Designing Regulatory Policies for Complex Externalities: The Case of Agricultural Pollution

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Abstract

The paper examines the issue of designing and implementing policy measures to control complex agricultural externalities. Complex externalities refer to the situation where a production (firm on firm) externality coexists with a detrimental (firm on society) externality. The paper identifies the optimal solution for complex externalities, which is a combination of spatially differentiated taxes. However, severe information requirements render the first-best policy infeasible. Finally, a likely voluntary scheme based on firm self-report is examined which may enforce firm compliance with the optimal policy.

Keywords: *complex externalities, state dependent linear ambient tax, voluntary schemes, moral hazard, incentive compatibility, spatial externalities.*

JEL Classification: *H23, Q19, Q28*

Introduction

This paper examines the issue of regulating complex externalities via a set of zonal (differentiated) economic instruments. We coin the term "complex externalities" to describe the situation in which the actions of one economic agent affect the production possibility of other agents (firm on firm or production externality), while at the same time the actions of all agents adversely affect social welfare (firm on society or detrimental externality). A typical example of complex externalities is the case of an upstream pollution generating farm which releases emissions that affect, either positively or negatively, the production process of a downstream farm. At the same time, the pollution generated by both firms adversely affects the quality of a water body imposing damages to society.

To the best of our knowledge, the issue of controlling complex externalities has not attracted considerable attention in the relevant literature. Xepapadeas (1997) examines the issue of a production externality but restricts his analysis only to the case of a nega-

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tive production externality, while he does not include the likely firm on society externality. There is an abundance of papers which deal with the issue of a firm on society externality. Very useful summaries of that literature are given by Hanley *et al*. (1997) and Russell (2001). Our paper attempts to fill this gap by showing that the optimal control of complex (agricultural) externalities requires a set of spatially differentiated emission taxes, appropriately defined to reflect the kind of interactions, either positive or negative, between firms.

In most of the cases, it is reasonable to argue that the benefits to the downstream farm (due to the elevated nitrate level in the irrigation water) are not considerable enough to offset the damages that such nitrates impose to society. As a result, the optimal policy for controlling complex externalities turns out to be a set of spatially differentiated emissions taxes. In general, the latter is a well known result, in the sense that when there is transportation of pollutants from one region to another, the optimal tax in a region reflects the marginal damages of the pollutants that remain in a region, and the marginal damages of the transported pollutants to other regions (Siebert 1985; Xepapadeas 1992). What distinguishes the solution derived for complex externalities is that there may be regions (land zones) in which the optimal instrument is not only defined by those marginal damages but also includes a tax or a subsidy to account for the farm on farm externality.

A serious limitation of the first best solution is the intensity of the informational requirements, which along with the likely administrative costs might turn such as solution not applicable. Consequently, the regulator needs to design a surrogate set of policy measures which mimic the rationale of the fist-best solution while being easy to apply. Such a system which utilises self-reporting is briefly described in section 2.

The paper is organized as follows. The next section describes a model of complex externalities and derives the optimal solution under perfect information. Section 3 discusses the main limitations of the optimal policy and examines a likely voluntary scheme implementing a system of zonal taxes, while section 4 gives the conclusions.

1) The Model

Consider an agricultural catchment which is split into two zones, where the land quality in each zone is assumed to be heterogeneous. The two zones assumption substantially simplifies the model's notation without sacrificing its generality (Siebert 1985). The upstream zone, A, is suitable for non-irrigated crops, such as barley or oat, while the downstream zone, B, is suitable for irrigated crops, such as cotton or corn. For simplicity we assume that there is a single farm (firm) in each zone producing a single product, the amount of which is q_1 and q_2 respectively. A typical side-effect of the firm production is the release of nitrate emissions. These nitrate emissions end up to the adjacent river, through run-off, and deteriorate its water quality imposing damages to these individuals that use the river water. In addition to the damages imposed to society by the released emissions, the firms are linked through a production externality since the downstream firm uses irrigation water, the quality of which is affected by the emissions released by the upstream firm.

In turn, assume that both farms are risk neutral. The upstream farm produces a product q_1 according to a strictly increasing and concave function of the input used, x_1, that is $q_1 = f_1(x_1)$, $\frac{\partial f_1}{\partial x_1} > 0$ and $\frac{\partial^2 f_1}{\partial x_1^2} < 0$. The unregulated optimal input choice of the upstream firm is given by $x_1^* = \arg\max \pi_1(x_1)$, where $\pi(x_1) = p_1 f(x_1) - w x_1$, p_1 is the price of the product q_1 and w is the input price. At the same time, the expected emissions released by the upstream farm, e_1, are given by $e_1 = e_1(x_1, \gamma_1)$, where x_1 denotes the input choices, and γ_1 represents a vector of farm specific characteristics (e.g. soil type, technology employed).[4] The emission function is assumed to be a strictly increasing and convex function in the input used, that is $\frac{\partial e_1}{\partial x_1} > 0$ and $\frac{\partial^2 e_1}{\partial x_1^2} > 0$.

In line with Shortle and Horan (2001) and DiToro (2001) we assume that the nitrate concentration in the upper zone of the river, c_1, is a function of the nitrate emissions released by farm 1 and transferred to the river by runoff, e_1; a vector of physical characteristics of zone A (e.g. relief), ω_1; and the initial concentration of the background level of nitrates in the upper zone, c_0, $c_1 = c_1(e_1, \omega_1, c_0)$.

The farm in zone B produces a product q_2 using two inputs, nitrate fertilizer, x_2, and irrigation water, r_2. Equally, the production function of the downstream farm is specified as: $q_2 = f_2(x_2, r_2, c_1)$ with $\frac{\partial f_2}{\partial x_2} > 0, \frac{\partial^2 f_2}{\partial x_2^2} < 0$, $\frac{\partial f_2}{\partial r_2} > 0$, $\frac{\partial^2 f_2}{\partial x_2 \partial r_2} < 0$, $\frac{\partial f_2}{\partial c_1} \gtrless 0$, $\frac{\partial^2 f_2}{\partial c_1^2} < 0$ and $\frac{\partial^2 f_2}{\partial x_2 \partial c_1} < 0$. The emissions released by the downstream farm directly depend on the input used by the farmer, x_2, and indirectly on the nitrogen concentration of irrigation water c_1, and a vector of farm specific characteristics as in the case of the upstream farm, that is: $e_2 = e_2(x_2, c_1, \gamma_2)$ with $\frac{\partial e_2}{\partial x_2} > 0$, $\frac{\partial^2 e_2}{\partial x_2^2} > 0$, $\frac{\partial e_2}{\partial c_1} > 0$, $\frac{\partial^2 e_2}{\partial c_1^2} > 0$ and $\frac{\partial^2 e_2}{\partial x_2 \partial c_1} > 0$. The nitrate concentration in zone B is given by $c_2 = c_2(e_2, \omega_2, c_1)$, where equally ω_2 denotes the vector of physical characteristics of zone B.

In addition, we assume that there is one village in each zone that is supplied with water by the river. The water is potable if the nitrate concentration is lower than a critical value denoted as $\hat{c}$. Consequently, the elevated level of nitrates in the river waters imposes cleanup costs if $c_i > \hat{c}$. These costs are given by the function $D = \delta_1 d_1(c_1) + \delta_2 d_2(c_2)$, where $d_1(c_1)$ represents the cost of cleaning the water in the upstream zone, while $d_2(c_2)$ denotes the analogous costs in the downstream zone. We

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[4] To simplify the analysis, we ignore the fact that pollution is inherently stochastic and all random variables are replaced by their expected values.

also assume that the cleanup cost increases at a decreasing rate as nitrate concentration rises, $d_i' > 0$ and $d_i'' < 0$ due to standard arguments related to economies of scale in water treatment. The scalars $\delta_i \;\; i = 1,2$ are defined as $\delta_i = 1 \;\; if \;\; c_i > \hat{c}$ and $\delta_i = 0 \;\; if \;\; c_i \leq \hat{c}$. Note that if $\delta_1 = 1$ then $\delta_2 = 1$ and if $\delta_1 = 0$ then $\delta_2 = \begin{cases} 0 & c_2 \leq \hat{c} \\ 1 & c_2 > \hat{c} \end{cases}$. The specific choice of a threshold damage function, instead of a generic societal damage from polluting activities, captures the limiting case where polluting emissions impose no financial burden on society. A typical example of such a limiting case is when the prevailing level of pollution in one site is lower than the safety limit set by the regulator. In such a case, no action is required and hence the clean-up costs are zero. In addition, apart from institutional reasons there are also physical reasons that may justify the choice of a threshold damage function. For instance, due to the assimilative capacity of environmental media, low levels of pollution may have no impact on environmental quality and consequently no requirements for clean-up costs. It is clear that the case where $\delta_1 = \delta_2 = 0$ can not be qualified as a complex externality and therefore we always assume that $\delta_2 = 1$.

We consider the water clean-up costs to be a proxy of external damages, since in the absence of regulation farms do not take them into account. Presumably external costs may be defined in a much broader fashion. In particular, external costs may include damages imposed to individuals who use the river for recreational purposes, or in general any damages imposed to society by the presence of high nitrate concentration in the river. In terms of this study, we restrict our attention to the water clean-up costs.

It is known that when the effects of emissions from various sources differ, Pareto optimality requires instruments individually tailored to each source (Baumol and Oates 1988). As a result, we assume that the regulator opts for a system of zonal (differentiated) instruments to influence farm behaviour. The regulator's problem is to choose the appropriate rates of instruments to maximize social welfare subject to the constraint that firms maximize profits. Social welfare is expressed by the net surplus (quasi-rents less the monetary damages from the released emissions) so the regulator's problem can be written as:

$$\max_{x_1, x_2} \left(p_1 f_1(x_1) - w x_1 \right) + \left(p_2 f_2(x_2, r_2, c_1) - w x_2 \right) - \delta_1 d_1\left(c_1\right) - d_2(c_2) \tag{1}$$

Subject that both firms maximizing profits after a tax/subsidy, t_i, imposed on emissions

$$\max_{x_1, e_1 > 0} \left(p_1 f_1(x_1) - w x_1 \right) - t_1 e_1 \tag{2}$$

$$\max_{x_2, e_2 > 0} \left(p_2 f_2(x_2, r_2, c_1) - w x_2 \right) - t_2 e_2 \tag{3}$$

Note that taxes and subsidies are transfer payments between firms and the regulator and thus are not included in the social welfare specification. The problem (1)-(3) is a typical mathematical programming problem with equilibrium constraints. One possible solution is to replace the optimization problems from the constraints (the inner problems) with the Karush-Kuhn-Tucker (KKT) conditions (Luo et al, 1996). Assuming then that both inner problems have interior solutions the KKT conditions are:

$$\left(p_1\frac{\partial f_1}{\partial x_1}-w\right)=t_1\frac{\partial e_1}{\partial x_1} \tag{4}$$

$$\left(p_2\frac{\partial f_2}{\partial x_2}-w\right)=t_2\frac{\partial e_2}{\partial x_2} \tag{5}$$

The Lagrangean function of the problem (1), (4) and (5) is

$$\begin{aligned} L=&\left(p_1 f_1(x_1)-wx_1\right)+\left(p_2 f_2(x_2,r_2,c_1)-wx_2\right)-\delta_1 d_1\left(c_1\right)-d_2(c_2)\\ &+\lambda_1\left(t_1\frac{\partial e_1}{\partial x_1}-p_1\frac{\partial f_1}{\partial x_1}+w\right)+\lambda_2\left(t_2\frac{\partial e_2}{\partial x_2}-p_2\frac{\partial f_2}{\partial x_2}+w\right)\end{aligned} \tag{6}$$

Assuming once again that an interior solution exists, the optimality conditions for the new problem are:

$$\begin{aligned}\frac{\partial L}{\partial x_1}=&\left(p_1\frac{\partial f_1}{\partial x_1}-w\right)+\left(p_2\frac{\partial f_2}{\partial c_1}\frac{\partial c_1}{\partial e_1}\frac{\partial e_1}{\partial x_1}\right)-\delta_1 d_1'\left(\frac{\partial c_1}{\partial e_1}\frac{\partial e_1}{\partial x_1}\right)-d_2'\left\{\frac{\partial c_1}{\partial e_1}\frac{\partial e_1}{\partial x_1}\left(1+\frac{\partial c_2}{\partial e_2}\frac{\partial e_2}{\partial c_1}\right)\right\}\\ &+\lambda_1\left(t_1\frac{\partial^2 e_1}{\partial x_1^2}-p_1\frac{\partial^2 f_1}{\partial x_1^2}\right)+\lambda_2\frac{\partial c_1}{\partial e_1}\frac{\partial e_1}{\partial x_1}\left(t_2\frac{\partial^2 e_2}{\partial x_2\partial c_1}-p_2\frac{\partial^2 f_2}{\partial x_2\partial c_1}\right)=0\end{aligned} \tag{7}$$

$$\frac{\partial L}{\partial x_2}=\left(p_2\frac{f_2}{\partial x_2}-w\right)-\delta_2 d_2'\left(\frac{\partial c_2}{\partial e_2}\frac{\partial e_2}{\partial x_2}\right)+\lambda_2\left(t_2\frac{\partial^2 e_2}{\partial x_2^2}-p_2\frac{\partial^2 f_2}{\partial x_2^2}\right)=0 \tag{8}$$

$$\frac{\partial L}{\partial t_1}=\lambda_1\frac{\partial e_1}{\partial x_1}=0 \tag{9}$$

$$\frac{\partial L}{\partial t_2}=\lambda_2\frac{\partial e_2}{\partial x_2}=0 \tag{10}$$

$$\frac{\partial L}{\partial \lambda_1}=t_1\frac{\partial e_1}{\partial x_1}-p_1\frac{\partial f_1}{\partial x_1}+w=0 \tag{11}$$

$$\frac{\partial L}{\partial \lambda_2}=t_2\frac{\partial e_2}{\partial x_2}-p_2\frac{\partial f_2}{\partial x_2}+w=0 \tag{12}$$

From (9) & (10) we can see immediately that $\lambda_1=\lambda_2=0$ since $\frac{\partial e_i}{\partial x_i}>0$. In turn, substituting (11) into (7) we obtain the optimal instrument for the upstream firm:

$$t_1\frac{\partial e_1}{\partial x_1}+\left(p_2\frac{\partial f_2}{\partial c_1}\frac{\partial c_1}{\partial e_1}\frac{\partial e_1}{\partial x_1}\right)-\delta_1 d_1'\left(\frac{\partial c_1}{\partial e_1}\frac{\partial e_1}{\partial x_1}\right)-d_2'\left\{\frac{\partial c_1}{\partial e_1}\frac{\partial e_1}{\partial x_1}\left(1+\frac{\partial c_2}{\partial e_2}\frac{\partial e_2}{\partial c_1}\right)\right\}=0 \tag{13}$$

Given that $\frac{\partial e_1}{\partial x_1}\neq 0$, (13) reduces to

$$t_1=\delta_1 d_1'\frac{\partial c_1}{\partial e_1}+d_2'\left\{\frac{\partial c_1}{\partial e_1}\left(1+\frac{\partial c_2}{\partial e_2}\frac{\partial e_2}{\partial c_1}\right)\right\}-p_2\frac{\partial f_2}{\partial c_1}\frac{\partial c_1}{\partial e_1} \tag{14}$$

According to (14) the optimal emission instrument for the upstream firm comprises three components. The first component is the marginal damages in zone A by a unitary increase in the nitrate concentration c_1, $\delta_1 d_1' \frac{\partial c_1}{\partial e_1}$. The second component is the total (direct and indirect) marginal damages in zone B by a unitary increase in the nitrate concentration c_1, $d_2' \left\{ \frac{\partial c_1}{\partial e_1} \left(1 + \frac{\partial c_2}{\partial e_2} \frac{\partial e_2}{\partial c_1} \right) \right\}$. The direct marginal damages in zone B brought about by an increase of c_1 is $d_2' \frac{\partial c_1}{\partial e_1}$, and the indirect marginal damages brought about by an increase of c_2 (as a result of a unitary increase in c_1) is $d_2' \left\{ \frac{\partial c_1}{\partial e_1} \frac{\partial c_2}{\partial e_2} \frac{\partial e_2}{\partial c_1} \right\}$. The rationale behind the second component is the interregional diffusion of pollutants, a flow of pollutants from zone A to zone B, which results in a set of spatial differentiated emission taxes. The latter is a well-known result in the relevant literature (see Tietenberg (1974), Siebert (1985) and Xepapadeas (1992)). The first and second components of (14) capture the issue of firms on society externality. By contrast, the third component of (14) reflects the value of the marginal product of the downstream farm brought about by a unitary increase in the nitrate concentration c_1 in zone A, $p_2 \frac{\partial f_2}{\partial c_1} \frac{\partial c_1}{\partial e_1}$, which captures the issue of production (firm on firm) externality.

It is noteworthy, that whether the instrument defined by (14) reduces to a pure tax or a pure subsidy is an empirical issue. The reason is that the last component, $p_2 \frac{\partial f_2}{\partial c_1} \frac{\partial c_1}{\partial e_1}$, may be positive or negative depending upon the sign of $\frac{\partial f_2}{\partial c_1}$. If we assume that the production function $q_2 = f_2(.)$ has a polynomial form, the sign of $\frac{\partial f_2}{\partial c_1}$ is positive if the total nitrogen applied is less than the nitrogen requirements for maximum yield, $r_2^* c_1 + x_2^* \leq \hat{x}_2$.[5] On the other hand, if $r_2^* c_1 + x_2^* > \hat{x}_2$ then $\frac{\partial f_2}{\partial c_1} < 0$, since the indirect nitrogen shifts downstream farm to the third stage of production.

If the nitrate concentration in zone A is greater than the critical value, $c_1 > \hat{c} \Rightarrow \delta_1 = 1$, it is clear from equation (14) that the optimal instrument in zone A is a tax as long as the emission from the upstream farm reduces the productivity of the downstream farm $\frac{\partial f_2}{\partial c_1} < 0$. In such a case, the optimal tax in zone A comprises the spatially differentiated

[5] Note that the total nitrogen applied to zone B consists of the direct nitrogen applied through fertilizers, x_2^* where $x_2^* = \arg\max \pi_2(x_2)$, and the indirect nitrogen transferred through irrigation water, $r_2^* c_1$. The amount of nitrogen which maximizes yields is $\hat{x}_2 = \arg\max f_2(x_2)$, $\hat{x}_2 > x_2^*$ since $w > 0$.

impacts of the upstream farm to zone A and zone B (firm on society negative externality) and the negative interaction of the upstream farm to the downstream farm (firm on form negative externality).

By contrast, if $\frac{\partial f_2}{\partial c_1}>0$ the optimal instrument in zone A is likely to be a tax as long as the value of marginal product of the downstream farm brought about by a unitary increase in c_1, $p_2\frac{\partial f_2}{\partial c_1}\frac{\partial c_1}{\partial e_1}$ is lower than the sum of the marginal damages brought about by a unitary increase in the nitrate concentration c_1 in zone A, $d_1'\frac{\partial c_1}{\partial e_1}$, and in zone B, $d_2'\left\{\frac{\partial c_1}{\partial e_1}\left(1+\frac{\partial c_2}{\partial e_2}\frac{\partial e_2}{\partial c_1}\right)\right\}$.

In the extreme case where $d_1'\frac{\partial c_1}{\partial e_1}+d_2'\left\{\frac{\partial c_1}{\partial e_1}\left(1+\frac{\partial c_2}{\partial e_2}\frac{\partial e_2}{\partial c_1}\right)\right\}<p_2\frac{\partial f_2}{\partial c_1}\frac{\partial c_1}{\partial e_1}$ the optimal instrument in zone A is a subsidy.

By the same rationale, If the nitrate concentration in zone A is lower than the critical value, $c_1<\hat{c}\Rightarrow\delta_1=0$, equation (14) reduces to:

$$t_1=d_2'\left\{\frac{\partial c_1}{\partial e_1}\left(1+\frac{\partial c_2}{\partial e_2}\frac{\partial e_2}{\partial c_1}\right)\right\}-p_2\frac{\partial f_2}{\partial c_1}\frac{\partial c_1}{\partial e_1} \tag{15}$$

From equation (15) it is clear that the optimal instrument in zone A is a tax as long as $\frac{\partial f_2}{\partial c_1}<0$. Equally, if $\frac{\partial f_2}{\partial c_1}>0$ the regulator may impose a tax or a subsidy in zone A depending on the relative magnitude of the impacts that the upstream farm has on the clean-up costs and the productivity of the downstream farm. Specifically, if $d_2'\left\{\frac{\partial c_1}{\partial e_1}\left(1+\frac{\partial c_2}{\partial e_2}\frac{\partial e_2}{\partial c_1}\right)\right\}<p_2\frac{\partial f_2}{\partial c_1}\frac{\partial c_1}{\partial e_1}$ then the optimal instrument in zone A is a subsidy, which is the case where the positive firm on firm externality dominates the negative firm on society externality. By contrast, if $d_2'\left\{\frac{\partial c_1}{\partial e_1}\left(1+\frac{\partial c_2}{\partial e_2}\frac{\partial e_2}{\partial c_1}\right)\right\}>p_2\frac{\partial f_2}{\partial c_1}\frac{\partial c_1}{\partial e_1}$ the optimal instrument in zone A is a tax.

By contrast, if we assume that the production function $q_2=f_2(.)$ has a von Liebig form then the farm to farm interaction cannot be negative since $\frac{\partial f_2}{\partial c_1}\geq 0$. The rationale behind such a specification is the "law of minimum", which states that crop output increases linearly with the availability of the limiting factor (nutrient) until it reaches a maximum. Beyond that point nutrient's availability has no effect on crop output and therefore the case where $\frac{\partial f_2}{\partial c_1}<0$ is not possible (Grimm, et al. 1987). The next table summarizes the previous results.

Table 1: *Optimal Policy Instrument for the Upstream Zone*

		Polynomial		von Liebig
		$\frac{\partial f_2}{\partial c_1}<0$	$\frac{\partial f_2}{\partial c_1}>0$	$\frac{\partial f_2}{\partial c_1}>0$
$\delta_1=1$	$\left\lvert d_1'\frac{\partial c_1}{\partial e_1}+d_2'\left\{\frac{\partial c_1}{\partial e_1}\left(1+\frac{\partial c_2}{\partial e_2}\frac{\partial e_2}{\partial c_1}\right)\right\}\right\rvert<\left\lvert p_2\frac{\partial f_2}{\partial c_1}\frac{\partial c_1}{\partial e_1}\right\rvert$	Tax	Subsidy	Subsidy
	$\left\lvert d_1'\frac{\partial c_1}{\partial e_1}+d_2'\left\{\frac{\partial c_1}{\partial e_1}\left(1+\frac{\partial c_2}{\partial e_2}\frac{\partial e_2}{\partial c_1}\right)\right\}\right\rvert>\left\lvert p_2\frac{\partial f_2}{\partial c_1}\frac{\partial c_1}{\partial e_1}\right\rvert$	Tax	Tax	Tax
$\delta_1=0$	$\left\lvert d_2'\left\{\frac{\partial c_1}{\partial e_1}\left(1+\frac{\partial c_2}{\partial e_2}\frac{\partial e_2}{\partial c_1}\right)\right\}\right\rvert<\left\lvert p_2\frac{\partial f_2}{\partial c_1}\frac{\partial c_1}{\partial e_1}\right\rvert$	Tax	Subsidy	Subsidy
	$\left\lvert d_2'\left\{\frac{\partial c_1}{\partial e_1}\left(1+\frac{\partial c_2}{\partial e_2}\frac{\partial e_2}{\partial c_1}\right)\right\}\right\rvert>\left\lvert p_2\frac{\partial f_2}{\partial c_1}\frac{\partial c_1}{\partial e_1}\right\rvert$	Tax	Tax	Tax

Although the choice of the appropriate functional form is controversial (Paris and Knapp 1989; Berck and Helfand 1990) we can arguably rule out the case $\frac{\partial f_2}{\partial c_1}<0$ by assuming that the quantity of indirect nitrogen is practically negligible for shifting production to the third stage of production $r_2^* c_1 << \hat{x}_2 - x_2^*$, and therefore the farm to farm interaction is always positive $\frac{\partial f_2}{\partial c_1}>0$. Consequently, the optimal control policy for zone A consists of a set of tailored emission taxes, $\delta_1 d_1'\frac{\partial c_1}{\partial e_1}+d_2'\left\{\frac{\partial c_1}{\partial e_1}\left(1+\frac{\partial c_2}{\partial e_2}\frac{\partial e_2}{\partial c_1}\right)\right\}$, to account for the firm on society externality and a subsidy, $p_2\frac{\partial f_2}{\partial c_1}\frac{\partial c_1}{\partial e_1}$, to account for the production externality.

In principle, the firm on firm externality may be positive or negative. As a result, the instrument defined by (14) may be reduced to a pure tax or a pure subsidy depending on the sign and the magnitude of the last component, $p_2\frac{\partial f_2}{\partial c_1}\frac{\partial c_1}{\partial e_1}$, which is an empirical issue.

In most of the cases, however, it is reasonable to argue that the benefits to the downstream farm (due to the elevated nitrate level in the irrigation water) are not considerable enough to offset the damages that such nitrates impose to society. Such a claim is reasonable either the upstream farm imposes damages in both zones, $\delta_1=1$, or only in zone B, $\delta_1=0$. Put differently, the subsidy component in (14) is actually a downward adjustment of a tax on the upstream farm's emissions. As a result, the optimal tax in zone A is less than the sum of marginal damages that the upstream farm imposes to both zones.

Equally, substituting (12) into (8) we get:

$$t_2 = d_2' \frac{\partial c_2}{\partial e_2} \tag{16}$$

That is to say, the optimal instrument for the downstream firm is an emission tax which equals to marginal damages imposed by a unitary increase in the nitrate concentration c_2. Hence, we derive the following proposition:

Proposition 1:

The optimal policy for the control of complex agricultural externalities requires a combination of spatially differentiated emission taxes. In particular, the optimal tax for the upstream farm is lower than the sum of the marginal damages imposed to both zones because such a tax takes into account the firm on firm (positive) externality. By contrast, the optimal tax for the downstream farm is equal to the marginal damages imposed to the downstream zone.

The previous analysis can be extended for the case of multiple firms and multiple zones at the expense of complex notations without altering the very meaning of proposition 1.

2) Implementing a Zonal System of Taxes for Controlling Complex Externalities.

It is self-evident that the system of differentiated taxes suffers from severe informational problems. In order for the regulator to be able to specify such a system he/she needs information related to the production functions, the input use of firms, reliable estimators of nitrate emission functions and the transfer coefficients.

It is a typical assumption in the related literature (see Shortle and Horan (2001)) that part of that information can be retrieved through the use of simulation models, which under specific conditions can provide reliable estimators about farms' production and pollution possibilities, $f_i(.)$ and $e_i(.)$. An indispensable input in such models, however, is the farm input use, which is private information. As a result, the crucial policy question is to design feasible control schemes that provide farmers with the appropriate incentives to report their input use.

To this end, we consider a possible scheme that induces farmers to self-report their input use and in turn the regulator estimates tax liabilities for each farm according to (14) & (16). Such a policy is often classified as a voluntary scheme (Segerson and Micheli 1998; Khanna 2001). We make the assumption that farms are willing to participate in this scheme as long as the non-participant farms are expected to pay higher taxes than the participant farms. Segerson and Wu (2006) have shown that the background threat of an ambient tax is sufficient to induce voluntary compliance. An example of a threat policy for the non-participant farms is the following:[6]

$$\begin{aligned} T_1 &= d_1'(c_1) + d_2'(c_1) + \alpha d_2'(c_2 - c_1) \\ T_2 &= (1-\alpha) d_2'(c_2 - c_1) \end{aligned} \tag{17}$$

[6] For the sake of brevity and clarity, we have assumed $\delta_2 = 1$.

The scheme in (17) is essentially a version of the damage based tax mechanism initially proposed by Hansen (1998) accordingly adjusted to reflect the spatial impacts of pollution in the case of a complex externality. It is suffice to say that the marginal damages of pollution are evaluated at the cut-off level of pollution, $d_i' \equiv d_i(\hat{c})$. According to (17) the farm in zone A is liable for the direct damages imposed to zone A and zone B by $d_1'(c_1)+d_2'(c_1)$, and the indirect damage imposed to zone B, $\alpha d_2'(c_2-c_1)$, with $0<\alpha<1$. On the other hand, the farm in zone B is liable only for a share of the damages imposed to that zone, $(1-\alpha)d_2'(c_2-c_1)$ given that the farm in zone A is liable for the rest $\alpha d_2'(c_2-c_1)$.

To recapitulate, the proposed scheme works in two stages. First, the farms are requested to report the amount of input used and the realized output. Then, the regulator using the formulas (14) and (16) estimates the tax liabilities for each farm and announces the tax payments that the farms have to make. At the same time, the regulator threatens the likely cheating farms with a damage based tax if the observed nitrate concentration exceeds the estimated nitrate concentration in each zone, $c_i^r \le c_i$, where c_i^r denotes the observed ambient concentration of nitrates in each zone, while c_i denotes the estimated (expected) concentration of nitrates in each zone.

The regulator's estimations can either be *ex-ante* or *ex-post* after the realization of all random variables. Although the *ex-ante* estimations can be made on the basis of historical weather data, type I and II errors cannot be ruled out. Type I error refers to the probability of charging a damage based tax on a compliant farm. This may happen under extremely wet weather conditions, under which the observed nitrate concentration in the river can be higher than the estimated concentration which is based on a compliant farm's self-reports. By contrast, a type II error refers to the probability of not finding a noncompliant farm guilty, which may happen under very dry weather conditions. On the other hand, *ex post* estimations are immune to such errors while the incentives provided can be ex ante. Horan *et al* (1998) argue that the implementation of a state-dependent (*ex-post*) tax is not likely to be significantly more demanding than that of a state-independent (*ex-ante*) tax, while it bears a resemblance to the kind of tax that everybody is (or should be) familiar with, that of graduated income taxes.

The self-selection of the proposed scheme requires that the tax liability under the damage based scheme has to be higher than the tax liability estimated on the basis of farmers' reports. In other words, incentive compatibility constraints which describe individual rationality must hold. These can be written as:

$$\pi\left(x_1^{**}\right)-t_1e_1^{**} \ge \pi\left(\tilde{x}_1\right)-T_1 \tag{18}$$

$$\pi\left(x_2^{**}\right)-t_2e_2^{**} \ge \pi\left(\tilde{x}_2\right)-T_2 \tag{19}$$

with $x_i^{**}=\arg\max\left[\pi_i(x_i)-t_ie_i\right]$, $\tilde{x}_i=\arg\max\left[\pi(x_i)-T_i\right]$, $e_1^{**}=e_1\left(x_1^{**},\gamma_1\right)$, $e_2^{**}=e_2(x_1^{**},c_1^{**},\gamma_2)$ and $c_1^{**}=c_1\left(e_1^{**},\omega_1,c_0\right)$. Plugging (14) into (18) and (16) into (19) yields:

$$\begin{aligned}&\pi\left(x_1^{**}\right)-\pi\left(\tilde{x}_1\right)+d_1'\left\{c_1-\frac{\partial c_1}{\partial e_1}e_1^{**}\right\}+d_2'\left\{c_1-\frac{\partial c_1}{\partial e_1}e_1^{**}\right\}\\&+d_2'\left\{\alpha\left(c_2-c_1\right)-\frac{\partial c_1}{\partial e_1}\frac{\partial c_2}{\partial e_2}\frac{\partial e_2}{\partial c_1}e_1^{**}\right\}+p_2\frac{\partial f_2}{\partial c_1}\frac{\partial c_1}{\partial e_1}e_1^{**}\geq 0\end{aligned}\tag{20}$$

$$\pi\left(x_2^{**}\right)-\pi\left(\tilde{x}_2\right)+d_2'\left\{(1-\alpha)c_2-c_1-\frac{\partial c_2}{\partial e_2}e_2^{**}\right\}\geq 0\tag{21}$$

A close examination of (20) & (21) reveals the sufficient condition under which the incentive compatibility constraints (18) & (19) hold. Specifically, provided that $d_2'>0$ and $\pi\left(x_i^{**}\right)-\pi\left(\tilde{x}_i\right)\geq 0$ (see Appendix for a proof), (21) reduces to $\frac{\partial c_2}{\partial e_2}\frac{e_2^{**}}{c_2}\leq 1$ since $(1-\alpha)\frac{c_2-c_1}{c_2}<1$. Such a condition is always valid since it implies that the elasticity of nitrate concentration with respect to the emissions released by the downstream farm is less than unity, which is a reasonable assumption. As a result, the proposed scheme is incentive compatible.

Equally, since $d_i'>0$, $\frac{\partial f_2}{\partial c_1}\geq 0$ the sufficient conditions for equation (20) to hold are :

a) $1\geq\frac{\partial c_1}{\partial e_1}\frac{e_1^{**}}{c_1}$ and b) $1\geq\frac{\partial c_1}{\partial e_1}\frac{\partial c_2}{\partial e_2}\frac{\partial e_2}{\partial c_1}\frac{e_1^{**}}{c_2}=\varepsilon_{c_2e_2}\varepsilon_{c_1e_1}\varepsilon_{e_2c_1}$

since $\frac{c_2-c_1}{c_2}<1$ and $0<\alpha<1$. Given that the values of elasticities involved in the previous conditions are likely to be less than one then it is obvious that the value of the product $\varepsilon_{a_2e_2}\varepsilon_{a_1e_1}\varepsilon_{e_2a_1}$ is also less than one. As a result, equation (20) is always valid and hence the proposed scheme is incentive compatible.

3) Conclusions

Our paper presents the first, to our best knowledge, treatment of complex agricultural externalities that emerge in cases where firm on firm and firm on society interactions simultaneously exist. It has been shown that the optimal control policy for the upstream farm is a combination of emission taxes and a subsidy. In most of the cases, however, the benefits to the downstream firm (which determine the subsidy component) are not considerable enough to offset the damages brought about the released nitrates by the upstream firm. As a result, the overall effect of combining emission taxes with a subsidy for the upstream firm is a pure emission tax which is lower than the sum of marginal damages imposed to both zones. By contrast, the optimal control instrument for the downstream farm is an emission tax which equals the marginal damages in the downstream zone.

A serious limitation of the first best solution is the intensity of the informational requirements, which along with the likely administrative costs might turn such as solution infeasible. Finally, we have examined the possibility of a voluntary scheme, based on firms self-reports, to overcome the information burden that abounds complex agricultural externalities. Compliance with such a scheme is enforced via threatening firms with a state dependent linear ambient tax system. The analysis has shown that such a scheme is incentive compatible under very reasonable assumptions.

A final word about the main caveats of our analysis is needed. First, we have ignored any strategic interactions between firms by assuming Nash equilibrium. Second, we have not examined the possibility of firms' cheating via false reports; and finally, in line with the majority of the related literature on self-reports we have ignored the likely cost imposed on the firms for gathering information and submitting reports. All these issues can be easily included in future extensions.

Appendix:

Proof of $\pi\left(x_i^{**}\right)-\pi\left(\tilde{x}_i\right)\geq 0$

Defining $\pi\left(x_i^{**}-\tilde{x}_i\right)=\pi\left(x_i^{**}\right)-\pi\left(\tilde{x}_i\right)$ and using the mean value theorem we have:

$$\pi\left(x_i^{**}-\tilde{x}_i\right)=\frac{d\pi}{dx}\left(x_i^{**}-\tilde{x}_i\right) \tag{i}$$

From the envelope theorem we know that $\frac{d\pi}{dx_i}=p_i\frac{df_i}{dx_i}-w$, and since in the rational stage of production the value of the marginal product is greater or equal to the price of input, $p_i\frac{df_i}{dx_i}\geq w$ (Hirshleifer and Glazer 1992) it is immediately apparent that $\frac{d\pi}{dx_i}\geq 0$. As a result $\pi\left(x_i^{**}\right)-\pi\left(\tilde{x}_i\right)\geq 0$ always holds if $\left(x_i^{**}-\tilde{x}_i\right)\geq 0$. The latter is proved below. The farms are assumed to maximize profits after tax, $\max\left[\pi_i\left(x_i\right)-t_ie_i\right]$. The first order conditions of the previous maximization problem are:

$$\frac{\partial\pi}{\partial x}-t\frac{\partial e}{\partial x}=0 \tag{ii}$$

Total differentiation of (ii) gives:

$$\left[\frac{\partial^2\pi}{\partial x^2}-t\frac{\partial^2 e}{\partial x^2}\right]dx-\frac{\partial e}{\partial x}dt=0 \tag{iii}$$

and rearranging (iii) we get:

$$\frac{dx}{dt}=\frac{\partial e}{\partial x}\Big/\left[\frac{\partial^2\pi}{\partial x^2}-t\frac{\partial^2 e}{\partial x^2}\right] \tag{iv}$$

given that $\frac{\partial^2\pi}{\partial x^2}\neq t\frac{\partial^2 e}{\partial x^2}$, which is satisfied from the second-order condition of the firm's optimality , $\frac{\partial^2 \pi}{\partial x^2} - t\frac{\partial^2 e}{\partial x^2} < 0$. Consequently (iv) implies that $\frac{dx}{dt} < 0$ given that $\frac{\partial e}{\partial x} > 0$. Since we wish the damaged based tax to be stricter than the first best emission tax, $T_i > t_i e_i^{**}$, the equivalent emission tax of the damage based tax, $\hat{t}_i$, will be higher than the first best emission tax $\hat{t}_i > t_i$ and hence $\tilde{x}_i \leq x_i^{**}$. The equivalent emission tax of the damage based tax is, by definition, the tax that results in the same optimal input use as the damage based tax, $\tilde{x}_i = \arg\max\left[\pi(x_i) - T_i\right] \equiv \arg\max\left[\pi(x_i) - \hat{t}_i e_i\right]$. Therefore we have $\left(x_i^{**} - \tilde{x}_i\right) \geq 0$, which means that $\pi\left(x_i^{**}\right) - \pi\left(\tilde{x}_i\right) \geq 0$ is always true. Q.E.D.

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