NETWORK FLOW MODELS: USE IN RURAL FREIGHT TRANSPORTATION ANALYSIS AND A COMPARISON WITH LINEAR PROGRAMMING

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The logistical efficiency and viability of the rural transportation system is a growing concern of agricultural producers, transportation regulators, and policy makers. Railroad facility limitations, abandonment of railroad track, deterioration of the rural road system, and changing transportation rate structures are examples of issues for which solutions and evaluations are required[1] [14, 15]. To plan and evaluate transportation alternatives, decision makers need a research tool with an ability to answer numerous "what if" types of questions. Often the model must include a spatial and/or temporal dimension, in addition to microscopic detail of the transportation and marketing system. Therefore, large models are needed and efficiency of computer solution procedures becomes a relevant consideration.[2] The flexibility of the research tool to include necessary system realism may also become an important concern.

Linear programming has been used successfully by applied economists to solve a variety of models which include a spatial and/or temporal dimension. Many of these models have what is termed a network structure, the type of structure characteristic of the transportation and transshipment models. Published works by Fedeler and Heady [3], Leath and Martin [11], Holroyd [6], and King and Logan [8] are examples from an extensive list of research products in which linear programming is used to solve models with a network structure. Recently, several articles have alluded to the superiority of network flow algorithms in comparison with linear programming for solution of models with a network structure. Fuller et al. [4] used a network flow model in combination with implicit enumeration for a plant location problem involving fixed charges. Their approach involved partial enumeration in conjunction with a network flow algorithm which was found to be more efficient for the solution of network structured subproblems than linear programming codes. Wright and Meyer [16] employed a network model to analyze development of a Brazilian rail corridor and indicated superiority of network flows models to linear programming for transportation planning. Although network flow models appear to be superior for certain types of problems, no systematic analysis has been attempted to identify these situations.

The objective of this article is to illustrate and compare network flow and linear programming models for the solution of network structured problems representative of rural freight transportation systems.[3] In particular, the purposes of this article are to (1) demonstrate the potential use of network flow models in rural transportation research and (2) compare linear programming and network flow models with respect to computer solution efficiency and flexibility to incorporate alternative transportation system features. The comparative analysis is carried out on a linear minimum-cost capacitated transshipment model representative of a rural freight transportation system. Transportation system models often

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[1]The Regional Rail Reorganization Act of 1973 authorizes appropriation of federal funds for financial assistance to specific rail line operations given ICC abandonment approval However, to qualify, the state must establish an overall state transportation plan. The rail portion of the plan identifies service required on particular lines consistent with economic goals of the state.

[2]Baumel et al.'s work at Iowa State University is an example of meaningful transportation research and is illustrative of the analytical model's extensive data requirements [1, 10]. Their system model included spatial and temporal characteristics of commodity flows that originated at the farm and terminated at port locations. The model included cost and constraint information on the rural road system, grain assemblers' plants, rail line segments, multiple-car rail shipments, and truck and barge activities. The Iowa State model was designed to answer rail abandonment questions.

[3]There are two principal classes of network models. One class is directed toward selecting a single chain through a network that optimizes an objective function. They are characterized by project planning (critical path method), shortest route, and maximum flow models. The second class are network flows models and are characterized by the transportation and transshipment models. The solution of these models yields sets of chains which when superimposed will form least-cost flows through the network.

have a network structure and include large numbers of variables and constraints; accordingly, this representation is useful to facilitate the comparative analysis [13].

Transportation planning and policy evaluation generally include several phases. The initial phase involves an inventory of a region's transportation system and transportation-dependent industries with an estimation of their cost or rate parameters and constraints. On the basis of historical data of commodities' spatial and temporal flow patterns, a flow generation model is developed. The purpose of such a model is to predict magnitude and direction of future commodity flows or transportation demand. The final phase involves planning and policy evaluation. It is for this phase of transportation planning that linear programming and network flow models are analyzed.

THE MODEL

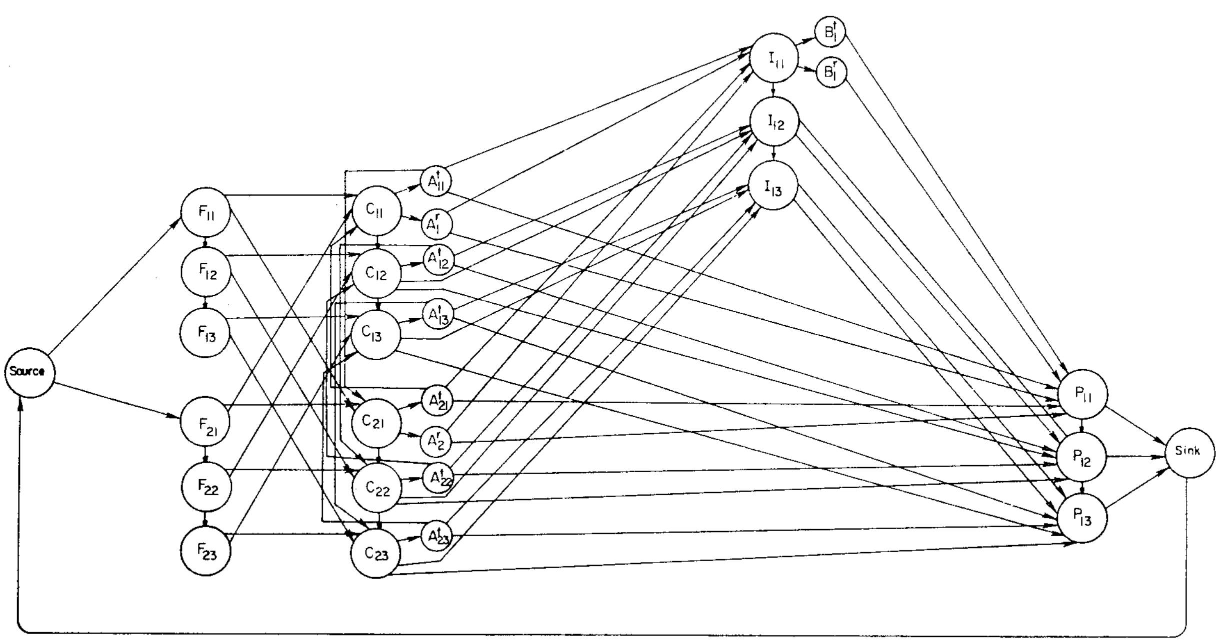
The comparative analysis is carried out on a model which represents a wheat marketing/transportation system that involves grain flows from production locations (farms) through country elevators and inland terminals to port terminal destinations. The developed model represents a crop year; however, to facilitate a temporal analysis, the year is divided into time periods of different lengths. Wheat harvest is carried out in the initial time period. A small portion of the harvested supply is exported during harvest; accordingly, the remainder must be stored at farms, country elevators, or inland terminals for later shipment to port terminals. Wheat is assembled to country elevators prior to further movement through the system. Rail and truck transportation is available for all movements except those involving assembly of wheat to country elevators, in which case farm trucks are used. The model includes grain storage and transportation capacity constraints; linear costs are assumed for the grain handling, storage, and transportation activities. Quantity of wheat harvested at each farm and amount demanded by each port per time period are predetermined. It is assumed that all harvested grain will be shipped to port terminals for export during the crop year. Given these assumptions and system constraints, the objective is to determine that flow pattern through the system which minimizes total annual cost.

RURAL FREIGHT SYSTEM AS A NETWORK FLOW MODEL

To illustrate the construction of a network flow model, a prototype of the above-described model is developed. The network model is constructed with nodes and interconnecting arcs. Nodes represent elements of the system. Arcs connect nodes and include information about lower and upper bounds associated with the magnitude of arc flow, in addition to the unit cost of this flow (Figure 1).

The prototype model includes two farms (F=2), two country elevators (C=2), one inland terminal (I=1), and one port terminal (P=1). In addition, the model includes two time periods (T=2). To represent two time periods,

FIGURE 1. PROTOTYPE NETWORK MODEL OF RURAL FREIGHT SYSTEM



three points in time must be established. Grain stored from point 1 in time to point 2 in time will have been stored through the first time period. Similarly, grain stored from point 2 in time to point 3 in time will have been carried through the second period. The nodes in the network are identified as:

F_{ik}: represents production location i (farm) at point k in time

$i = 1, 2, ..., F$
$k = 1, 2, ..., T+1$

C_{ik}: represents country elevator i at point k in time

$i = 1, 2, ..., C$
$k = 1, 2, ..., T+1$

I_{ik}: represents inland terminal i at point k in time

$i = 1, 2, ..., I$
$k = 1, 2, ..., T+1$

P_{ik}: represents port terminal i at point k in time

$i = 1, 2, ..., P$
$k = 1, 2, ..., T+1$

The quantity shipped from the country elevators and inland terminals by truck and rail is constrained by the amount of transportation service available. Hence, the following set of artificial nodes are created.

A^t_{ik}: represents the truck node associated with country elevator i at point k in time

A^r_i: represents the rail node associated with country elevator i during harvest

B^t_i: represents the truck node corresponding with inland terminal i during harvest and

B^r_i: represents the rail node associated with inland terminal i during harvest, where

$i = 1, 2, ..., C$, or
$i = 1, 2, ..., I$, and
$k = 1, 2, ..., T+1$, as appropriate.

Each arc connecting the nodes includes three parameters which identify a lower bound or minimum flow, an upper bound or maximum flow, and a cost of unit flow through the arc. The cost parameters on appropriate arcs represent a facility's cost of grain receiving, storing, and loading-out activities in addition to costs of transporting grain between alternative locations by either truck or railroad.

The lower bounds on all arcs are set equal to zero except those linking the source node with the farm nodes and the port terminal nodes with the sink node. The lower and upper bounds on the arcs terminating at the farm nodes are set equal to the quantity of grain harvested at the farm. The arcs terminating with the sink node have their lower and upper bounds set equal to the exogenously estimated foreign grain demand. The upper bound on arcs linking the F_{ik} (farms) and C_{ik} (country elevators) nodes is set equal to infinity to reflect the lack of upper bound on this flow. Arcs beginning at the C_{ik} notes and terminating at the A^t_{ik} or A^r_i notes have upper bounds set equal to the respective quantity of truck and rail service available. Similarly, arcs originating at inland terminal i at harvest and terminating at the B^t_i or B^r_i nodes have upper bounds on flow set equal to the available quantity of truck or rail service. All remaining arcs have their upper bound set equal to infinity, except those involving storage activities.

The multiperiod characteristics of the model are introduced through the use of storage arcs which link a storage facility through different points in time. For example, the vertical arc connecting node F_{11} (farm 1, time point 1) and F_{12} (farm 1, time point 2) connects time point 1 with time point 2 and represents the first time period. The upper bound on this arc reflects that farm's storage facility's capacity. The vertical storage arcs at country elevators and the inland and port terminals contain similar upper bounds.

The solution technique requires that a return arc be created, originating at the sink node and terminating at the source node. The lower and upper bounds on this arc are set equal to the total grain supply which is identical to grain demand. After construction of the network flow model, a network flow algorithm is applied to resolve the least-cost solution.

Any network flow model can be formulated as a linear programming model. In a linear programming model, each node is represented by a row (constraint) and each arc by a column (activity). For example, the foregoing prototype model formulated in a linear programming format would include 28 rows and 70 columns. The direction of flow on an arc linking node i to node j is indicated in the linear programming table as a $+1$ coefficient in the row for node i and -1 in the row for node j. Linear programming and network flow models yield identical least-cost solutions.

SOLUTION EFFICIENCY AND FLEXIBILITY OF LINEAR PROGRAMMING AND NETWORK FLOW MODELS

The comparative analysis was carried out on a model of the wheat marketing/transportation system that included the following characteristics: 29 production locations (farms), 17 country elevators, three inland terminals that are outside the research area, two Gulf port terminals, and three time periods.

To determine the effect of model size on core requirements and solution times, several variations of the grain transportation model were solved. The initial solution was the full-scale model and involved 4851 activities (arcs) and 578 constraints (nodes). The solution of this model by the linear programming code required 334.64 seconds of execution time and 768K bytes of core. In contrast, the out-of-kilter code required 448K bytes of core and 6.94 seconds of execution time (Table 1). The network flow code was 46-48 times faster than the linear programming code and required about 58 percent as much core. As model size increased, the network code's solution speed increased in relation to that of the linear programming code.[4]

In some cases, models include more complex features than those illustrated. The foregoing model was restricted to the assumption of linearity in all cost relationships; however, some problems may require inclusion of convex or concave cost functions. Concave functions are representative of economies of scale relationships, whereas convex functions are useful for representation of transportation congestion costs. Unfortunately, network flow models are restricted to convex functions that can be approximated by piecewise linear segments. The network flow algorithm guarantees a global optimum for models that include convex forms. Linear programming, via the separable programming extension, attains a global minimum for models with convex functions, but a local minimum for those with concave relationships [2]. Even though the conditions for an optimum are not attained for models which include concave relationships, the separable programming technique is recognized as producing acceptable results [7].

Figure 2 illustrates a piecewise linear convex

FIGURE 2. PIECEWISE LINEAR CONVEX COST FUNCTION (a) WITH NETWORK MODEL REPRESENTATION (b)

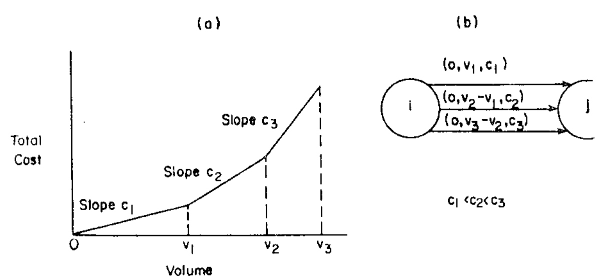


TABLE 1. COMPUTER SOLUTION EFFICIENCY OF LINEAR PROGRAMMING AND NETWORK FLOW MODEL

Problem Characteristics [a]		Solution Method [b]					
		Linear Programming [c]			Network Flow [d]		
Activities (arcs)	Constraints (nodes)	Core (bytes)	Solution Time (sec)	Cost ($)	Core (bytes)	Solution Time (sec)	Cost ($)
4851	578	768K	334.64	66.93	448K	6.94	1.39
3047	362	488K	163.89	32.78	284K	3.46	0.69
1543	222	368K	67.49	13.50	200K	1.45	0.29

[a]In a linear programming model, each network node is represented by a row (constraints) and each arc by a column (activity).

[b]All solutions obtained on an Amdahl 470V/G

[c]Revised simplex code.

[d]Revised out-of-kilter code.

[4]A version of the wheat marketing/transportation model involving 320 constraints and 2911 activities was run on the out-of-kilter code, Kuester and Mize's LP code, and on IBM's software package, Mathematical Programming System Extended (MPSX). MPSH' CRASH startup procedure was used. The respective solution times in seconds were 1.41, 63.25, and 75.05.

cost function and presents the network representation necessary to include the three piecewise segments as three arcs. Because the cost function is convex, $c_1<c_2<c_3$, flow begins to pass through the arc with the lowest cost, c_1. When the capacity of this arc (v_1) is reached, flow commences through the second arc. Flow begins to pass through the third arc only if the second arc's capacity is reached. It follows that convex relationships can be introduced into a network model by simply adding appropriate nodes and arcs. Accordingly, computer solution costs increase modestly. The grid linearization technique of separable programming permits inclusion of convex cost. With this technique the powerful simplex or revised simplex algorithm is used; therefore, solutions for these models are obtained with little increase in computational difficulty [12]. Accordingly, for models characterized by convex costs, the network flow algorithm is about 46-48 times faster than the grid linearization technique of separable programming, i.e., the comparative advantage is about that associated with linear models. It follows that convex functions are most efficiently accommodated by network flow models, whereas concave functions by necessity must be handled in a separable programming framework.

In some problems, it may be necessary to maintain the identity of various commodities simultaneously shipped between supply, transshipment, and demand nodes. With network models the identity of each commodity cannot be preserved. Obviously, for those models which require inclusion of a multicommodity dimension, this limitation is a major shortcoming. In some cases, transportation planning may not require this information because the principal interest is aggregate volume (tons) of flow. The single-commodity characteristic of network models prohibits accurate representation of transit rate structures. In general, a transit rate structure requires that the quantity transported into a storage location (transit point) by a particular mode must be equal to the quantity shipped out by that mode. Because the network procedure seeks the least-cost routing through the network, there is no assurance that this condition is met. For those models which include a multicommodity dimension, linear programming is the necessary solution procedure.

Through the introduction of slack activities, linear programming can easily accommodate those models in which demand and supply are unequal. Similarly, unequal demand and supply can be handled with network flow models. When supply exceeds demand, no changes are required in the network flow model's construction. The network algorithm allocates quantities from selected supply locations so as to minimize total cost and meet demand requirements; quantities in excess of demand are simply unallocated. If demand is in excess of supply, a dummy or artificial supply node must be created to represent an additional supply equal to excess demand. This node is connected with transshipment and/or demand nodes via arcs that include very large costs. *Ex post* the solution, flows on the added arcs must be identified and removed to conclude the optimal flow pattern. In addition, the objective function value must be reduced by the magnitude of costs associated with the flow on the added arcs.

In many cases, it is desirable to know the range over which parameter values can vary without causing violent changes in the least-cost solution. This investigation is termed postoptimality analysis. Many linear programming computer codes provide ranging analysis for objective function coefficients and right-hand side constants. To the authors knowledge, no network flow codes have this capability; accordingly, an analogous investigation can be accomplished only by changing parameters and then solving the model. Dual values provide information about the effect of changing constraints and are available from both the linear programming and network flow model solutions.

CONCLUDING REMARKS

This article illustrates the potential use of network models in rural transportation research and compares linear programming and network flow models with respect to computer solution efficiency and flexibility. Comparative analysis is restricted to minimization of a model with a network structure, e.g., transportation and transshipment models. In general, network flow models are limited to problems with this structure.

The comparative analysis shows the network flow algorithm to be substantially faster (46-48 times) than the linear programming code. In general, for heavily constrained large models, the network code is clearly more computer efficient. An additional attractive feature of network models is their ability to attain a global minimum when a convex relationship is included. This task is accomplished with little complication or increase in computer time. Linear programming, via the separable programming extension, can accommodate either a convex or concave relationship, albeit with some complication and no guarantee of a global minimum for problems characterized by concave relationships. If the problem situation requires a concave relationship, linear programming must be selected. In contrast to linear programming, network models cannot handle a multicommodity problem or a problem that requires the researcher to preserve more than one commodity's identity throughout the system. Network models do not provide information about the range over which parameter values may vary, although both tools yield duals. Neither tool requires that demand and supply be equal.

In summary, network flow models are superior research tools to analyze the rural transportation system if the problem does not include concave costs or require preservation of more than one commodity's identity throughout the system.

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