AN APPLICATION OF SAFETY-FIRST PROBABILITY LIMITS IN A DISCRETE STOCHASTIC FARM MANAGEMENT PROGRAMMING MODEL

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Abstract

A sequential decision-making model was developed, and data from farm-raised catfish production were used to demonstrate its use. Outcomes of sequences of decisions which satisfied chance constraints on ending cash balances were traced through a specified time period. Discrete choice variables were specified due to the fixed nature of pond facilities. Recourse actions specified were sale of production in excess of endogenously determined transfer levels or purchase of inputs to supplement needs of the next production stage. Production activities cannot be changed during the planning period. Only yield variability was considered due to its impact on relative competitiveness among growth stages. Deviations were calculated from endogenously determined target levels based on goal and probability limits.

Key words: chance constraints, discrete stochastic programming, limited recourse.

Recognition of the importance of uncertainty in farm management decision making has led to the replacement of deterministic linear programming models with probabilistic risk programming models. An essential element of this model transition has been the use of distributions of values as opposed to expected values or mean values. Hazell made a major contribution to this effort to incorporate uncertainty into farm planning models by developing a linear alternative to quadratic or semi-variance programming. His technique, Minimization of Total Absolute Deviations (MOTAD), could be used with rather simple modifications of widely used linear programming algorithms. Farm planning models under uncertainty became significantly more tractable and increasingly accessible to the farm management research community.

The model presented in this paper employs a method whereby certain multiple stage problems can be specified to obtain feasible plans for each stage while satisfying chance constraints on ending cash balances. It traces through the outcome of a sequence of decisions, but it is not a purely stochastic model in the sense that recourse actions are limited based on results of a previous stage. Production activities cannot be changed; once a sequence is selected, the decision maker is "locked into" that strategy for the planning period. Recourse actions specified are sale of production in excess of endogenously determined transfer levels or purchase of inputs to supplement the next production stage. Variability considered is based on yield. Price variability was not analyzed in this study because the relative competitiveness among growth stages is dominated by yield variability associated with mortality.[1]

The use of endogenously determined "risk reference" levels for calculation of deviations follows the method developed by Atwood. Although the MOTAD technique represents a major step in the development of farm planning models, the interpretation of deviation levels is not intuitive. Chance constraints are capable of addressing this shortcoming. The logic underlying chance constraint models—

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[1]The October producer price for farm-raised catfish varied from only $.61 to $.70 per pound over the period of 1985 to 1987 (Alabama Crop Livestock Reporting Service). Yield variability is considerably higher as indicated in Table 1.

the selection of a probability of not reaching a specified goal—is more operational than the selection of a deviation limit as used in MOTAD models. Survey results from Mao and Patrick et al. tend to support the existence of safety-first type decisions.

The objective of this study is to develop a methodology for assessing the economic viability of alternative sequential production strategies using chance constraint programming. The research is important to agricultural economists because many agricultural commodities have alternative sequential production stategies with different associated risks. The use of chance constraints in deciding among these production stategies provides a tractable method for dealing with these risks.

Aquacultural production activities are used to demonstrate this sequential decision making model with chance constraints. Catfish producers face the decision of producing eggs, fry, fingerling, or food-fish. The fixed nature of pond production facilities dictates the use of integer solutions. This paper will proceed by providing a brief overview of farm planning models under uncertainty, followed by model development and results.

BRIEF REVIEW OF SEVERAL UNCERTAINTY MODELS

Modeling decision making in an environment of uncertainty continues to attract research efforts of a number of agricultural economists. Many of these efforts have been directed toward single-period, two-attribute (usually income or profits and risk) risk models. An example of such efforts is the MOTAD model developed by Hazell and used by Mapp et al. or Gebremeskal and Shumway in analyzing potential responses to risk management. More recently, Tauer introduced a model in which risk is measured as the occurrence of events which fall below some fixed target or goal of the firm. Tauer demonstrated that a fixed target model (Target-MOTAD) could be used to generate second-degree stochastically-efficient activity mixes.

While such results are interesting, they are primarily static in nature in that such models attempt to find equilibrium-type activity mixes. These activity mixes usually represent a mix for which annual expected income-risk trade-offs are examined. These models do not examine the possible consequences of a sequence of events. Alternative models are available for modeling risk dynamics but have not attracted the level of efforts exerted on the static risk models. One such alternative approach has been termed "stochastic" or "stochastic dynamic" programming (Anderson et al.; Kim et al.). Although applications have been limited in agricultural economics, an early series of papers was published by Rae in which he discusses potential methods to implement stochastic programming including objective functions and constraints. He discusses the use of expected utility, lexicographic utility, and income-variance alternatives for stochastic programming.

The methods discussed by Rae in implementing expected utility maximization across time suffer from many of the problems of the static expected utility model. A major problem is the difficulty of eliciting accurate utility functions and the potential for specification error. This paper presents an alternative which is similar to the lexicographic options discussed by Rae. The alternative is related to the static models known as safety-first models in which constraints are placed upon the probability of failing to achieve certain goals of the firm. The paper specifically presents a method whereby the expected ending income at the end of a sequence of events is maximized subject to probabilistic constraints imposed upon potential ending states. Before proceeding further, a brief discussion of safety-first programming methods is in order.

Several forms of safety-first models have been proposed as alternatives to expected utility maximization. Roy proposed that decision makers might select activities which minimize the probability of failing to achieve a certain goal for income, i.e.,

(1) Minimize $\Pr(Z < g)$,

where Pr(.) is the probability of event (.), Z is an income random variable, and g is an income goal.

Other forms of safety-first criterion have been discussed by Kataoka and Telser. Telser's criterion is of particular interest. It maximizes expected income subject to probabilistic constraints on failing to achieve income goals and can be written as

(2) Maximize $E(Z)$

(3) Subject to $\Pr(Z < g) < L$,

where E(.) is the expected value of (.) and L is

an upper limit on Pr (Z < g). (Telser's criterion can be viewed as a special case of Charnes and Cooper's chance-constrained programming.)

The safety-first approach to decision making has been contrasted to expected utility maximization by Pyle and Turnovsky. Their method requires the assumption of normally distributed returns. Discussion and applications of safety-first decision making in agricultural economics include the papers by Barry and Robison; Kennedy and Francisco; Musser et al.; and De Janvry.

The methods used to implement safety-first programming vary depending upon the assumptions of the researcher. Pyle and Turnovsky assumed a normal distribution allowing the use of E-V analysis. Other researchers, including Telser in his original paper, have used stochastic inequalities which place sharp upper limits on the probability of failing to achieve firm goals. Several stochastic inequalities have been used, such as Chebychev's mean-standard error or mean absolute deviation (Anderson et al.).

These inequalities tend to generate upper bounds which are quite conservative or are difficult to implement (Sengupta). Recently Atwood presented a stochastic inequality which often generates considerably less conservative upper bounds than Chebychev's inequality. A special case of the inequality uses linear lower partial moments. Atwood showed that the linear lower partial moment could be used in a linear programming model to impose chance or probabilistic constraints. Atwood et al. demonstrated that a modified Target-MOTAD model could be used to implement Telser's criterion. The method requires only that a finitely discrete vector Z of possible end states can be computed and a goal, g, determined. The following linear constraints guarantee that $Pr(Z < g) < L$:

(4) $Z - 1t + Id > 0$, and

(5) $1t - (1/L)\, P\, d < g$,

where Z is a vector of possible end states, 1 is a vector of ones, t is an endogenously determined risk reference level,[2] I is an identity matrix, d is a vector of deviations below t (or 0 if $Z_i \geq t$), 0 is a vector of zeros, and P is a transposed vector of probability levels. In a more detailed discussion by Atwood et al., modified linear stochastic programming techniques are used to compute a vector of potential endstates after a series of decisions and stochastic events. The above constraints are then used to impose constraints upon the probability of losses at the end of the sequence of events.

DATA

Much of the data were obtained through a survey (Hatch et al.). The survey included over one-third of diversified aquaculture producers in Alabama, and selection was based on availability of sufficient farm records.

Egg production was accomplished in brood ponds averaging approximately four acres. The average diversified firm had approximately 10 acres of brood ponds. Fry were produced in a hatchery building of approximately 1,200 square feet. Eggs were incubated using wire baskets suspended in troughs with a continous flow of water. The average producer had 16 acres of fingerling production.

A frequency distribution for survival at each stage was developed from the survey and consultation with aquacultural specialists. Using these survival rates, a yield for each interval was calculated (Table 1). Net returns associated with these yields and the probability of each state were used in the programming matrix.

MODEL

This stochastic recourse model satisfies chance constraints for meeting the goal of covering fixed costs, while maximizing ending expected income. The goal is endogenously determined depending upon the activities undertaken. In a chance constraint model, the risk reference level (t) below which deviations are measured is internally chosen so as to be that which is least constraining (Atwood et al.). Production commitments are made for all stages before any production begins. Recourse actions are allowed so that appropriate adjustments can be made depending on the outcome of the previous stage.

This application uses catfish aquaculture as the example. Integer programming is employed because of the discrete nature of ponds and pond production. There are four sequential stages of production, from first to

[2]The risk reference level is endogenously determined given the target goal, the mean income, and the probability level. It indicates the risk/income reference point which is used to compute deviations below the reference level.

	Decision Variables																						
	Produce 1	Produce 11	Transfer	Sell 1-1	Sell 1-2	Sell 11-1	Sell 11-2	Buy 1-1	Buy 1-2	Save Open	Save 1	Save 2	Borrow Open	Borrow 1	Borrow 2	Tlevel	Dev 11	Dev 12	Dev 21	Dev 22	Ttheta	Total Fixed	Constant Values
Acre limit	1	1																				≤	X
Output 1-1	-D		1	1				-1														≤	0
Output 1-2	-E		1		1				-1													≤	0
Input 1-11		K	-1																			≤	0
Output 11-1		-L				1																≤	0
Output 11-2		-M					1		{I}													≤	0
Fixed Cost	A	B							{II}	{III}						{Chance Constraint Module}						-1 =	0
Cash Open	C									1			-1									≤	0
Cash 1		H		-J				U		-G	1		F	-1								≤	0
Cash 2		H			-J				U	-G		1	F		-1							≤	0
Cash 11						N					V			-W		-1	1					≥	0
Cash 12							N				V			-W		-1		1				≥	0
Cash 21						N						V			-W	-1			1			≥	0
Cash 22							N					V			-W	-1				1		≥	0
Theta																	.25	.25	.25	.25	-1	=	0
Suf. Con.																1					-q* -1	≥	0
Exp. Cash						R	R				S	S		-T	-T							max	

[a]Alphabetic characters in the matrix represent specific coefficients used in the analysis. D, E, L and M are physical outputs. K is the input requirement for the next stage, A and B are fixed costs, G and H are the variable cost cash requirements for one unit of production activity in stages 1 and 11, respectively. J and N are the sale prices for the products of stages 1 and 11, respectively. U is the buy price for the output of stage 1 (equals J plus marketing margin). R is the sale price of stage 11 outputs times the probability of the state of nature. G and V are the amount saved plus interest accrued during the period of the production stage. F and W are the amount borrowed plus interest payments accrued at the end of the stage. S is the value of savings times the probability of the occurrence of the outcome of the states on which the savings activity is based. T is the value of borrowing times the probability of the occurrence of the borrowing activity level. I, II and III are quadrants of the matrix.

q = 1/L where L is the probability limit.

Figure 1. Illustration of Sequential Decision Making Model with Chance Constraints (Two Stages and Two States of Nature).[a]

last: egg, fry, fingerling, and food-fish. Out of each production stage there are three possible states of nature or outcomes (Table 1). Examining aggregate production, this results in 81 possible end states of nature existing at the end of a production cycle.

The stages are independent. The ability to buy input before the initiation of each stage, after the first, allows each stage to be independent of the physical output of the previous stage. However, output from a preceding stage can be used as input in a succeeding stage and excess output from preceding stages can be sold. There is also a cash impact depending on the outcome of the previous stage, but this can be compensated for by additional borrowing or saving.

The only constraints, other than cash flow, are a total pond acreage constraint and a limit of one egg hatchery for fry production. The hatchery does not use pond acreage. Ponds not used may be rented out before initiating the first stage and that cash made available for production activities. In developing the matrix coefficients, a marketing margin was included in the buy and sell prices; and the borrowing rate was slightly above the savings rate.

Figure 1 illustrates a simpler model in which there are two stages and two states of nature in each stage. The letter variables in the following discussion refer to the coefficients of that tableau. The tableau can be thought of as consisting of three quadrants and a chance constraint module. The northwest quadrant, I, models the physical production and movement of input and output. These movements include selling output, buying additional output (inputs for the next stage) and transferring output to initiate the next stage.

The southwest quadrant, II, lies directly below the northwest quadrant. It translates all production-related activities into their impacts on the cash flow.

The southeast quadrant, III, is the cash flow quadrant which calculates interest rates over the periods of the production phases and transfers cash from one production stage to the next. It includes the borrowing and savings activities which provide cash for the production requirements registered in quandrant II.

The chance constraint module is attached to the ending cash rows of quadrant III. This section uses methods similar to Target-MOTAD to compute mean deviations below risk reference level (t). The selection of the risk reference level is endogenized to allow the selection of the least constraining linear stochastic inequality and probability bounds (Atwood).

Initiating production in stage 1 creates a need for cash in quadrant II (C) which must be satisfied by borrowing in quadrant III, Borrow Open. This is reflected in a single initial cash situation row. Stage 1 production activities also result in two levels of output (D and E) dependent on the state of nature at the end of stage 1.

Output from the first stage may be transferred to the next stage serving as an input there. Because the production levels for all stages were fixed by commitments made before production began, if a commitment was made to produce in stage 11, that production must proceed regardless of the outcome in stage 1. This means input requirements will be the same and hence the transfer level the

TABLE 1. STATES OF NATURE AND SUBJECTIVE PROBABILITIES OF STOCHASTIC FARM MANAGEMENT PROGRAMMING MODEL FOR FARM-RAISED CATFISH

Event	Description	Probability
Stage 1	Egg production—poor (33.6 lb/acre)[a]	.20
	—average (79.2 lb/acre)	.70
	—good (134.4 lb/acre)	.10
Stage 2	Fry production—poor (1.913 million)	.50
	—average (2.975 million)	.40
	—good (4.038 million)	.10
Stage 3	Fingerling production—poor (10.9 thous/acre)	.15
	—average (34.5 thous/acre)	.55
	—good (63.9 thous/acre)	.30
Stage 4	Food-fish production—poor (2.9 thous lb/acre)	.20
	—average (3.9 thous lb/acre)	.70
	—good (4.5 thous lb/acre)	.10

[a]Numbers in parentheses are yields associated with respective states of nature.

TABLE 2. SUMMARY OF RESULTS OF DISCRETE STOCHASTIC FARM MANAGEMENT PROGRAMMING MODEL FOR FARM-RAISED CATFISH

Expected Income ($000)	Probability Limit (%)	Rent (acres)	Egg (acres)	Fry (unit)	Fingerling (acres)	Food (acres)
41.5	50.00	0	0	1	40	0
41.4	25.00	0	10	1	30	0
41.3	21.50	0	15	1	25	0
41.1	20.00	0	20	1	20	0
40.9	10.00	0	25	1	15	0
40.4	5.00	0	25	1	10	5
40.3	4.00	0	20	1	10	10
39.8	2.00	0	20	1	5	15
37.1	1.00	5	20	1	5	10
33.8	0.01	10	20	1	0	10
33.8	0.03	10	20	1	0	10

same for all output levels from the previous stage. For each state of nature, independent recourse decisions may be made to buy and sell output. A decision may not be made to change the production level in the next stage.

The buy-and-sell activities (J and U) are translated onto two rows, Cash 1 and 2, in quadrant II. These two rows represent cash situations existing after the stage 1 outcomes are available for starting stage 11. In quadrant III, the debt plus interest on the single borrowing activity which initiated stage 1 is transferred to Cash 1 and 2 (F). Any saving would be treated similary (G).

Independent recourse for each of the two cash situations is allowed through further borrowing and saving before the initiation of stage 11. This recourse complements the buy/sell recourse in the production stage. It provides cash to meet the production commitment for stage 11 regardless of the stage 1 outcome.

If production occurs in stage 11, the cash needs for production will be registered in each of the two cash rows. There will again be two states of nature resulting in two output levels. And again, buying and selling activities will be translated into cash in quadrant I. The two output levels from stage 1 in addition to the two from stage 11 result in four possible combinations of states of nature. These are reflected in the four cash situation rows existing at the end of stage 11. The two independent borrowing-and-saving activities which initiated stage 11 are transferred, plus interest, each to one of the sets of two cash situations. Sell, save, and borrow activities associated with stage 11 are registered in the objective function. The coefficients are the cash value of the activities multiplied by the probabilities of their occurrence.

The four end-state cash rows feed into the chance-constraint module. The four cash situations plus any deviation below the risk-reference level must be greater than or equal to the target. The risk-reference level (TLEVEL) is internally determined.

In the THETA row the negative deviation is multiplied by the probability of its occurrence. In the two-by-two model these probabilities were equal. In the catfish model each of the 81 end states had a different probability. These probabilities were determined by the probability of the outcomes associated with each production stage (Table 1).

The TTHETA column sums the THETA row and multiplies the result (theta = *Pd*) by $-q^*$ in the SUFCON row, where $q^* = 1/L$ (Atwood). The goal, meeting fixed costs, is transferred into the SUFCON row from the production activities. The SUFCON row imposes the constraint that the risk reference level (t) satisfies equation (5). This guarantees that the probability of returns over variable costs failing to cover fixed costs does not exceed $L = 1/q^*$.

RESULTS

Results of the model are presented in Table 2. The top row is the linear programming (LP) solution that arises when deviation is unconstrained. The bottom row is the expected income that can be achieved with least nonzero risk allowed in the model. The intercept is the option of renting all pond acres.

The risk-income tradeoff is apparent as one moves down the rows of Table 2, with declining risk associated with declining income. The rank of alternatives by expected income is precisely the reverse of rank by risk. Fingerlings are most profitable, but the dispersion of

expected income represented by the total absolute deviations is also the highest or least desirable. Eggs rank second in both categories; food-fish has the least dispersion, but also the lowest returns per acre. Fry production has no risk in the sense that negative returns are never experienced under the states of nature considered in the model. It does not compete for acreage and is constrained only by hatchery capacity. Thus, important considerations not addressed in the model that limit fry production are the technical skills and intensive labor requirements.

The LP solution allocates all acreage to fingerlings, the "no deviations" solution allocates all acreage to rental, and the least non-zero risk solution uses all acreage for food-fish production.

As the probability limit of falling below the goal is decreased, the solution initially replaces one fingerling pond with a brood pond. As the chance constraint is further decreased, the brood pond will become a food-fish pond. With additional reduction in allowable probability of not reaching the goal, the solution will eventually have to drop another fingerling pond, replacing it with eggs. Each time the fingerling activity is decreased, the egg activity is able to come into solution at a higher level. The cycle of dropping one fingerling pond with initial replacement by eggs and then increasing substitution of food-fish for eggs repeats itself until all acres are used for food-fish.

The middle rows (4%–20%) approximate the average mix of activities of diversified farmers obtained in the survey discussed earlier. The average diversified fish farmer in Alabama had approximately 10 acres of brood, a hatchery operating at capacity, 20 acres of fingerlings, and 10 acres of food-fish. According to the estimates in the model, he would be making in the range of $41.1 thousand to $40.4 thousand annually (not including repayment of land purchase cost). The least risky non-rental option is to produce only food-fish, a strategy chosen by approximately two-thirds of Alabama catfish producers (Hatch et al.).

As with all modeling efforts, this study is not without its limitations. The study only examines production variability while using fixed product and factor prices. Future studies examining price-yield risk might generate differing results or income-risk tradeoffs.

In many areas of Alabama, markets for the catfish products are limited. The prices used were primarily from the western part of the state where catfish production is concentrated. In other areas, uncertainty as to market availability and net prices will also likely affect production decisions. Modeling uncertainties about market availability will likely be difficult given the limited market information available.

SUMMARY AND CONCLUSIONS

A discrete stochastic farm management model with chance constraints was developed using data from farm-raised catfish in Alabama. It was designed to assess the risk-income trade-offs associated with buying, selling, and producing at alternative fish growth stages (eggs, fry, fingerling, or food-fish). The mix of activities selected by the model for the intermediate levels of risk (5%–20%) approximated the average mix of diversified farmers (those producing more than one stage) obtained in a farm survey. Two-thirds of all catfish farmers chose the least non-zero risk solution, producing only food-fish.

The model traced through outcomes of sequential decisions while satisfying safety-first probability limits on ending cash balances. Recourse actions between stages were limited to sale of production in excess of endogenously determined transfer levels or purchase of inputs to supplement the next production stage. Discrete choice variables were used to reflect the fixed nature of pond facilities. Deviations were based solely on yield variability and calculated from target levels based on probability limits and an endogenously determined goal.

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