Weather-Based Crop Insurance Contracts for African Countries

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Introduction

Weather constitutes the major production risk in agriculture. Floods and droughts can result in complete crop failures and severe financial stress for growers. This is especially true in most developing countries where crop insurance products are virtually non-existent and where the government’s ability to provide disaster relief is very limited.

Recent years have witnessed the emergence of weather derivatives that allow traders to securitize correlated risks. Weather-based insurance, although rarely used in the agricultural sector, have recently received considerable attention in the literature as potential agricultural risk management tools (Mahul; Martin et al.; Miranda and Vedenov; Turvey; Dischel). Vedenov and Barnett recently addressed the efficiency of weather derivatives as risk management instruments for corn, soybean and cotton productions in the US. They considered a few weather indices and found the basis risk between the indices and the area yields are significant.

In a global effort to mitigate agricultural production risk in developing countries, the World Bank, in collaboration with other international development agencies, governments, and/or local financial institutions, has embarked on pilot weather-based insurance programs in a number of countries, such as India, Mexico, Mongolia, Morocco, and Nicaragua (Skees, et al; Skees and Ayurzana). However, most of these pilot projects are either rainfall-based or temperature-based. While rainfall alone, for example, may suffice in regions such as India (monsoon rains) as a single source of crop yield variations, it may not adequately explain yield variations in other regions where ‘agricultural drought’ is the main problem. Thus there is a need to exploit multivariate weather indices, which incorporate more than one weather event.

In this paper, we first analytically examine farmers’ demand for weather-index insurance
within the expected utility framework, and empirically apply the model in a developing country context. The specific objectives of this paper are to: a) explore indices constructed by multiple weather variables, b) investigate the optimal insurance coverage decisions from representative producers with alternative risk preferences and premium levels, and c) evaluate the efficiency of alternative index-based insurance using the producers’ certainty equivalent income.

In the following sections, we will discuss the development of weather-based insurance contracts. Next we present the expected utility model for producers’ insurance decision and conduct comparative static analysis using the mean-variance (M-V) framework. The empirical background presents the data used in the simulations. Finally, results and conclusions are given.

**Insurance Contracts**

Weather derivatives are commonly indexed using one weather variable, such as rainfall (R), temperature (T), or growing degree-days (GDD). Indices can also be constructed as a joint distribution of multiple weather variables. In order to choose a mixture of practical single-variable and multiple-variable indices that can best correlate with yield, the following seven indices are selected. R, T and GDD are single variable indices. RQ_RT index is a quadratic index in rainfall and temperature, and RQ_RG is quadratic in rainfall and growing degree-days. They are reduced quadratic forms because the cross term of the two variables is omitted. Then we have the full quadratic indices, i.e. Q_RT and Q-RG that include the interaction terms. Table 1 lists the functional forms of the selected indices.

Assume the grower only faces production risk and that weather-based insurance contracts are the only risk management instruments at his disposal. Then an indemnity function similar to the European options payment structure can be constructed (Skees and Zeuli; Turvey; Martin et
al.). Put options insurance are selected for weather factors when the concern is on insufficiency, and call option type insurance is considered when the concern is on excessiveness of the weather factor.

Thus the indemnity functions are defined as:

(1) \[ I(\tilde{\omega}) = \alpha \max(\omega_e - \tilde{\omega}, 0), \]

for put options, and

(2) \[ I(\tilde{\omega}) = \alpha \max(\tilde{\omega} - \omega_e, 0), \]

for call options,

where \( I(\tilde{\omega}) \) is the stochastic indemnity, \( \alpha \) is the tick, \( \tilde{\omega} \) is the weather index, and \( \omega_e \) is the critical weather index value that would trigger a payment. The tick can be expressed as currency or output per unit of index, depending on the denomination of the indemnity schedule.

If production costs are assumed constant and ignored from the risky income, the grower’s with \( n \) shares of the insurance contract has an income per unit of land as:

(3) \[ \pi = \bar{y} + n[I(\tilde{\omega}) - (1 + \lambda)P], \]

where \( \bar{y} \) is the stochastic yield, the output price is normalized to unity, and the tick of the insurance is normalized accordingly so that the income is equal to the production denomination.

For an actuarially fair contract, the premium will be the expected indemnity, i.e \( P = EI(\tilde{\omega}) \). A premium loading is considered to account for transaction costs, with \( \lambda \) as the loading factor.

When the risky output is linearly dependent on the weather index, we have

(4) \[ \bar{y} = \mu + \beta (\tilde{\omega} - \overline{\omega}) + \tilde{e}, \]

where \( E(\bar{y}) = \mu \); \( Var(\bar{y}) = \sigma_y^2 \), \( E(\tilde{\omega}) = \overline{\omega} \); \( Var(\tilde{\omega}) = \sigma_{\tilde{\omega}}^2 \), \( E(\tilde{e}) = 0 \), \( Var(\tilde{e}) = \sigma_{\tilde{e}}^2 \), and \( Cov(\tilde{\omega}, \tilde{e}) = 0 \).

The beta coefficient, \( \beta = \frac{Cov(\bar{y}, \tilde{\omega})}{\sigma_{\tilde{\omega}}^2} \), is a commonly used measure of systematic risk. In the context of weather, it represents the undiversifiable risk of yield due to weather. Because
is influenced by the grower’s choice of the weather index, $\omega$, it is referred to as the index basis risk coefficient. For a put-option-type indemnity structure $\beta$ is positive because of the positive co-variation between yield and the weather event, but negative for the call option contract.

**Expected Utility Model**

Given the profit function in (3), consider a representative grower who chooses the number of contracts to maximize his expected utility of final wealth at harvest, i.e.

$$\max_n E[U(w_0 + \bar{\pi})]$$

where $w_0$ is the grower’s initial per hectare wealth at planting, and $U(\cdot)$ is the utility function representing the grower’s risk preference.

The value of the insurance is measured by its Certainty Equivalent, $CE$, defined as:

$$E[U(w_0 + \pi(n^*))] = E[U(w_0 + \bar{\gamma} + CE)]$$

where $n^*$ is the optimal number of contracts. Using the insurance at its optimal hedge level to mitigate the production risk is equivalent to giving the producer the $CE$ income. The constant relative risk aversion (CRRA) utility function in (7), which has been widely used in crop insurance literature (Wang et al; Coble, et al), is used in the empirical analysis.

$$U(\bar{w}) = (1 - \theta)^{-1} \bar{w}^{1-\theta}$$

where $\theta$ is the CRRA coefficient.

Comparative static analysis is performed with respect to variables of interest, namely the basis risk coefficient, the relative risk aversion coefficient, and the premium loading-factor. For this purpose, we use the M-V model in (8) as an approximation to draw analytical results.

$$\max_n U^n = E(\bar{w}) - \frac{\theta}{2E(\bar{w})} Var(\bar{w})$$
where $E(\tilde{w}) = \mu + w_0 - n\lambda P$, and $Var(\tilde{w}) = \sigma^2 + n^2\sigma^2_{I(\omega)} + 2n\text{Cov}(\tilde{y}, I(\omega))$. If from equation (4) we assume that $\tilde{e}$ and $\tilde{w}$ are conditionally independent (given that they are uncorrelated by definition), then $\tilde{e}$ and $I(\tilde{w})$ are uncorrelated. We can then write $\text{Cov}(\tilde{y}, I(\tilde{w})) = \beta \text{Cov}(\tilde{w}, I(\tilde{w}))$.

The first order conditions to this maximization is given by

$$
\lambda Pn^2 - 2(\mu + w_0)n + \frac{\lambda P\sigma^2 + 2\lambda P(\mu + w_0)^2 + 2(\mu + w_0)\theta \text{Cov}(\tilde{y}, I(\tilde{w}))}{2\lambda^2 p^2 - \theta \sigma^2_{I(w)}} = 0.
$$

For actuarially fair insurance contracts, the loading factor $\lambda$ is set to zero, and the solution to (9) becomes

$$
n^* = -\frac{\text{Cov}(\tilde{y}, I(\tilde{w}))}{\sigma^2_{I(\omega)}} = -\frac{\beta \text{Cov}(\tilde{w}, I(\tilde{w}))}{\sigma^2_{I(\omega)}}.
$$

This directly implies that,

**Proposition 1.** Under actuarially fair premiums, the risk aversion levels do not affect the optimal insurance demand.

This proposition is also consistent with Lapan and Moshini who asserted that the M-V solution implies that risk attitudes have no effect on the optimal hedge under unbiased prices.

Next, we consider the effect of changes in the index-specific basis risk coefficient, $\beta$.

$$
\frac{\partial n^*}{\partial \beta} = -\frac{\text{Cov}(\tilde{w}, I(\tilde{w}))}{\sigma^2_{I(\omega)}} \geq 0.
$$

**Proposition 2.** If the weather index and the indemnity function are positively (negatively) correlated, the optimal insurance demand decreases (increases) as the beta increases.

This finding is consistent with the numerical results presented in Table 2. If we use $R^2$ as a proxy for $\beta$, we see that there is no pattern developing with $n^*$ as $R^2$ increases across the

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1 It is difficult to compare $\beta$ across the different indices because it depends on the magnitude of the choice index. A
indices. Therefore we can only determine the relative efficiencies of the alternative weather indices by measuring the grower’s certainty equivalent income across the indices.

When a premium loading is considered, the solution to (9) becomes

\[ n^* = \frac{\mu + w_0}{\lambda P} \left( \frac{\text{Var} \left( \frac{\mu + w_0}{\lambda P} I(\omega) + \tilde{y} \right)}{\sigma^2_{I(u)} - \frac{2\lambda^2 P^2}{\theta}} \right)^{\frac{1}{2}}, \text{then}^{2} \]

\[ \frac{\partial n^*}{\partial \theta} = \frac{\lambda^2 P^2 \left( \frac{\mu + w_0}{\lambda P} I(\omega) + \tilde{y} \right)}{\theta^2 \left( \sigma^2_{I(u)} - \frac{2\lambda^2 P^2}{\theta} \right)} > 0. \]

It follows that

**Proposition 3.** In the presence of a premium loading, the marginal increment in the grower’s relative risk aversion will lead to a corresponding increase in the optimal number of insurance contracts.

Proposition 3 says that the more risk averse the grower is the higher the insurance coverage he would need to hedge his production risk. This is because when the premium is loaded, the grower reduces his coverage in order to restrict the extra premium payment. Now that the grower is more risk averse, he is willing to make a tradeoff between the certain unfair premium payment and the risk reducing effects by increasing his coverage.

Next, we consider the effects of changes in the premium on insurance demand. Using the normalized and easy to use measure in regression is $R^2$. Although $R^2$ is defined as $1 - \frac{\sum e_i^2}{\sum y_i^2}$, it is an estimator of

\[ \mathcal{R} = 1 - \frac{\sigma_e^2}{E(\tilde{y}^2)} = 1 - \frac{\sigma_e^2}{\beta^2 \sigma_{y_i}^2 + \sigma_e^2 - \mu^2}. \] Therefore, \( \beta = \sqrt{\frac{\sigma_e^2}{\sigma_{y_i}^2 (1 - \mathcal{R})} + \frac{\mu^2 - \sigma_e^2}{\sigma_{y_i}^2}}. \)
same equation (12), we have

\[
\frac{\partial n^*}{\partial \lambda} = -\frac{\mu + w_0}{\lambda^2 P} - \frac{1}{2} \left( \frac{\text{Var} \left( \frac{\mu + w_0}{\lambda P} I(\bar{\omega}) + \tilde{y} \right)}{\frac{\sigma^2_{I(\bar{\omega})}}{\theta} - \frac{2\lambda^2 P^2}{\theta}} \right)^{\frac{1}{2}} \right)
\]

(14)

\[
= \begin{cases} 
\sigma^2_{I(\bar{\omega})} - \frac{2\lambda^2 P^2}{\theta} & \text{if } \frac{\partial \text{Var} \left( \frac{\mu + w_0}{\lambda P} I(\bar{\omega}) + \tilde{y} \right)}{\partial \lambda} < 0
\end{cases}
\]

where

\[
\text{Var} \left( \frac{\mu + w_0}{\lambda P} I(\bar{\omega}) + \tilde{y} \right) = -2 \frac{\mu + w_0}{\lambda^2 P} \sigma^2_{I(\bar{\omega})} \left( \frac{\text{Cov}(\tilde{y}, I(\bar{\omega}))}{\sigma^2_{I(\bar{\omega})}} + \frac{\mu + w_0}{\lambda P} \right).
\]

It is not possible to unambiguously sign equation (14) since it would depend on the farmer’s initial wealth, and the level of premium loading, among other factors. However, a priori, we expect that the demand for insurance will be inversely related to the price of insurance. This will be achieved if the numerator of the last brackets in (14) is positive.

Therefore,

\[
\frac{\partial n^*}{\partial \lambda} < 0 \text{ iff } \frac{\partial \text{Var} \left( \frac{\mu + w_0}{\lambda P} I(\bar{\omega}) + \tilde{y} \right)}{\partial \lambda} > \frac{-4\lambda P^2 \text{Var} \left( \frac{\mu + w_0}{\lambda P} I(\bar{\omega}) + \tilde{y} \right)}{\theta \sigma^2_{I(\bar{\omega})} - 2\lambda^2 P^2}.
\]

It follows that:

**Proposition 4.** Under conditions of sufficiently low initial wealth, higher transaction costs will make weather-based insurance less attractive as a risk-reducing instrument.

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\(^2\)To ensure real roots for the quadratic expression, the denominator in the brackets in (12) is positive.
Empirical Background and Data

We choose South Africa (SA) for a number of reasons. Firstly, SA is one of the largest economies in Africa and has a strong agricultural sector. It is among the first emerging markets to conclude some weather derivative transactions. Secondly, its weather conditions are similar to other surrounding African countries such as Botswana and Namibia, who also have high-risk agricultural production but limited yield records.

The most important factor limiting SA agricultural production is the availability of water. Rainfall is distributed unevenly across the country. The two provinces chosen for this study, namely Northwest and Free State, are the main grain producing regions of the country in terms of planted acreage. The principal town of Vryburg in the Northwest Province is the centre of a large agricultural district. Corn is the main crop produced. Free State Province is situated in the center of the country. It is generally hot, making it suitable for growing corn. Figure 1 shows the map of the corn growing areas of SA.

The required yield and weather data were obtained from two government agencies. The National Department of Agriculture provided the provincial yield data for corn for the period 1980 - 2003. Two centrally located weather stations were selected, one in each province. The SA Weather Service provided the daily data for rainfall and temperature for the two selected weather stations. The data are then accumulated into annual data for each growing season to match with the yield data. The growing (rainy/planting) season is from November/December to April/May. To construct the GDDs, a base temperature of 20 °C , which is a daily mean temperature, was chosen for its best predictive power on corn yield.

The relationship is explored between the linearly detrended yield and weather variables. Table 1 presents the weather-yield functional forms used in this analysis. The indices in all tables
are listed in ascending order of $R^2$. In general, the univariate indices are not as good fitting as the bivariate indices. The best index to predict production is the quadratic model with rainfall and temperature for Northwest, and the quadratic model with rainfall and GDD for Free State, respectively.

After the detrended yield, rainfall and temperature passed the normality test, we simulated a sample of 2,000 normally distribution random observations for each of the rainfall, GDD, temperature and yield based on the estimated model.

**Results**

Results from the expected utility maximization model (5) are obtained numerically. Based on the utility function (7), the optimal number of insurance contracts is calculated for different values of the CRRA coefficient (0.5, 1, 3, and 5) and premium-loading factor (0 for actuarially fair and 0.1). The trigger weather index is set at the mean level of each weather index. Similarly, certainty equivalents of the grower’s final wealth are obtained from model (6).

Results are presented in tables 2. When no premium loading is considered (left side), the expected utility model yields an almost constant optimal coverage as in Proposition 1, although slightly influenced by the risk aversion level. This is because the mean variance model is only an approximation of the expected utility model. The representative grower studied with different risk preferences will buy about 1.4 shares of weather-indexed insurance per hectare of cropland.

Across alternative indices, the optimal coverage does not show a particular trend even though the $R^2$ is increasing, as per Proposition 2. This is because a change in indices leads to an associated change in the underlying indemnity function. Therefore, it is not surprising that the optimal coverage level is not monotonically increasing across the indices. However, if we hold
the underlying indemnity function constant, grain growers with yields more positively correlated with the weather index should buy more of this type of insurance.

Table 2 also presents the CE values associated with each index. The CE’s change both across risk aversion levels and across the alternative indices. For example, consider the rainfall index (R) in the Northwest Province. The CE value increases from 1.7 kg/ha when the risk aversion level is .5, to 16.8 kg/ha when the risk aversion level is 5. Thus, at optimal levels of insurance, the more risk-averse grower will value the same insurance contract more highly.

When comparing across alternative indices, the CE increases as R² increases. Again, taking the Northwest Province in table 2 as an example, the CE increases from 1.7 kg/ha for the rainfall index (R) to 7.5 kg/ha for the rainfall-temperature quadratic index (Q_RT) when the CRRA coefficient is .5. Since the indices are arranged in ascending order of their R², the corresponding CE values rise in the same pattern. These results show the superiority of multivariate weather indices, in terms of their relative efficiencies, as potentially viable hedging instruments. Meanwhile, the GDD is better than temperature, and both are better than rainfall for both provinces.

However, we also observe a slight variation in the ordinal ranking of the CE’s across the indices as the risk aversion level increases. As the CRRA coefficient increases, the ranking of the CE values changes only slightly. As a result, the results still follow the pattern of the R² ranking for the actuarially fair cases. Table 4 shows the discrepancies in the ordinal ranking across the different indices.

Although a higher R² indicates a higher correlation between the yield and the weather index, it does not always guarantee a higher correlation between the yield and the indemnity payment, which is a truncated weather index. Furthermore, in contrast to the M-V model, the
expected utility model takes into account the correlation and other relations based on higher moments between yield and the indemnity payment. As a result, higher CE value is not always achieved for a higher $R^2$.

The right side of Table 2 allows a premium loading of 10 percent into the system. For low values of the CRRA coefficient, the optimal insurance coverage is negative when no restrictions are imposed on the choice. This means that, as the price of insurance becomes more expensive, a low risk-averse grower becomes a net “seller” of insurance contracts. The size of the selling is larger for the poorer indices. This is because for the poorer indices that are less correlated to the yield risk, selling those contracts will not result in amplifying the risks from the production very much. However, as the grower’s risk aversion is increases, the grower won’t offer such insurance for sale for the given certainty equivalent income. He would still buy such insurance, although in lesser quantities compared to the no loading case. As a result, the optimal contract share is increasing as the risk aversion increases, as suggested by Proposition 3.

When the growers offer the insurance for sale, the ranking in CE values across the alternative indices at the optimal contacts is reversed. Since the grower is a net seller to obtain the extra mean revenue, he increases his risk by offering the weather index insurance. The higher the basis risk between his own yield and the weather index, the less total risk he accepts by offering such insurance, thereby increasing his utility. When the grower buys loaded insurance, his CE is less than for the no load case for the higher cost and lower risk protection, which is consistent with Proposition 4. The ranking of the weather indices becomes similar to the no load case.

**Summary and Conclusion**

In contrast to previous work that suggests that a single-variable weather index suffices to
develop an insurance contract, this study shows that the insured grower achieves a higher utility from multivariate weather indices. The most important single weather index we found in the study area was GDD, and the combination of rainfall and either temperature or GDD outperformed the single variable indices by a large margin.

Depending on the growers risk preference, he may choose to buy or offer such insurance for sale if the price is not actuarially fair. The risk protection value of weather-indexed insurance follows the predictive power of the index on yield in general, though not exactly. There is a trade off between choosing an index with a large number of weather variables that can improve on the efficiency of the contract, and choosing a single-variable index that is easily understood by the growers.

Therefore further research could look into the construction of an appropriate weather index or indices, which not only would improve the goodness of fit (or any other measure of correlation) on yield, but also is easily understood by the market participants.
References


Paper No. 55.


<table>
<thead>
<tr>
<th>Weather Index</th>
<th>Model</th>
<th>$R^2$</th>
<th>Adj$R^2$</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>North West</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$R$</td>
<td>$\hat{y}_{det} = -1.104 + 0.003R$</td>
<td>.224</td>
<td>.187</td>
</tr>
<tr>
<td>$T$</td>
<td>$\hat{y}_{det} = 16.37 - 0.004T$</td>
<td>.673</td>
<td>.658</td>
</tr>
<tr>
<td>$GDD$</td>
<td>$\hat{y}<em>{det} = 3.392 - 0.007GDD</em>{20}$</td>
<td>.775</td>
<td>.764</td>
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<td>$RQ__RT$</td>
<td>$\hat{y}_{det} = 72.889 + 0.0114R - 1.28\times10^{-5}R^2 - 0.0354T + 4.11\times10^{-6}T^2$</td>
<td>.786</td>
<td>.739</td>
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<tr>
<td>$RQ__RG$</td>
<td>$\hat{y}<em>{det} = 3.757 + 0.0083R - 8.68\times10^{-6}R^2 - 0.0165GDD</em>{20} + 1.05\times10^{-5}GDD_{20}^2$</td>
<td>.852</td>
<td>.819</td>
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<td>$Q__RG$</td>
<td>$\hat{y}<em>{det} = 5.277 + 0.0056R - 8.2\times10^{-6}R^2 - 0.0202GDD</em>{20} + 1.25\times10^{-5}GDD_{20}^2 + 4.29\times10^{-6}R*GDD_{20}$</td>
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<td>.811</td>
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<td>$Q__RT$</td>
<td>$\hat{y}_{det} = 354.013 - 0.1019R - 4.34\times10^{-6}R^2 - 0.1686T + 1.99\times10^{-5}T^2 + 2.74\times10^{-5}RT$</td>
<td>.863</td>
<td>.823</td>
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<td><strong>Free State</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$R$</td>
<td>$\hat{y}_{det} = -0.537 + 0.002R$</td>
<td>.290</td>
<td>.256</td>
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<tr>
<td>$T$</td>
<td>$\hat{y}_{det} = 11.62 - 0.003T$</td>
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<td>.509</td>
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<td>$GDD$</td>
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<td>.598</td>
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<tr>
<td>$RQ__RT$</td>
<td>$\hat{y}_{det} = -73.473 + 0.0069R - 5.69\times10^{-6}R^2 + 0.0435T - 6.52\times10^{-6}T^2$</td>
<td>.652</td>
<td>.574</td>
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<td>$Q__RT$</td>
<td>$\hat{y}_{det} = -102.553 + 0.0228R - 6.21\times10^{-6}R^2 + 0.0578T - 8.27\times10^{-6}T^2 - 4.32\times10^{-6}RT$</td>
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<td>.647</td>
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<tr>
<td>$Q__RG$</td>
<td>$\hat{y}<em>{det} = -1.576 + 0.0066R - 5.0\times10^{-6}R^2 + 0.0069GDD</em>{20} - 1.76\times10^{-5}GDD_{20}^2 - 1.74\times10^{-6}R*GDD_{20}$</td>
<td>.712</td>
<td>.627</td>
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### Table 2: Optimal Coverage and Certainty Equivalent Values

<table>
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<tr>
<th>Weather Index</th>
<th>( \theta = 0.5 )</th>
<th>( \theta = 1 )</th>
<th>( \theta = 3 )</th>
<th>( \theta = 5 )</th>
<th>( \theta = 0.5 )</th>
<th>( \theta = 1 )</th>
<th>( \theta = 3 )</th>
<th>( \theta = 5 )</th>
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<td></td>
<td>( n^* )</td>
<td>CE (^2)</td>
<td>CE (^3)</td>
<td>CE</td>
<td>CE (^*)</td>
<td>CE (^*)</td>
<td>CE (^*)</td>
<td>CE (^*)</td>
</tr>
<tr>
<td><strong>Northwest</strong></td>
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<td></td>
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<td></td>
</tr>
<tr>
<td>R</td>
<td>1.37</td>
<td>1.7</td>
<td>1.36</td>
<td>3.3</td>
<td>1.36</td>
<td>10.0</td>
<td>1.35</td>
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<td>T</td>
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<td>5.7</td>
<td>1.44</td>
<td>11.4</td>
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<td>34.3</td>
<td>1.42</td>
<td>57.5</td>
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<td>1.42</td>
<td>6.2</td>
<td>1.42</td>
<td>12.4</td>
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<td>37.5</td>
<td>1.39</td>
<td>63.1</td>
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<td>1.28</td>
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\(^1\)Loading factor  
\(^2\)CRAA coefficient  
\(^3\)Optimal number of insurance contracts  
\(^4\)The certainty equivalent income, denominated in production units of kg/ha.
Figure 1 Map of South Africa major corn growing areas.