Contracting, Signaling of Uncertain Quality, and Price Volatility?

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Abstract

Theoretical and simulation results clarify the role of forward procurement contracting as a determinant of spot price levels and volatility. A stylized model determines market share across quality when procurers forward contract to manage quality risk. Actual supply is specified as price dependent and stochastic. Simulation examines sensitivity of spot price level and volatility to extent of forward contracting, risk aversion, and ability to adjust spot market demand (recontracting). The results show that as forward contracting increases mean spot price decreases and variance increases. This effect increases as risk aversion decreases and as the extent of recontracting adjustment in spot demand decreases.
1. Introduction

Contracting is a method for coordination that is a substitute for anonymous, just-in-time procurement through agricultural spot markets. The use of forward contracts in food and agricultural supply chains has spread rapidly during the past decades. For many commodities such as poultry, pork, or veal nearly all production is forward contracted in some countries. The motivations for forward contracting vary across products and economic settings. In this paper, we focus not on these motivations, but on the implications of forward contracting for cash, or spot market price volatility. When forward contracting increases in supply chains that retain substantial transaction volume coordinated by markets, it is of interest to examine the implications of such contracting for spot price performance. When detrimental effects exist, they constitute an economic externality that the behavior of agents using contracts impose of those that remain oriented toward the spot market. It is the objective of this paper to examine the possibility of one such effect of increased forward contracting, namely increased price volatility.

In an economy, there are two functions that contracts provide. First, contracts can be regarded an insurance/risk-smoothing scheme that may improve performance in markets characterized by uncertainty and risk-averse agents. Second, contracts enhance performance by reducing transaction costs when asymmetry in information exists. The implications for prices depend on contract specifications. When information changes with time, contracting may precede actual transactions. Contracts may allow parties to write *ex ante* contracts before particular information is available though that can be revised, or more specifically defined *ex post*. Where such recontracting is not allowed, forward-contracted supply is often called “captive supply”. Such supply is captive in the sense that has been taken out of the market and priced by within a contract.

Only limited past literature has considered the relationships between prices and the extent of contracting. Almost entirely, past work has considered the effects on contracted transaction or spot price levels and the volume of forward contracted cattle. Ward, et al. (1999) found that reduction in the supply of available fed cattle due to contracting led to a change in the distribution of available cattle from feedlots to packers and, potentially, a change in the relative bargaining position of feedlots and packers. Jointly, these changes would were speculated as affecting changes in price. Elam (1992), Schroeder et al. (1993) and Ward, et al. found evidence that the spot price is inversely related to the extent of contracting. Schroeder et al. further concluded that change in forward contract percentage had greater impact on cash transaction prices than did contract volume. Results of Ward et al. (1996) and of Williams et al. (1996) both indicated that price paid for cattle procured through forward contracting is less than that available in the spot market price. The intuition is that forward contracting provides risk sharing, but the packer or processor does not have complete control over production decisions. Feeders or producers may be willing-to-accept a lower price to have some of the production risk assumed by the processors (Love and Burton 1999).

Game theoretic simulation has been used, though not to consider price implications. Zhang and Sexton (2000) developed a duopsony game model with a spatial market and demonstrated that exclusive contracts can be used in some market settings to reduce competition among buyers, and hence enhance oligopsony power for buyers. This result partly explains that rising concentration and vertical control in the livestock sector.
2. Contracting, Captive Supply, and Price Volatility

Two kinds of players are specified in fed cattle market: the feeder and the packer. Two type of transactions, forward contract and spot/cash market, are assumed to occur between feeders and packers. Feeders play the role of the suppliers of animals, whereas the downstream packers are buyers. We simplify the model to a two feeder and one packer problem. We assume that the packer has no direct way of knowing what quality of products a feeder will supply. This would especially be the case in the cash market where animals are traded on auction markets or purchased by roving buyers that would have little insight to the condition of the animals. However, feeders may be able to signal their types, or say quality, and packers can draw the inferences from the actions of feeders. For example, feeders can provide some assurance concerning quality to a potential buyer by showing a certificate which includes breed, feeding and watering records, vet references, and other information that would describe production practices that could predict quality or by putting labels on their products.

Within this setting, we specify a general multi-stage game with observed actions and incomplete information. There are four players in the market, two feeders and one packer. The other is a hypothetical player, Nature. We assume Nature has the first move and determines feeder’s type. Here, we assume that type represents the quality of product supplied and is determined by an exogenous process. We formalize the idea that information is unknown at the outset though becomes predictable during the course of the contract, and further becomes known after the contract.

To proceed, define a signaling model across four successive time periods. In the first, Nature generates a random draw that decides the true type for each of two feeders. We assume type indicates the quality of output produced by the feeder. Each feeder has a type \( w \) in a finite set \( \Omega \). Further, we assume that types are independent. At the beginning of the game, each feeder knows his type but is given no information about opponents’ type. The packer has no information about these two feeders. In the second period of the process, feeders and the packer make their investment decisions, setting their planned supply and demand of animals, respectively. We assume each quality of meat has a market outlet.

In the third period, the forward contract market is opened. Here, the value of the forward contract market to feeders is assumed to include the improvement of financing potential and the locking in of buyers. From the packer perspective, control of input supply quantity and quality is of interest. Although the packer has no knowledge of feeder type, the feeders prefer that high quality meat will sell at a higher price. Thus, the high-quality feeder has an incentive to “signal” or reveal type to avoid inaccurate assessment of quality in the market. At the same time, low quality feeders would benefit from offering the false signal that their type is high rather than low. Within this specification, the existence of the forward contract market follows from its ability to differentiate prices by quality. It follows that the buyer, packer will be assumed to be able to determine type after delivery. However, ex ante, the packer faces risk of accepting false signals from low quality feeders. It follows that packers have an incentive to differentiate across quality by paying higher forward contract prices for high-quality meat when a truthful signal is offered.

Given both feeder and packer have incentives to differentiate prices by quality, the success of the forward market relies on finding a mechanism that discourages low quality
feeders from falsely signaling. One approach is for high quality feeders to signal quality by selling their products with a warranty such as a label or certificate on the meat.

To convince packers that their signals are truthful, feeders will be assumed to indemnify their warranty on their signal or claim of quality. Based on this specification the problem dissolves into a contracting problem in which feeders of both quality levels negotiate with packers and sign contracts that specify quantity of animals (number and live weight), price, and the indemnity mechanism that is triggered when products do not satisfy the signaled quality.

In the fourth period, all animals have been fed to their market weight and uncertainty over available of total supply is resolved. Meanwhile, the packer might adjust his planned demand as information such as the spot price evolves. We assume that the production of quantity is stochastic. Meat not contracted in earlier periods is marketed in a competitive spot market. In this initial model, we assume the probability of either feeder type animal going to the spot market is equal. That is, the feeders have diffuse priors concerning the data generating process we call Nature. The spot market is assumed to be an anonymous auction market where animal type is not observable at a reasonable cost. Thus, the only information available to the packer concerning quality is that which can be elicited through effective contracts that encourage truthtelling concerning quality.

As cast, we have introduced a typical feature of signaling models following Spence (1973). Specifically, we suppose that feeders producing high quality have an incentive to signal quality to buyers through offering a warranty on announced quality. This policy is, of course, not costless. At a minimum, labeling and specification of the warranty will incur costs. At the extreme, quality may not be fully observable or predictable even to the feeders if further uncertainty were introduced. For now, we do not allow for such secondary processes. Once quality is determined by Nature, it is not conditional on feeder’s behavior. Quality of products is not stochastic during the transaction period. By comparison to Spence’s problem, we replace the worker’s type with the quality of the meat, the diploma signal with a quality label or certificate, and the employee’s wage with the forward contract price of the meat.

Generally, signaling games are leader-follower games in which the leader has private information. The leader moves first, then the follower observes the leader’s action, but not the leader’s type, before choosing his own action. In our model, the leader is a feeder who knows his product quality and must choose a level of label or certificate. The follower, a packer, observes the feeder’s certificate but not his quality and then decides whether participate forward contracting.

Separating and Pooling Equilibrium

Our model is developed in a simple version of Perfect Bayesian Equilibrium\(^1\) (PBE) that is limited to multi-stage games with observed actions and incomplete information. PBE in pure strategies is considered. The spirit of our model is that, for any certificate

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\(^1\) PBE results from combining the ideas of subgame perfection, Bayesian equilibrium, and Bayesian inference: Strategies are required to yield a Bayesian equilibrium in every continuous game given the posterior beliefs of the players, and the beliefs are required to be updated in accordance with Bayes’ law whenever it is applicable (Funenberg and Tirole, 1993, Ch.8).
level the feeder chooses, the offered contract should be reasonable in the sense of being consistent with equilibrium play in the continuation game. The reasonable contract to offer will typically depend on the packer’s beliefs about the feeder’s quality production, which in turn can depend on the feeder’s observed level of certificate. If this level is one to which the equilibrium assigns positive probability, the posterior distribution of the feeder’s quality production can be using Bayes’ rule, and the reasonable contract will depend on which posterior distribution is specified.

Two equilibrium concepts are introduced. One is separating equilibrium, which is an equilibrium in which the different types of players choose different actions. In our model, the different quality feeders choose to invest in different certificate levels as a signal. The packer can therefore infer the feeder’s quality by examining his certificate. As long as the feeders can convince the packer that signals are credible, the feeders can earn different forward contract prices based on their quality. The other equilibrium concept employed is that of a pooling equilibrium. In the spot market a pooling equilibrium is assumed in which each quality type feeder, in effect, chooses the same certificate level. Effectively, no warranty is offered in the spot market. In the absence of a warranty that signals quality, the packer offers the same spot price to each of the two type feeders. At this point in the scenario, forward contract deliveries have occurred and only a spot market exists.

The goal of our model is to focus on the functions and the effects of forward contracting in transactions and price levels and volatility. A continuous-type model is introduced and followed by a two-type model. We assume that only two kinds of quality exist: high or low quality. By applying the "intuitive criterion" of Cho-Kreps (1987), there is only one equilibrium survived among all equilibria, which is a separating equilibrium: the high-quality feeder chooses a least-cost certificate that allows the signaling of type while at the same time not attracting the low-quality feeder to pretend he is of high quality, whereas the low-quality one chooses no signal at all.
Theoretical Specification and Analysis

The following diagram summarizes the game structure of this model.

To proceed, we consider the case where type is continuous. Consider a feeder’s behavior function first. The profit is defined as:

\[
\pi^F = p^{w'}_f (w') \cdot q^{w'}_f + p_s q_s - C^w(q^{w'}_f, q'_s) - I(w, w') \cdot q^{w'}_f - h(x|\omega),
\]

where \(w\) is feeder’s true quality and \(w'\) is the announced quality. \(p^{w'}_f\) and \(q^{w'}_f\) are the price and quantity based on the announced type, respectively, in forward contract market. \(q_s\) is the quantity in the spot market, and \(p_s\) is the price in spot market. We note that no superscript appears since the spot market does not provide for quality-based differentiation. Moreover, \(C^w(\cdot)\) is the cost function based on the true type. \(h(x|\omega)\) is the signal cost to feeders and is conditional on the true type and controlled by \(x\). \(x\) is a
vector of variable inputs used by feeders to “produce” the warranty certificate or label. We assume that quality does not depend on the certificate, though we suppose that signaling with a certificate costs low quality feeders more, i.e. $\frac{\partial^2 h}{\partial x \partial w} < 0$. Further, we suppose that $x$ is publicly observable though $w$ is only known to feeders. $I$ is defined as a unit indemnity that is due to buyers at delivery if true type deviates from announced type.

Before analyzing the feeder’s optimization problem, several issues deserve brief clarification. First, the indemnity, $I(w, w')* q_f^w$, paid on warranty is assumed to satisfy:

\begin{equation}
I(w < w') > 0 \quad \text{and} \quad I(w > w') \leq 0.
\end{equation}

Equation (2) indicates that indemnity is implemented only if the announced quality is higher than actual one. To ensure that indemnity results in the signal being interpreted as credible, feeders design $I(\cdot)$ to satisfy two constraints. One is the feeder’s incentive compatibility constraint (IC) (3). This simply requires that each feeder prefers the contract that was designed for him and assures that the type is truthfully announced (truthtelling).

\begin{equation}
(IC): \left[ p_f(w') - p_f(w) \right] q_f^w - I(w, w') q_f^w' \leq 0.
\end{equation}

The other is the individual rationality constraint (IR) (4). This constraint guarantees that each type of feeder voluntarily accepts his designated contract. This is ensured by the requirement that contracting fulfills or satisfies the objective of profit maximization.

\begin{equation}
(IR): \pi_f^w (p_f^w, q_f^w, p_s, q_s) \geq \pi_f^w (p_f^w, q_f^w).
\end{equation}

Further, before proceeding, we characterize the cost of signaling $h(x|w)$.

Positive marginal cost and convexity, $\frac{\partial h}{\partial x} > 0$ and $\frac{\partial^2 h}{\partial x^2} > 0$, respectively, are assumed to be satisfied. No cost occurs if no labeling or certificate. $h(\cdot)$ is known to feeders only. The packer infers a feeder’s true type from the observed action, $x$. Third, we assume the contract price is a quality-based price, $p_f = p_f(w')$ and $\frac{\partial p_f}{\partial w'} > 0$.

To accommodate uncertainty in quality, we specify mean-variance representations for the feeder expected utility functions. The expected utility function of a feeder with quality $w$ can be expressed:

\begin{equation}
EU_f^w (p_f^w, \tilde{p}_s; q_f^w, q_s^w, w, x) = p_f(w)q_f^w + \tilde{p}_s q_s^w - \lambda \text{var}(p_s - c^w q_s^w) - c^w (q_f^w + q_s^w)^2 - I(w, w') * q_f^w - h(x|w), \text{where}
\end{equation}

\begin{equation}
I(w, w') = R + \gamma (w' - w) \quad \text{if} \quad w \neq w'
\end{equation}

\begin{equation}
= 0 \quad \text{if} \quad \text{otherwise}
\end{equation}

Equation (5) indicates the feeder’s expected utility with quality $w$. We assume that feeders hold a subjective distribution about $p_s$ with mean $\tilde{p}_s$ and variance $\sigma_s^2$. Profits are defined as total expected revenue from the sale of cattle by contract and on the spot

\begin{itemize}
\item $\frac{\partial^2 h}{\partial x \partial w} < 0$
\end{itemize}

\begin{itemize}
\item Notice here, a low-quality feeder offers a quality warranty only if $[p_f(w') - p_f(w)] - I(w, w') \geq 0$
\end{itemize}
market less a quadratic production cost function, label cost \( h(x|w) \), and the costs associated with the spot price volatility as reflected by risk aversion characterized by \( \lambda \), the relative risk aversion parameter. Indemnity is defined in equation (6) as derived from an IC constraint and involves a constant and a weighted differential between announced type and true type. First order differentiation yields the planned supply for contract and spot markets, the optimal level of certificate, and the optimal announced type.

The main value of the continuous case we have discussed is as follows. Each type feeder produces some quality animals such that \( p^+ > p^- \), where \(-w\) is the next lower quality. Price differentials establish incentives for feeders to signal. However, low quality feeders have an incentive to provide higher quality signals. Thus, the indemnity must be included as a deterrent.

**A Simple Example**

Next, we turn to a two-type model and examine the separating equilibrium. The fact that we can separate the feeders in our model follows from the existence of a signal and the variation of its cost by type of the feeder. In two-type case, the low quality is the “base” quality and the high quality feeder uses an indemnity to enforce/assure the incentive constraint to deter the low quality one to falsely signal. Therefore, the low-quality feeder chooses not to signal, and faces no risk of indemnity payment, accepting the low quality contract price. The high-quality feeder, however, chooses to signal and reveals his true type. This unique equilibrium is the most efficient separating perfect Bayesian equilibrium and entails the least cost warranty. Thus, high-quality feeder will be assured to earn the higher contract price.

Two results of Spence’s model also follow from our model. First, the only equilibrium concept that is intuitively appealing in this specification is that of a separating equilibrium. Such an equilibrium results in \( h(x|w^-) = 0 \) and \( p^+_f \) for the low-quality feeder and \( h(x|w^+) = h^+ \), the optimal level, and \( p^+_H \) for the high-quality feeder, with \( \{ p^+_f (w^+) - p^+_f (w^-) \} - I(w^-,w^+) = 0 \). In other words, only one incentive compatibility constraint is binding. This follows from the fact that prevents the low-quality feeder from posing as a high-quality one. Second, only the low-quality feeder receives the efficient allocation associated with no signaling cost. In other words, the high-quality feeder pays the price of incomplete information in our model.

**Feeder Behavior**

In the first period, Nature assigns randomly each feeder’s type. Both feeders face the same probabilities of each type occurring. Suppose that we face a situation that one is of high quality and the other is of low quality. After each feeder knows his type as private information, the one with high quality who wants to trade in forward contract market sends a signal to the packer to reveal his type. Then, both feeders decide independently their supply according to the expected utility \( E[U(\pi)] \). If a feeder is of high quality, his expected utility is as follows:
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\[ EU^{FH}(p^H_f, \bar{p}_s, q^H_f, q^H_s, w^H) = p^H_f(w^H)q^F_{fh} + \bar{p}_s q^H_s - \lambda \text{var}((p_s - c^H)q^H_s) \]
\[ -c^H(q^F_{fh} + q^H_s)^2 - I(w, w')q^F_{fh} - h(x|w), \text{where} \]
\[ I(w, w') = R + \gamma(w' - w) \quad \text{if} \quad w \neq w' \]
\[ = 0 \quad \text{if} \quad \text{otherwise} \]

By contrast, the low-quality feeder’s expected utility can be represented:
\[ EU^{FL}(p^L_f, \bar{p}_s; q^L_f, q^L_s, w^L) = p^L_f q^F_{lf} + \bar{p}_s q^L_s - \lambda \text{var}((p_s - c^L)q^L_s) - c^L(q^F_{lf} + q^L_s)^2 \]

Notice here, since the spot price reflects the price for average quality, in general, it would be expected that \( p^f_f - p_s < 0 \). So, the low quality feeder in general would not participate in forward contracting. However, we assume that the low quality feeder may participate in contracting hoping to deceive the buyer or due to strong risk aversion.

Recall that the forward contract market coordinates the transactions between two feeders and one packer as they negotiate to determine the contract price, quantity, and indemnity based on their planned production, \( q^{FW} (w=H, L) \), and planned demand, \( q^{FWR} (w=H, L) \). A unique separating equilibrium occurs in the forward contract market. Consider the feeders’ optimization problem. Both high- and low-quality feeders maximize their expected utility by allocating the total quantity of fed cattle between contract and spot market. The high quality feeder maximizes equation (7) subject to (IC) and (IR) constraints:

\[ \max_{q^F_{fh}, q^F_{lf}, w^H, w^L} EU^{FH}(p^H_f, \bar{p}_s, q^H_f, q^H_s, w^H, x) = p^H_f(w^H)q^F_{fh} + \bar{p}_s q^H_s \]
\[ -\lambda \sigma_s^2 q^F_{fh}^2 - c^H(q^F_{fh} + q^H_s)^2 - (I + \gamma(w' - w))q^F_{fh} - h(x|w) \]

Solution of the first order conditions, yields the optimal supply to contract and spot markets, the optimal level of certificate, and the optimal announced type:

\[ q^F_{fh} = \frac{\bar{p}_s - p^H_f}{2\lambda \sigma_s^2} \quad \text{[Supply of high quality meat in spot market]} \]
\[ q^F_{lf} = \frac{p^L_f}{2c^H} + \frac{\bar{p}_s - p^H_f}{2\lambda \sigma_s^2} \quad \text{[Supply of high quality meat in forward contract market]} \]
\[ -\frac{\partial h(x|w)}{\partial x} = 0 \quad \text{[Optimal certificate level]} \]
\[ \frac{\partial p^H_f(w^H)}{\partial w^H} * q^F_{fh} - \gamma q^F_{fh} = 0 \quad \text{[Optimal announced type]} \]

On the other hand, the low quality feeder maximizes his expected utility (8) subject to his production constraint, and yields the supply to these markets:

\[ q^F_{lf} = \frac{\bar{p}_s - p^L_f}{2\lambda \sigma_s^2} \quad \text{[Supply of low quality meat in spot market]} \]
\[ q^F_{lf} = \frac{p^L_f}{2c^L} + \frac{\bar{p}_s - p^L_f}{2\lambda \sigma_s^2} \quad \text{[Supply of low quality meat in forward contract market]} \]
Equation (13) clarifies that if contract market exists, $q^{FH}_f > 0$, then $\frac{\partial p^H_f(w^s)}{\partial w^s} = \gamma > 0$.

This means that announcing the high type earns the high contract price and provides the incentive for the high quality producer to signal. Otherwise, no signaling is necessary. On the other hand, the feeders’ incentive constraint, equation (3), together with equation (12) restricts cheating behavior and supports truth-telling.

**Packer Behavior**

Define $q^{PH*}$ and $q^{PL*}$ as the target level of slaughter for a given time period. Without loss of generality, we assume that the packer will purchase the proportion $\beta$ of cattle from contract market for both high- and low-quality meat. In other words, the proportion $1-\beta$ trades in spot market. Note that $\beta$ is exogenously determined and represents the optimal hedge ratio. Hence, the packer’s demands in forward contract market for either quality and in spot market are as follows:

$$q^{PH}_f = \beta q^{PH*}, w = H, L,$$

$$q^{PL}_f = (1-\beta)(q^{PH*} + q^{PL*})$$

**Market Equilibrium**

After forward contracts are signed, the production shock occurs. As the delivery date approaches, the spot market absorbs all remaining transactions. To proceed, note that the spot market equilibrium must be considered both from an expectational perspective as well as from an actual perspective. That is, during the forward market transactions period, an expectational spot market equilibrium occurs determining the expected spot price that equates expected supply from equation (10), (11), (14) and (15) and demand from equation (16) and (17) for either quality in forward contract and spot markets, as in equations (18) and (19).

$$q^{PH}_f = \frac{p^H_f}{2c^w} + \frac{\tilde{p}_s - p^H_f}{2\lambda \sigma^2_s} = \beta q^{PH*}, w = H, L \quad \text{[Forward Contract Market equilibrium]}

$$\frac{\tilde{p}_s - p^H_f}{2\lambda \sigma^2_s} + \frac{\tilde{p}_s - p^L_f}{2\lambda \sigma^2_s} = (1-\beta)(q^{PH*} + q^{PL*}) \quad \text{[Spot Market Equilibrium]}

The resulting contract price function for either quality meat derived from equation (18) is

$$p^H_f = \frac{2c^w \lambda \sigma^2_s q^{PH*} - \tilde{p}_s}{\lambda \sigma^2_s - c^w} - \frac{c^w}{\lambda \sigma^2_s - c^w} \tilde{p}_s, w = H, L.$$  

From equation (20), it shows that the forward contract price for either quality is affected by the packer’s hedge ratio, $\beta$, and the expected spot price. Moreover, equation (19) represents total expected supply equating total demand without considering actual quality that occurs in spot market. The left hand side and the right hand side of equation (19) are the planned supply and the planned demand subtracting to the contract portion, respectively. The partial reduced form for the rational expected spot price derived from equation (19) is:

$$\tilde{p}_s = \frac{p^H_f + p^L_f}{2} + \lambda \sigma^2_s(1-\beta)(q^{PH*} + q^{PL*})$$

Substituting equation (20) into (21) results in the final reduced form for rational expected spot price:
\( p_i = \frac{\beta}{A}[e^H \lambda s^2 (\lambda s^2 - c^L) q^{PH^*} + e^L \lambda s^2 (\lambda s^2 - c^H) q^{PL^*}] + \frac{(1-\beta)}{A}[2\lambda s^2 (\lambda s^2 - c^H) (\lambda s^2 - c^L) (q^{PH^*} + q^{PL^*})], \)

where \( A \equiv 2(\lambda s^2 - c^H) (\lambda s^2 - c^L) + c^H (\lambda s^2 - c^L) + c^L (\lambda s^2 - c^H) \). Notice that the hedge ratio, \( \beta \), does affect the expected spot price.

Next, we solve the actual equilibrium for the spot price, \( p_s \). As we mentioned earlier, all uncertainty becomes certain when the deliver date approaches. At that time, the production shock \( v \), is realized which is assumed to affect quality only, not quality, and meanwhile, the packer adjusts his planned demand due to the spot price comes out. Therefore, the spot price is derived from the physical balance of the spot market:

\[ \frac{\tilde{p}_i - p^H_i}{2\lambda s^2} + \frac{\tilde{p}_i - p^L_i}{2\lambda s^2} + v = (1-\beta)(q^{PH^*} + q^{PL^*}) \delta p_s, \]

Equation (23) is the actual spot market equilibrium. The left hand side and the right hand side are the actual supply and the actual demand, respectively. The term, \( \delta p_s \), reflects the packer’s adjusted demand, i.e. \( \delta \) is used here as a balanced mechanism to adjust planned spot demand to current spot price. The interpretation of \( \delta \) is as follows:

\[ q^P_s = (1-\beta)(q^{PH^*} + q^{PL^*}) \delta p_s = (1-\beta)(q^{PH^*} + q^{PL^*})(1 + \frac{\partial q^P_s}{\partial p_s}) \]

\[ \delta = [1 + (\partial q^P_s / \partial p_s)](1 / p_s) \]

where \( \partial q^P_s / \partial p_s \leq 0 \)

Thus, as spot market demand is more sensitive to current spot price, \( \delta \) decreases.

In actual equilibrium, the spot price is:

\[ p_s = \frac{1}{\delta (1-\beta)(q^{PH^*} + q^{PL^*})}[\tilde{p}_i - p^H_i + \frac{\tilde{p}_i - p^L_i}{2\lambda s^2} + v]. \]

Substituting equations (20) and (22) into (24) yields:

\[ p_s = p_s(q^{PH^*}, q^{PL^*}, \tilde{\sigma}_s^2 | \beta, \lambda, \delta, v, \lambda^H, \lambda^L) \]

3. Simulation Studies

Given the packer’s planned demand, equation (25) is a function of packer’s target demand and feeder’s subjective spot price variance conditional on the hedge ratio, risk aversion parameter, demand adjusted parameter, supply shock, and production cost. We are particularly interested in determining the effects of forward contract characteristics such as the hedge ratio, \( \beta \), on the level and volatility of fed cattle transaction prices.

Whereas many empirical studies have suggested \( \frac{\partial p_s}{\partial \beta} < 0 \), based on simulation we hope to determine the robustness of this type of result. Our other interest is in the effect on the spot price volatility, i.e. the sign of \( \frac{\partial^2 \tilde{\sigma}_s^2}{\partial \beta^2} \). Intuitively, when packer’s hedges increase, more forward contracting will occur reducing spot demand. Decreased demand in spot
market would likely drive spot prices down. The effect on variance is more difficult to motivate intuitively, and we try to get some understanding from simulation results.

**Simulation Setup**

To simplify the simulation model, 500 simulated trading periods are considered. The main focus here is to examine the spot price level and its variance and how they are related to hedge ratio, $\beta$. To consider robustness of these results, we also consider sensitivity to the risk aversion parameter, $\lambda$, and the demand adjustment term, $\delta$.

The simulation procedure is as follows. At the beginning, say period 0, appropriate initial values for the parameters ($\beta, \lambda, \sigma_\delta^2, \delta, q_{PH}^*, q_{PL}^*$) are set. We assume that the random shock, $v_1$, follows the standard normal distribution with zero mean and unitary variance. The anticipated spot price is defined by equation (22). In the period 1, the forward contract prices for each quality are calculated from equation (20). After the production shock, $v_1$, is drawn, the spot price for period 1 is derived from equation (25).

From period 2 on, the anticipated spot price variance is defined as:

$$\sigma_s^2 = (p_{s,t-1} - \bar{p}_{s,t-1})^2.$$  

This shows that feeders adopt most newly information to subjective spot price. The myopic expectation is applied. From period 2 to period 500, in general,

$$p_n = p_n(q_{PH}^*, q_{PL}^*, \sigma_\delta^2, p_{s,t-1}| \beta, \lambda, c^w), w = H, L$$

After 500 periods, the mean and the variance of spot price are calculated. In theoretical consideration, the spot price is affected by the hedge ratio, $\beta$, risk aversion, $\lambda$, and the adjusted demand term, $\delta$.

**Experimental Design and Results**

The simulation results in general show that as hedge increases mean of spot price falls, and variance increase. This effect increases as $\lambda$ decreases and as $\delta$ decreases.

Case 1: $\lambda = 0.1, 0.5, 0.9$ and $\delta = 0.1$ vs. spot prices

Figure 1 shows that as risk aversion, $\lambda$, decreases, concavity of $p_s$ with respect to the hedge ration, $\beta$, increases, i.e. $p_s$ is more sensitive to $\beta$. Although we specify $\beta$ exogenous, in further work we allow it to be endogenous. In that case, as $\lambda$ increases, it is intuitive that more would be hedged, so less would be supplied to the spot market. For a given hedge, $\beta$, it would follow that an increase in $\lambda$ would increase the spot price.

Case 2: $\lambda = 0.1$ and $\delta = 0.1, 0.5, 0.9$ vs. spot prices

Figure 2 shows that spot prices have much milder response to the hedge ratio as $\delta$ increases, i.e. concavity of $p_s$ decreases with increases in $\delta$. Note that $\delta$ reflects the adjustment of planned spot demand to current spot price, so as $\delta$ decreases, the current spot adjustment needed to balance any change in the actual excess demand is accentuated. Give an example, when $\delta$ is small, which means the adjustment in spot price to a change in excess demand increases. Recall the equation (23), the shrunken spot demand drives the mean of spot prices up. Moreover, this response with respect to the hedge ratio becomes a trivia as $\delta$ increases.

Case 3: $\lambda = 0.1, 0.5, 0.9$ and $\delta = 0.1$ vs. the variance of spot prices
Figure 3 shows in general, the volatility (contemporary variance) of the spot price increases with $\beta$. That is, as more is hedged out of the spot market, spot prices become more volatile. Further, for a given $\beta$, the variance of spot price decreases as $\lambda$ increases. This is consistent with theoretical results above. As risk aversion increases, more supply would be hedged, leaving less supply, and a larger proportion of supply that is stochastic, in the spot market.

Case 4: $\lambda = 0.1$ and $\delta = 0.1, 0.5, 0.9$ vs. the variance of spot prices

Figure 4 shows a result that is analogous to those found for case 2. First, as $\beta$ increases, the volatility of spot prices increases. Plot 4 illustrates that as $\delta$ decreases (larger adjustment of demand to current spot price), the variance in spot price would increase.

4. Conclusions

Three objectives are met in this paper. First, our model shows that procurement contracting indeed plays an important role in spot price discovery and spot price volatility. In our model setting, forward contracting is used as an insurance/risk-smoothing instrument to facilitate market transactions faced with quality uncertainty and involving risk-averse agents. Due to asymmetric information, the existence of forward contracting enhances transaction performance by information-sharing and reduction of transaction costs. Furthermore, our results illustrate that contracting can lead to reduce feeder prices received, not due to market power of packers, but instead due to the residual nature of spot markets that operate in conjunction with forward contracting. We find that as contracting increases, spot prices levels decrease and spot price volatility increases.

Secondly, our results clearly illustrate that spot price effects are conditional on contract characteristics. With contracting, spot and forward price effects depend on the specific market conditions. Spot price levels could be increased, decreased, or left unchanged. Based on our illustrative simulation specification, an inverse relationship between the spot market price and the forward contracting is found. Intuitively, forward contracting provides risk-sharing, and sellers may be willing to accept a lower price to have some of the production risk assumed by the purchasing firm, especially while the packers have more market power in noncompetitive market. Further, both our theory and our simulations illustrate increased forward contracting can induce increased spot price volatility. This result is not a universal one, instead it is due to the particular parameterization of the market setting.

Third, we consider the risk aversion, $\lambda$, and the adjusted demand term, $\delta$, as representing market structure to examine the sensitivity of spot price performance. According to the simulation outcomes, the negative relationship between the spot price and forward contracting is amplified as $\lambda$ decreases and diminished as $\delta$ increases, whereas the positive relationship between the variance of spot price and forward contracting is amplified as $\lambda$ decreases and diminished as $\delta$ increases. These results are intuitive though also dependent on parameterization. Finally, one policy implication is worthy of mention. In our case 4, as demand adjustment response to the current spot price increases (as $\delta$ increases), the variance of spot price decreases. This suggests the importance of allowance for demand adjustment in the spot market. Alternatively, we interpret this as recontracting. That is, as contracts or other regulations disallow recontracting, spot price volatility will be accentuated.
References


Figure 1: Case 1: $\lambda = 0.1, 0.5, 0.9$ and $\delta = 0.1$ vs. spot prices

Figure 2: Case 2: $\lambda = 0.1$ and $\delta = 0.1, 0.5, 0.9$ vs. spot prices
Figure 3: Case 3: $\lambda = 0.1, 0.5, 0.9$ and $\delta = 0.1$ vs. the variance of spot prices

Figure 4: Case 4: $\lambda = 0.1$ and $\delta = 0.1, 0.5, 0.9$ vs. the variance of spot prices