Prior to World War II, labor's share in the U.S. manufacturing and agricultural sectors was relatively constant. Keynes [9] called this "a bit of a miracle." Several studies [4, 10] have shown that labor's share in the U.S. manufacturing sector has increased in the post-war period. The opposite appears to have been the case for U.S. agriculture. Two studies [11, 14] indicate that labor's relative share in the U.S. agricultural sector has declined in the post-war period.

There has been a substantial substitution of inputs, capital (K) and labor (L), in both the manufacturing and agricultural sectors in the post-war period. The secular increase in the wage-rental ratio has encouraged substitution of capital for labor. However, while this argument alone might explain the observed decline in labor's share in the agricultural sector, it does not explain what has occurred in the manufacturing sector. Moreover, this argument excludes another important characteristic of both sectors in the post-war period: technological change.

Adoption of labor-saving technology has been quite rapid in the U.S. agricultural sector during the last several decades [5]. The most rapid substitution rate of machinery for labor in the agricultural sector has occurred in the South. Kaneda [8] notes that the high rate of technical change in the Southeast and Delta regions since World War II is a reflection of cotton production mechanization.

This article has two objectives: (a) to indicate why both elasticity of factor substitution and bias of technical change must be known in order to determine labor's relative share of output value, and (b) to illustrate empirically the importance of these two parameters for the case of U.S. cotton production mechanization.

**ELASTICITY OF FACTOR SUBSTITUTION AND TECHNOLOGICAL CHANGE**

Mathematical analysis in this section is based on two assumptions: (a) the production function is homogeneous of degree one with two homogeneous inputs, capital (K) and labor (L), and (b) perfect competition prevails in both input and output markets. Output for a given industry over time (t) is represented by a production function, \( Y = f(K, L, t) \). Over the relevant range of production, both marginal products are strictly positive \( (f_K > 0, f_L > 0) \) and both decrease monotonically \( (f_{KK} < 0, f_{LL} < 0, f_{KL} > 0) \).

Changes in factor shares are dependent on two important parameters: (a) elasticity of factor substitution, and (b) bias of technology being adopted.

Elasticity of factor substitution refers to ease of substitution of one input for another for a given output level. Elasticity of factor substitution may be defined as proportionate rate of change in the factor ratio divided by the proportionate rate of change in the factor price ratio. Mathematically, elasticity of factor substitution \( (\sigma) \) may be expressed as:

\[
\sigma = \frac{f_L f_K}{Y f_{KL}}
\]

Hicks [6] classified technical change according to its initial effect on the marginal physical product of capital and labor. Technical change which leaves
the marginal physical products of capital and labor unchanged is neutral. If the marginal physical product of labor increases more (less) relative to the marginal physical product of capital, it is a capital-saving (labor-saving) technological change. The bias of the technological change (\( \beta \)) can be defined as:

\[
\begin{align*}
\beta &= 1 \text{ Hicksian neutral} \\
\beta > 1 &\text{ Labor-saving (capital-using)} \\
\beta < 1 &\text{ Capital-saving (labor-using)}
\end{align*}
\]

If each factor of production is paid its marginal physical product such that total output is just exhausted, Euler's Theorem holds, \( Y = f_L L + f_K K \). From Euler's Theorem it follows that absolute shares of capital and labor are \( Kf_K \) and \( Lf_L \), respectively, and relative shares of capital and labor would be \( R_K = Kf_K/Y \) and \( R_L = Lf_L/Y \), respectively.

By differentiating labor's relative share with respect to time, and after some algebraic manipulation, the change in labor's relative share can be expressed as a function of capital and labor's absolute shares, rate of technological change, bias of the technological change, and elasticity of factor substitution. Differentiating labor's relative share with respect to time gives:

\[
\frac{d(R_L/Y)}{dt} = \frac{d(Lf_L/Y)}{dt}
\]

Substituting for \( Y \) and \( dY \) from Euler's Theorem, expanding, and rearranging terms yields:

\[
\frac{d(R_L/Y)}{dt} = \frac{1}{Y^2} \left[ Kf_K (f_L \frac{dL}{dt} + Lf_{LL} \frac{dL}{dt} + Lf_{LK} \frac{dK}{dt} + Lf_{KL} \frac{dL}{dt} + Kf_{KK} \frac{dK}{dt} + Kf_{KL} \frac{dL}{dt}) \right]
\]

Johnson's [7] definition of technical progress over time is:

\[
\beta \lambda = 1/L \cdot dL/dt = \beta \cdot 1/K \cdot dK/dt
\]

where \( \lambda \) equals time derivative of technical change.

Partially differentiating Euler's Theorem with respect to \( L \) yields:

\[
\frac{\partial Y}{\partial L} = Kf_{KL} + Lf_{LL} + f_L
\]

Substituting definitions (1) and (5) and the derivations from (6) into (4), with some rearranging of terms, yields:

\[
\frac{d(R_L/Y)}{dt} = R_L R_K \lambda (\beta - 1) \left( \frac{a - 1}{o} \right).
\]

Equation (7) expresses labor's relative share as a function of five parameters. By definition labor and capital's absolute shares (\( R_L \) and \( R_K \)) are always positive. Also, \( \lambda \), the proportional increase in the effective quantity of capital (\( K \)) per unit of time, is positive. Hence, changes in labor's relative share are determined by two parameters: (a) bias of the technological change (\( \beta \)), and (b) elasticity of factor substitution (\( \sigma \)). Once values of these two parameters are known, changes in labor's relative share can be ascertained.

If either \( \beta \) or \( \sigma \) equals one, any change in quantity of labor used will have no effect on labor's relative share. However, if \( \beta \) is greater than one (labor-saving technological change), substitution of capital for labor will increase labor's relative share only if \( \sigma \) is greater than one. If \( \beta \) is greater than one and \( \sigma \) is less than one, a decrease in use of labor will increase labor's relative share! The converse would be true when \( \beta \) is less than one (capital-saving technological change). Table 1 summarizes the various possible changes in labor's relative share for different values of \( \beta \) and \( \sigma \), assuming a decline in use of labor in a given economic sector or industry.

<table>
<thead>
<tr>
<th>( \sigma )</th>
<th>( \beta )</th>
<th>Change in Labor's Relative Share with a Decrease in Labor Use Over Time</th>
</tr>
</thead>
<tbody>
<tr>
<td>( &gt; 1 )</td>
<td>( &gt; 1 )</td>
<td>No Change</td>
</tr>
<tr>
<td>( &gt; 1 )</td>
<td>( &lt; 1 )</td>
<td>Increase</td>
</tr>
<tr>
<td>( &lt; 1 )</td>
<td>( &gt; 1 )</td>
<td>Increase</td>
</tr>
<tr>
<td>( &lt; 1 )</td>
<td>( &lt; 1 )</td>
<td>Decrease</td>
</tr>
</tbody>
</table>

\(^1\) For more detail on algebra involved in this derivation see Johnson [7] and Martin [13].
The key point is that displacement of labor with a labor-saving technology in a given industry would only decrease labor's relative share in that industry if elasticity of substitution is greater than one. This is precisely what Lianos [11] found to be the case for the U.S. agricultural sector in the aggregate since World War II.

Ferguson and Moroney [4], however, found that despite adoption of labor-saving technology in most industries in the U.S. manufacturing sector, capital deepening accompanied by an elasticity of factor substitution less than one resulted in an increase in labor's relative share. This implies that there has been greater ease of substitution of capital for labor in the U.S. agricultural sector than in the manufacturing sector. Hence, in spite of the introduction of labor-saving technology, labor's relative share increased in the industrial sector. In the agricultural sector, however, labor was more easily displaced by capital-intensive, labor-saving technology and consequently, labor's relative share declined.

U.S. COTTON LABOR'S RELATIVE SHARE: AN EMPIRICAL EXAMPLE

Rather extensive investigations [4, 9, 10] have been made of changes in factor shares for selected industries within the U.S. manufacturing sector. This has not been the case for the crop or livestock components of the U.S. agricultural sector.

Mechanization of cotton production in the U.S. has been quite rapid. Cotton production prior to the post World War II period was one of the most labor-intensive major crops. The majority of labor input for cotton production during the pre-war period was required for the harvesting operation.2 Introduction of mechanical cotton harvesters after World War II reduced labor requirements in harvesting by approximately 95 percent [12].

Rate of adoption of mechanical harvesters was quite rapid. In 1946 only one percent of U.S.-grown cotton was mechanically harvested. By 1970 virtually all (97 percent) cotton produced in the U.S. was picked mechanically.

For most family and hired workers who had been employed in cotton production this meant the end of agricultural employment, and eventually compelled many to go to towns and cities, (mostly in the North) to live and seek employment. The resulting rural-urban migration led to difficult socio-economic adjustment problems for both migrants and affected cities. For cotton farmers, the capital-intensive nature of the new technology drastically altered farm organization and operation.

The question addressed in this section, however, is: What happened to labor's relative share within the cotton sector? Real farm wages in the South increased 50 percent from 1952 to 1969, the period of the most rapid rate of adoption of cotton pickers, while man-hours devoted to cotton production fell over 80 percent. Furthermore, real value of cotton production, including acreage diversion transfer payments, also fell by nearly 60 percent. Moreover, labor's relative share (S_L) in the cotton sector fell from 39 percent in 1952 to 22 percent in 1969, a decline of 44 percent (Table 2).

The mathematical derivation in the previous section suggests that, given a labor-saving technological change in the cotton sector which displaces

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### TABLE 2. WAGE RATES, MAN-HOURS, VALUE OF PRODUCTION AND LABOR'S RELATIVE SHARE FOR U.S. COTTON PRODUCTION, 1952-1969

<table>
<thead>
<tr>
<th>Year</th>
<th>Real Wage Rate/Hours</th>
<th>Man-Hours Cotton-Labor (millions)</th>
<th>Real Value Output Including Acreage Diversion Payments ($ millions)^b</th>
<th>Labor Share^c</th>
</tr>
</thead>
<tbody>
<tr>
<td>1952</td>
<td>0.5710</td>
<td>1655</td>
<td>2446.4</td>
<td>0.3863</td>
</tr>
<tr>
<td>1953</td>
<td>0.5978</td>
<td>1609</td>
<td>2736.8</td>
<td>0.3555</td>
</tr>
<tr>
<td>1954</td>
<td>0.5905</td>
<td>1269</td>
<td>2407.3</td>
<td>0.3133</td>
</tr>
<tr>
<td>1955</td>
<td>0.6043</td>
<td>1233</td>
<td>2655.2</td>
<td>0.2821</td>
</tr>
<tr>
<td>1956</td>
<td>0.6335</td>
<td>1074</td>
<td>2389.8</td>
<td>0.2847</td>
</tr>
<tr>
<td>1957</td>
<td>0.6253</td>
<td>818</td>
<td>1787.8</td>
<td>0.2861</td>
</tr>
<tr>
<td>1958</td>
<td>0.6220</td>
<td>769</td>
<td>2015.1</td>
<td>0.2374</td>
</tr>
<tr>
<td>1959</td>
<td>0.6320</td>
<td>911</td>
<td>2386.2</td>
<td>0.2230</td>
</tr>
<tr>
<td>1960</td>
<td>0.6433</td>
<td>851</td>
<td>2861.7</td>
<td>0.1868</td>
</tr>
<tr>
<td>1961</td>
<td>0.6540</td>
<td>772</td>
<td>2677.6</td>
<td>0.1886</td>
</tr>
<tr>
<td>1962</td>
<td>0.6603</td>
<td>629</td>
<td>2648.6</td>
<td>0.1693</td>
</tr>
<tr>
<td>1963</td>
<td>0.6720</td>
<td>647</td>
<td>2816.0</td>
<td>0.1544</td>
</tr>
<tr>
<td>1964</td>
<td>0.6958</td>
<td>573</td>
<td>2614.0</td>
<td>0.1525</td>
</tr>
<tr>
<td>1965</td>
<td>0.7155</td>
<td>483</td>
<td>2212.3</td>
<td>0.1562</td>
</tr>
<tr>
<td>1966</td>
<td>0.7405</td>
<td>399</td>
<td>1324.6</td>
<td>0.1272</td>
</tr>
<tr>
<td>1967</td>
<td>0.7610</td>
<td>364</td>
<td>1397.3</td>
<td>0.1574</td>
</tr>
<tr>
<td>1968</td>
<td>0.8008</td>
<td>275</td>
<td>1446.7</td>
<td>0.1579</td>
</tr>
<tr>
<td>1969</td>
<td>0.8615</td>
<td>279</td>
<td>1062.3</td>
<td>0.1230</td>
</tr>
</tbody>
</table>

SOURCE: United States Department of Agriculture publications [16, 17, 18].

^aAverage of four major cotton regions: South Atlantic, East South Central, West South Central and Pacific. Deflated by Prices Paid by Farmers Index, 1947-49 = 100.

^bDeflated by Wholesale Price Index, 1947-49 = 100. Price used is a composite of market price and support price based on cotton program participation.

^cColumn one multiplied by column two divided by column three.

2The other labor-intensive activity was "chopping" cotton. This operation has also been largely mechanized.
labor ($\beta > 1$), a decline in cotton labor's relative share would require an elasticity of factor substitution greater than one for the U.S. cotton sector (Table 1). An elasticity of factor substitution greater than one would reflect relative ease of substitution of capital for labor in response to a secular increase in the wage-entail ratio.

Time series data on the stock of capital invested in machinery used in cotton production are not available. Thus, it is not possible to estimate the elasticity of factor substitution ($\sigma$) based on a CES production function where the capital-labor ratio and the wage-ental ratio are used as explanatory variables. However, it is feasible with available data, to use two other alternative approaches to estimate $\sigma$.

The CES production function may be expressed as:

$$ Y = \left[ (\alpha_o \tau K)^{-\rho} + (\beta_o \tau L)^{-\rho} \right]^{-1/\rho} $$

where:

- $Y =$ output
- $K =$ capital
- $L =$ labor
- $\rho =$ substitution parameters
- $\alpha_o, \beta_o =$ distribution parameters, and
- $\tau_k, \tau_l =$ rate of factor augmentation for capital and labor respectively [11].

Differentiating (8) with respect to labor (L) yields:

$$ \frac{\partial Y}{\partial L} = (Y/L)^{1+\rho} (\beta_o \tau)^{-\rho} $$

Assuming the real wage rate ($w$) is equal to the marginal physical product of labor ($\partial Y/\partial L$), re-arranging and substituting terms in (9), and converting to logarithms gives:

$$ \log S_L = (\sigma-1) \log \beta_o + (1-\sigma) \log w + \gamma \tau (\sigma-1) \log t $$

Based on data in Table 2, the following estimates were obtained by ordinary least squares.\(^3\)

$$ \log S_L = -0.473 - 0.509 \log w - 0.336 \log t $$

\(^3\)Standard errors are contained in parentheses under their respective regression coefficients.

Although the Durbin-Watson ($d'$) is in the inconclusive range and the coefficient of the real wage variable is significant only at the 0.15 level, statistical results are consistent with a priori expectations. Given the estimated coefficient for the wage variable, elasticity of factor substitution is 1.5.

An alternative method of estimating elasticity of factor substitution is suggested by R. G. D. Allen [1, p. 373].

$$ E_L = - (1 - S_L) (\sigma) + (S_L) (\eta) $$

where

- $E_L =$ price elasticity of demand for labor
- $S_L =$ labor's relative share
- $\eta =$ price elasticity of product demand, and
- $\sigma =$ elasticity of factor substitution.

Tyrchniewicz and Schuh [15] report a long-run price elasticity of demand for hired farm labor in the United States of $-0.49$ and for unpaid family labor of $-3.0$. Wallace and Hoover [19] estimated a price elasticity of demand for hired and family farm labor of $-1.433$. Unpaid family labor and operator labor represent a major portion of the traditional sharecropper cotton labor force which has been replaced with the modernization of cotton production.\(^4\) Hence, long-run price elasticities of demand for cotton labor of $-1.0$ and $-1.5$ appear to be reasonable estimates.

Blakeley's [2] and Martin's [13] estimates of the price elasticity of demand for cotton are $-0.86$ and $-0.89$, respectively. Cotton labor's average relative share for the period 1952-1969 is 0.23.

Using these parameter estimates, the Allen formula gives values between 1.0 and 1.7 for the elasticity of factor substitution. These estimates are consistent with the previous estimate of $\sigma$.

**SUMMARY AND CONCLUSIONS**

Although knowledge of bias of the technological change occurring in a given economic sector may be indicative of how labor's relative share of output value may change, knowledge of elasticity of factor substitution of capital for labor is required before any conclusive statement can be made about how labor-saving technology may be affecting labor's relative share. If a capital input can be easily substituted for labor, then labor's relative share will tend to decline. If, however, the ease of substitution of capital for

\(^4\)In 1959 about 15 percent of the U.S. Cotton crop was grown by 65 percent of the cotton producers. These farms relied heavily on family and operator labor [3].
labor is more limited, labor’s relative share, even though a labor-saving technology is being adopted, can increase.

U.S. cotton production has been rapidly mechanized in the post World War II period. Given the relative ease of substitution of capital for labor \((\alpha > 1)\) and the labor-saving bias \((\beta > 1)\) of modern capital inputs such as cotton pickers, U.S. cotton labor’s relative share of output has tended to decline since World War II.

Knowledge of elasticity of factor substitution can be especially important for policy-makers in developing economies where labor tends to be relatively abundant. If elasticity of factor substitution is less than one, then adoption of a labor-saving technology can actually lead to an increase in labor’s relative share. However, if elasticity of factor substitution is greater than one, adoption of a labor-saving technology will not only displace labor but moreover labor’s relative share will decline.

One concluding caveat is in order. A decline in labor’s relative share in a particular industry, or within a given sector, does not necessarily mean that those workers who left the industry or sector are worse off. Workers may be able to obtain employment in another sector. Furthermore, a decline in labor’s relative share implies only that the portion of the value of total output going to labor employed in a given sector or industry has declined. The labor share analysis presented in this article is based only on functional distribution of income—it does not explain personal income distribution.

REFERENCES
