + \gamma_7 A C R_{t-1} + \epsilon_3. \]

Here, \( P_t \), \( \sigma^2_t \), and \( \sigma^3_t \) denote, respectively, the rational expectations of the first, second, and third central moments of the (real) effective producer price distribution as defined in (14)–(16). Also, \( D A \) denotes corn acres idled under government programs and \( C P_t \) is per acre cost of production for corn. Initial real wealth of corn producers, denoted by \( W L T_{t-1} \) and constructed in a manner similar to that described in Chavas and Holt, is included as well. Acres idled under government programs are included to account for the reduction in supply associated with set-aside programs (Holt 1992). Costs of production are used as a measure of input prices. Initial wealth is included to facilitate a wider range of risk response, i.e., to allow for risk response that is not necessarily constrained to constant absolute risk aversion (Pope and Just). Last, lagged acres are included to allow for partial adjustments in attaining desired acres planted to corn. Wealth and cost variables are deflated by the one-period lagged CPI.

The yield equation is specified as

\[ Y L D_t = \delta_0 + \delta_1 D A_t + \delta_2 W I_t + \delta_3 t + \epsilon_4 t. \]

Positive square roots of diverted acres are included to account for yield slippage induced by acreage set-asides (Love and Foster). The variable \( W I_t \) is an index of weather conditions, computed as the first principal component of temperature (in degrees Fahrenheit) and precipitation (in inches) during the growing season (June–August) in the Corn Belt. A linear trend \( t \) is included to reflect technological advances in corn yields over time.

To complete the rational expectations model, second-order autoregressive (AR) models were specified for the exchange rate and the first difference of disposable income. A first-order AR model with first- and second-order lags on corn price was specified for grain-consuming animal units. Finally, because producers don't know the real support price at the time planting decisions are made, a second-order AR model was specified for the real support price.
Table 1. Coefficient Estimates

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Log Likelihood Value = -288.831

Notes: The $\nu$, coefficients denote the parameters of the AR(2) model for the exchange rate, $\lambda$, coefficients denote the parameters of an AR(2) model for the first difference of real income, $\theta$, coefficients denote the parameters of an AR(1) model for grain-consuming animal units with first- and second-order lags on corn price, and $\omega$ denotes the parameters of an AR(2) model for the support price for corn.

Estimation Results

The above framework is used to estimate a rational expectations model with risk and bounded prices for the U.S. corn market from 1950-90. All data on prices—including average farm prices, target prices, and loan rates; total disappearance; grain-consuming animal units; and all production data (i.e., total production, acres planted, acres set aside and diverted, and average yields)—were obtained from various U.S. Department of Agriculture (USDA) issues of Feed Situation and Outlook Report. Exchange rates were obtained from various issues of the USDA's Agricultural Outlook. Weather variables were obtained from Teigen and Singer. Disposable income and CPI data were obtained from various issues of the U.S. Department of Commerce's Survey of Current Business.

FIML estimation results are reported in table 1. Preliminary estimation results indicated the presence of first-order autocorrelation in the residuals of the demand and production equations. These two equations were subsequently estimated in quasi-difference form with corresponding autocorrelation parameters $\rho_1$ and $\rho_2$, respectively. All estimated supply and demand parameters have theoretically correct signs and many of the coefficients associated with economic variables are significant at usual levels. Importantly, estimated coefficients for expected price $P_t^e$ and for the expected third central moment of price $\sigma_3^t$ are both positive, and the coefficient for $\sigma_3^t$ is significant at the .05 level. Likewise, the estimated coefficient for expected price variance $\sigma_2^t$ is negative and significant at the .05 level. The estimated coefficient for initial wealth $WL_{t-1}$ is positive and significant, indicating that constant absolute risk aversion (CARA) does not hold. A positive wealth effect also is consistent with corn producers exhibiting decreasing absolute risk aversion (Sandmo).

Because a riskless model (similar to those considered by Shonkwiler and Maddala and by Holt and Johnson) is nested within the present model, a likelihood ratio test of the hypothesis that $\gamma_2 = \gamma_3 = 0$ is appropriate (e.g., a test of the hypothesis that risk effects
in the acreage supply equation are zero). The resulting test statistic is 13.09, which well exceeds the critical value of 5.991 in the asymptotic \( \chi^2(2) \) distribution at the .05 level. Based on these results, it is my conclusion that risk effects associated with second and third moments of the truncated price distribution have a significant impact on corn acreage planting decisions.

Wald tests for constant relative risk aversion (CRRA) and constant partial relative risk aversion (CPRRA), evaluated at the means of the sample data, were conducted in a manner similar to that described by Pope and Just. Test statistics obtained are 9.598 and 16.316, respectively. Both critical values are extreme in the asymptotic \( \chi^2(1) \) distribution, indicating that neither CRRA nor CPRRA are supported by the data. These results are consistent with those obtained by Chavas and Holt for corn and soybean acreage decisions.

The estimation results reveal that the effective producer price distribution is right-skewed at all data points. Positive skewness indicates the mean of the producer price distribution is above the mode, implying that prices are more likely to be below the mean. Consequently, even though the underlying market price distribution is normal, price support programs have resulted in producer price distributions which are no longer symmetric.

Given the above results, the signs on the coefficients associated with higher-order moments in the supply equation are plausible. For instance, the negative sign associated with \( \sigma_p^2 \) is justified under the assumption of risk aversion because risk-averse agents prefer decreasing variance (Meyer). The positive sign for \( \sigma_q^3 \) also is reasonable because agents exhibiting decreasing or constant absolute risk aversion prefer positive skewness (Tsiang). Hence, the empirical evidence is consistent with corn producers exhibiting downside risk aversion, a result which has not been established previously in the literature.

The own-price elasticity of demand is \(-.554\) at the means of the data—an estimate that compares favorably with prior estimates (e.g., Shonkwiler and Maddala; Holt and Johnson). Implied acreage elasticities with respect to the mean, variance, and third central moment of price are \(.052\), \(-.018\), and \(.015\), respectively. The acreage elasticity with respect to initial wealth is \(.074\), a value which is consistent with that obtained by Chavas and Holt. The mean price acreage elasticity is somewhat below previous estimates for corn (e.g., Lee and Helmerger; Chavas and Holt; Holt 1992). This discrepancy may be due to the fact that many prior studies either have not included risk terms or have not included a direct measure of acreage set-asides.\(^\text{12}\)

The estimated model was simulated historically, and for each endogenous variable the actual value was regressed on the predicted value and the computed \( R^2 \) obtained. The results are reported in table 2. In all cases, the \( R^2 \)'s are high. In fact, the \( R^2 \) values for all endogenous variables except \( PC_t \) and \( GCU_t \) are above .90, thus indicating the model does a good job of predicting quantities supplied and demanded. The \( R^2 \) for \( PC_t \) is somewhat lower, about .86, but still suggests the model does a good job of explaining historical price adjustments in the corn market.

**Price-Band Stabilization Experiments**

The estimated rational expectations model provides a rich framework within which to examine alternative price support and stabilization strategies in a market equilibrium.

<table>
<thead>
<tr>
<th>Endogenous Variables</th>
<th>( QD_t )</th>
<th>( QS_t )</th>
<th>( PC_t )</th>
<th>( ACR_t )</th>
<th>( YLD_t )</th>
<th>( EXR_t )</th>
<th>( INC_t )</th>
<th>( GCU_t )</th>
<th>( P_t )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( .9287 )</td>
<td>(.9061)</td>
<td>(.8690)</td>
<td>(.9396)</td>
<td>(.9135)</td>
<td>(.9455)</td>
<td>(.9964)</td>
<td>(.7473)</td>
<td>(.9482)</td>
<td></td>
</tr>
</tbody>
</table>

Notes: All \( R^2 \) estimates were obtained from auxiliary regressions where the observed variable is regressed on the corresponding simulated value. The \( R^2 \) for \( PC \) was computed by using the unconditional expectation of price.
Table 3. Simulations of Corn Supply/Demand Model with Alternative Minimum and Maximum Price Levels

<table>
<thead>
<tr>
<th>Prices</th>
<th>Min.</th>
<th>.30</th>
<th>1.70</th>
<th>.756</th>
<th>.756</th>
<th>2.436</th>
<th>.056</th>
<th>4.755</th>
<th>.000</th>
<th>.000</th>
<th>3.59</th>
<th>3.59</th>
<th>.000</th>
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<tbody>
<tr>
<td></td>
<td>Max.</td>
<td>.35</td>
<td>1.65</td>
<td>.750</td>
<td>.751</td>
<td>2.421</td>
<td>.138</td>
<td>4.770</td>
<td>.000</td>
<td>.001</td>
<td>3.55</td>
<td>3.55</td>
<td>.001</td>
</tr>
<tr>
<td></td>
<td>Market</td>
<td>.40</td>
<td>1.60</td>
<td>.738</td>
<td>.739</td>
<td>2.378</td>
<td>.331</td>
<td>4.804</td>
<td>.001</td>
<td>.004</td>
<td>3.54</td>
<td>3.54</td>
<td>.004</td>
</tr>
<tr>
<td></td>
<td>Producer</td>
<td>.45</td>
<td>1.55</td>
<td>.707</td>
<td>.710</td>
<td>2.237</td>
<td>.785</td>
<td>4.891</td>
<td>.003</td>
<td>.016</td>
<td>3.47</td>
<td>3.46</td>
<td>.005</td>
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<tr>
<td></td>
<td></td>
<td>.50</td>
<td>1.50</td>
<td>.650</td>
<td>.664</td>
<td>1.799</td>
<td>1.479</td>
<td>5.051</td>
<td>.014</td>
<td>.071</td>
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<td>1.45</td>
<td>.621</td>
<td>.654</td>
<td>1.305</td>
<td>1.582</td>
<td>5.131</td>
<td>.033</td>
<td>.170</td>
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<td>3.19</td>
<td>.051</td>
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<td></td>
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<td>1.40</td>
<td>.616</td>
<td>.671</td>
<td>.935</td>
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<td>.055</td>
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<td>1.35</td>
<td>.619</td>
<td>.698</td>
<td>.650</td>
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<td></td>
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<td>1.30</td>
<td>.624</td>
<td>.732</td>
<td>.428</td>
<td>.712</td>
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<td>.107</td>
<td>.550</td>
<td>3.75</td>
<td>3.20</td>
<td>.147</td>
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<td></td>
<td></td>
<td>.75</td>
<td>1.25</td>
<td>.629</td>
<td>.770</td>
<td>.261</td>
<td>.440</td>
<td>5.108</td>
<td>.140</td>
<td>.717</td>
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<td>3.21</td>
<td>.182</td>
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<td></td>
<td></td>
<td>.80</td>
<td>1.20</td>
<td>.632</td>
<td>.811</td>
<td>.143</td>
<td>.238</td>
<td>5.099</td>
<td>.179</td>
<td>.912</td>
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<td>1.15</td>
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<td>.856</td>
<td>.069</td>
<td>.107</td>
<td>5.097</td>
<td>.223</td>
<td>1.136</td>
<td>4.36</td>
<td>3.23</td>
<td>.260</td>
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<td></td>
<td></td>
<td>.90</td>
<td>1.10</td>
<td>.632</td>
<td>.903</td>
<td>.028</td>
<td>.035</td>
<td>5.101</td>
<td>.271</td>
<td>1.382</td>
<td>4.60</td>
<td>3.22</td>
<td>.300</td>
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<tr>
<td></td>
<td></td>
<td>.95</td>
<td>1.05</td>
<td>.629</td>
<td>.951</td>
<td>.007</td>
<td>.004</td>
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<td>1.646</td>
<td>4.86</td>
<td>3.21</td>
<td>.339</td>
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</tbody>
</table>

Notes: All results were obtained by simulating the rational expectations model at the means of the sample data. All prices are in real terms, and are expressed in dollars per bushel. Production is in billion bushels. Gross farm revenues are expressed in (real) billions of dollars and are defined as production times price.

a All second moment values are multiplied by 100. Also, the variance of the market price distribution is 2.4443.

b All third-moment values are multiplied by 1,000.

c Percent denotes the proportion of gross farm revenue (including subsidies) that is attributed to government support.

Importantly, expressions for moments of the effective producer price distribution in (14)–(16) can be readily modified to allow for maximum price limits (e.g., truncation of the upper tail). See the appendix for details. Operationally, a price stabilization program that uses a system of minimum and maximum prices could be administered in a fashion similar to that described in previous sections. That is, producers receive a subsidy if the market price falls below the minimum price and pay a tax if the realized price is above the maximum price.

The adjustments in production, expected market and producer prices, and higher-order moments of the producer price distribution are examined by simulating the model at the data means over a wide range of minimum and maximum prices. Selected results are reported in table 3. Note that low levels of minimum price accompanied by high levels of maximum price have little impact on expected prices, risk variables, or production. This implies that the market price distribution and the producer price distribution virtually coincide for relatively wide minimum-maximum price bands.

As the range over which prices can adjust freely is reduced, the second moment of the producer price distribution declines and the third central moment increases initially, thus providing a direct incentive to expand corn acreage. However, higher production levels translate into lower expected market prices and, in turn, lower expected producer prices (fig. 1). In other words, the market price effect initially dominates the truncation effect, and the expected producer price falls. But for minimum price levels exceeding $.55 and maximum price levels below $1.45, the truncation effect outweighs the market price effect, and the expected producer price increases (fig. 1).

Expected production increases initially as the minimum and maximum price limits are squeezed (fig. 2). This positive production response occurs even though expected producer prices decline at first as the price bands are narrowed (fig. 1). The implication is that the risk reduction induced by the system of minimum and maximum prices more than offsets the mean price response over a wide range of price bands. This result highlights the importance of risk response in analyzing partial price stabilization policies because production clearly would decline in the absence of risk effects.

The plots in figures 3 and 4 illustrate the effects of alternative price bands on, respec-
Figure 1. Relationship between expected producer price and minimum and maximum prices

Figure 2. Relationship between expected planted acreage and minimum and maximum prices
tively, the second and third central moments of producer price. The variance of the truncated price distribution clearly has an inverse relationship with the minimum price (fig. 3). Overall, there appears to be only limited response to the maximum price. As Eeckhoudt and Hansen note, there is a spread effect and a location effect that work against each other when the maximum price is varied. Consequently, in a rational expectations model, the response to the maximum price in general will be less pronounced than the corresponding response to the minimum price.

Referring to figure 4, note that the third central moment increases initially with the minimum price and begins to decline at the $.50 minimum price level. An intuitive explanation for the non-monotonic relationship between the third central moment and the minimum price is not apparent. This is because not only is the third moment a highly nonlinear function of the price limits, but it is also a complicated function of the first and second moments of the truncated price distribution as well. This lack of intuition notwithstanding, the above results serve to underscore the observation that price stabilization, as implemented through a system of minimum and maximum price bands, can affect both the shape and the position of the producer price distribution.

Table 3 also provides an illustration of the level of government involvement for selected minimum and maximum price bands. The expected (real) government subsidy, or the difference between the expected producer price and the expected market price, initially is negligible, but increases to $.22 per bushel when producers are allowed to face only a $.30 price band. Of interest is that expected gross farm revenues decline initially as the minimum-maximum price band is reduced from the $.30-$1.70 level; however, beyond $.50-$1.50 band widths, expected gross farm revenues increase. The proportion of expected gross farm revenues derived from government sources increases from zero initially to over 20% as the minimum-maximum band widths are reduced below $.40 (table 3).

Because interactions between minimum and maximum prices and the model's endog-
enous variables are highly complex, it is useful to investigate their marginal effects on the rational expectations equilibrium. To this end, short-run elasticities and flexibilities with respect to the minimum price are reported in table 4 for a wide range of price bands. Planted acreage elasticities are initially small and positive, but increase rapidly in mag-

Table 4. Elasticities for Selected Variables with Respect to the Minimum Price

<table>
<thead>
<tr>
<th>Prices</th>
<th>Higher Moments</th>
<th>( \Phi(K_1) )</th>
<th>( 1 - \Phi(K_2) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Min.</td>
<td>Max. Market</td>
<td>Producer</td>
<td>Second</td>
</tr>
<tr>
<td>.30</td>
<td>1.70</td>
<td>-.026</td>
<td>-.026</td>
</tr>
<tr>
<td>.35</td>
<td>1.65</td>
<td>-.073</td>
<td>-.067</td>
</tr>
<tr>
<td>.40</td>
<td>1.60</td>
<td>-.203</td>
<td>-.196</td>
</tr>
<tr>
<td>.45</td>
<td>1.55</td>
<td>-.601</td>
<td>-.542</td>
</tr>
<tr>
<td>.50</td>
<td>1.50</td>
<td>-.754</td>
<td>-.490</td>
</tr>
<tr>
<td>.55</td>
<td>1.45</td>
<td>-.225</td>
<td>.122</td>
</tr>
<tr>
<td>.60</td>
<td>1.40</td>
<td>.000</td>
<td>.410</td>
</tr>
<tr>
<td>.65</td>
<td>1.35</td>
<td>.097</td>
<td>.580</td>
</tr>
<tr>
<td>.70</td>
<td>1.30</td>
<td>.128</td>
<td>.690</td>
</tr>
<tr>
<td>.75</td>
<td>1.25</td>
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<td>.911</td>
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<td>.90</td>
<td>1.10</td>
<td>-.079</td>
<td>.953</td>
</tr>
<tr>
<td>.95</td>
<td>1.05</td>
<td>-.111</td>
<td>.978</td>
</tr>
</tbody>
</table>

Notes: All results were obtained by simulating the rational expectations model at the means of the sample data. Variable \( \Phi(K_1) \) denotes the probability that the expected market price will be below the minimum price, and \( 1 - \Phi(K_2) \) is the probability that the expected market price will be above the maximum price.
nitude as the band width is narrowed to the $.50–$1.50 level. Over the same range of price bands, expected producer price flexibilities are negative and fall in magnitude. This is because truncation effects associated with the minimum and maximum price limits cause reductions in risk (as captured by the second and third central moments), with the result that planted acres expand. A positive production response subsequently places downward pressure on expected market and producer prices.

Table 4 also illustrates that for band widths above the $.50–$1.50 level, acreage elasticities with respect to the minimum price decline in magnitude and actually become negative at some point. Over the same range, expected producer price flexibilities are everywhere positive and increasing in magnitude. This pattern of response is interesting because, as noted in table 4, variance elasticities are everywhere negative and decline monotonically as band widths are narrowed. Consequently, “shape” effects, as measured by the third moment, play an important role in determining both the direction and magnitude of production responses. For narrow band widths (above the $.85–$1.15 level), acreage elasticities are again positive, indicating that the expected producer price response dominates the risk response. Overall, these results help underscore the importance of risk effects in determining the outcome of a partial price stabilization scheme.

Conclusions

Previous research has not adequately addressed the relationship between government price support programs and subjective price and risk expectations of producers in agricultural markets. To explore this issue, conceptual and empirical models were developed that included minimum and maximum price limits, risk-averse producers, and rational expectations. Following Eeckhoudt and Hansen, a system of minimum and maximum prices was assumed to truncate producers' subjective density function of price. The empirical analysis was based on a bounded price variation model that included rational expectations of the first three central moments of the effective producer price distribution.

The empirical framework was used to obtain FIML estimates of a model of the U.S. corn market. Among other things, the results show that wealth effects are important in aggregate supply decisions and that producers behave in a manner consistent with downside risk aversion. This is among the first known studies to incorporate a measure of risk beyond the second moment into an aggregate agricultural supply model. Overall, the results indicate that the third-moment term is both statistically and economically significant. Future research into the effects of risk on aggregate supply decisions—especially when government programs are present—thus should attempt to incorporate third-moment measures of risk.

The estimated model was used to investigate equilibrium (reduced-form) impacts of alternative minimum and maximum price levels on the U.S. corn market. It was found, for instance, that because of market price feedback, expected producer prices actually decline over a wide range of minimum and maximum price bands; however, production levels generally increased, even when the expected producer price declined. This result is attributed to the risk reduction arising from the truncation of the producer price distribution, and highlights the importance of analyzing government price support and stabilization programs in a market equilibrium—rational expectations context.

[Received December 1992; final revision received June 1994.]

Notes

1 Indeed, a frequent justification for government intervention in agricultural markets is that risk is reduced and overall market efficiency subsequently is improved (Newbery and Stiglitz).

2 Innes (1990b) does explore the implications of a target price—deficiency payment program along with production controls in a market with multiplicative production uncertainty and risk-averse producers; however,
his analysis is conducted at a very aggregate level (e.g., the market for "food") and, as such, may be of limited use for analyzing the effects of government intervention in specific markets. More importantly, his model allows for only two states of nature (e.g., good weather versus bad weather) and key parameters were not estimated econometrically.

This hypothetical system of minimum and maximum price bands is similar to the target price–deficiency payment scheme used in the U.S. and elsewhere. The policy considered here is more general, in that an upper price bound also is included.

The notation used for firm-level decisions implies the price distribution \( g(p \mid Q) \) can be characterized adequately by a finite number of its central moments. While this assumption may not always be theoretically valid, Kendall and Stuart show that a probability distribution can be approximated to the \( n \)th degree by an \( n \)th degree polynomial whose coefficients are functions of the first \( n \) moments of the distribution. This moment-based approach also is consistent with the procedure used in many empirical studies where expected utility is approximated by a Taylor series (Antle 1983).

Quiggin and Anderson obtain essentially the same results by using a stochastic dominance approach.

This can occur, for instance, if the minimum and maximum prices are positioned in the tails of the distribution \( f(p \mid Q) \). In this case, small changes in \( p_m \) and \( p_u \) would have little direct effect because the probabilities \( F(p_m \mid Q) \) and \( 1 - F(p_u \mid Q) \) would not change much.

The estimation framework focuses on a situation where only minimum prices apply. This is because government programs in the U.S. historically have relied only on price supports (e.g., minimum prices). As illustrated in the appendix, the model is easily modified to accommodate a situation where maximum prices also apply.

When implementing the model, the parameters in \( \psi_t, \psi_y, \) and \( \Sigma \) used in deriving (18) are estimated simultaneously with the parameters of the structural model (table 1). Operationally, this requires an unconcentrated likelihood function be used to obtain FIML estimates of the model’s parameters. The result is that the (fixed) variance of the market price distribution (18) is estimated, in accordance with the rational expectations hypothesis, by using the error process associated with the “true model.”

A linear approximation to the (obviously) nonlinear production identity in planted acres and average yields is used here because the effects of truncation on the moments of the effective producer price distribution derived in previous sections are valid only when the model’s endogenous variables are linear transformations of underlying random error terms.

When solving the rational expectations model, it is necessary to account for the fact that producers do not know with certainty what idled acres will be. Participation in U.S. commodity programs is voluntary, and hence the amount of land retired is, in essence, endogenous. However, complications arise when attempting to formally endogenize acres diverted under government programs because acreage set-asides have not always been a precondition for receiving price support protection. Rather than tackle the additional problems associated with endogenizing land retirement programs for corn, I have instead linked acres diverted to price support levels and effective diversion payments by using a quadratic specification. The resulting OLS estimates were then used to predict acres idled when solving the rational expectations model.

In the Fair and Taylor (1983) iterations, the expected value for \( W_I \), in the yield equation was set equal to zero (the average value for \( W_I \)). The assumption is that producers expect “normal” temperature and precipitation conditions to prevail when forming price expectations.

For example, Lee and Helmberger report corn acreage own-price elasticities ranging from .118 for free market years to .249 for farm program years. Similarly, Chavas and Holt report a corn acreage own-price elasticity of .158 and a corn acreage price risk (e.g., variance) elasticity of .02. Holt (1992) reports an own-price elasticity of corn supply of .223.

The range used is $1.81 to $1.02 per bushel for the (real) maximum price and $.21 to $1 per bushel for the minimum price. The prices were incremented in $.01 intervals, resulting in 6,400 model simulations.

References


### Appendix

Using an approach similar to that described in the text, it can be shown that the expressions relating the expectations of the first three central moments of the truncated normal price distribution to both minimum and maximum price limits are:

(A1) \[ P_t = \bar{P}_t \Phi(K_{\bar{P}}) + \sigma_2 \Phi(K_{\bar{P}} - \phi(K_{\bar{P}})) + P_t \Phi(K_t) - \Phi(K_t)] + \bar{P}_t[1 - \Phi(K_{\bar{P}})], \]

(A2) \[ \sigma_2^2 = \bar{P}_t \Phi(K_{\bar{P}}) + \sigma_2^2 \Phi(K_{\bar{P}} - \phi(K_{\bar{P}})) + 2 \sigma_2^2 \Phi(K_{\bar{P}} - \phi(K_{\bar{P}})] + \bar{P}_t[1 - \Phi(K_{\bar{P}}) - P_t^2], \]

(A3) \[ \sigma_2^3 = \bar{P}_t[\bar{P}_t - 3P_t \Phi(K_{\bar{P}}) + \sigma_2^2 K_{\bar{P}} \Phi(K_{\bar{P}})] + 3 \sigma_2^2 [P_t - 3P_t P_t] \Phi(K_{\bar{P}} - K_t \phi(K_{\bar{P}})) + [P_t^3 - 3P_t^2 P_t + 3 \sigma_2^2 (P_t - P_t)^2] \Phi[K_t - \Phi(K_{\bar{P}})] + \bar{P}_t[\bar{P}_t - 3P_t[1 - \Phi(K_{\bar{P}})] + 2P_t^3, \]

where

(A4) \[ K_{\bar{P}} = [\bar{P}_t - (\sigma_t)^2 (\Phi(K_{\bar{P}}) + \sigma_t P_t + \sigma_t^2 \sigma_2^2 + \sigma_t^2 \sigma_2^2)]/\sigma_t, \]

(A5) \[ K_{\bar{P}} = [\bar{P}_t - (\sigma_t)^2 (\Phi(K_{\bar{P}}) + \sigma_t P_t + \sigma_t^2 \sigma_2^2 + \sigma_t^2 \sigma_2^2)]/\sigma_t, \]

and \( P_t \) is the price of the commodity, \( \bar{P}_t \) is the price barrier, \( \sigma_t \) is the standard deviation, \( \phi(K_{\bar{P}}) \) is the price barrier, and \( K_t \) is the price of the commodity.
(A6) \( 1 - \Phi(K_1) = \text{Prob}[(a^* - 1)^{-1}(u_{1t} - u_{1}) > \bar{P}_{1t} - (a^* - 1)(\beta'e_1X_{1t} + \beta'_fP_t + \beta'_s\sigma_{a_t} + \beta'_g\sigma_{a} - \alpha'X_{11})]\),

and

(A7) \( 1 - \Phi(K_2) = \text{Prob}[(a^* - 1)^{-1}(u_{2t} - u_{1}) > \bar{P}_{2t} - (a^* - 1)(\beta'e_1X_{1t} + \beta'_fP_t + \beta'_s\sigma_{a_t} + \beta'_g\sigma_{a} - \alpha'X_{11})]\).

Here, \( \bar{P}_u \) denotes the minimum guaranteed price and \( \bar{P}_u \) denotes the maximum guaranteed price. All other variables are as defined in the text. For purposes of simulating the model with both minimum and maximum prices, the expressions in (A1)–(A7) are substituted for those in (14)–(18).