DEMAND THEORY: Time Allocation and Outdoor Recreation

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INTRODUCTION

The concept of demand has evolved through the centuries, enriched by the natural-value, exchange-value controversy and the diamond, water paradox of the classicists [11], the intuitive insights of Marshall [8], and the mathematical rigor of Slutsky [10], and Hicks and Allen [6]. These developments led to the conceptualization of a demand function as a solution function to a constrained extremum problem [5, 6, 9, 10].

This contemporary theory seems to apply rather well to many textbook examples. Commodities to which it seems particularly inapplicable include those that require a high degree of consumer assembly (i.e., may not be purchased in a simple package) and those that entail the expenditure of blocks of time. Leisure activity is a class of commodity possessing such difficulties.

The famous letter of Professor Hotelling [7] and the "Clawson Model" [4] were apparent attempts to apply contemporary theory to commodities with a high degree of consumer assembly and significant time requirements. Both suggested the use of travel distance, or distance of the facility from the residence of the consumer, as a surrogate for recreation prices.

Burt and Brewer [2] have carried forth this suggestion by generating a method of empirically computing direct recreational benefits. Burt and Brewer computed consumer's surplus by using distance to the recreational site from the residence as a surrogate for the price of a visit.

This discussion presents a comparative summary of several extensions of contemporary theory that investigate restrictions imposed by available time and the assembly of commodities from time and goods. Some implications of these theories for the evaluation of outdoor recreation facilities and activities are pointed out. However, the evaluation of time is not considered.

NAIVE MODELS

Contemporary Theory

Contemporary consumer theory assumes the maximization of a strictly quasi-concave utility function subject to a linear budget constraint [5, 6, 9, 10] and that goods are purchased with prices and income determined exogenously. In symbols,

$$\text{Maximize} \quad U = U(x_1, \ldots, x_n)$$

$$\text{Subject to} \quad p_1x_1 + p_2x_2 + \ldots + p_nx_n = I$$

The $x_1, \ldots x_n$ are regarded as positive flows of commodities[1] and the prices $p_1, \ldots, p_n$ and income I are non-negative. In case certain mathematical conditions hold,[2] the results presented are that the demand functions implied by the first order conditions for utility maximization are single valued, differentiable and homogeneous of order zero in all prices and income. In addition, the change in each good with respect to a compensated change in its own price (substitution effect), is negative for all (compensated) price changes in a neighborhood of the price-income point under consideration. It is apparent that implied

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[1] In case non-negativity is assumed, the existence of solutions may always be mathematically assured by using the Kuhn-Tucker theorem. Similarly, inequality in the constraint may be easily handled.

[2] Mathematical conditions include strict quasi-concavity of the utility function and continuous first and second order partial derivatives of the utility and constraint functions.

hypotheses about time allocation and consumer assembly do not arise from such a model.

Adam in Eden

The Judeo-Christian tradition has provided us with a description of sorts of the complete outdoor recreationist. It seems that Adam was surrounded by vast abundance of "fruits of nature" in the Garden of Eden and was commissioned to utilize them as he saw fit, with one well known exception. Since there was no scarcity in the Garden of Eden and he was alone, there was no exchange.

It is apparent that Adam's days were of limited length and that he, as with most of us today, could experience only a limited number of the "fruits of nature" at a time. If we suppose that Adam had a strictly quasi-concave utility function with arguments as quantities of "fruits of nature," that he could enjoy "fruits of nature" one at a time, and that he maximized utility each day subject to the exhaustion of available time, we could express Adam's choice problem as follows:

$$\text{Maximize} \quad U = U(x_1, \ldots, x_n)$$

$$\text{Subject to} \quad t_1x_1 + t_2x_2 + \ldots + t_nx_n = \tau$$

the $x_1, \ldots, x_n$ in this case are non-negative quantities of "fruits of nature" and the per unit time requirements $t_1, \ldots t_n$ and length of day τ are non-negative and given.[3] The implications of such a model in terms of "demands" for "fruits of nature" are identical to those for goods in the contemporary model except that time parameters have assumed the allocative role of money parameters.

Time and Money Allocation with Fixed Proportions

After the creation of Eve, interpersonal utility comparisons resulting in barter arrangements came about, and increased in incidence as commerce developed following the banishment from Eden. The descendents of Adam and Eve, however, must continue to engage in the allocation of time.

By defining an activity as a combination of time and goods for consumption as a unit and assuming that participation in all activities could be obtained for a fee, the choice problem of a typical individual could be specified as:

$$\text{Maximize} \quad U = U(x_1, \ldots x_n)$$

$$\text{Subject to} \quad \sum_{i=1}^{n} p_ix_i = I$$

$$\sum_{i=1}^{n} t_ix_i = \tau$$

where $x_1, \ldots, x_n$ are positive quantities of consumption activities, the p_i are prices or fees paid to participate or wages received for participating in activity i, I is a residual wealth parameter, t_i is a parameter representing the units of time required to produce one unit of x_i, τ is the length of planning period. Note that each $t_i > O$ because x_i is an activity.[4] This model, its implications, and its origins are reviewed in Wilson [13, 14].

A MORE REFLECTIVE MODEL

A review of "naive" consumer models has focused attention upon time allocation. However, it will be useful to pursue a more comprehensive model that may better reflect the decision processes of a consumer. In the present section a refinement and generalization of the naive models will be made through alteration of certain of the functions.

Time and Money Allocation with Variable Proportions

The linear time constraint with fixed coefficients in previous models may be altered to allow both fixed and variable time proportions in the production of activities.[5] Furthermore, relationships associated with certain parameters in the implicit production function may be derived and interpreted. The specification is as follows:

$$\text{Maximize} \quad U = U(x_1, \ldots, x_n)$$

$$\text{Subject to} \quad F(z_1, \ldots, z_n, y_1, \ldots, y_m, v_1, \ldots, v_n,$$

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[3]Others, notably Becker [1], might assume several kinds of time. This assumption would only replicate the time constraint for each kind: for example; daytime, night-time, weekday, weekend, holiday, etc.

[4]The utility function is defined for a specified planning horizon τ. Changing τ implies changing U so that questions as to changes in the optimum implied by changing τ may not be well posed.

[5]The assumption that activities are consumed one at a time is retained.

$$s_1, \ldots, s_r, Y_1 \ldots Y_m) = 0$$

$$x_i = w_i + z_i, i = 1, \ldots, n$$

$$\sum_{i=1}^{n} q_i w_i + \sum_{i=1}^{m} p_i y_i = I$$

$$\sum_{i=1}^{n} t_i w_i + \sum_{i=1} v_i = \tau$$

where $x_1, \ldots, x_n$ are work and consumption activities with $w_1, \ldots, w_n$ purchased and $z_1, \ldots, z_n$ produced, $y_1, \ldots, y_m$ are goods, $q_1, \ldots, q_n$ and $p_1, \ldots, p_m$ are exogenous prices of purchased activities and input goods respectively, $s_1, \ldots, s_r$ and $Y_1 \ldots Y_m$ are exogenous production parameters, $t_1, \ldots, t_n$ are exogenous time coefficients for purchased activities, $v_1, \ldots v_n$ are variable non-negative endogenous time inputs for the produced activities and I and τ are as defined previously.[6]

Under appropriate conditions, the first-order Lagrange conditions for this problem may be solved for each of the variables $z_1, \ldots z_n, w_1, \ldots w_n, y_1, \ldots y_m, v_1, \ldots v_n$ to obtain locally differentiable generalized demand and supply functions dependent on the parameters $s_1, \ldots, s_r, Y_1, \ldots, Y_m, q_1, \ldots, q_n, p_1, \ldots, p_m$, I, $t_i, \ldots, t_n$ and τ.[7] The 3n+m generalized demand and supply functions are all homogeneous of degree zero in the money parameters $q_1, \ldots, q_n, p_1, \ldots, p_m$, and I. However, all of them cannot be homogeneous of degree zero in the time parameters.[8]

Certain points should be noted about the functions obtained from the first order conditions. The produced activities do not possess market prices, but their generalized demand functions are well defined and depend on the other prices and parameters. Furthermore, the prices in the system are all attached to either inputs or purchased activities. Time, as a variable factor input in the production of an activity, behaves as a good in that a generalized derived demand function for its use in each activity is deduced. However, the different time demands do not have associated market prices.

The production parameters $s_1, \ldots, s_r$ have an interesting interpretation in the case of outdoor recreation. Amongst these parameters are included such items as the minimum distance that must be traveled if a particular recreation site is to be visited (an activity). The actual travel distance is an activity jointly demanded with the site visit. Also included as parameters would be the minimum required values of travel time, total travel expenditure, total time expenditure, total expenditure, outfitting expenditure, etc. Such production parameters are obviously exogenous, but the actual levels chosen in the allocation process for these items are either endogenous activities or activity total costs, as the case may be. Neither the production parameters, nor the values of related activities, nor their total costs in money or time, would appear to be surrogates for prices for produced activities on theoretical grounds.

Another set of parameters arising from the production relationships, $Y_1, \ldots, Y_m$, have found their place in recreation demand analysis. These parameters are related to the latent demand hypothesis [15], attraction hypothesis [12], or learning by doing hypothesis [3], as it has been variously termed. Regardless of the terminology, the gravity model [12] and the econometric studies [3, 15] employ recreation production input (supply) parameters in the "demand" relationships. An attempt will be made to rationalize such procedures and demonstrate their consistency with time allocation demand theory. In a period sufficiently short to have relevance in a consumer's time allocation process, it would seem reasonable to regard the existing stocks of recreational facilities, environmental attributes (crowding, quality, etc.) and the degree and diversity of recreational development as parameters. These facility input supply parameters would then represent constraint parameters for the aggregated production of recreation by all consumers recreating in a given geographic region. As the consumer performs his utility calculus, these parameters could enter his computations as parameters in his recreation production function that reflect his knowledge of aggregate behavior. That is, they might be viewed as micro-surrogates for macro-constraints on aggregate recreation production; thus, they logically would appear as parameters in the generalized demand functions.

It has been shown in Wilson [13, 14] that the compensated rates of change of demands for activities

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[6]Mathematically the utility function U is strictly quasi-concave. All functions possess continuous first and second-order partial derivatives.

[7]Solutions will not exist in general using the Lagrange method for nonpositive variables. Here it is assumed that all variables are positive. Solutions for cases in which some of the variables have zero values may be obtained using the Kuhn-Tucker theorem. Solutions similar to these for the Lagrange multipliers λ, γ, and δ can also be obtained.

[8]Statements about homogeneity in $s_1, \ldots, s_r$ and $Y_1, \ldots Y_m$ depend on the form of F as in the time constraint.

and production inputs with respect to their own money and time parameters are negative. These rates of change provide a set of hypotheses to be tested in empirical demand investigations. The algebraic signs of compensated rates of change of activities or inputs with respect to other parameters cannot be deduced. As in contemporary consumer behavior theory, uncompensated rates of change may be positive, zero or negative, depending on the magnitude and direction of the residual wealth effects and time effects in the Slutzky equations. In addition, it is not generally possible to deduce the algebraic sign of either the compensated or the uncompensated rates of change in the total activities x_1 (sum of purchased, w_i, and produced, z_i, activities) with respect to changes in any of the parameters. These qualitative results correspond closely with those of contemporary theory.

Knowledge of the production function F should allow derivation of certain of the rates of change in produced activities z_i, goods y_i, and variable time v_i with respect to the own price of goods p_i. Thus, hypotheses about the system of demand functions may be more completely developed than in models discussed previously. In case a new recreation facility does not provide the capability for new activities, the facility effects only the constraints in the problem in known ways and does not disturb the utility relationship. Changes in demand parameters for goods and time inputs in this case can be deduced from the changes in the production function. Directions of changes in activities are not usually deducible. All other propositions deducible from the fixed proportions model are also deducible for this model [13, 14].

It should be mentioned that with produced activities, such as recreation, the activity quantities may be measured in amounts of time spent. In such circumstances, the fixed time parameters will be equal to 1 and the variable time for such an activity will be identical to the quantity of activity. For activities measured in time units, demand functions for associated time inputs will be redundant.

Example

Suppose that the typical consumer has available to him three activities, working x_1, dining x_2, and recreation x_3. He may obtain recreation in either of two ways; by the purchase of a fixed recreation package w_3, or by production of recreation, utilizing variable amounts of a services recreation facility y and time v. The production parameter d might represent distance to the facility, while Y might represent the size of the facility and e a positive constant. The consumer's choice problem is characterized as follows:

$$\text{Maximize} \quad U = U(x_1,x_2,x_3)$$

$$\text{Subject to} \quad x_3 = w_3 + z_3$$

$$z_3 = av^2 + by^2 - cvy - d + eY$$

$$x_1p_1 + x_2p_2 + w_3p_3 + yp_y = 0$$

$$x_1t_1 + x_2t_2 + w_3t_3 + v = \tau$$

where it is assumed that $p_y = 0$.

The Lagrangean function

$$L = U(x_1,x_2,w_3 + z_3) + \lambda(av^2 + by^2 - cvy - d + eY - z_3) + \gamma(p_1x_1 + p_2x_2 + p_3w_3 + p_yy) + \delta(t_1x_1 + t_2x_2 + t_3w_3 + v - \tau)$$

yields first order conditions for a relative constrained maximum of U which, under certain conditions, may be solved for $x_1,x_2,w_3,z_3,y,v,\lambda,\gamma,\delta$, in terms of the parameters $a,b,c,d,e,Y,p_1,p_2,p_3,p_y,t_1,t_2,t_3$ and τ. The solutions (generalized demand or supply functions) may be expressed as:

$$v_i = h^i\ (a,b,c,d,e,Y,p_1,p_2,p_3,p_y,t_1,t_2,t_3,\tau),$$

$$\text{for } i = 1, \ldots, 9.$$

The demand (supply) functions h^i are each differentiable, unique and homogeneous of degree zero in the prices p_1,p_2,p_3, and p_y provided that U is strictly quasi-concave. They are not homogeneous in a,b,c, d,e, and Y nor in t_1,t_2,t_3 and τ.

The demand function for x_3 is $h^3 + h^4$. Its rate of change cannot be deduced for compensated changes in p_3 or p_y.

The sign and magnitude of certain of the compensated and uncompensated rates of change in demand can be deduced.

It should be noted that public and private policy makers could control Y, the quantity (stock) of a facility on hand and manipulate it at their will. Similarly, p_y could be manipulated.

DISCUSSION AND CONCLUSIONS

Consumer behavior theories have been summarized and some relevant implications pointed out. The variable proportions time allocation model appears to describe the manner in which activities, goods, and variable time inputs are related to prices and other known money, time, and production parameters. It has intuitive appeal as a decision framework representing consumers of outdoor recreation.

There should be little doubt concerning the meaning of a demand function for a produced activity. Such demand functions are well defined whether or not the activities or goods each have money prices that can be nonzero. The demand functions have as arguments all parameters in the problem.

If an activity is both purchased and produced, the price of the activity as purchased does not hold an equivalent relationship to the activity as produced and to the total of purchased and produced. This is evidenced by the indeterminateness in the response of the produced activity and, consequently, total activity to a change in the purchase price. Thus, purchase price may be no surrogate for a money price for a produced activity. Similar statements may be made about time parameters.

Recreational facilities are themselves physical inputs for which a derived demand function is obtainable. In the event that the facilities are public goods they are often accorded zero prices by fiat. The application of contemporary theory to recreational problems has led to a lack of appreciation for the distinct roles of facility inputs and activity outputs. Indeed, none of the models provide insight into possible surrogates for prices for the use of non-priced recreational facilities or activities.[9]

It has been suggested for many years [7], and again recently [2], that a proper surrogate for the price of a recreational facility (input) or facility visit (activity) paid by a visitor might be the distance from the residence of the visitor to the recreational site. Confusion exists, of course, as to whether this distance should be accorded as a price to the visit or to the facility. The variable proportions time allocation model puts this problem in focus. The distance from the residence to the recreational site is a parameter in the production of activities from a facility. As such, it is a parameter in the consumer's demand functions, both for the facility and for activities associated with it.

There is no evidence that distance is properly a surrogate for price except that as distance diminishes, one would expect both the amounts of activities and facility use to increase via time substitution. The distance parameter may be viewed as a lower bound for recreational travel, an activity demanded jointly with activities at each recreational site. Travel cost is the total cost of the recreational travel activity.

Samuelson [9] has pointed out that consumer's surplus as a tool for the measurement of welfare is both superfluous to the analysis and expressible in at least a half dozen mutually inconsistent forms in contemporary theory. Burt and Brewer [2], on the other hand, accept these shortcomings and point to the usefulness of such a measure. It appears that such positions are justified for commodities for which contemporary theory appears adequate. Such commodities are purchased rather than produced, have prices with a nonzero range, and have minimal time allocation effects. At present, a companion consumer's surplus theory for the variable-proportions time-allocation demand theory has not been developed. Therefore, any relationship of the quantities computed by Burt and Brewer [2] to utility changes is unknown and, furthermore, may be coincidental.

The point cannot be overemphasized. The computation of recreational benefits as consumer's surplus by using distance or total travel cost as a price may have been intuitively appealing to Hotelling [7], Clawson [4], and Burt and Brewer [2], but its meaning is at best nebulous and, at worst, nonsense. Such measures were suggested before a sufficiently reflective demand theory was developed, and now appear spurious. With an appropriate demand theory at hand, it is now apparent that there is no companion theory of consumer's surplus for produced activities. At such time as economic theory provides a consumer's surplus framework for produced activities, the benefits question may be settled.

The variable proportions time allocation theory provides a rationale for the use of supply variables in a demand function. Aggregate stocks of goods or recreation facilities might appear as parameters in the individual consumer's production function as indicators of perceived productivity. As production function parameters, they appear as parameters in the generalized demand functions. Such supply stocks as demand parameters could be extremely useful instruments in a public planning process.

Such a public planning process might be easily conceived. A possible objective function to optimize that might be regarded as a surrogate for a social welfare function might be aggregated recreation activity demand. Such a function could be optimized using as controls changes in the aggregate supply stocks, and subject to public budgetary limitations. This procedure could be used until there is available some defensible method of estimating benefits to recreation investments.

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[9] Indeed, the demand functions are well defined without some prices. The question of proxies for prices arises only with respect to the computations of benefits via consumer's surplus. Since at this point there is little reason to suspect that the conventional consumer's surplus approach is applicable, it may be that the question of proxies for prices is irrelevant.

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