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FORMULATION OF BROILER FINISHING RATIONS BY QUADRATIC PROGRAMMING

Bill R. Miller, Ronaldo A. Arraes, and Gene M. Pesti

Abstract

Least cost feed mix by linear programming (LP) is a standard economic analysis in the poultry industry. A significant body of nutrition knowledge is now contained in the constraint set of industry LP models. This knowledge might be merged into an improved economic model that contains production response information. Analysis using a quadratic programming model indicated that a leading broiler firm could have improved economic efficiency by increasing protein density and reducing energy density of broiler finisher feed. If applicable industry wide, similar savings could be as high as \$120 million per year.

Key words: quadratic programming, production function, broilers, experimental design, feed mix.

In general, the main objective of a firm is to maximize profits thereby implying that costs should be minimized for the output produced. This means that a firm may need to improve technical and/or economic efficiency in production (Seitz). The broiler industry is not an exception to this general rule as seen by the evolutionary improvement of efficiency in broiler production (Henson). Yet, there are continuing problems that hinder improvements.

One potential problem is that the industry has concentrated its nutrition efforts around finding the least cost per pound of feed. Concepts of basic nutritional requirements have been used to set right-hand-sides of linear programming (LP) problems designed to find proportions of alternative ingredients in a least cost feed mix. Little thought has been given to the concept that nutritional requirements might be set in relation to growth response to various nutritional levels

and relative to costs of nutrients. Such trade-offs can be determined in a quadratic programming (QP) framework.

The primary objective of this paper is to demonstrate that a model including the production response to basic nutrients may replace LP as a standard economics analysis in the broiler industry. The QP model presented revolves around least cost of broiler output in response to protein and energy input. Feed containing these nutrients is the major input in broiler production.

The proportion of feed cost to total cost of broiler production is about 73 percent (Arraes). Hence, for any given forward price for broilers, minimizing feed cost per pound of broiler gain is of primary concern to the broiler industry. Forward contracting is common in the industry as is specification of the average size bird to meet contract demands. Thus, how to derive the array of feedstuffs (feed formulas, diet, or ration) for least cost production while maintaining the minimum nutrient requirements for maximum technical efficiency continues as the main problem of economic efficiency in poultry nutrition. There is a large variety of feedstuffs that can be used as sources of protein and metabolizable energy which are the fundamental nutrients needed for chicken growth. An appropriate choice of feedstuffs is essential to achieve efficiency.

For example, corn and soybean meal have been the two principal feedstuffs used in feed-mix formulas due to their high nutrient density, relatively low prices, and availability. In general, they represent more than 80 percent of the ration composition as currently derived by the industry. However, prices of corn and soybean meal have recently shown increased variation within short periods of time (*Georgia Agricultural Facts*).

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These feedstuff prices are crucial determinants of current least cost feed rations using LP. Choosing amounts of corn and soybean meal in a QP ration according to their price levels and marginal productivities of nutrients might lead to more economically efficient broiler production. It is an hypothesis of this study that the broiler industry does not produce broilers as efficiently as it could. Allison and Baird, and Chao, using different techniques have found ration formulas with lower costs per unit of gain than LP rations employed by the broiler industry.

Whether the specifications of current LP models lead to the least cost per pound of gain for broilers is not known. Certainly the LP technique succeeds in getting lower feed cost, but certainly it fails by not taking into account the performance of the bird. Brown and Arscott state that marginal analysis of production economics theory would seem to afford a better approach than the LP cost model. The way marginal analysis has been applied to livestock production has caused major problems because of the type of feedstuffs that were prespecified for the analysis. For instance, the work done by Heady and Dillon, using marginal analysis on broilers, specified corn and soybean as the feedstuffs. Consequently, the optimum solution is a function of the feedstuffs and their prices only. This method is inaccurate because the use of feedstuff does not consider response to the fundamental nutrients required by the boiler for growth. Protein (P) and energy (E) are fundamental nutrients but typically, since they are a component of each feedstuff, their prices are not available. This study overcomes that problem.

MODELING APPROACH

Two important transformations were required to use the production response data. First, since there are no price data for the P and E determinants of growth, the response function was transformed from P and E space into feed ingredient space (see Appendix). Second, the Appendix explains the central argument of how the poultry industry's standard LP analysis of feed mix is used as a set of constraints on the production response data. Thus, appropriate experimental designs can produce new information for a new economic model of least cost production. The new model (QP) retains all of the currently known nutrition knowledge in LP but incor-

porates price and productivity data thereby pointing to improvements in economic efficiency.

MODEL TESTING

Quadratic response surfaces for various protein and energy levels may be derived by fairly common experiments. Feather sexed day-old male chicks were used in an experiment in Georgia for this purpose. These chicks were randomly assigned to 55 pens with 42 chicks per pen. The birds were fed *ad libitum* (as needed) with eleven different diets made up of five protein densities (17.5, 18.63, 19.57, 20.88, and 22.0 percent) and five metabolizable energy densities (1,315, 1,372, 1,429, 1,486, and 1,542 kcal./lb.), Figure 1. The experiment was designed so that there were five replicates for each ration.

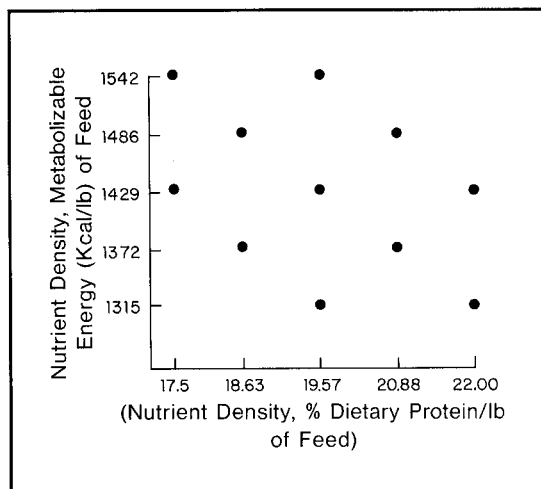


Figure 1. Eleven Diet Combinations Used in an Experiment to Estimate Response of Broiler Live-weight to Protein and Metabolizable Energy, Georgia, Spring, 1982.

TABLE 1. QUADRATIC RESPONSE OF BROILER TO CUMULATIVE AMOUNTS OF PROTEIN AND ENERGY, GEORGIA, 1982

Source of variation	Coefficient estimate	Standard error
a ₀ : Intercept	0.041988	0.0189
a ₁ : Protein intake (kg)*	1.457695	0.0118
a ₂ : Energy intake (MJ)	0.026180	0.0016
a ₃ : (Protein intake) ²	-1.758822	0.0027
a ₄ : (Energy intake) ²	-0.000423	0.0002
a ₅ : Protein-energy interaction	0.039050	0.0200
R ² =		.99

* Protein and energy intakes include those during the first 3 weeks of the broilers' lives (.206 kg and 11.70 MJ, respectively). For example, predicted weight for chicks fed 220 g protein/kg and 12.13 KJ M.E./g:

$$= 0.042 + 1.457695 (0.531 + 0.206) - 1.758822 (0.531 + 0.206)^2 + 0.02618 (29.29 + 11.70) - 0.000423 (29.29 + 11.70)^2 + 0.039050 (0.531 + 0.206) (29.29 + 11.70) = 1.703 \text{ kg: (Observed} = 1.751 \pm 0.008).$$

Dillon has argued that autocorrelation is usually present in experiments of this type and preliminary tests suggested this. A first order autocorrelation process using Durbin's method (Kmenta, p. 289) was used to correct the OLS model, Table 1.

The quadratic response is a concave function and the statistical evidence is strong that this is a good description of broiler growth. The signs of the coefficients, significance of the coefficients, and magnitude of R^2 are consistent with expected results and from the standpoint of economic theory.

The production response, when transformed into the QP objective function, implies that liveweight is a function of cumulative nutrition intake. Intake is expected to be a function of size of bird (growth) and thereby implying some possible joint dependence between growth and feed consumption (Burt). Models specifying joint dependence were not estimated but a check was made to see if there were differences in consumption rates by birds on different rations. In all cases, consumption rates and days to market of birds on experiment were within industry expectations. Thus, if there was significant joint dependence, it did not result in production response parameters that produced birds outside the time frame ordinarily expected for a given size bird.

An appropriate QP model was constructed by setting a cost constraint equal to 71 cents per bird, which was an average in a North Georgia broiler firm (equation 16a in the Appendix). Average feedstuff prices in the cost equation were collected from the same firm. Nutrition constraints were constructed from the broiler firm data set up for LP and transformed to appropriate constraints for maximizing the transformation of production response in Table 1. Restrictions on nutrient density of mixed feed were set in ranges reflecting the actual specifications used in the broiler feeding experiment (The transformation of nutrient density is explained in the Appendix). Typical industry restrictions on density of protein and energy were revised to be greater than or equal to zero. The P and E levels were then determined by ingredients found in solution of the model. Production response was explicit in the feed formulation and the least cost of production analysis required only the LP data, the response equation, and the appropriate transformations.

QP PROGRAMMING RESULTS

Since the problem is formulated in feed ingredient space, the results in terms of ration mix (or diet formulation) are similar to linear programming. The feed ingredients that maximize broiler weight at 71 cents per bird are compared in Table 2 with the linear programming feed mix constructed from the data used in the QP model. The QP mix used the same ingredients and satisfied the same nutrient density constraints as LP except it used the amounts of protein and energy that would maximize growth at the 71 cent level. It is apparent from Table 2 that the QP model is showing some trade-off between protein and energy. Less corn was used in the QP solution but more soybean meal was used in comparison with LP. Other sources of energy and protein were likewise affected. Fat as a source of energy was decreased; protein supplement and feather meal (protein source) were increased in relation to the LP solution. Prices of alternative sources of energy and protein played a part. Dried whey, for example, was reduced as other protein sources increased. Wafer meal (energy source) increased while other energy sources decreased.

The maximum weight of a bird produced for 71 cents on the diet in Table 2 was projected by the growth response equation to be 1.84 kg or just more than 4 pounds liveweight, Table 3. Data were not available to compare directly with LP but average feed costs per bird by the firm using the ration in Table 1 were 72 to 75 cents for about the same size bird. Such comparisons could be

TABLE 2. OUTPUTS (DIET FORMULATIONS) FROM LINEAR (LP) AND QUADRATIC (QP) PROGRAMMING MODELS

Ingredient	LP ^a QP	
	(g/kg of Mixed feed)	
Corn	584	540
Soybean meal	144	185
Animal fat	18	10
Protein supplement	31	38
Blood meal	7	8
Ground limestone	7	8
Deflourinated phosphate	8	5
Choline chloride (350 g/kg)	20	20
Methionine (MHA)	1	2
Feather meal	58	78
Dried whey	43	23
Wafer meal	92	97
Vitamin premix	1	1
Fixed ingredients ^b	4	4
Trace mineral premix	1	1

^a Based on National Research Council (1977) constraints for 3 to 6 week old broilers, QP satisfies the same constraints except for P and E.

^b Antibiotics and anticoccidial drug.

TABLE 3. BROILER RESPONSE EXPECTED FROM QUADRATIC PROGRAMMING OF RATION FORMULATION, GEORGIA, SPRING, 1982*

Item	Response
Average liveweight (kg.)	1.84
Feed efficiency (kg. feed/kg. bird) (Conversion)	1.91
Feed consumption (kg. feed/kg. bird)	3.52
Total feed cost (cents/broiler) or C (Constraint)	71
Feed cost per kg. of broiler (cents/kg.)	38.546
Days to market	44.2
Protein	
Density (percent)	23.5
Intake (kg./bird)	.827
Metabolizable energy	
Density (Kcal./lb.)	1,437
Intake (MJ)	46.217

* Price levels for April 12, 1982, were 12.94 cents/kg. for corn and 23.76 cents/kg. for soybean meal.

questionable, however, because the LP results were achieved under average farm conditions while the QP results were from birds grown on an experiment station farm.

Feed efficiency (conversion) for the QP experiment was judged as excellent, 1.91 in comparison to an industry expectation of less than 2.0, Table 3. Days on feed (44.2) were projected from the experimental data and were about the average of the industry for this size of bird. The percentage of protein

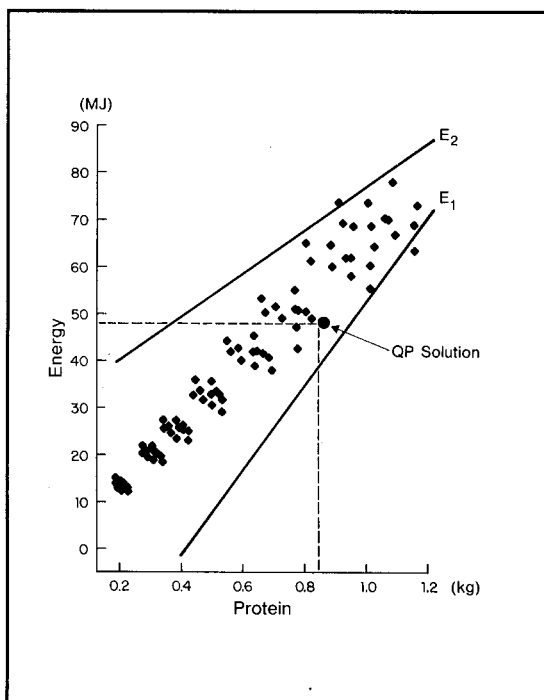


Figure 2. Data Set of Cumulative Amounts of Protein and Energy from Broiler Growth Response Experiment, Isoclines from the Resultant Quadratic Growth Function and the Economic Optimum Combination of Protein and Energy in One QP Solution of a Broiler Diet Problem.

in the QP ration was higher (23.5) than that used in April, 1982 by the industry (21.7). As expected, because of substitution possibilities, the density of energy was lower in QP (1,437 Kcal./lb.) compared to 1,480 Kcal./lb. that the industry was using. The combination of energy and protein predicted by QP was just outside of the range of experimental combination, Figure 1, and was well within the region of technical feasibility, Figure 2, where the nutrient densities in the experiment are shown on a cumulative basis of nutrient intake. The region of technical feasibility was defined by the E_1 and E_2 isoclines, Figure 2, calculated from the growth response function estimated for the study.

SUMMARY AND CONCLUSIONS

Broiler growth response functions portrayed in quadratic form in relation to cumulative protein and energy intake are highly descriptive of broiler growth. Furthermore, this response function is simply one more piece of information that is easily additive to data now being used by the broiler industry in linear programming of broiler rations. Adding the response function requires feed formulation to be constructed as a quadratic programming (QP) model which should be well within the capabilities of computers now in use. Results, in terms of identifying the least cost of production feed ingredients, will appear to the user to be the same from QP as from linear programming (LP) of least cost feed. However, the least cost of production by QP will be constructed in relation to explicit growth response expectations as opposed in implicit or unstated growth response underlying the use of LP.

Expected benefit from QP is broiler production at least cost per pound of gain as identified by knowledge of how to set maximum growth specifications of protein and energy levels in relation to feed prices. Lowering the cost of broiler production by even 1 cent per pound would represent a cost saving of \$120 million annually for the industry. Further research is needed to verify the expected performance of broilers on high protein rations. The economically efficient level of energy projected in this study (1,437 Kcal./lb.) was slightly lower than that used by the poultry industry (1,480 Kcal./lb.), but the percent protein in poultry diets that produced the least cost per pound of gain was much higher (23.5 percent versus 21.7) than that fed by the industry.

Transforming the growth response (W) into feed ingredient space allows all of the information derived from price and nutrition work with LP to be applied directly in the expanded format of QP. This approach opens the door to much future research. Two immediate areas are:

(1) replication of the production function response, and

(2) use of simultaneous equation methods in evaluating possible joint dependence between growth and consumption.

Other areas include development of additional production functions by sex of bird, quality of bird (carcass fat), temperature conditions, and bird density of housing. Additional economic modeling will be needed on least cost per pound of gain per unit of time and to include all inputs to production.

APPENDIX

Broiler Production Functions

The production response of broilers to protein and energy was derived for this study by Arraes for a two-input quadratic production function. Heady and Dillon and Brown and Arscott, early pioneers in using marginal analysis of production response to crops and livestock, define a typical two-input quadratic production function as:

$$(1) W = f(P, E) = a_0 + a_1P + a_2E + a_3P^2 + a_4E^2 + a_5PE,$$

where, in this study: W is output (liveweight broiler) and P and E are nutrient inputs (protein intake and metabolizable energy intake), respectively.

The linear and quadratic part of the quadratic production response accounts for the diminishing marginal productivity of each input. Also, an interaction term (PE) appears in the equation to incorporate the effect of the marginal physical product of one input being a function of the level of the other input. The marginal product in broiler weight from a small increment in protein may depend on the level of energy that the broiler is consuming. That is:

$$(2) (MPP)_p = f_p(P, E) = a_1 + 2a_3P + a_5E = \text{marginal product of protein, and}$$

$$(3) (MPP)_e = f_e(P, E) = a_2 + 2a_4E + a_5P = \text{marginal product of energy.}$$

The function needs to be concave in the region of P and E used to define consistent points of technical and economic efficiency.

The conditions for concavity of the function are given by;

$$(4) f_{pp} = 2a_3 < 0,$$

$$(5) f_{ee} = 2a_4 < 0, \text{ and}$$

$$(6) f_{pp} \bullet f_{ee} - f_{pe}^2 = 4a_3a_4 - a_5^2 > 0.$$

Equations (4) and (5) imply that the coefficients a_3 and a_4 must be negative. From equation (6), no expectation can be inferred concerning the sign of a_5 , unless prior information is provided. The absolute value of a_5 would depend on the magnitude of a_3 and a_4 and satisfy equation (6), or, $|a_5| \leq 2\sqrt{a_3a_4}$. Given the conditions stated for equation (6), a concave broiler response can be found with either a negative or positive sign for a_5 . It is also reasonable and necessary to expect that a_1 and a_2 are positive for the expectation of positive marginal productivities. Otherwise, increased consumption might not produce growth.

By fixing output level at W_0 and rearranging equation (1), the isoquant equation for W_0 is derived:

$$(7) a_3P^2 + (a_1 + a_5E)P + (a_2E + a_4E^2 - W_0 + a_0) = 0.$$

Equation (7) can be described as a simple quadratic equation in P (or, similarly in E). By solving equation (7);

$$(8) P = [-(a_1 + a_5E) + ((a_1 + a_5E)^2 - 4a_3(a_2E + a_4E^2 - W_0 + a_0))^{1/2}] / 2a_3.$$

Isoquants of a concave function described by equation (8) are convex to the origin only in a diamond-shape area of technical efficiency shown in Figure 3, between the P and E axis defined by the lines $E_1 = -(a_1 + 2a_3P) / a_5$ and $E_2 = -(a_2 + a_5P) / 2a_4$. Outside of this area, production could be increased by reducing one or more inputs. In the region of technical efficiency, a point of economic efficiency can be found on an isoquant tangent to a given total feed cost. Since there are a number of possible levels of total feed cost, an investigation of the expansion path is required.

A point of economic efficiency occurs where the marginal rate of technical substitution equals the input price ratio, i.e.,

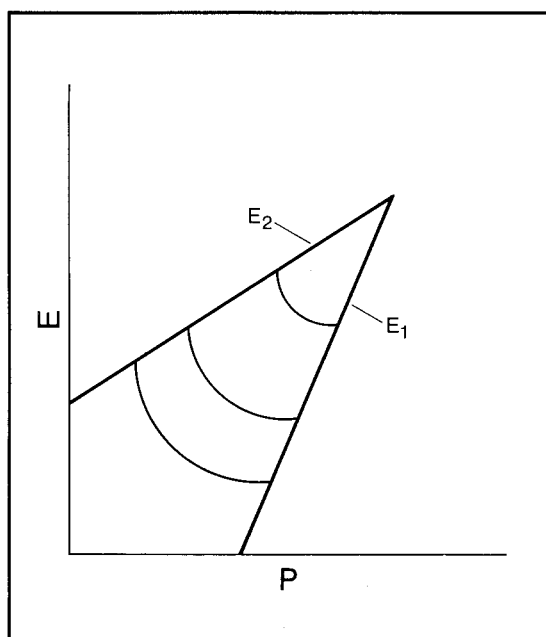


Figure 3. Possible Isoclines of Quadratic Production Function for Broiler Response.

$$(9) (MRTS)_{pe} = \frac{(MPP)_p}{(MPP)_e} = \frac{r_p}{r_e},$$

where r_p and r_e are the prices of protein and metabolizable energy, respectively. By substituting the marginal products of P and E into equation (9), it follows that,

$$(10) \frac{a_1 + 2a_3P + a_5E}{a_2 + a_5P + 2a_4E} = \frac{r_p}{r_e}.$$

Solving equation (10) for P,

$$(11) P = \frac{(a_2r_p - a_1r_e)}{(2a_3r_e - a_5r_p)} + \frac{(2a_4r_p - a_5r_e)}{(2a_3r_e - a_5r_p)} \cdot E$$

or simply,

$$(11a) P = K_1 + K_2E.$$

Equation (11) shows all combinations of P and E to achieve economic efficiency for alternative levels of total fixed feed cost. In other words, the expansion path from the quadratic production function is a positively sloped straight line not passing through the origin ($K_1 \neq 0$, $K_2 > 0$). The ratio of P and E is different for every level of output. This indicates there must be a trade-off between

protein and energy as the bird gets heavier (higher isoquants). This is an important concept because the industry practice of using linear programming for broiler feed formulation assumes a fixed protein-to-energy ratio within any given formulation. The current industry use of LP least cost feed mix does not allow the protein energy ratio to be a function of feed prices or the productivity of protein and energy. There are other optimization models, including dynamic optimization and LP models of the block diagonal type, that might incorporate variability of protein to energy in the analysis. However, it is not within the scope of this paper to compare all models. Rather, the goal is to make an improvement in industry practices by adding growth response information to the wealth of linear programming data that are currently the industry standard.

Transformation to Ingredient Space

Finding market prices for protein intake (r_p) and metabolizable energy (r_e) is a difficult, if not impossible, task. Thus, the proposed model needs to be transformed. It is suitable to write the quadratic equation (1) as a function of all available feedstuffs that may be used to provide protein and metabolizable energy for broilers.

In a matrix format, the quadratic form of equation (1) becomes:

$$(12a) W = a_0 + [a_1 \ a_2] \begin{bmatrix} P \\ E \end{bmatrix} + [P \ E] \begin{bmatrix} a_3 & 1/2a_5 \\ 1/2a_5 & a_4 \end{bmatrix} \begin{bmatrix} P \\ E \end{bmatrix}$$

$$(12b) w = a_0 + w^*$$

$$(12c) w^* = A_1P + P' A_2P.$$

The transformation from the nutrient space (P, E) into the feed ingredient (X) space is made through the coefficients (content) of protein (M_p) and energy (M_e) in each ingredient (x_j) in X where: $j = 1, \dots, n$; $p = 1, \dots, n$; $e = 1, \dots, n_1$; and

$$(13) \begin{bmatrix} P \\ E \end{bmatrix} = \begin{bmatrix} M_p \\ M_e \end{bmatrix} \begin{matrix} X \\ (n \times 1) \end{matrix} \quad \begin{matrix} (2 \times 1) & (2 \times n) \end{matrix}$$

¹ Coefficients of the vectors M_p and M_e , or the amounts of P and E per unit of feed ingredient, can be supplied by the National Research Council, Nutrient Requirements of Domestic Animal, No. 1, Poultry, 1977, or any major feed mixing firm.

Substituting this relationship into equation (12c), it follows that:

$$(14) \quad w^* = [a_1 a_2] \begin{bmatrix} M_p \\ M_c \end{bmatrix} + X' \begin{bmatrix} M_p \\ M_c \end{bmatrix} X.$$

The production response function is thus expressed as a transformed function of the n feed ingredients (X). Since the prices (r_j) of all x_j in X are well defined, equation (14) can be analyzed to find an exact point of economic efficiency in the feasible region of growth, Figure 3.

Conceptual Source of QP Feed Formulation

When two or more inputs (n -feeds) are used in a production process, the efficiency problem could be solved by means of the Lagrange technique. The problem might be formulated as follows:

Maximize transformed production response:

$$(15a) \quad W^* = f(x_1, x_2, \dots, x_n),$$

subject to a given feed cost:

$$(15b) \quad \bar{C} = \sum_{j=1}^n r_j x_j.$$

The Lagrangean function is:

$$(16) \quad L = f(x_1, x_2, \dots, x_n) + \lambda(\bar{C} - \sum_{j=1}^n r_j x_j) \text{ and}$$

$$\begin{matrix} B & X & \geq & K \\ (m \times n) & (n \times 1) & & (m \times 1) \end{matrix}$$

$$\begin{matrix} T & X & \geq & 0 \\ (m-1 \times n) & (n \times 1) & & (m-1) \end{matrix}$$

the Kuhn-Tucker sufficiency conditions for optimality are well known (Chiang).²

The preceding approach, which at this point is a trivial broiler diet problem, does help to specify a more general QP problem of diet formulation. The Lagrangean approach formulates an important economic question: if a broiler producing firm has available only \bar{C} dollars to spend on feed cost/broiler, what is the least cost combination of feed inputs (x_j) to formulate a ration and what is the expected maximum broiler liveweight (W)? Solution of the problem within the region of the concave function specified in Figure 1 shows maximum total broiler liveweight that can be obtained for cost \bar{C} which readily translates into least cost per pound of broiler at cost level \bar{C} . Further, since broiler producers generally assume constant forward contract prices, the solution also translates into maximum net returns above feed cost for the specified level of cost. Parametric change in \bar{C} would trace out a range of technically feasible costs, liveweight, net returns, and, perhaps most importantly, the feed mix and associated specifications of P and E . Specifications of P and E would be a function of feedstuff prices and expected growth response. This suggests an improvement over current *ad hoc* methods of specifying P and E levels in LP.

However, translating this simple approach into a complete and general feed formulation problem for the feed industry requires careful attention to additional concepts of technical feasibility related to nutrients required for growth. The literature and history of poultry nutrition require that additional restrictions (other than cost) must be specified to produce maximum growth response to P and E . In other words, other nutritional requirements and growing conditions must be fixed, at least within specified ranges, when the production response to P and E is determined. These additional restrictions have been easily incorporated into LP least cost feed mix programming and must also be applied in feed formulation by a QP model.

QP Feed Formulation for Least Cost of Production

Perhaps the best way to observe how production response is constrained by nutrients

² The sufficiency conditions show the solution to be maximum when W is a concave function and the constraint set is convex.

other than P and E is to briefly examine other nutrients in linear programming currently used in poultry nutrition. In feed formulation for broilers, linear programming is used to minimize feed cost per pound of feed subject to a set of nutrient requirements:

$$(16a) \text{ Minimize: } \sum_j r_j x_j \quad j=1 \dots n,$$

$$(16b) \text{ subject to: } \sum_j b_{ij} x_j \begin{matrix} \geq \\ < \end{matrix} k_i \quad i=1 \dots m, \\ x_j \geq 0,$$

where b_{ij} is the amount of the i^{th} nutrient in a pound of the j^{th} feed (x_j) and k_i is the requirement of the i^{th} nutrient per pound of mixed feed. Current feed formulation practices include more nutrient specifications (values of K_i) than just protein and energy. Methionine, lysine, sodium, and fiber are a few values of k_i that are common.

Since k_i is a rate or ratio, poultry nutritionists discuss this rate as "nutrient density." Moreover, in recent years, they have formulated variations of the standard LP (equations 16a and 16b) to allow nutrient density of a pound of mixed feed to be a function of various notions of feed processing and distribution costs (Pesti et al.). However, none of the variations include the concept that the nutrient density of protein and energy in feed should be a function of their relative prices and productivities. In fact, nutrient density (of P and E) is generally desired to be fixed within ranges by nutritional concepts. Pesti et al. have shown that the feed mix industry appears to have no recognition that some LP variations allow nutrient density to be a function of economic variables. Thus, one result of this report should be a more general understanding of the role of price in setting nutrient levels (density). With very little change in current procedures, the QP analysis proposed will go well beyond LP but use all of the data and nutrition specifications of LP except P and E density.

To construct a QP model to maximize equation (14), a simple transformation needs to be made on the right-hand-side (RHS of equation 16b) of the LP formulation. All of the nutrient densities, except protein and energy, must remain fixed in specified ranges by the least cost of production model. The densities (nutrient per unit of feed) are easily main-

tained by making the amount of mixed feed (MF) an endogenous variable in the QP model. A unique feature of the LP diet problem is that the right-hand-side (RHS) of the problem contains the coefficients of the MF variable. The RHS vector for LP defines the required density coefficients of one unit of the mixed feed (MF). Thus, substituting the MF variable into equation (16b) results in a reduced

form. The system $\begin{matrix} \beta & X \\ (m \times n) & (n \times 1) \end{matrix} \begin{matrix} \geq \\ < \end{matrix}$
 $\begin{matrix} K \\ (m \times 1) \end{matrix}$ is transformed into

$$\begin{matrix} T & X \\ (m-1 \times n) & (n \times 1) \end{matrix} \begin{matrix} \geq \\ < \end{matrix} \begin{matrix} O \\ (m-1) \end{matrix}$$

As an example consider:

$$(17) \begin{matrix} b_{11}X_1 + b_{12}X_2 \geq k_1 \\ b_{21}X_1 + b_{22}X_2 \geq k_2 \\ X_1 + X_2 = 1 \end{matrix} \quad \text{or } BX \geq K,$$

but the right-hand-side defines one unit of MF. Thus,

$$(18) \begin{matrix} b_{11}X_1 + b_{12}X_2 \geq k_1 MF \\ b_{21}X_1 + b_{22}X_2 \geq k_2 MF \\ X_1 + X_2 = MF, \end{matrix}$$

and by substitution of the last equation into all others:

$$(19) \begin{matrix} (b_{11} - k_1)X_1 + (b_{12} - k_1)X_2 \geq 0 \\ (b_{21} - k_2)X_1 + (b_{22} - k_2)X_2 \geq 0 \\ \text{or } TX \geq 0. \end{matrix}$$

While the reduced form is of little use in LP, it greatly facilitates the QP model by allowing the levels of X to be a function of their prices, productivity of P and E, and all other required nutrient densities. Now, if the objective function of the LP model, equation (16a), is constrained to some constant cost per bird (\bar{C}) and appended as the last row following $TX \geq 0$, the result is a set of con-

straints that allows bird growth transformed to feed ingredient space to be maximized subject to nutrient density restrictions on the feed mix and a constraint on feed cost.

Thus, an appropriate least cost of production model is:

$$(20) \text{ Max: } w^* = A_1 \begin{bmatrix} M_p \\ M_e \end{bmatrix} + X' \begin{bmatrix} M_p M_e \end{bmatrix}$$

$$A_2 \begin{bmatrix} M_p \\ M_e \end{bmatrix} X$$

$$\text{subject to: } TX \geq 0 \text{ and}$$

$$rX = \bar{C}.$$

The density coefficients of protein (P) and energy (E) may be greater than or equal to any required density in the mixed feed and this will be reflected in the reduced form matrix (T). Solution of the QP problem with transformed growth response as the objective function will then yield the amounts of ingredients from which densities for P and E can be calculated that produce maximum liveweight for the specified cost \bar{C} , i.e., the economically efficient contribution of P and E. The result is a general QP problem for least cost of broiler production and associated feed formulation.

Thus, the objective function and nutrient constraints of any current industry linear programming model for a feed mix form the

major part of the constraint set for examining the quadratic objective which is growth response to the basic nutrients consumed. Maximizing liveweight gain subject to parametric changes in the right-hand-side value of \bar{C} traces out a restricted expansion path of economic efficiency. The classical expansion path results indicated by equations (9), (10), and (11) are not generally obtained because of the additional nutrient density restrictions of the type described in $TX \geq 0$ that arise from

usual technical restrictions used by the poultry industry. These additional restrictions mean that each solution of the quadratic programming problem finds a point on a restricted expansion path. Perhaps more importantly, the optimum levels of P and E and the feed mix associated with least cost per pound of broiler output can be estimated for any feasible cost (\bar{C}) per bird. Furthermore, the optimum levels of P and E are a function of feed prices and the observed production response (W). An economic trade-off between nutrient densities of P and E occurs whenever feedstuff prices change.

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