Complementarity of Labor Market Institutions, Equilibrium Unemployment and the Propagation of Business Cycles

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Michael C. Burda    Mark Weder*

Abstract

This paper evaluates complementarities of labor market institutions and the business cycle in the context of a stochastic dynamic general equilibrium model economy. Matching between workers and vacancies with endogenous time spent in search, Nash–bargained wages, payroll taxation, and differential support for unemployed labor in search and leisure are central aspects of the model. For plausible regions of the policy and institutional parameter space, the model exhibits more persistence than standard RBC models and can exhibit indeterminacy of rational expectations paths without increasing returns in production. Furthermore, labor market institutions act in a complementary fashion in generating these effects.

1 Introduction

The high level of unemployment in Europe remains a difficult subject for economists. The number of candidate explanations seems to have risen faster in recent years than the phenomenon itself. Since the publication of the 1994 O.E.C.D. Job Report, it is received wisdom that government policies and more generally ”institutions” have played a central role in the European unemployment problem. Here one includes unemployment insurance, collective bargaining mechanisms, social assistance, job protection and other labor market regulation, labor and capital taxation, and product market regulation. A central difficulty remains, however, that these institutions

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have been around since the late 1960s, when unemployment in Europe was strikingly low: why are things so different now compared to then? For this reason, more promising approaches have stressed interactions of institutions with exogenous events, or have invoked models with increasing returns to scale and multiple equilibria.

Recent empirical work by Blanchard (1999) and Blanchard and Wolfers (2000) attributes a large component of variance in unemployment to interactions of institutions with shocks such as the oil price increases of the 1970s and the slowdown in total factor productivity. The theoretical underpinning for these interactions is lacking, however.

In related theoretical papers, Coe and Snower (1997) and Orszag and Snower (1998) have stressed labor market institutions in affecting unemployment. An important conclusion of this research is that modest reforms in isolation may have little impact, as long as other institutions remain unchanged. As these analyses are static, the dynamics of such complementarities and their interactions with the rest of the macroeconomy – including the business cycle – are not well-understood. It is seldom noted that rises in European unemployment have generally occurred at business cycle frequencies, making it a cyclical as well as a structural phenomenon. In any case it would appear imperative to model unemployment as the outcome of an equilibrium process, given that the largest increases occurred at least two decades ago. More recently, Prescott (1999) has echoed the view that labor market institutions and the “rules of the game” may be essential to explaining highly persistent phenomena like the Great Depression in the United States, the current bust in Japan or high unemployment in Europe.

Our paper investigates potential complementarities of labor market institutions in the context of a dynamic stochastic general equilibrium model. We base the investigation on the expectation that the functioning of the labor market – as clearinghouse for the most important factor of production – will affect business cycle dynamics. Our paper takes this issue seriously by giving up assumptions that labor markets are perfectly competitive or can costlessly replicate the social optimum, as in Merz (1995) and Andolfatto (1996). We study a number of institutions which are suspected of influencing the steady state and the dynamics of economies, including the generosity of unemployment insurance, the subsidy of non-search unemployment (social welfare or unconditional unemployment assistance), the efficiency of job matching, labor taxation, and wage bargaining. In particular, we explicitly distinguish between unemployment insurance – a subsidy to job search – and social assistance, which is a subsidy to leisure (time spent not working and not searching). In doing so, we stress the difficulty of distinguishing between those who search and those who do not. One key finding is that this “misclassification rate” of leisure as search unemployment - or in another interpretation the rate of moral hazard -

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1For examples see Blanchard and Summers (1987), and Ljungqvist and Sargent (1998).
plays a central role in determining the dynamic properties of the model (namely the possibility of endogenous cycles). In addition, we study the role of bargaining power of workers and capital in wage determination, deviating from competitive remuneration. Finally, we study the model under both constant returns in production conditions usually employed in real business cycle (RBC) analysis as well as under increasing returns, to determine whether complementarities are operative between these aspects.

It is already known that taxation can induce multiple equilibria in static (for examples, see Blanchard and Summers, 1987 or Burda, 1994) and indeterminacy in dynamic (Schmitt-Grohe and Uribe, 1997) settings. By considering labor taxation, we explicitly allow for this interaction as well, in particular its effect on endogenous search intensity by influencing the net (after tax) gain to work. Our model allows us to analyze explicitly the consequences of labor market reform as has been already done in the context of the Mortensen-Pissarides model (Millard and Mortensen, 1997, or Mortensen and Pissarides, 1996).

Intimately related to this issue is the nature of unemployment. As Lucas (1977, 1978, 1980), Pissarides (1990) and others have argued, the distinction between ”voluntary” and ”involuntary” unemployment is ill-defined if not vacuous. The old notion of involuntary unemployment as a state in which workers are ready to work at some going wage (Keynes) does not do justice to the nature of the phenomenon. There are both involuntary and voluntary aspects of all unemployment. This paper is agnostic as to the actual unemployment we observe, endorsing a concept of equilibrium unemployment which relies on matching frictions and search, but at the same time recognizes that the vast majority of unemployed do not choose this state.2

The central results of the paper can be summarized as follows. We find that important regions of the parameter space in this general equilibrium model are associated with both high output persistence as well as indeterminate dynamics or sunspot equilibria. In particular, a higher ”misclassification rate”, or the fraction of leisure which is compensated as search unemployment, is associated with indeterminacy3. At the same time, model indeterminacy can be generated by higher replacement ratios, holding the misclassification rate constant. Our findings of complementarity extend also to the division of match surplus in the economy. Further, we show that allowing the presence of modest increasing returns in production makes indeterminacy more likely and improves the match of the model’s times series to those of actual economies.

The paper is organized as follows. In the next section, the main features of our dynamic general equilibrium model are described. In particular, the maximization problem of representative households and firms are used to derive aggregate demand

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2The variable unemployment is absent from most work in the RBC literature, which generally assumes clearing labor markets.

3For examples of this indeterminacy literature, see Benhabib and Farmer (1994), Farmer (1993), and Weder (2000).
and supply relationships and market clearing conditions for general equilibrium. In the third section, we study the equilibrium dynamics of the model economy, in particular the possibility of sunspot equilibria. This is followed by an analysis of the stochastic properties of the model’s variables, which we compare to those of contemporary time series of the German economy. Section 5 discusses some results on marginal versus global effects of altering labor market institutions, and extends the complementarity of labor market institutions highlighted by Coe and Snower (1997) to a dynamic setting. Section 6 concludes.

2 Model

The model is based on a standard representative agent growth model of a closed economy. The household sector owns all inputs and rents its services to the firms. Firms produce a homogeneous final good which is then sold to households to be consumed or invested as physical capital. There is uncertainty in the model, especially as regards the instantaneous employment of labor resources; following the literature, we assume the existence of complete contingent markets, so that agents can insure themselves against this idiosyncratic risk.4

2.1 Preferences

The economy consists of a large number of identical households of measure one.5 Every household has access to a complete set of frictionless asset markets. Preferences at time 0 are defined over sequences of consumption $c_t$, and leisure $l_t$:

$$E^0 \sum_{t=0}^{\infty} \beta^t U(c_t, l_t) \quad \beta \in (0, 1).$$

(1)

$\beta$ stands for the subjective discount rate. $E^0$ denotes the expectations operator conditional on time 0 information. Instantaneous utility is specialized to the functional form

$$U(c_t, l_t) = \log c_t + \frac{A}{1 + \chi} l_t^{1+\chi} \quad A > 0, \quad \chi \leq 0.$$  

(2)

4It should be stressed that this is a standard simplification and should not be interpreted as a trivialization of unemployment. Indeed, imperfect information and moral hazard aspects of unemployment are likely to lead to co-insurance. Thus, even if the unemployed were indifferent in equilibrium, resource costs of providing that insurance is motivation alone for treating unemployment as a serious economic problem.

5Small (large) letters indicate individual (aggregate) variables. Since households are identical in equilibrium, we omit the identifying index.
The time endowment can be divided between leisure, employment $l_t$, and time engaged in search activity, $s_t$:

$$l_t + l_t + s_t = 1$$

The explicit modeling of time spent neither in work nor in leisure (i.e. nonmarket activities) is a novel aspect in the dynamic general equilibrium literature with search. Most existing models simply define search unemployment as the complement to time spent working, and capture the costliness of search in terms of lost output.\(^6\)

Employment evolves according to

$$l_{t+1} = (1 - \delta_L)l_t + f_t s_t \quad \delta_L \in (0, 1].$$

(3)

where $f_t$ is the job finding rate per unit time expended in search, and $\delta_L$ represents an exogenous wastage of employment matches. In this model on-the-job search is ruled out. Capital is accumulated by the households according to

$$k_{t+1} = (1 - \delta_K)k_t + i_t \quad \delta_K \in (0, 1].$$

(4)

where $k_t$ and $i_t$ are the capital stock and investment expenditure respectively. Physical capital depreciates by $\delta_K$ per period.

### 2.2 Firms and Technology

In the final goods sector, output is supplied by a fixed large number of competitive firms. Each operates under technology that constant returns to scale at the firm level

$$x_{i,t} = Z_t k_{i,t}^\alpha l_{i,t}^{1-\alpha} (K_t^\alpha L_t^{1-\alpha})^\theta$$

with $\alpha \in (0, 1), \quad \theta \geq 0$.

(5)

where $k_{i,t}, l_{i,t}$ are the firm’s $i$ input in capital and labor. $K_t$, and $L_t$ are aggregate inputs in capital and labor. $\theta$ denotes externalities that can imply increasing returns. Total factor productivity $Z$ evolves as a stationary stochastic process

$$\log Z_{t+1} = (1 - \rho_z) \log Z + \rho_z \log Z_t + z_{t+1} \quad Z \equiv 1, \quad \rho_z \in [0, 1).$$

$z_{t+1}$ is i.i.d. normal with mean zero and variance $\sigma^2_z$. In choosing its optimal employment level, each firm has to take into account current aggregate states and that it may take time to fill vacancies. Firms can post vacancies $v_{i,t}$ at cost $a$. Each firm chooses sequences of capital services to hire, and how much to invest in finding new workers by posting vacancies. In addition, the firm’s decision implicitly reflects the aggregate savings decisions of households, since the interest rate will depend on the availability of loanable funds for firms’s investments.

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2.3 Labor Market Institutions

2.3.1 Matching and Search in the Labor Market

A hallmark of the model we study is that it explicitly incorporates several important deviations of labor markets from the Walrasian paradigm. In a model of equilibrium unemployment, search and trade frictions preclude immediate wedding of production factors. Incremental changes in labor supply and the level of employment cannot take place instantaneously but require time and resources to match advertised vacancies by firms and search effort by agents. As a reaction to trade frictions, households must use part of their time endowment for labor market search activities. Similarly, firms post vacancies to signal workers their willingness to hire; these vacancies and the search activity of firms has resource costs which are either explicit (in the form of job placement agencies) or implicit (the opportunity cost for the firm engaged in search). Aggregate employment $L_{t+1}$ evolves according to

$$L_{t+1} = (1 - \delta_L)L_t + M_t \quad \delta_L \in [0, 1]$$

where $\delta_L$ is an exogenous separation probability, $M_t$ is the measure of job matches which occur in period $t$. The timing indicates a time-to-match lag. It has become standard to employ a functional form of Cobb-Douglas type

$$M_t = V_t^g S_t^{1-g} \quad g \in (0, 1).$$

Denote by $V_t$ are the announced vacancies and $S_t$ is the time spent by households in search. This functional form derives its popularity from Blanchard and Diamond (1989) who report a reasonable fit with U.S. labor market data. There is a technical problem which has been noted by Den Haan, Ramey and Watson (2000) and motivates the following modification of the matching function:

$$M_t = \Theta S_t V_t^{\frac{1}{1+g}} \quad \Theta > 0, \quad g \leq 1.$$  \hfill (7)

The constant $\Theta$ is a scaling factor. Every agent and firm is defined to be small in relation to the economy so that actions do not affect aggregates and take matching rates as given. The rate that a given searching individual agent is matched with a vacant job during the unit interval – appearing in (3) – is given by

$$f_t = \frac{M_t}{S_t}.$$  \hfill (8)

\footnote{Specifically, the Cobb-Douglas form does not guarantee matching probabilities between zero and one. In contrast, it is easy to verify that (7) satisfies the usual functional properties of a matching function while generating plausible matching probabilities.}
For constant \( f \), the mean duration of a completed unemployment spell is given by \( f^{-1} \). Similarly, the rate at which vacancies are filled is defined as
\[
q_t = \frac{M_t}{V_t},
\]
with mean steady-state vacancy duration given by \( 1/q \). As the respective transition rates depend on the aggregate number of the two types of traders, the matching process generates trading externalities.

Merz (1995), Andolfatto (1996), and Den Haan, Ramey and Watson (2000) also examine the role of search in dynamic general equilibrium model. The current paper differs from these because it explicitly admits the existence of relevant labor market institutions and abandons the assumptions that perfect competition obtains everywhere and that fluctuations are caused only by shocks to the economy’s fundamentals (as a rule, total factor productivity). We will discuss these aspects in what follows.

2.3.2 Payroll Taxes, Unemployment Insurance and Social Assistance

Our model economy possesses a simple government sector which collects taxes and redistributes them to currently unemployed agents. Let \( \tau_t \) be the payroll tax rate on labor income at time \( t \). We assume that the government runs a balanced budget, so the government must set the tax rate period by period to satisfy the constraint
\[
\tau_t w_t L_t = b S_t + \varepsilon b (1 - S_t - L_t) = (1 - \varepsilon) b S_t + b \varepsilon (1 - L_t)
\]
where \( w_t \) is the wage rate and \( b \) is a transfer payment per unemployed agent who is engaged in search. Unemployed agents not engaged in search enjoy a transfer equal to \( \varepsilon b \) with \( \varepsilon \in [0, 1] \). \( \varepsilon \) has the interpretation of a misclassification error of leisure as search. If \( \varepsilon = 1 \) then leisure and search are treated symmetrically; if \( \varepsilon = 0 \) there is no subsidy of leisure. The balanced budget restriction renders the tax rate \( \tau \) endogenous, while the unemployment insurance benefit \( b \) and \( \varepsilon \) are constant policy parameters.

It should be stressed that unemployment in the model has two forms: Time not spent working but spent in search for a new job, and leisure (time withheld from market activities). The government compensates both forms of unemployment in different ways. First, an unemployment insurance scheme compensates search time \( S \) at constant fraction of the steady-state net wage, \( b \). Second, non-search, non-market activity is also subsidized by the state: the parameter \( \varepsilon \) thus captures the generosity of the social safety net. In particular, it helps to determine the gains from job search in unemployment, and admits the possibility of a “poverty trap” in which agents are indifferent between work and the dole. A value of \( \varepsilon \) close to 1 reduces the net gains to search by increasing the value of leisure or other nonmarket
activities relative to search. Ceteris paribus, a high value of \( \varepsilon \) also increases the countercyclicality of taxes, since in times of low employment transfers increase and must be financed.

### 2.3.3 Wage Determination and Bargaining Strength

In this section we model the wage bargaining process, which represents the second deviation from a standard labor market. Matches give rise to surplus which can be shared arbitrarily between paired agents and firms, so that the wage will generally deviate from marginal productivity remuneration characteristic of the neoclassical model. Wages in the present model are determined at the match level as a Nash bargaining solution where the constant \( \xi \) can be interpreted as the bargaining strength of the firms. In particular, the wages are derived as the solution of

\[
\arg \max w_t = \left[ (1 - \alpha) \frac{Y_t}{L_t} + \frac{aV_t}{S_t} - w_t \right]^{\xi} \left[ (1 - \tau_t) w_t - \frac{A(1 - L_t - S_t)^\lambda}{\lambda_t} - \varepsilon b \right]^{1-\xi}.
\]

Implicitly, the fallback position for labor excludes any subsidy to search in case of failure to reach an agreement; this could be interpreted as an institutional arrangement which penalizes refusal to accept job offers after a match occurs. The wage then obeys

\[ w_t(1 - \tau_t) = (1 - \tau_t)(1 - \xi) \left[ (1 - \alpha) \frac{Y_t}{L_t} + \frac{aV_t}{S_t} \right] + \xi \left[ \frac{A(1 - S_t - L_t)^\lambda}{\lambda_t} + \varepsilon b \right] \tag{11} \]

where \( \xi \in [0, 1] \). According to (11), the net wage rate is a weighted average of the marginal product of labor net of advertising costs per number of unemployed agent and the disutility that arises from work corrected for forgone search costs. The expression \( \frac{A(1 - S_t - L_t)^\lambda}{\lambda_t} + \varepsilon b \) is the minimum compensation that workers require to work. In the extreme case \( \xi \to 1 \), the firm collects the entire surplus and the wage rate reaches its allowable minimum. This wage rule is similar in spirit to Merz (1995) and Andolfatto (1996), who show that for a particular value of \( \xi \), this rule is equivalent to the optimal rule chosen by a social planner and the market solution and can be considered an optimal contract. This correspondence is only possible, however, in economies with a one-to-one relationship between the market outcomes and social (planner’s) optima since no compelling reason is given why this should be the case. We think of deviations from the optimum as ”labor market imperfections” which give excessive power either to labor or to management. Under a bargaining setup, it is easy to obtain suboptimal equilibria since the marginal product of labor is distorted away from its social opportunity cost, and agents do not necessarily internalize the effects of their presence in the market (Hosios, 1990).
2.4 Optimal Behavior

2.4.1 Households

The household maximizes lifetime utility \((1)\) by choosing a sequence of consumption, labor and physical capital subject to the constraints \((3), (4)\) and the period-by-period resource restriction

\[
c_t + i_t = (1 - \tau_t)w_t l_t + bs_t + r_t k_t + \varepsilon b (1 - s_t - l_t)
\]

where \(w_t\) and \(r_t\) are respectively the wage and the capital rental rate. We follow the standard procedure and assume that workers pool their incomes (and implicitly their unemployment) and make their policy decisions in a manner of a representative agent whose decisions are denoted by lower case letters.\(^8\) Economic profits in this economy are zero.

Denote the aggregate state of the economy by \(\Omega_t \equiv \{K_t, L_t, Z_t\}\). The value function \(v: \mathbb{R}^2_+ \times \Omega \rightarrow \mathbb{R}_+\) for the representative agent’s dynamic programming problem satisfies the functional equation

\[
v(k_t, l_t; \Omega_t) = \max_{c_t, s_t, l_{t+1}, k_{t+1}} U(c_t, l_t) + \beta E_t v(k_{t+1}, l_{t+1}; \Omega_{t+1})
\]

subject to \((3), (4)\) \((12)\) and given the aggregate laws of motion for \(\Omega_t\). Assuming the differentiability of the value function, the first order conditions can be written as \(\forall t \geq 0:\)

\[
\frac{1}{c_t} = \lambda_t
\]

\[
\lambda_t = \beta E_t \lambda_{t+1} [r_{t+1} + 1 - \delta]
\]

\[
\mu_t f_t = A(1 - s_t - l_t)^\chi - (1 - \varepsilon)\lambda_t b
\]

\[
\mu_t = \beta E_t \left[ \lambda_{t+1} [ (1 - \tau_{t+1}) w_{t+1} - \varepsilon b] - A(1 - s_{t+1} - l_{t+1})^\chi + (1 - \delta L) \mu_{t+1} \right]
\]

where \(\lambda_t\) and \(\mu_t\) denote the Lagrangian multipliers associated with the capital (physical wealth) and employment constraints. The first two equations \((14)\) and \((15)\)

---

\(^8\)See, for example, Danthine and Donaldson (1995). To date, no easily implementable algorithm is available allowing the modeller to track easily individual wealth profiles, unemployment durations, and other state variables; our procedure masks potential effects created by agent heterogeneity. Alternatively, one may think of each household being a family which receives income from having a fraction \(l_t\) of its members at work.
describe the optimal savings sequence. Equations (16) and (17) characterize the household’s optimal search and labor supply policies. They imply that the household equalizes the marginal disutility of searching today to its expected payoff, which is given by the wage payments (in terms of consumption units) and minus disutility from working and plus the expected value of foregone search costs.

2.4.2 Firms

The problem of the representative firm can be defined as choosing values of vacancies, capital, and future employment such to maximize the expected sum of discounted profits, taking the path of wages, interest rates, and intertemporal marginal rates of substitution as given. The households’ optimal behavior implies an asset pricing kernel — the price at time \( t \) of a certain claim to one unit of period \( t+1 \) consumption — equal to

\[
\rho_{t+1} \equiv \beta \frac{\lambda_{t+1}}{\lambda_t}
\]

which is the intertemporal marginal rate of substitution in consumption. It is assumed that \( \rho_{t+1} \) is the discount factor employed by firms and is taken by them as given.\(^9\)

Define the firm’s value function \( W : \mathcal{R}_+ \times \Omega \rightarrow \mathcal{R}_+ \). Assuming that \( W \) is unique, it can be characterized as the solution to the functional equation

\[
W(l_i; \Omega_t) = \max_{v_{i,t}, k_{i,t+1}, l_{i,t+1}} x_{i,t} - w_l l_{i,t} - r_t k_{i,t} - a v_{i,t} + E_t \rho_{t+1} W(l_{t+1}; \Omega_{t+1})
\]

subject to the production function and the firm transition equation for employment

\[
l_{i,t+1} = (1 - \delta) l_{i,t} + q_t v_{i,t}.
\]

Profit maximization requires for each firm to set its capital, employment, and vacancy sequences such that for all \( t = 0, 1, \ldots \)

\[
r_t = \alpha x_{i,t} k_{i,t}^{-1}
\]

and

\[
\frac{a}{q_t} = E_t \rho_{t+1} \left[ (1 - \alpha) \frac{x_{i,t+1}}{l_{i,t+1}} + \frac{a(1 - \delta_L)}{q_{t+1}} - w_{t+1} \right].
\]

(20) indicates that firm hires capital up to the equality of the rental rate and the marginal product of capital. It is also optimal for the firm to advertise vacancies such that the marginal cost of posting an opening (per unit probability of filling the vacancy) is equal to expected profits plus the firm’s costs of foregone search, conditional on that the job is filled (21).

\(^9\)In the absence of the representative agent or a complete contingent claims markets assumption, we would encounter difficulties in determining the firms’ criterion function. See for example Radner (1974).
2.5 Symmetric equilibrium

An equilibrium in this economy is a set of functions \((C_t, M_t, V_t, S_t, w_t, r_t, p_t, K_{t+1}, L_{t+1})\) defined on the aggregate state \(\Omega_t\), which satisfy (i) profit and value maximization of firms, (ii) utility maximization of agents, and (iii) market clearing. The symmetry of the environment and market clearing implies that \(K_t = k_t\), \(L_t = l_t\), \(C_t = c_t\), \(V_t = v_t = v_{t,t}\), \(\frac{M_t}{W_t} = q_t\), \(\frac{M_t}{S_t} = f_t\). In what follows we will study the attributes of a symmetric equilibrium.

3 Dynamic Behavior

3.1 Rational expectations solution

As is the case for most RBC models, the present model cannot be solved analytically. Following King, Plosser and Rebelo (1988), we loglinearize the model around its balanced growth path in the absence of shocks. Let us denote percentage deviations from the steady state by \(b_X\), the rational expectations solution of the model reduces to the following stochastic matrix difference equation:

\[
\begin{bmatrix}
\hat{S}_{t+1} \\
\hat{\mu}_{t+1} \\
\hat{\lambda}_{t+1} \\
\hat{L}_{t+1} \\
\hat{K}_{t+1} \\
\hat{Z}_{t+1}
\end{bmatrix} =
\begin{bmatrix}
E_t\hat{S}_{t+1} - \hat{S}_{t+1} \\
E_t\hat{\mu}_{t+1} - \hat{\mu}_{t+1} \\
E_t\hat{\lambda}_{t+1} - \hat{\lambda}_{t+1} \\
0 \\
0 \\
E_t\hat{Z}_{t+1} - \hat{Z}_{t+1}
\end{bmatrix}
+ M
\begin{bmatrix}
\hat{S}_t \\
\hat{\mu}_t \\
\hat{\lambda}_t \\
\hat{L}_t \\
\hat{K}_t \\
\hat{Z}_t
\end{bmatrix}
\]

where we note that search time, \(\hat{S}\), the shadow value of wealth, \(\hat{\lambda}\), and the shadow value of employment, \(\hat{\mu}\), are endogenous and nonpredetermined. The presence of imperfect competition and matching externalities implies that it is not possible to solve for the market equilibrium as the solution to the social planner’s problem. More importantly, the usual Arrow-Debreu welfare theorems cannot be invoked to rule out irregular rational expectations equilibria. In particular, if the matrix \(M\) has more than three eigenvalues inside the unit circle, the rational expectations path is no longer unique. Such dynamic systems are said to be irregular. Indeterminacy in rational expectations of this type implies that equilibria are possible in which fluctuations in economic activity can be driven by arbitrary and self-fulfilling changes in people’s expectations. It should be stressed that such sunspot equilibria are not based on agent irrationality - under the circumstances it is perfectly rational to follow such nonfundamental signals. Rational expectations business cycle models with indeterminacy represent a workable equilibrium interpretation of Keynes’ animal
spirits hypothesis.\textsuperscript{10}

\section*{3.2 Calibration}

In this subsection we describe the parametric specification of our model and assign parameter values such that the long run properties of our model economy correspond to the growth path of postwar Germany. This calibration methodology is now common procedure in modern dynamic general equilibrium theory. The fundamental period in the model is the quarter, so we will compare our model economy performance to quarterly German time series.

In the absence of stochastic disturbances, the model is in its steady state. In steady state, we assume that the agents spend 20 percent of their time endowment working and searching. We set the rate of unemployment equal to 7.5 percent which implies a value for $S$ of 0.016; this implies that unemployed agents spend about 20 minutes per day (on average) in search. The parameter $\delta_L$, which is also the ratio of unemployment rate to the employment rate, is set to 0.081. We assume $\rho = 0.78$ as in Den Haan, Ramey and Watson (2000). Labor and firms are assigned equal bargaining strength so that $\xi = 0.50$. We set the steady state tax rate $\tau$ such that the unemployed agents receive 50 percent of the steady state net wage, which is realistic for Germany. The parameters ($\alpha, \beta, \delta_K$) are standard in RBC models (see for example Cooley and Prescott, 1995).

Table 1 summarizes the benchmark model. Remaining parameters will be calibrated in the next section. Depending on the particular values taken by those parameters, the implied consumption share is roughly 75 percent and the fraction $aV/Y$ assumes very small positive values (less than one percent of output).

\begin{table}[htp]
\centering
\begin{tabular}{|l|c|}
\hline
Parameter & Calibrated Value \\
\hline
$\alpha$  & 0.30  \\
$\beta$   & 0.99  \\
$S$      & 0.016 \\
$\xi$    & 0.50  \\
$\delta_K$ & 0.025 \\
$\delta_L$ & 0.081 \\
$\rho$   & 0.78  \\
$L$      & 0.20  \\
\hline
\end{tabular}
\caption{Calibration values for the model economy}
\end{table}

\textsuperscript{10}See Farmer (1993) for extensive discussions of these issues.
3.3 Indeterminacy

We have fixed all parameters a priori except the degree of increasing returns ($\theta$), the replacement rate ($rr \equiv b/(1 - \tau)w$), the welfare generosity parameter ($\varepsilon$) and the labor supply elasticity ($-\chi^{-1}$). Our strategy is now to look at the behavior of the economy for various combinations of these parameters. In particular, we are interested in those combinations which contribute to yield irregular equilibria.

For this purpose we look at the eigenvalues which result from our preferred calibrations. We first consider combinations of $\theta$ and the slope of the labor supply which yield indeterminacy of rational expectations paths. To focus attention on a benchmark case with the least institutional detail, we set both $rr$ and $\varepsilon$ — and thereby taxes — equal to zero. Figure 1 shows that at one extreme, the minimum increasing returns required to obtain indeterminacy is 1.51 in Hansen's (1985) case of infinitely elastic aggregate labor supply, and is slightly higher than in the Benhabib and Farmer (1994) model which uses a standard labor market. At the other extreme, the lower bound on increasing returns necessary for indeterminacy rises at low labor supply elasticities. Again, this pattern is similar to models with a Walrasian labor market. We conclude that in this parametrization, dynamic equilibria with indeterminacy can only obtain at implausibly high returns to scale.

In Figure 2 we take a different tack, assessing combinations of $rr$ and $\varepsilon$ while assuming constant returns to scale and $\chi = 0$. Evidently, once tax distortions are introduced, indeterminacy is more likely to obtain. In fact, if the net replacement ratio exceeds roughly 50 percent (which is the case for unemployment compensation

\[ \text{Figure 1: Indeterminacy regions in (\chi, \theta) space} \]

\footnote{To facilitate comparison with Merz (1995) and Andolfatto (1995), we consider in this case a Cobb-Douglas matching function with an elasticity of matches with respect to vacancies of 0.4, an estimate reported by Blanchard and Diamond (1989).}
in Germany and most Western European countries) increasing returns in production are no longer necessary to induce the indeterminacy result. If we allow for welfare payments \((\varepsilon > 0)\), the model exhibits indeterminacy at all reasonable calibrations for European economies. A numerical example that implies indeterminacy at constant returns underscores the plausibility of our argument: assuming German values for the gross quarterly wage income of 7500 Euro, a replacement rate of 58 percent, and \(\varepsilon\) equal to 0.30, the calibration implies that the social security payments are 954 Euro per quarter, a plausible value for Germany’s current welfare system.

We offer the following economic intuition for indeterminacy: suppose that agents expect the future real return to labor (and capital) to be high: they start investing and searching today to realize these returns tomorrow. Under normal conditions of decreasing returns, a higher level of employment implies a lower future wage rate and thus cannot be a rationally expected equilibrium since the wage must increase rather than decrease. That is exactly where increasing returns enter the picture. Future output and marginal products must rise in order to validate expectations of a higher marginal product. Any given labor (and capital) input generates a larger marginal product and the initial optimistic belief of higher returns is self-fulfilling. This can only happen if increasing returns in production allow it, or if taxes on labor income are countercyclical. But this is exactly what the balanced budget requires. Since increased employment implies a lower equilibrium tax rate, the after-tax return to labor can be increasing with labor even in the absence of increasing returns to scale. Expectations on higher returns can again be self-fulfilling. Furthermore, the procyclical reduction in distortions shifts out the effective production possibility frontier, generating a wealth effect that spurs additional spending. A complementary effect arises from the additional fiscal relief due to lower take-up of welfare in good
times.

We suspect that the model’s (fiscal and productive) increasing returns are only partly responsible for the result and that the model’s pseudo two-sector structure – induced by the delay between increased search of firms and workers and increased employment – is also responsible.\textsuperscript{12} That is, an increase in search activity need not coincide with employment reductions. Rather, additional resources can be drawn out of leisure. The equilibrium return schedules to labor and to search shift as a result of the agents composition of time allocation. This behavior will also be important in explaining the model’s output persistence.

4 Moments

4.1 Population moments

In Table 2 and 3 we present business cycle statistics in the form of population moments for macroeconomic variables from the U.S. and West Germany. We choose Hodrick-Prescott filtering as our lens for viewing the data. Like many O.E.C.D. countries, one finds a confirmation of the usual business cycle facts: consumption is (slightly) smoother than GDP whereas investment is much more volatile than GDP. Business cycle persistence refers to the fact that when the growth rate of undetrended output is above average, it tends to remain high for a few quarters. One measure of persistence is the autocorrelation statistic; if the data follow a random walk, the autocorrelation is zero; if there is strong mean reversion in the levels, the first differenced data would evidence negative autocorrelation. For U.S. and German GDP data, the autocorrelation statistic displays positive serial correlation positive for lags one to three. The U.S. economy display similar statistics. The only important differences between the two economies appear to be that the U.S. cycle is less persistent than Germany’s, and that labor lags the cycle in Germany rather than being coincident.

\textsuperscript{12}One might think of two separate technologies with output which is either physical goods or job matches.
Table 2: U.S.A. 1954:I-1991:II

<table>
<thead>
<tr>
<th>Variable x</th>
<th>Rel. Volatility</th>
<th>Correlation of Output (real GDP) with</th>
<th>(x(t-3))</th>
<th>(x(t-2))</th>
<th>(x(t-1))</th>
<th>(x(t))</th>
<th>(x(t+1))</th>
<th>(x(t+2))</th>
<th>(x(t+3))</th>
</tr>
</thead>
<tbody>
<tr>
<td>Output</td>
<td>1.00</td>
<td>0.38</td>
<td>0.63</td>
<td>0.85</td>
<td>1.00</td>
<td>0.85</td>
<td>0.63</td>
<td>0.38</td>
<td></td>
</tr>
<tr>
<td>Consumption</td>
<td>0.50</td>
<td>0.55</td>
<td>0.68</td>
<td>0.78</td>
<td>0.77</td>
<td>0.64</td>
<td>0.47</td>
<td>0.27</td>
<td></td>
</tr>
<tr>
<td>Investment</td>
<td>3.10</td>
<td>0.43</td>
<td>0.63</td>
<td>0.82</td>
<td>0.90</td>
<td>0.81</td>
<td>0.60</td>
<td>0.35</td>
<td></td>
</tr>
<tr>
<td>Labor</td>
<td>0.97</td>
<td>0.14</td>
<td>0.39</td>
<td>0.67</td>
<td>0.88</td>
<td>0.91</td>
<td>0.80</td>
<td>0.63</td>
<td></td>
</tr>
<tr>
<td>Wage share</td>
<td>0.45</td>
<td>-0.44</td>
<td>-0.53</td>
<td>-0.51</td>
<td>-0.46</td>
<td>-0.13</td>
<td>0.13</td>
<td>0.32</td>
<td></td>
</tr>
<tr>
<td>Unemployment</td>
<td>7.08</td>
<td>-0.51</td>
<td>-0.68</td>
<td>-0.83</td>
<td>-0.83</td>
<td>-0.76</td>
<td>-0.59</td>
<td>-0.42</td>
<td></td>
</tr>
<tr>
<td>Vacancies</td>
<td>8.08</td>
<td>0.55</td>
<td>0.74</td>
<td>0.87</td>
<td>0.93</td>
<td>0.85</td>
<td>0.68</td>
<td>0.47</td>
<td></td>
</tr>
<tr>
<td>Output growth</td>
<td>1.00</td>
<td>0.03</td>
<td>0.22</td>
<td>0.37</td>
<td>1.00</td>
<td>0.37</td>
<td>0.22</td>
<td>0.03</td>
<td></td>
</tr>
</tbody>
</table>


Table 3: Germany 1970:I-1994:IV

<table>
<thead>
<tr>
<th>Variable x</th>
<th>Rel. Volatility</th>
<th>Correlation of Output (real GDP) with</th>
<th>(x(t-3))</th>
<th>(x(t-2))</th>
<th>(x(t-1))</th>
<th>(x(t))</th>
<th>(x(t+1))</th>
<th>(x(t+2))</th>
<th>(x(t+3))</th>
</tr>
</thead>
<tbody>
<tr>
<td>Output</td>
<td>1.00</td>
<td>0.60</td>
<td>0.78</td>
<td>0.89</td>
<td>1.00</td>
<td>0.89</td>
<td>0.78</td>
<td>0.60</td>
<td></td>
</tr>
<tr>
<td>Consumption</td>
<td>0.66</td>
<td>0.62</td>
<td>0.75</td>
<td>0.85</td>
<td>0.94</td>
<td>0.92</td>
<td>0.73</td>
<td>0.60</td>
<td></td>
</tr>
<tr>
<td>Investment</td>
<td>1.98</td>
<td>0.55</td>
<td>0.74</td>
<td>0.86</td>
<td>0.96</td>
<td>0.89</td>
<td>0.79</td>
<td>0.65</td>
<td></td>
</tr>
<tr>
<td>Labor</td>
<td>0.51</td>
<td>0.09</td>
<td>0.28</td>
<td>0.48</td>
<td>0.65</td>
<td>0.78</td>
<td>0.84</td>
<td>0.85</td>
<td></td>
</tr>
<tr>
<td>Wage share</td>
<td>0.57</td>
<td>-0.54</td>
<td>-0.53</td>
<td>-0.46</td>
<td>-0.36</td>
<td>-0.18</td>
<td>0.02</td>
<td>0.21</td>
<td></td>
</tr>
<tr>
<td>Unemployment</td>
<td>9.49</td>
<td>-0.30</td>
<td>-0.48</td>
<td>-0.69</td>
<td>-0.78</td>
<td>-0.85</td>
<td>-0.86</td>
<td>-0.80</td>
<td></td>
</tr>
<tr>
<td>Vacancies</td>
<td>10.51</td>
<td>0.45</td>
<td>0.64</td>
<td>0.70</td>
<td>0.71</td>
<td>0.65</td>
<td>0.54</td>
<td>0.39</td>
<td></td>
</tr>
<tr>
<td>Output growth</td>
<td>1.00</td>
<td>0.28</td>
<td>0.21</td>
<td>0.35</td>
<td>1.00</td>
<td>0.35</td>
<td>0.21</td>
<td>0.28</td>
<td></td>
</tr>
</tbody>
</table>

Variable definitions: Private Final Consumption Expenditures (C), I=Fixed Capital Formation (I), Output =C+I Total Employment, Standardized Unemployment Rate. All variables (except wage share and unemployment) were logged and detrended by applying the Hodrick-Prescott filter. National income and product accounts data are measured in 1991 DM terms. Source of Data: OECD National Accounts and IMF Economic Indicators, Statistisches Bundesamt (output growth).

4.2 Model Moments

4.2.1 Search Model with Determinate Solution Paths (RBC)

We begin by exploring the implications of an economy not subject to any imperfections except for labor market search. In particular, we consider an economy with output elasticity of capital \(\alpha\) equal to 0.36, \(\varepsilon = rr = 0\), constant returns \(\theta = 1\), equal bargaining power, \(\xi = 0.5\), and \(\chi = -1\). This economy is determinate. It is well-known that when productivity shocks are serially uncorrelated, the model is largely incapable of replicating business cycle behavior, and so we abstain from
belaboring this point. When subject to persistent technology shocks \( (\rho = 0.95) \), the model generates the following business cycles:

| Table 4: RBC with Search \( (\varepsilon = r\tau = 0, \theta = 1, \xi = 0.5, \chi = -1) \) |
|-----------------|-----------------|
| Variable \( x \) | Rel. Volatility | Correlation of Output with Correlation of Output with |
|                 |                 | \( x(t-3) \) | \( x(t-2) \) | \( x(t-1) \) | \( x(t) \) | \( x(t+1) \) | \( x(t+2) \) | \( x(t+3) \) |
| Output          | 1.00            | 0.30         | 0.53         | 0.81         | 1.00         | 0.81         | 0.53         | 0.30         |
| Consumption     | 0.34            | 0.10         | 0.36         | 0.67         | 0.91         | 0.79         | 0.66         | 0.55         |
| Investment      | 3.57            | 0.36         | 0.57         | 0.82         | 0.99         | 0.78         | 0.47         | 0.21         |
| Labor           | 0.37            | 0.20         | 0.45         | 0.63         | 0.84         | 0.91         | 0.53         | 0.24         |
| Wage Share      | 0.11            | -0.22        | -0.28        | -0.32        | -0.16        | 0.41         | 0.33         | 0.28         |
| Unemployment    | 0.39            | -0.06        | -0.10        | -0.17        | -0.36        | -0.77        | -0.48        | -0.26        |
| Vacancies       | 4.55            | 0.25         | 0.31         | 0.35         | 0.18         | -0.36        | -0.31        | -0.27        |
| Output growth   | 1.00            | -0.06        | -0.07        | 0.26         | 1.00         | 0.26         | -0.07        | -0.06        |

Table 5: RBC with Search \( (\varepsilon = r\tau = 0, \theta = 1, \xi = 0.5, \chi = -1) \)

Statistics are based on 2000 artificial realizations of the model.

As already established by Andolfatto (1996), the RBC model with search is able to replicate major stylized business cycle facts, in both qualitative terms and as regards relative volatilities and comovements of output with other macroeconomic aggregates. Moreover, the persistence of output and output growth is significantly greater than in standard RBC models with Walrasian labor markets.\(^{13}\) Ostensibly, sluggishness resulting from labor market matching generates positive autocorrelations (at lag one). Another unusual feature is that labor lags the cycle by one quarter as in the U.S. economy. Unlike standard RBC models, the wage share is nonconstant and follows a countercyclical pattern as reported in Tables 2 and 3. In addition, it is noteworthy that our model generates a slight phase shift in the wage share which is also found in the data, a fact which is not replicated by Merz’s (1995) RBC model.

4.2.2 Search Model with Indeterminacy deriving from Institutions

We now turn to model economies with indeterminacy induced by institutional features, modest increasing returns, or both. We assume constant returns in production and some imperfections due to the government intervention. We first consider a model in which indeterminacy arises due to institutions only. To focus discussion on the innovations in this paper, we consider \( \varepsilon = 0.3, \xi = 0.4, \) and \( r\tau = 0.5 \) with otherwise standard features: \( \chi = 0 \) (Hansen, 1985), constant returns, and \( \alpha = 0.3 \). The model is driven by white noise sunspot activity only. The resulting economy is described compactly in Table 5.

\(^{13}\)Cogley and Nason (1995) show the absence of persistence for a wide variety of RBC settings.
Table 5 Sunspots ($\varepsilon = 0.3, \xi = 0.4, rr = 0.5, \chi = 0, \theta = 1$)

<table>
<thead>
<tr>
<th>Variable x</th>
<th>Rel. Volatility</th>
<th>Correlation of Output with</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>$x(t-3)$</td>
</tr>
<tr>
<td>Output</td>
<td>1.00</td>
<td>0.11</td>
</tr>
<tr>
<td>Consumption</td>
<td>0.16</td>
<td>-0.24</td>
</tr>
<tr>
<td>Investment</td>
<td>3.89</td>
<td>0.06</td>
</tr>
<tr>
<td>Labor</td>
<td>1.58</td>
<td>0.15</td>
</tr>
<tr>
<td>Wage Share</td>
<td>0.15</td>
<td>0.37</td>
</tr>
<tr>
<td>Unemployment</td>
<td>1.64</td>
<td>0.03</td>
</tr>
<tr>
<td>Vacancies</td>
<td>12.73</td>
<td>0.59</td>
</tr>
<tr>
<td>Output growth</td>
<td>1.00</td>
<td>-0.22</td>
</tr>
</tbody>
</table>

The first remarkable qualitative result is that consumption is procyclical without any need of scale economies or technology shocks. This can be regarded as an improvement of indeterminacy models with low (here constant) returns to scale. The economic intuition for this result is as follows. First, in the presence of fiscal increasing returns, distortions are countercyclical and thereby shift the effective (net-of-tax) production possibility frontier. Secondly, the resources that the economy allocates towards vacancies - less than one percent of output - are countercyclical. In a sense, the economy has a two-sector character with firms’ search as the absorbing sector. Further, output is persistent as evinced by the significant positive autocorrelation of output growth. The wage share and the rate of unemployment are countercyclical. In contrast to most models of this genre, labor input is more volatile than in the data.

Two problems with the sunspot variant of our model should be noted. First, the wage share displays the wrong phase shift: it peaks a few quarters before output rather than lagging it. Second and more significantly, while vacancies lead the cycle, their contemporaneous negative correlation with output is at odds with what is observed in actual economies. An intuition for this result is the increasing attractiveness of posting vacancies as the economy approaches the peak in the cycle. Recall that the attractiveness of vacancies can be derived from their effectiveness in generating employment matches. As unemployment in the form of search and leisure declines, the return from vacancies declines as well. Consequently, only improving returns from employment can rationalize posting more vacancies. Clearly, this points to a feature missing from our model, either the need for shifts in the production function (due either to technology shocks or to increasing returns) or to heterogeneity in labor markets which could explain on-the-job search and job ladder phenomena. The latter would be an interesting aspect as it is well-known that mobility and vacancies are strongly procyclical, while the reporting of vacancies is countercyclical. Yet it is not clear that this feature hinders our ability to account for the lion’s share of labor market facts.
4.2.3 Search Model with Indeterminacy Deriving from Increasing Returns in Production

The third case we consider assumes increasing returns to scale. We set $\theta = 1.2$ which is at the upper bound of recent studies for the U.S. economy and appears even more plausible for Europe.\footnote{See for example Basu and Fernald (1997), and Röger (1999).} Table 6 reports the statistics associated with the increasing returns model. As with the previous example, we continue to suppress technological shocks and assume white noise sunspot shocks only.

\begin{table}[h]
\centering
\caption{Increasing returns and sunspots ($ibid$ and $\theta = 1.2$)}
\begin{tabular}{|c|c|c|c|c|c|c|c|c|}
\hline
Variable & Rel. Volatility & $x(t-3)$ & $x(t-2)$ & $x(t-1)$ & $x(t)$ & $x(t+1)$ & $x(t+2)$ & $x(t+3)$ \\
\hline
Output & 1.00 & 0.41 & 0.62 & 0.85 & 1.00 & 0.85 & 0.62 & 0.41 \\
Consumption & 0.33 & 0.41 & 0.68 & 0.88 & 0.79 & 0.71 & 0.61 & 0.51 \\
Investment & 4.01 & 0.03 & 0.46 & 0.68 & 0.96 & 0.83 & 0.59 & 0.36 \\
Labor & 1.19 & 0.46 & 0.67 & 0.87 & 0.99 & 0.81 & 0.55 & 0.32 \\
Wage Share & 0.15 & 0.28 & 0.30 & 0.16 & -0.33 & -0.21 & -0.19 & -0.15 \\
Unemployment & 1.19 & -0.30 & -0.46 & -0.68 & -0.96 & -0.84 & -0.61 & -0.38 \\
Vacancies & 8.09 & 0.43 & 0.48 & 0.35 & -0.19 & -0.36 & -0.35 & -0.30 \\
Output growth & 1.00 & 0.02 & 0.08 & 0.35 & 1.00 & 0.35 & 0.08 & 0.02 \\
\hline
\end{tabular}
\end{table}

This variant can replicate a business cycle characteristics slightly better than the first: consumption becomes more volatile and more procyclical. Similarly to the U.S. data, it slightly leads the cycle. When compared to the model version with constant returns in production, vacancies lead the cycle and are only mildly negatively correlated at lag zero.

4.3 Impulse response dynamics

Next, we will illustrate the endogenous propagation mechanism in the model by turning to the impulse response dynamics. We focus on the response of the economy to innovations to technology and to sunspots shocks, and we limit this exercise to the case of constant returns in production. Figure 3 traces the responses associated with a one-time technology shock. Output, consumption, investment and labor all rise on impact. The variables exhibit the typical hump-shape responses and they return to their steady state only very slowly. In Figure 4 we track the response of the same economy to a white-noise sunspot shock. Output does not respond on impact; it is predetermined since total factor productivity is constant. As modelled, the sunspot shock therefore reduces some output component – in our case
investment. This corresponds to the discussed countercyclical behavior that arises for the constant returns version of the model. Yet, starting in the second period, the variables show a positive comovement. Again, the striking feature of the model is the degree of persistence imparted by a transitory shock: all the persistence is due to endogenous propagation for the white noise shock\(^\text{15}\). Output needs more than seven years to return to its steady state.\(^\text{16}\) Overall, the model is able to significantly propagate temporary shocks which contributes in explaining stylized facts which we have identified.

5 Complementarities in Policy Interventions: Partial versus Global Reforms

In widely noted papers, Coe and Snower (1997) and Orszag and Snower (1998) discuss the inefficacy of partial labor market reforms which marginally alter only a subspace of existing labor market interventions without addressing interactions the institutions might have on incentives. This complementarity of policy reform means, for example, that a piecemeal reduction of unemployment benefit will deliver only modest labor market effects, if taxation, union bargaining, or the social welfare system are not reformed at the same time. Here we argue that in a well-articulated dynamic general equilibrium model, this conclusion extends to the persistence of

\(^{15}\)Contrast this to the productivity shock in the standard RBC model, which in standard calibrations carries an autocorrelation coefficient of \(\rho > 0\).

\(^{16}\)Under increasing returns in production, the model exhibits even greater persistence.
output as well.\textsuperscript{17} This is because the elimination of sunspot equilibria which arise in economies such as these may require simultaneous reforms, i.e. that several parameters be changed at the same time.

To see this, return to Figure 2, which depicts combinations of the steady-state replacement rate $rr$ and welfare generosity $\varepsilon$ that yield either determinacy or indeterminacy while holding the rest of the model parameters at standard settings. The previous section associated high output persistence with the indeterminacy regime, and moreover that indeterminacy in our setup can also arise from ”bad policy” configurations, not just increasing aggregate returns in production as in the standard literature (e.g. Farmer and Guo 1994). Furthermore, Figure 1 shows that in the absence of government policies, the model economy we consider is determinate, unless increasing returns in production are implausibly large. By continuity, there will exist a set of liberalizations of labor market policies which can eliminate sunspot equilibria and thereby reduce economic instability. If reforms are only partial, however, this may not be the case. For example, if the replacement ratio exceeds a threshold of 50 percent (which is below the relevant rate in Germany and most other Western European countries), the complete elimination of social welfare payments does not alter the qualitative behavior of the economy; the model remains indeterminate and subject to nonfundamental shocks. Equivalently, if the misclassification rate or gen-

\textsuperscript{17} It is possible to show that by moving out of the indeterminacy region and towards the perfect market economy is welfare-improving. A similar conclusion applies when sunspot fluctuations are suppressed while remaining in the indeterminacy region, with constant returns in production. In this case, gains from intertemporal bunching of economic activity are small in relation to the agents' incentive to smooth consumption. A complete welfare analysis is beyond the scope of the current paper.
erosity parameter $\varepsilon$ is large, a stronger cutback in replacement rates is needed to rule out sunspot equilibria. Overall, our results support the notion that global reforms of the social welfare system are more appropriate than small, piecemeal changes for reducing volatility in the macroeconomy.

Whatever incentives may be in a laissez-faire economy to substitute intertemporally, they are flattened out by the social system. This increases persistence in general. Then the self-fulfilling story kicks in: if agents believe times are bad, spending time searching becomes less lucrative. Thus the taxes are high and there is no reason to invest, remuneration remains low, so there is no reason to search. The key aspect is that the social system aggravates the situation, because one needs to search today to get the job tomorrow.

6 Conclusion

Most equilibrium business cycles models, while long on microfoundations, are hard-pressed to replicate key business cycle facts. In this paper we show that progress is possible on this front by considering explicit imperfections in the labor market as well as their complementarity. Although the labor market is the economy’s ”boiler room”, accounting for two-thirds of all factor payment transactions, the quality and demand for what is traded is in fact remarkably heterogeneous and trade highly decentralized. Actions of agents may exert external effects on the evolution of the aggregate quantity of labor transacted. Finally, the wage is an instrument of distribution of surplus as well as an indicator of labor productivity. The size of the surplus as well as its division may be influenced by non-market factors, primarily labor market institutions. As a result, factor remuneration may deviate significantly from neoclassical marginal cost principles, at least in the short to medium term. Surprisingly, such imperfections and their interaction only occupy a subsidiary role in the literature on dynamic equilibrium macromodels. The introduction of labor market imperfections, wage bargaining, and endogenous search combine and interact with a distortionary government to create persistence in the simulated time series.

A central finding of this paper is that, for a plausible region of the parameter space, labor market institutions can induce both strong persistence as well as nonuniqueness of rational expectations equilibria. Furthermore, it is no longer necessary to impose increasing returns in production to induce this result. In recent years, a number of researchers have formulated models with sunspot equilibria and self-fulfilling prophecies as an alternative to the technology-driven, real business cycle literature. This paper contributes to this literature by bringing together both sunspot and technology-driven business cycle models with a non-Walrasian labor market. In particular, it is to our knowledge the first attempt within the general equilibrium indeterminacy literature that departs from the assumption of perfect labor markets. It is shown that with these simple changes, the model can reproduce
a number of key stylized facts of the business cycle. Some of these elusive stylized facts include the persistence and countercyclical behavior of the labor share and unemployment, as well as cyclical patterns of output and its components.
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