

**VALUE AT RISK: AGRICULTURAL PROCESSOR
PROCUREMENT AND HEDGING STRATEGIES**

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ABSTRACT

Agricultural firms that use Value at Risk (VaR) tend to be the large diversified corporations. The benefits of VaR in the agricultural industry are not limited to large conglomerates; however, and this study provides empirical examples of how mid to large sized commodity end-users can use VaR to quantify price risk exposure. By reporting price risk in terms of dollars as a single summary statistic, VaR provides a more intuitive measure of risk for decision makers, especially when the distribution of portfolio value changes is non-normal. VaR also separates downside from upside potential by focusing on the left-hand tail of a portfolio's distribution of returns. The purpose of this study is to demonstrate how VaR can be applied to the portfolio of a hypothetical U.S. bread baking company. Six of the bakery's prominent commodity inputs were considered, including flour, bakery shortening, and sugar. Mill feed price risk was also included since it is commonly a component of flour pricing agreements. In one case, a portfolio of costs and the risk of procurement cost changes are measured. In another case, output price risk was included and represented white pan bread prices as a price risk variable. This portfolio contains both cost and revenue items, and the risk of payoff changes resulting from input and output price changes is considered. In another case, the VaR for a Mexican flour milling company was modeled inclusive of the effect of foreign exchange risk.

In each case, different hedging instruments were considered for use in various hedging strategies. Though VaR can be utilized by decision makers for numerous management aspects, in the cases analyzed in this study VaR estimates are used to quantify the price risk associated with different hedging strategies.

While risk reduction is the primary reason for hedging, it is not the only aspect that management must consider. Numerous other aspects enter into the decision, which are not represented in the VaR statistic. Therefore, VaR is a valuable tool for measuring the risk exposure of these firms, but in no way does it tell the whole story.

Key Words: Value at Risk, Hedging, Processor Futures, Options

Value at Risk: Agricultural Processor Procurement and Hedging Strategies

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INTRODUCTION

Price risk management is a crucial function in Agribusiness firms involved in production, trading, and processing all face commodity price risks. The agricultural processor's hedging decision is complicated by several facets, including the fact that these firms are exposed to the price risks for both inputs and outputs.

Value at Risk (VaR) offers an advantage over other analysis methods in that it is able to separate the potential of large profits from the risk of large losses. The traditional mean variance framework does not make this distinction and characterizes all deviations from expected returns, positive or negative, as risk. Since managers and decision makers do not consider the potential of realizing large profits as true risk, VaR is considered by many to be much more intuitive than traditional risk measures.

VaR has acquired an ever-increasing number of advocates and practitioners in both the financial and energy sectors of the economy. These users apply VaR for internal risk management as well as employing it as a tool for reporting risks to government regulators when required. The agricultural sector, however, has lagged behind the financial and energy sectors in the adoption of this relatively new risk measurement methodology. Currently, only a few of the largest agricultural conglomerates use VaR in their risk management and reporting divisions. Use of VaR by a few agricultural firms has primarily resulted from a crossover of techniques utilized by the companies' financial and energy desks. While the largest agricultural companies likely have the most to gain from using VaR, the potential benefits of applying VaR in other mid to large sized agricultural firms are promising.

A limited number of studies have incorporated VaR into the context of agricultural hedging strategies. Using VaR in this application would lead to two distinct advantages. First, VaR would allow processor's risks to be expressed to management and decision makers as a single, summary statistic, which is more easily understandable. VaR is also able to separate potential profits from the risk of large losses. The traditional mean variance framework, as well as other well-used analysis tools, cannot make that distinction and express all deviation from the expected return as risk.

In this study, VaR is used to illustrate a measure of corporate price risk exposure for an agricultural processing firm. VaR offers an alternative to the traditional mean variance framework. The implementation of VaR allows for the construction of a model that accurately measures the price risk exposure of agricultural processors. The main objective of this study is to develop a VaR model that incorporates the various aspects of corporate price risk management and to illustrate the uses of VaR in the context of agricultural processors. The model includes

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common risk management tools and allows decision makers to compare the risk-reducing effects of different hedging and procurement strategies used to manage a firm's price risk exposure. The second objective involves constructing a VaR model using *@Risk*TM (Palisade Corporation, 2000) that incorporates forward, futures, and options contracts. The third objective is to apply the model to two domestic bread baking company situations and then to the case of a Mexican flour milling company.

Value at Risk

The development of Value at Risk (VaR) can be traced to the late 1980s, when major financial firms began to adopt VaR as a measure of the risks inherent to their trading portfolios. The release of *RiskMetrics*TM, by the risk management group at J.P. Morgan in October of 1994, provided a catalyst to VaR's growth by attempting to standardize the use of VaR throughout the industry (Linsmeier and Pearson, 2000). VaR's popularity as a risk measurement tool has risen dramatically in the last decade to include firms from nearly every sector of the economy (Mina and Xiao, 2001). VaR has also received increasing literary attention from the areas of finance and agricultural economics (Manfredo and Leuthold, 2001a).

VaR can be formally defined as a single, summary statistic that measures the worst expected losses during a given time period, with a specified level of confidence, under normal market conditions (Jorion, 2001). A common example of VaR considers a portfolio with a VaR measure of \$1 million, during a holding period of one day, at the 95% confidence level. This states that the portfolio will not experience one-day losses exceeding \$1 million, more than 5% of the time under normal market conditions (Manfredo and Leuthold, 2001a).

VaR offers an attractive alternative to traditional risk measurement tools, such as the mean-variance framework and delta-gamma-vega analysis (Hull, 2000). First, VaR summarizes portfolio risks in terms of potential dollar or percentage losses, as opposed to classifying risk with respect to standard deviations above or below the expected portfolio returns. Although measuring risk in terms of standard deviations provides accurate estimates of risk exposure for normally distributed random outcomes, managers and decision makers think of risk in terms of dollars. VaR provides managers a summary statistic that expresses risk in easily understood terms (Manfredo and Leuthold, 2001a). Second, VaR focuses on downside risk, as opposed to traditional risk measures that classify both upside and downside potential equally.

Although realized only in the two full valuation VaR methodologies, a third advantage offered by VaR is the ability to capture the non-linear payoffs of portfolios that contain options or option-like instruments. One of the fundamental assumptions of most traditional risk measures, including analytical VaR, is that returns of a given amount above or below those expected occur with equal likelihood. This assumption can hold for portfolios that contain only physical assets, forward contracts, and futures contracts. The presence of options in a portfolio invalidates this assumption by introducing non-linear payoffs. The ability to provide accurate estimates of risk exposure for portfolios that contain options gives VaR a significant advantage over other measures.

The literature contains countless warnings that VaR should not be construed as a panacea (Beder, 1995; Jorion, 2001; Duffie and Pan, 1997; Manfredo and Leuthold, 2001a; Linsmeier and Pearson, 2000). VaR describes only the loss that will be exceeded with some level of confidence. However, it says nothing about the absolute worst possible losses. VaR also assumes the portfolio remains constant over the entire time horizon. As the composition of the portfolio changes due to normal trading activity within the time horizon that VaR is measured, the accuracy of the VaR estimate declines. VaR relies on historical price data, and the price risk associated with assets for which historical data is not available is difficult to quantify with VaR. VaR position limits can also lead traders to “game” the system, trading in markets where the historical data resulting in low VaR estimates does not accurately represent the current situation (Jorion, 2001).

Value at Risk Methodologies

The objective of risk valuation is to provide an accurate estimate of market risk. Different methodologies have developed for computing VaR and Manfredo and Leuthold (2001a) describe how these vary with respect to accuracy, ease of implementation, time requirements, and ease of explanation to management.

One of the methods is known as Monte Carlo simulation which is based on the same principal as historical simulation, in that portfolio returns are actually generated for numerous possible scenarios. However, instead of subjecting the current portfolio to actual historical price changes over the previous N time periods, Monte Carlo simulation requires the user to assign an appropriate statistical distribution to each price risk variable that adequately approximates its possible changes (Linsmeier and Pearson, 2000).

Once statistical distributions have been assigned to each price risk variable, pseudo-random values are generated for each, constructing N possible overall return values for the portfolio in question. Linsmeier and Pearson (2000) describe that N is a significantly large number greater than 1,000 and, in some cases, greater than 10,000. These N possible returns are then treated just as those in historical simulation. Each of the N simulated portfolio values is subtracted from the marked-to-market value of the actual current portfolio. These hypothetical profit and loss values are then ranked in order of magnitude and the loss that is exceeded no more than X percent of the time is selected as the VaR statistic.

Although various forms of these methodologies are used, Jorion (2001) stresses that Monte Carlo simulation is a much more powerful tool than the other VaR methods. It offers more flexibility and overcomes many of the problems associated with parametric methodologies and historical simulation. Instruments with non-linear payoffs, such as options, do not present a problem for Monte Carlo simulation. The ability to vary parameter distributions and evaluate “what-if” scenarios offers another advantage (Linsmeier and Pearson, 2000). These aspects, coupled with its ability to incorporate fat tails in the distribution, unlikely extreme scenarios, and the passage of time, lead Jorion (2001) to suggest Monte Carlo simulation’s technical superiority over other methodologies.

The relatively long computational times for large portfolios makes Monte Carlo simulation comparatively expensive to implement and is perhaps its most significant disadvantage. The actual implementation is not overwhelming when off-the-shelf software is available. Alternatively, under circumstances where the necessary software does not exist, developing Monte Carlo models from scratch can be very time consuming. Manfredo and Leuthold (2001a) also discuss how the valuable freedom to choose statistical distributions to represent each price risk variable can result in adverse affects. Distributions chosen by the designer of the model may not accurately represent the variability of each of the price risk variables, which increases model error.

Value at Risk in Agricultural Economics

Although the study and application of VaR has received considerable attention in the financial literature, its implementation in the agricultural economics literature is limited.¹ Manfredo and Leuthold (2001b) examined the relationship between the prices of fed cattle, which are a cattle feeders' output, and corn and feeder cattle prices, which are inputs for this type of firm. Sanders and Manfredo (1999) use a hypothetical food service company to demonstrate VaR implementation for a commodity end-user, but consider only the risk of the commodity inputs in their analysis. This study uses the portfolio of a hypothetical bread baking company and a flour milling firm to provide examples of VaR in a commodity processor application. The scope of the basic commodity end-user application is expanded in this study by including both input and output price risk. This is further complicated by the fact that prices of consumer goods, such as bread, are exposed to forces that differ significantly from those that affect prices of raw commodities. Another extension considers the effect of foreign exchange risk for a Mexican flour milling company that purchases its input in a foreign currency. These case studies use VaR to quantify the price risk associated with the portfolios of hypothetical agricultural processing firms and to evaluate the risk-reducing effects of various hedging strategies.

THEORETICAL MODELS OF VALUE AT RISK

The historical foundations of VaR can be traced back half a century to the works of Markowitz and Roy, both published in 1952. Markowitz (1952) was the first to formally define the tradeoffs that investors faced between risk and expected returns, which explained the logic behind the practice of diversification. The mean-variance framework that he used, however, is only accurate when the portfolio returns are normally distributed or the utility function of the investor is quadratic.

¹ Manfredo and Leuthold (2001a) discuss several agricultural areas in which VaR could provide substantial benefits. They indicate that one of the main uses has to do with the fact that "Publicly traded agribusiness firms must comply with SEC regulations concerning the reporting of positions in highly market sensitive assets, including spot commodities, futures, and options positions" (p. 110). They also express that, had elevator managers and producers been using VaR, the hedge-to-arrive crisis of 1996 could possibly have been averted. Agricultural lenders could also apply VaR in credit scoring as well as using it to determine the magnitude of price risk they are exposed to indirectly through their borrowers.

Roy (1952) argued that the objectives behind portfolio selection and diversification are concerned much less with stabilizing expected returns than with avoiding economic disasters. His “Safety First” criterion indicates that the ultimate goal of portfolio selection is to minimize the probability that a disastrous loss will be incurred. The similarities between the ideas of Roy and Markowitz become apparent when Roy (1952) indicated that the definition of a disastrous loss is likely to vary as the expected return of an investment changes. In other words, low levels of expected return lead to relatively small losses being considered to be disastrous. As expected returns increase, he states that the investor’s definition of a disastrous loss that must be guarded against likely increases as well.

The first support for a confidence-based risk measurement criterion comes from Baumol (1963). He points out that the absolute value of the risk measure, standard deviation, is much less important than the value of the standard deviation relative to the value of the expected return. A confidence-based selection criterion is then offered as a method of incorporating both risk and expected return into one number which captures the relationship between the two. This is the fundamental basis for VaR. The equation Baumol (1963) uses to represent the lower confidence limit (L) is $L = E - K\sigma$ where E is the expected portfolio return, σ is the standard deviation of portfolio returns, and K is the number of standard deviations from the expected return that corresponds to the desired confidence level. This equation is essentially equivalent to the equation representing the cutoff return (R^*), used in the VaR models explained in the following sections.

VaR is formally defined as a single, summary statistic indicating the portfolio loss that will be exceeded with a probability of $1 - c$, during a given time period (t), under normal market conditions, where c is the specified confidence interval. The first step is to mark-to-market the current portfolio of assets. This figure, W_0 , represents the current value of the portfolio and is commonly referred to as the initial portfolio value. The second step is collecting historical price data associated with each relevant price risk variable, which are used to determine the variability caused by each of the various factors or, in the case of historical simulation, is used to construct the actual simulations themselves.

Finally, selection of the time horizon (t) is made and the confidence interval (c) to be used needs to be specified.

Value at Risk Computation

The three different VaR approaches can be separated into two basic categories. Models in the general distributions category, consisting of the two full valuation approaches, use simulation techniques to calculate VaR statistics for portfolios with returns that exhibit any distribution. The model in the alternative category uses an analytical approach. Also called the variance/covariance method, the analytical approach assumes that possible returns take the form of a parametric distribution. The theoretical model for both of the categories will be described in detail in the following sections.

General Distributions

Jorion (2001) begins his explanation of VaR by defining W as the end of period portfolio value, W_0 as the initial portfolio value, and R as the rate of return on the portfolio such that $W = W_0 (1 + R)$. W^* is defined as $W^* = W_0 (1 + R^*)$, or the portfolio value when R^* , the critical rate of return associated with the confidence level c , is realized. The confidence level c indicates that a return equal to, or lower than, the critical rate of return R^* is only expected to occur with a frequency of $1 - c$ during normal market conditions. Jorion (2001) continues to explain that, in its broadest context, the VaR statistic for a future portfolio W with a probability distribution $f(W)$ can be derived from the integral equation:

$$1 - c = \int_{-\infty}^{W^*} f(W) dW$$

This equation represents the probability that end of period portfolio value will be less than or equal to W^* , the critical portfolio value. It states that the area in the far left hand tail of the probability distribution between $-\infty$ and W^* must sum to the probability $1 - c$ (Jorion, 2001). Therefore, W^* is a quantile of the distribution and the item of particular interest in this procedure.

Once the quantile W^* is read from the distribution of future portfolio values, the VaR can be found in either absolute or relative terms. The absolute VaR refers to the distance between the dollar loss quantile and the initial portfolio value, without considering the expected portfolio value, and is represented by:

$$VaR(zero) = W_0 - W^* = -W_0 R^*$$

The absolute VaR can be helpful in cases where the expected portfolio value is difficult to calculate, but the relative VaR provides a more logical statistic in many applications due to the inclusion of the time value of money concept. The relative VaR is defined as:

$$VaR(mean) = E(W) - W^* = -W_0 (R^* - \mu)$$

where μ is the expected value of R and $E(W)$ represents the expected value of W .

The model of VaR for general distributions is versatile and can be applied to portfolios with any distribution of returns. As Holton (1998) explains, the problem that one immediately encounters is the fact that closed form solutions do not exist for most portfolios. This leaves two options. The first, numerical integration methods, are practical for portfolios with one or two dimensions or price risk variables. However, he states that as portfolios become larger and more diverse, the “curse of dimensionality” increases the complexity of numerical integration methods, challenging the computing power of today’s most advanced technology. For this reason, general distribution problems are solved using simulation techniques to create the distributions of portfolio returns, which allow the dollar loss amount corresponding to the desired quantile to be read from the generated distribution.

The first step Jorion (2001) illustrates is that of converting the actual portfolio distribution $f(W)$ to the standard normal distribution $\Phi(\epsilon)$, with a mean of 0 and standard

deviation of 1. He defines W^* as the cutoff end of period portfolio value corresponding to the rate of return R^* associated with a desired level of confidence c , and W_0 as the initial portfolio value. He uses μ and σ to represent the expected return and standard deviation of R^* , respectively. It then shows that $W^* = W_0 (1 + R^*)$ and that, since R^* is usually a negative number, it can be expressed as $-|R^*|$. The relationship between the standard normal deviate α and R^* is:

$$-\alpha = \frac{-[R^*] - \mu}{\sigma}$$

such that $-\alpha$ is set equal to the expected rate of return subtracted from the cutoff rate of return, divided by the standard deviation of the rate of return.

The leap from a general distribution to the standard normal distribution, as well as the fundamental difference between full valuation and analytical VaR techniques, can be illustrated by expanding the general distribution integral equation such that:

$$1 - c = \int_{-\infty}^{W^*} f(W) dW = \int_{-\infty}^{-[R^*]} f(R) dR = \int_{-\infty}^{-\alpha} \Phi(\varepsilon) d\varepsilon$$

Therefore, the primary result is that, while the VaR for general distributions is found by searching for W^* , the VaR for a portfolio with returns of a standard normal distribution can be found by solving for the standard normal deviate α instead. The value of W^* can be found through simulation; however, finding the α that makes the equation true is much easier. Jorion (2001) points out that the cumulative normal probability distribution, illustrated in Figure 1, is represented by:

$$N(d) = \int_{-\infty}^d \Phi(\varepsilon) d\varepsilon$$

By setting $1 - c = N(d)$ it is shown that d and $-\alpha$ are equivalent. This means that $-\alpha$ can be read off the cumulative normal probability distribution as the standard normal variable d resulting from a $N(d)$ value equivalent to $1 - c$. Figure 1 demonstrates this logic using 95% confidence interval.

Now that the value of $-\alpha$ corresponding to the chosen confidence interval has been found, Jorion (2001) explains that by rearranging the equation for $-\alpha$ described earlier, we can determine that the cutoff return must be:

$$R^* = -\alpha \sigma + \mu$$

when $R^2 < 0$. This formula is equivalent to that described by Baumol (1963) when he first introduced the idea of measuring risk using a confidence based criterion. Baumol's equation:

$$L = E - K\sigma$$

varies from Jorion's (2001) only in the notation used to describe each variable, as Baumol (1963)

uses L to represent the lower confidence limit, E to represent expected portfolio returns, and K as the number of standard deviations from the expected returns that corresponds to the desired confidence level.

Rearranging the equation for R^* and substituting it into the equation for the relative VaR for a general distribution results in the relative VaR for a parametric distribution:

$$VaR(mean) = -W_0 (R^* - \mu) = W_0 \alpha \sigma \sqrt{\Delta t}$$

When μ and σ are expressed annually, instead of over the desired horizon, it is necessary to include the $\sqrt{\Delta t}$ factor to scale the parameters down to the appropriate time horizon chosen to evaluate the VaR statistic. Therefore, if a one day VaR was to be calculated, Δt would be set equal to $1/252$ as t represents time in years. The same time horizon consideration is applied in the absolute VaR calculation given as:

$$Var(zero) = -W_0 R^* = W_0 (\alpha \sigma \sqrt{\Delta t} - \mu \sqrt{\Delta t})$$

Just as in the general distribution VaR equations, the way that the expected rate of return is accounted for is the difference between the absolute and relative VaR measures.

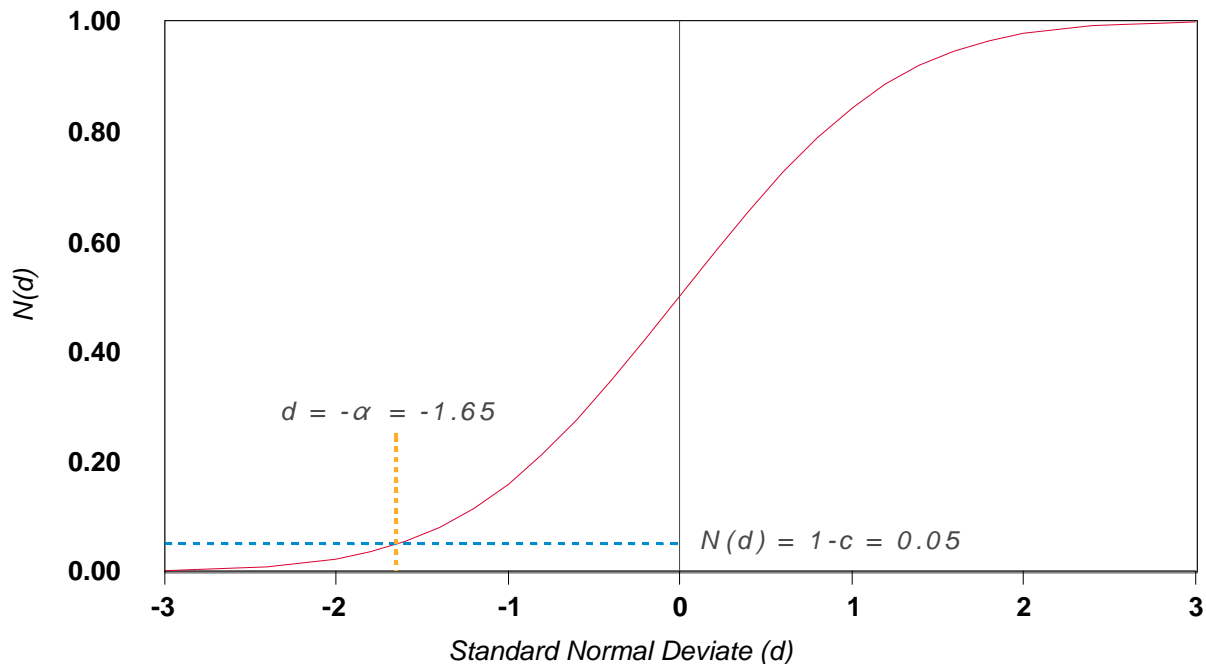


Figure 1. Cumulative Normal Probability Distribution

Source: Adapted from Jorion (2001).

Monte Carlo simulation is the most complex and powerful of the VaR methodologies. Computational time is also a significant detriment of Monte Carlo simulation as the need for large numbers of simulations, coupled with large portfolios, leads to lengthy time requirements.

Ease of Explanation to Management

Historical simulation is the most intuitive approach to VaR calculation, especially to those who are not trained in statistical techniques. Exposing the current portfolio to the market conditions experienced over the last N periods is a logical way to measure risk. Comprehension of the parametric method, which relies heavily on statistics, the normal distribution, and the variance/covariance matrix, requires significant knowledge of statistical techniques. Linsmeier and Pearson (2000) argue that explaining Monte Carlo simulation to management is even more difficult, as pseudo-random number generators and the fitting of distributions to data sets are foreign concepts to most individuals.

EMPIRICAL PROCEDURES

Three case studies are developed that demonstrate the VaR models. Case I is that of a U.S. bread baking company, where the procurement operations utilizes VaR independent of the rest of the company to evaluate the risks associated with procurement and hedging strategies. Case II, a U.S. bread baking company, demonstrates the impact of considering both input and output price risks simultaneously. Case III, a Mexican flour milling company, brings currency exchange risk into the equation for an entity that sells its output in the local currency and purchases its inputs in a foreign currency.

Case I: Procurement Operations of a U.S. Bread Baking Company

VaR computation for the procurement operations of a hypothetical U.S. bread baking company producing only white pan bread serves as the baseline scenario for this study. Inputs are divided into two categories. The first, ingredients, consists of flour, sugar, and bakery shortening. Mill feeds are also considered because the price of flour also depends on the price of mill feeds. Flour purchase agreements commonly require the pricing of mill feeds as well, which exposes the flour purchaser to mill feed price risk. The energy category is comprised of #2 diesel fuel, for use in the truck fleet, and natural gas, which fuels the bakery ovens. Although there are other inputs in the production of white pan bread, prices of these inputs account for the bulk of the price risks faced by a U.S. bread baker.

Case II: U.S. Bread Baking Company

This case study is as an extension of Case I. Instead of focusing on price risks strictly from the input side of the business, this illustration also considers output price risk. Another dimension of this scenario is that white pan bread, the firm's only output, is a consumer good. This is important because consumer goods' price movements differ from those of the firm's inputs, which are all commodities. This difference can have implications that must be considered when implementing hedging strategies. Two strategic aspects of the hedging

decision are outlined by Hull (2002). In most industries, input and output prices tend to move in tandem, though the degree to which price movements are related varies widely. This relationship is extremely important, as it relates to the level to which output prices adjust to compensate for changes in input prices. As the correlation between input and output prices increases, the demand for hedging instruments decreases. Hull (2002) also explains that deviating from the hedging strategies practiced by the firm's competitors can produce unwanted results. If a firm implements a hedging strategy when competing firms do not hedge, the firm will either develop an advantage or disadvantage. If the firm develops an advantage, either margins will be increased or prices may be decreased. If the opposite occurs, however, the firm will likely suffer reduced margins or be forced to increase prices. Price increases may not be followed by competitors since, in this situation, their input prices would not warrant the change. In some cases, implementing a hedging policy can actually increase the variability of returns. This result is the exact opposite of that desired and, therefore, the strategic aspects of hedging must be considered.

Case III: Mexican Flour Milling Company

In the third case study, the portfolio of a hypothetical Mexican flour milling company is used to illustrate use of VaR by an importer. Unlike the prior case, in which multiple inputs were used to produce a single output, this example considers a single input used to produce two outputs, flour and mill feeds. Another aspect of this case is the effect that the foreign exchange risk plays in risk exposure. The Mexican flour milling company purchases wheat, its only input, in U.S. dollars and sells its outputs of flour and mill feeds for Mexican pesos. This means that, not only do the price risks of the actual input and outputs themselves need to be considered, but that changes in exchange rates can effectively raise or lower the cost of the input, even when the actual input price remains stable.

Model Selection and Empirical Methods

The methodology selection decision was influenced by Jorion's (2001) statements that Monte Carlo simulation is "the most comprehensive approach to measuring market risk" (p. 226), as well as "the most powerful method to compute VaR" (p. 225). Although data and computing requirements are the most demanding for Monte Carlo simulation, the flexibility and power of the approach led to the selection of this methodology for the case study analyses. Since the VaR statistic will be used in a benchmark application in this study, the most common confidence interval, 95%, was selected. A confidence interval of 95% infers that losses are expected to exceed the VaR statistic in one out of every 20 time periods. This allows backtesting of the VaR model to be conducted more frequently, and with greater accuracy, than when higher confidence intervals are used.

The choice of time horizon offers less flexibility due to the frequency of observations with several of the data sets. While many of the data sets used contained daily or weekly data, several series were reported only as monthly averages. This restricted the potential time horizon choices to periods of 1-month or greater and, therefore, a 1-month time horizon was selected. Though dictated by the data, the 1-month time horizon performs well in these situations, due to the nature of an agricultural processor's portfolio. Despite the extensive markets for the

agricultural and energy inputs, these physical inputs are less liquid than those in the financial sector, where 1- and 10-day VaR calculations are common. An agricultural processor would tend to hold a much more stable portfolio than their financial sector counterparts, mitigating one of the most prominent disadvantages of longer VaR horizons. Although flour and mill feed prices change frequently, bread prices are much more stable.

Analytical Procedures

The fitting capabilities of @Risk™ (Palisade Corporation, 2000) was used to estimate the statistical distribution curve that provides the most accurate approximation to each actual, observed data set. Since all of the data sets used in this analysis were continuous sample data, @Risk™ estimated the parameters for each of the 21 possible distributions using the Maximum Likelihood Estimators (MLEs). The observed price level data was transformed to price change data before the statistical distributions were fitted or hedge ratios were calculated. Price change data for each period is calculated by subtracting the previous period's price from the current price (Hill and Schneeweis, 1981; Wilson, 1983). The ordinal rankings of distributions vary depending on whether the chi-squared, Anderson-Darling, or Kolmogorov-Smirnov fit statistic is used (Palisade Corporation, 2000). Chi-squared is the most commonly used of the three and, in this analysis, the distribution ranked highest by the chi-squared fit statistic is used to represent each data set, unless another distribution ranks higher than the best chi-squared distribution on both the Anderson-Darling and the Kolmogorov-Smirnov scales.

Model Details

The empirical model is constructed assuming a short spot position equivalent to a three-month supply of all inputs and outputs relevant to each particular case study. Price risks associated with these positions could be hedged using forward, futures, and options contracts. Transaction costs were not considered and margin requirements were assumed to be zero.

Options were available on each of the commodity futures contracts chosen to hedge the input price variables as well. While numerous strike prices could have been used in the model, nearest-the-money options were chosen for this analysis. Since these options were approximately at-the-money, the options' deltas were approximately 0.5.

Utility maximizing hedge ratios are calculated for each futures contract used in the analyses. These hedge ratios serve as the basis for the selection of hedging strategies that were evaluated in each case study. Futures price are unbiased.

The initial date for Cases I and II is October 1, 2002, while Case III uses an initial date of October 1, 2000. This is because output price data in Case III was only available through September of 2000. In each scenario analyzed, the initial market value of each spot, forward, futures, and options position was calculated. In Case III, an additional step of converting positions in foreign currencies to their present value in the local currency is undertaken. The sum of these positions represents the initial wealth, W_0 , of the procurement division of the bread baking firm being considered.

A correlation matrix was also compiled for each of the three case studies. The relationship between each price risk component and the futures contract used to hedge it is captured, as well as the broad relationships between each of the price risk variables relevant to the particular case study. The significance of each correlation coefficient was then evaluated using a t-test, and those that were not significant at the 5% confidence level were replaced with values of zero. The resulting matrix allowed the pseudo random number generator to emulate the historical relationships among the price risk variables.

Though all the options in this study are American options, Black's model is used here to provide a close estimate of the commodity option values. One currency option is also included in Case III, and the variant of the Black-Scholes option pricing model for currency options is used. The only additional data collected to include options in this analysis is the U.S. dollar risk-free rate observed in the form of 91-day U.S. Treasury bill rates.

The final step before the simulation can be run is setting up the formulas for the current portfolio value, W_0 , and the end of period values, W . In Case I, the initial portfolio value is represented as:

$$W_0 = \sum_{I=1}^6 (Q_I P_{I,0}) + \sum_{C=1}^5 (Q_C P_{C,0}) + \sum_{P=1}^5 (Q_P P_{P,0})$$

and the end of period portfolio value is defined as:

$$W = \sum_{I=1}^6 (Q_I \tilde{P}_{I,1}) + \sum_{GI=1}^6 Q_{GI} (\tilde{P}_{GI,1} - P_{GI,0}) + \sum_{F=1}^5 (Q_F (\tilde{P}_{F,1} - P_{F,0})) \\ + \sum_{C=1}^5 (Q_C \tilde{P}_{C,1}) + \sum_{P=1}^5 (Q_P P_{P,1})$$

where Q_X is the quantity of asset X in the portfolio and $P_{X,T}$ is the price of asset X at time T . In the equation for W_0 is the quantity of asset X in the portfolio and $P_{X,T}$. In the equation for W_0 , $T=0$ indicates current prices are used. In the equation for W , $T=1$ specifies that end of period prices are used. End of period prices are designated with a \sim indicating they are stochastic variables found through simulation and X is made up of five asset classes, I , GI , F , C , and P . These asset classes are defined as:

I = physical input commodities (1) flour, (2) natural gas, (3) diesel, (4) beet sugar, (5) soybean oil, and (6) mill feeds;

GI = forward contracts on the physical input commodities (1) flour, (2) natural gas, (3) diesel, (4) beet sugar, (5) soybean oil, and (6) mill feeds;

F = futures contracts (1) MGE wheat, (2) NYMEX natural gas, (3) NYMEX heating oil, (4) CBOT soybean oil, and (5) CBOT corn;

C = call options on futures contracts (1) MGE wheat, (2) NYMEX natural gas, (3) NYMEX heating oil, (4) CBOT soybean oil, and (5) CBOT corn; and

P = put options on futures contracts (1) MGE wheat, (2) NYMEX natural gas, (3) NYMEX heating oil, (4) CBOT soybean oil, and (5) CBOT corn.

Long positions result in positive Q_x values, short positions are represented with negative values, and a Q_x value of zero indicates no position in the asset.

In Case II, the portfolio value functions are extended to include the firm's output, or revenue component. The first term, however, is the only addition to the portfolio values in Case I. Portfolio values for CASE II are represented as:

$$W_0 = (Q_O P_{O,0}) + \sum_{I=1}^6 (Q_I P_{I,0}) + \sum_{C=1}^5 (Q_C P_{C,0}) + \sum_{P=1}^5 (Q_P P_{P,0})$$

and

$$W = (Q_O \tilde{P}_{O,1}) + \sum_{I=1}^6 Q_I \tilde{P}_{I,1} + \sum_{GI=1}^6 (Q_{GI} \tilde{P}_{GI,1} - P_{GI,0}) \\ + \sum_{F=1}^5 (Q_F \tilde{P}_{F,1} - P_{F,0}) + \sum_{C=1}^5 (Q_C \tilde{P}_{C,1}) + \sum_{P=1}^5 (Q_P \tilde{P}_{P,1})$$

In this case, the notation is equivalent to Case I with the addition of the subscript O , used to represent the firm's output, white pan bread. While $P_{x,T}$ represents the spot price at time T for all inputs and derivatives, it signifies the expected future spot price of the output. In this study, current spot, forward, and expected future spot prices are assumed to be equivalent.

The exposure to foreign currency exchange risk in Case III results in portfolio value functions slightly different than those used in Cases I and II. Here, the initial portfolio value is denominated in Mexican pesos and is represented as:

$$W_0 = \sum_{O=2}^2 (Q_O P_{O,0}) + (Q_1 P_{1,0} P_{FX,0}) + \sum_{C=1}^2 (Q_C P_{C,0} P_{FX,0}) + \sum_{P=1}^2 (Q_P P_{P,0} P_{FS,0})$$

and the Mexican peso denominated end of period portfolio value is:

$$\begin{aligned}
W = & \sum_{O=1}^2 (Q_O \tilde{P}_{O,1}) + (Q_I \tilde{P}_{I,1} \tilde{P}_{FX,1}) + \sum_{GO=1}^2 (Q_{GO} (\tilde{P}_{GO,1} - P_{GO,0})) + (Q_{GI} (\tilde{P}_{GI,1} - P_{GI,0}) \tilde{P}_{FX,1}) \\
& + \sum_{F=1}^2 (Q_F (\tilde{P}_{F,1} - P_{F,0}) \tilde{P}_{FX,1}) + \sum_{C=1}^2 (Q_C \tilde{P}_{C,1} \tilde{P}_{FX,1}) + \sum_{P=1}^2 (Q_P \tilde{P}_{P,1} \tilde{P}_{FX,1})
\end{aligned}$$

The basic notation remains constant for Case III; however, the asset classes making up X have changed. They represent O , S , GO , GI , F , C , P , and FX , where:

- O = physical outputs (1) flour and (2) mill feeds sold for Mexican pesos;
- I = physical input wheat;
- GO = forward contracts on the physical outputs (1) flour and (2) mill feeds;
- GI = forward contracts on the physical input wheat;
- F = futures contracts (1) KCBT wheat and (2) CME Mexican pesos;
- C = call options on futures contracts (1) KCBT wheat and (2) CME Mexican pesos;
- P = put options on futures contracts (1) KCBT wheat and (2) CME Mexican pesos; and
- FX = Mexican peso/U.S. dollar foreign currency exchange rate.

It is important to stress that short positions are represented with negative quantities in each of the three cases. The payoff function for all cases is represented as $\Pi = W - W_0$ such that the payoff is equal to the 1-month change in total portfolio value found by subtracting the end of period portfolio value from the initial portfolio value.

For each case, the equation for W_0 does not contain a term for either forward or futures contract valuation. Futures contracts are marked-to-market daily and, therefore, the value of futures contracts are reset to zero at the end of each day. While the value of a forward contract may be non-zero, its value at inception is always zero. All forward contracts used in these cases are initiated at time 0; therefore, the value of the forward contracts at time 0 is zero. The initial portfolio calculation reflects this by omitting the terms for the value of both forward and futures contracts, since these terms would be equal to zero. In calculating end of period portfolio values, the price change between time 0 and time 1 observed for each forward and futures contract is included, instead of using only the price of the asset at time 1, as represented for the spot asset and option values in the equation.

Simulation Procedures

Stochastic simulation using @Risk™ was used to derive the VaR. Latin hypercube sampling was used. This technique reduces the computing time necessary to assure accurate representation of the input distributions (Palisade Corporation, 2000).

Pseudo random values are drawn from the assigned distribution for the price change of each price risk variable, while maintaining the historical relationships expressed in the correlation matrix. The value drawn for each variable is added to the current price of the price risk variable to obtain end of period price levels. End of period portfolio value, W , is then calculated and the formula $W - W_0$ represents the absolute 1-month change in the value of the firm's portfolio, or profit, for one iteration. This process is repeated 10,000 times for each hedging strategy in each case study analyzed, and the fifth quantile of the distribution of $W - W_0$ is reported as the VaR of the portfolio for that specific scenario.

Empirical Data: Bread Baking Case Studies I and II

The data used for Cases I and II fall into four categories. The first, that of agricultural inputs, contains Minneapolis spring standard patent flour, Midwest beet sugar, and Decatur soybean oil. These price data series were taken from *Milling & Baking News* and aggregated into monthly average prices. Prices for flour, sugar, and soybean oil are reported in dollars per hundredweight. Table 1 presents the mean and standard deviation of the absolute, observed prices, and the mean and standard deviation of the absolute, observed prices, and the mean and standard deviation of changes in these variables. These statistics were calculated after daily and weekly observations were aggregated into monthly average data.

Table 1. Characteristics of Observed Date Series for Cases I and II

| Financial Variables | Absolute Price ¹ | | Price Change ¹ | | Start Date | End Date | Observation Frequency |
|-----------------------------------|-----------------------------|--------------------|---------------------------|--------------------|------------|-----------|-----------------------|
| | Mean | Standard Deviation | Mean | Standard Deviation | | | |
| Inputs | | | | | | | |
| Mpls Spring Standard Patent Flour | 10.2958 | 1.5232 | 0.0048 | 0.5570 | 4-Jan-85 | 27-Sep-02 | Weekly |
| Midwest Beet Sugar | 26.3644 | 2.4975 | -0.0046 | 0.6729 | 4-Jan-85 | 27-Sep-02 | Weekly |
| Decatur, Soybean Oil | 21.6101 | 5.5410 | -0.0127 | 1.4290 | 6-Apr-82 | 27-Sep-02 | Weekly |
| Midwest #2 On-Road Diesel Fuel | 1.2050 | 0.1642 | 0.0031 | 0.0528 | 4-Apr-94 | 23-Sep-02 | Weekly |
| Natural Gas - Industrial | 3.3283 | 0.8310 | -0.0022 | 0.2983 | Jan-84 | Sep-02 | Monthly |
| Futures | | | | | | | |
| MGE Hard Red Spring Wheat | 3.6562 | 0.6187 | 0.0026 | 0.1899 | 2-Jan-80 | 30-Sep-02 | Daily |
| CBOT Soybean Oil | 21.8240 | 4.4940 | -0.0146 | 3.5208 | 2-Jan-80 | 30-Sep-02 | Daily |
| CBOT Corn | 2.5673 | 0.5603 | -0.0003 | 0.1584 | 2-Jan-80 | 30-Sep-02 | Daily |
| NYMEX Heating Oil | 0.6365 | 0.1709 | -0.0001 | 0.0477 | 2-Jan-80 | 30-Sep-02 | Daily |
| NYMEX Henry Hub Natural Gas | 2.4331 | 1.1180 | 0.0133 | 0.4328 | 3-Apr-90 | 30-Sep-02 | Daily |
| Other | | | | | | | |
| Mpls, FOB Truck Mill Feeds | 64.8188 | 19.8753 | 0.1329 | 9.9500 | 4-Jan-80 | 27-Sep-02 | Weekly |
| U.S. 91-Day Treasury Bills (%) | 6.5047 | 3.0149 | -0.0394 | 0.6161 | Jan-80 | Sep-02 | Monthly |
| Output | | | | | | | |
| White Pan Bread | 0.7135 | 0.1611 | 0.0019 | 0.0095 | Jan-80 | Sep-02 | Monthly |

¹Calculated from monthly averages for each series; units given in text.

Energy input prices were the Midwest on-road #2 diesel fuel prices compiled from the U.S. Energy Information Administration (EIA). The natural gas price series, natural gas sold to industrial U.S. consumers, was also courtesy of the U.S. EIA. Diesel fuel prices were observed in dollars per gallon, and natural gas prices were in dollars per one million British thermal units (mmBtu). Futures were for Minneapolis Grain Exchange (MGE) hard red spring wheat futures contracts and are in dollars per bushel. Since Chicago Board of Trade (CBOT) corn futures had a higher correlation to Midwest beet sugar prices than either of the sugar futures contracts listed on the New York Board of Trade, corn futures were selected to hedge the price risk associated with sugar. Decatur soybean oil prices were hedged with soybean oil futures and mill feeds were hedged with corn futures, both traded on the CBOT. Corn futures prices were reported in dollars per bushel and soybean oil futures prices were in dollars per hundredweight. Both price data series were taken from the CBOT.

Number 2 diesel fuel was hedged with New York Mercantile Exchange (NYMEX) heating oil contracts. Data were available in dollars per gallon from the U.S. EIA. Price data for NYMEX Henry Hub natural gas contracts, used to hedge natural gas price risk exposure, were in dollars per mmBtu.

The final category of data used for both bread baking examples contains average auction high 91-day U.S. Treasury bill rates collected from the Federal Reserve. The 91-day Treasury bill rate was used only as a parameter in the option pricing models in estimating option values. The Treasury bill rate is not paired with a futures contract for hedging because its use in option valuation has a relatively small impact on the firm's total risk exposure. Minneapolis, FOB truck mill feeds price data were obtained from the industry publication *Milling & Baking News* and reported in dollars per ton. Mill feeds are considered a component of flour prices. The purchase of flour normally involves the pricing of the associated mill feeds, exposing the bread baking firm to another source of price risk. U.S. monthly average white pan bread prices were obtained from the U.S. Department of Labor, Bureau of Labor Statistics consumer price index. Data were reported in dollars per one-pound loaf.

Spot Input and Output Positions

The input demands and output quantities assumed for the hypothetical bread baking company in Cases I and II (Table 2) are consistent with that of a relatively large bakery. Minneapolis spring standard patent flour requirements serve as the basis for calculating other input quantities. Sugar and bakery shortening requirements for white pan bread recipes are approximately 8% and 2.75% of the total flour weight respectively (Faridi and Faubion, 1995). Mill feeds quantities are calculated based on a 72% wheat milling extraction rate, leaving 28% of each milled bushel of wheat as mill feeds and 72% as flour. Natural gas and #2 diesel fuel quantities are estimates for a large baking plant in the Midwest.

While VaR is based on a 1-month time horizon, it is assumed that the procurement operations considers itself short a 3-month supply of inputs. This is reasonable since consumer goods tend to be repriced no more frequently than monthly, with quarterly changes more common. Therefore, output prices are infrequently adjusted to reflect increased input prices. The output, white pan bread, is only considered in Case II.

Table 2. Input Quantities for Cases I and II and Output Quantity for Case II

| Months | Flour | Sugar | Bakery Shortening | Mill Feeds | #2 Diesel Fuel | Natural Gas | White Pan Bread |
|--------|---------|-------|-------------------|------------|----------------|-------------|-----------------|
| | (cwt) | (cwt) | (cwt) | (tons) | (gallons) | (mmBtu) | (1lb Loaves) |
| 1 | 40,000 | 3,200 | 1,100 | 778 | 65,000 | 38,700 | 6,000,000 |
| 3 | 120,000 | 9,600 | 3,300 | 2,333 | 195,000 | 116,100 | 18,000,000 |

Distributions and Correlations

A correlation matrix (Table 3) was created which captures the statistically significant relationships between the changes in absolute price for each of the price risk variables. The matrix is used to link the pseudo random variables, so that the observed, historical relationships between the price risk variables are considered. Only those correlations that were significantly different from zero at the 5% confidence level were included in the analysis. The correlation coefficients of particular interest represent the cash inputs and the futures contract used to hedge them. These include: flour/MGE wheat, sugar/CBOT corn, soybean oil/CBOT soybean oil, diesel fuel/NYMEX heating oil, natural gas/NYMEX natural gas, and mill feeds/CBOT corn. The flour/MGE wheat and diesel fuel/NYMEX heating oil correlations are both near 0.7, providing the highest levels of hedging effectiveness in this case. The other cash input/futures contract correlation coefficients are much lower, ranging from 0.44 to 0.2.

Table 3. Correlation Matrix for All Price Risk Variables in Case I and Case II

| Financial Variables ¹ | MF | BS | SOD | D | NGUS | MW | SO | CC | HO | NG | MLF | TB | B |
|----------------------------------|--------------|--------------|--------------|--------------|--------------|--------------|--------------|--------------|--------------|--------------|--------------|--------------|--------------|
| MF | 1.000 | 0.152 | 0.239 | | | 0.718 | 0.261 | 0.471 | | | | | |
| BS | | 1.000 | | | | 0.184 | | 0.200 | | | 0.143 | | |
| SOD | | | 1.000 | | | 0.152 | 0.403 | 0.200 | | | 0.169 | -0.160 | |
| D | | | | 1.000 | | | | | 0.674 | | | | |
| NGU | | | | | 1.000 | | | | | 0.442 | 0.177 | | |
| MW | | | | | | 1.000 | 0.332 | 0.540 | | | 0.266 | | |
| SO | | | | | | | 1.000 | 0.520 | | | 0.182 | | |
| CC | | | | | | | | 1.000 | | | 0.307 | | |
| HO | | | | | | | | | 1.000 | 0.229 | 0.145 | | |
| NG | | | | | | | | | | 1.000 | 0.237 | | |
| MLF | | | | | | | | | | | 1.000 | | |
| TB | | | | | | | | | | | | 1.000 | |
| B | | | | | | | | | | | | | 1.000 |

¹ MF = Minneapolis spring standard patent flour; BS = Midwest beet sugar; SOD = Decatur, soybean oil; D = Midwest on-road #2 diesel fuel; NGU = Natural gas sold to industrial U.S. consumers; MW = MGE hard red spring wheat futures; SO = CBOT soybean oil futures; CC = CBOT corn futures; HO = NYMEX heating oil futures; NG = NYMEX Henry Hub natural gas futures; MLF = Minneapolis, FOB truck mill feeds; TB = 91-day U.S. treasury bills; B = White pan bread.

The distributions used to estimate the 1-month changes in price risk variables are given in Table 4. These distributions and parameters were calculated according to the procedures described in previous sections. The mean and standard deviation of each distribution, as well as the parameters necessary to describe the location, scale, and shape of the distribution, are also revealed in the table.

Table 4. Distributions and Parameters for Price Change Data in Cases I and II

| Financial Variables | Distribution | Mean | Standard Deviation | γ^1 | α^2 | β^3 |
|-----------------------------------|---------------------|-------------|---------------------------|------------|------------|-----------|
| Inputs | | | | | | |
| Mpls Spring Standard Patent Flour | Logistic | 0.0158 | 0.5089 | | 0.0158 | 0.2806 |
| Midwest Beet Sugar | Log-Logistic | -0.0187 | 0.6127 | -4.9481 | 14.7280 | 0.4892 |
| Decatur, Soybean Oil | Normal | -0.0146 | 3.5208 | | | |
| Midwest On-Road #2 Diesel Fuel | Log-Logistic | 0.0023 | 0.0472 | -1.7166 | 66.0270 | 1.7182 |
| Natural Gas - Industrial | Log-Logistic | -0.0086 | 0.2508 | -4.7012 | 33.9990 | 0.4686 |
| Futures Contracts | | | | | | |
| MGE Hard Red Spring Wheat | Log-Logistic | -0.003 | 0.16791 | -1.9847 | 21.5550 | 1.9746 |
| CBOT Soybean Oil | Log-Logistic | -0.021 | 1.3107 | -17.9590 | 24.9020 | 17.8900 |
| CBOT Corn | Logistic | 0.0014 | 0.140326 | | 0.0014 | 0.0774 |
| NYMEX Heating Oil | Logistic | -5E-04 | 0.046229 | | 0.0005 | 0.0255 |
| NYMEX Henry Hub Natural Gas | Logistic | 0.0165 | 0.35448 | | 0.0165 | 0.1954 |
| Other | | | | | | |
| Mpls, FOB Truck Mill Feeds | Log-Logistic | 0.1882 | 9.9677 | -81.0870 | 14.8670 | 80.6590 |
| U.S. 91-Day Treasury Bills | Logistic | -0.02 | 0.42941 | | -0.0197 | 0.2368 |
| Output | | | | | | |
| White Pan Bread | Logistic | 0.0018 | 0.008856 | | 0.0018 | 0.0049 |

¹ γ represents the location parameter in log-logistic distributions.

² α represents the shape parameter in log-logistic distributions and the location parameter in logistic distributions.

³ β represents the scale parameter in logistic and log-logistic distributions.

Empirical Data – Case III: Mexican Flour Milling Company

For Case III, two data sets were collected from a Mexican flour milling firm of similar size to that used in this study. These two sets of price data were for the mill's outputs, flour and mill feeds, and both were reported in Mexican pesos per kilogram. Input price data, for hard red winter wheat 11% protein, FOB US Gulf, were available from U.S. Wheat Associates in dollars per bushel. Wheat requirements were hedged with Kansas City Board of Trade (KCBT) hard red winter wheat futures contracts. Price data for this instrument were observed from the KCBT in dollars per bushel.

The Mexican peso/U.S. dollar exchange rate was collected in the form of noon buying rates in New York City for cable transfers in foreign currencies, from the Federal Reserve Board of Governors. End of month prices for CME Mexican peso futures, used to hedge the currency exchange risk, were collected from Tradingcharts.com in dollars per peso. The 91-day U.S. Treasury bill rate data were also used in its same format for valuing options.

Table 5 presents the mean and standard deviation of the absolute, observed prices, as well as the mean and standard deviation of changes in price for all of the price risk variables utilized in the Mexican flour milling company case study. These statistics were calculated after daily and weekly observations and were aggregated into monthly average data as well. The table also describes the time period over which prices were observed and indicates the frequency of the observations. The hypothetical Mexican flour milling company in the case study is assumed to represent the average mill size as given in the *2000 Grain & Milling Annual* published by *Milling & Baking News*. The 1-day flour production of this mill is assumed to be 7,500 hundredweight. The firm considers itself short a 3-month supply of wheat and requires that 2.3 bushels of wheat be processed to yield one hundredweight of flour. The quantities of inputs utilized, and outputs produced, by this Mexican flour mill are given in Table 6, and based on the same 72% milling extraction ratio used in the previous cases.

Table 5. Characteristics of Observed Date Series for Case III

| Financial Variables | Absolute Price ¹ | | Price Change ¹ | | Start Date | End Date | Observation Frequency |
|--------------------------------|-----------------------------|--------------------|---------------------------|--------------------|------------|-----------|-----------------------|
| | Mean | Standard Deviation | Mean | Standard Deviation | | | |
| Inputs | | | | | | | |
| HRW 11% Protein, FOB U.S. Gulf | 3.6646 | 0.6633 | -0.0307 | 0.1888 | Sep-96 | Sep-00 | Monthly |
| Futures Contracts | | | | | | | |
| KCBT Hard Red Winter Wheat | 3.6084 | 0.6933 | -0.0055 | 0.1908 | 1-Jan-80 | 30-Sep-00 | Daily |
| CME Mexican Peso | 9.1091 | 0.8397 | 0.0367 | 0.3290 | Sep-96 | Sep-00 | Monthly |
| Other | | | | | | | |
| Peso/U.S. \$ Exchange Rate | 7.4698 | 2.1751 | 0.0757 | 0.3082 | Nov-93 | Sep-00 | Monthly |
| U.S. 91-Day Treasury Bills | 6.8309 | 2.9180 | -0.0242 | 0.6427 | Jan-00 | Sep-00 | Monthly |
| Output | | | | | | | |
| Flour sold for pesos | 2.3310 | 0.2686 | -0.0183 | 0.0759 | Sep-96 | Sep-00 | Monthly |
| Mill Feeds sold for pesos | 1.1919 | 0.1292 | -0.0008 | 0.0760 | Sep-96 | Sep-00 | Monthly |

¹Calculated from monthly averages for each series; units given in text.

Table 6. Input and Output Quantities Used in Case III

| Months | Wheat (bushels) | Flour (cwt) | Mill Feeds (tons) |
|---------------|---------------------------|-----------------------|-----------------------------|
| 1 | 523,250 | 10,340,909 | 3,992,709 |
| 3 | 1,569,750 | 31,022,727 | 11,978,128 |

Distributions and Correlations

A correlation matrix (Table 7) was created which captured the relationships between the changes in absolute price for each of the price risk variables described. The matrix was constructed in the same manner as the correlation matrix used in the previous cases. Omissions in Table 7 represent correlations not statistically different from zero, and are treated as zero values in the analysis. Correlations of particular interest include wheat/KCBT wheat, and Mexican peso/U.S. dollar exchange rate/CME Mexican pesos. The relationship between one of the outputs, flour, and both wheat and the exchange rate is also important.

Table 7. Correlation Matrix for Price Risk Variables in Case III

| Financial Variables¹ | GW | KCW | CME | EXC | TB | MLFP | FP |
|--|---------------|---------------|---------------|---------------|---------------|---------------|---------------|
| GW | 1.0000 | 0.8241 | | | | | 0.3989 |
| KCW | | 1.0000 | | | | | 0.3185 |
| CME | | | 1.0000 | 0.6342 | | | |
| EXC | | | | 1.0000 | | | 0.3039 |
| TB | | | | | 1.0000 | | |
| MLFP | | | | | | 1.0000 | |
| FP | | | | | | | 1.0000 |

¹ GW = Hard red winter wheat 11% protein, FOB U.S. Gulf; KCW = KCBT hard red winter wheat futures; CME = CME Mexican peso futures; EXC = Mexican peso/U.S. dollar exchange rate – noon buying rates in New York City for cable transfers in foreign currencies; TB = 91-day U.S. treasury bills; MLFP = Mill feeds sold for Mexican pesos; FP = Flour sold for Mexican pesos.

The distributions used to estimate the 1-month changes in price risk variables are given in Table 8. These distributions and parameters were calculated according to the procedures described in previous sections. The table also reveals the mean and standard deviation of each distribution, as well as the parameters required to describe the location, scale, shape, and lateral shift of the distribution.

Table 8. Distributions and Parameters for Price Change Data in Case III

| Financial Variables | Distribution | Mean | Standard Deviation | γ^1 | α^2 | β^3 | Shift⁴ |
|--------------------------------|---------------------|-------------|---------------------------|------------|------------|-----------|--------------------------|
| Inputs | | | | | | | |
| HRW 11% Protein, FOB U.S. Gulf | Logistic | -0.0326 | 0.1676 | | -0.0326 | 0.0924 | |
| Futures Contracts | | | | | | | |
| KCBT Hard Red Winter Wheat | Log-Logistic | -0.0065 | 0.18006 | -3.1115 | 31.3406 | 3.0998 | |
| CME Mexican Peso | Gamma | 0.0367 | 0.3264 | | 10.4820 | 0.1008 | -1.0201 |
| Other | | | | | | | |
| Peso/U.S. \$ Exchange Rate | Log-Logistic | 0.05547 | 0.23758 | -0.8093 | 6.89475 | 0.83515 | |
| U.S. 91-Day Treasury Bills | Logistic | -0.0042 | 0.44394 | | -0.0042 | 0.2448 | |
| Output | | | | | | | |
| Flour sold for pesos | Logistic | -0.0147 | 0.070521 | | -0.0147 | 0.0389 | |
| Mill Feeds sold for pesos | Logistic | -0.0003 | 0.07598 | | -0.0003 | 0.0419 | |

¹ γ

represents the location parameter in log-logistic distributions.

² α represents the shape parameter in log-logistic distributions and the location parameter in logistic distributions.

³ β represents the scale parameter in logistic and log-logistic distributions; ⁴Shift represents the magnitude to which distribution is shifted laterally.

RESULTS AND DISCUSSIONS

The section begins by reporting the results of Case I. Details about the portfolio, and the numerous hedge strategies evaluated for each portfolio, are presented. VaR statistics are shown, and strategies are ranked according the magnitude of the VaR. A discussion then follows analyzing the reasons for, and implications of, the results. The discussion then moves to stress testing, where several different stress events are presented to show the effects of the scenarios on the current portfolio. A section on variance stressing is also included to show the effects of periods of increased and decreased price variability.

Case II is then presented and analyzed in much the same manner described for Case I. The data from Case I is reused in Case II, with the addition of bread prices. Stress testing and variance stressing are not performed for this case, but a section on the impact that input/output correlation has on a firm's risk exposure is included instead. The section concludes with Case III, and a discussion of how foreign exchange risk is dealt with in the VaR model. The portfolio and its components differ significantly in this case, so portfolio details and hedging strategies are introduced.

Case I Results: Procurement Operations of a U.S. Bread Baking Company

Case I consists of the procurement operations of a U.S. bread baking company responsible for a portfolio consisting of six commodities, five of which are inputs used in producing white pan bread. Procurement operations are assumed to consider itself short a 3-month supply of inputs, which are flour, sugar, bakery shortening, #2 diesel fuel, and natural gas. Procurement operations also considers itself long mill feeds, since flour purchase agreements typically call for the pricing of the associated mill feeds.

The base case analysis takes place on the 1st of October, 2002, and each position is valued at the average monthly price which prevailed the previous month. Current prices are listed in Table 9, and the current value of the cash portfolio representing procurement costs at these prices is \$-2,376,547. The negative portfolio represents future expenditures and the risk considered is that input prices will increase, resulting in higher costs of procurement.

Table 9. Case I: Current Average Monthly Price as of October 1, 2002

| Financial Variables | Spot Position¹ | Current Price | Units | Position Value |
|----------------------------------|--------------------------------------|--------------------------|--------------|---------------------------|
| Inputs | | | | |
| Mpls Spring Standar Patent Flour | -120,000 | 12.61 | \$/cwt | -\$1,513,500 |
| Midwest Beet Sugar | -9,600 | 26.88 | \$/cwt | -\$258,000 |
| Decatur, Soybean Oil | -3,300 | 21.00 | \$/cwt | -\$69,300 |
| Midwest On-Road #2 Diesel Fuel | -195,000 | 1.40 | \$/gallon | -\$273,078 |
| Natural Gas - Industrial | -116,100 | 3.82 | \$/mmBtu | -\$443,502 |
| Futures Contracts | | | | |
| MGE Hard Red Spring Wheat | | 4.88 | \$/bushel | |
| CBOT Soybean Oil | | 20.10 | \$/cwt | |
| CBOT Corn | | 2.67 | \$/bushel | |
| NYMEX Heating Oil | | 0.75 | \$/gallon | |
| NYMEX Henry Hub Natural Gas | | 3.57 | \$/mmBtu | |
| Other | | | | |
| Mpls, FOB Truck Mill Feeds | 2,333 | 77.50 | \$/ton | \$180,833 |
| U.S. 91-Day Treasury Bills | | 1.63 | % points | |
| Cash Portfolio Value | | | | |
| Case I: Procurement costs | | | | -\$2,376,547 |

¹Positive values represent long positions and negative values represent short positions.

In hedging strategies involving forward or futures contracts, the current value of the cash portfolio is equivalent to W_0 , the initial portfolio value, because the current values of all futures contracts, and forward contracts at inception, are zero. When long positions in options are used, the premiums represent an initial outlay of funds that is added to, or subtracted from, the current value of the cash portfolio in the equation for initial portfolio value, W_0 . This model allows the firm to use different hedging tools. The firm's cash positions can be offset by positions taken in forward contracts, futures contracts, and options on futures contracts. Although the number of potential strategies that are available to this hypothetical firm to manage its risk exposure is immense, only some of these were analyzed in detail.

In Table 10, the first column numbers each strategy for reference. The next column reports the 1-month VaR statistics at the 95% confidence interval, indicating that, under normal market movements, the firm could expect portfolio losses under the particular strategy to exceed the VaR one out of every 20 months. Table 10 also reports the position taken in hedging instruments. This section lists the number of futures and options contracts taken. Positions in forward contracts are not given for Case I and Case II, since positions equal and opposite that of the cash portfolio are relatively straightforward.

The first strategy listed in Table 10 is the no hedge, or control portfolio (1). When the VaR of the cash portfolio is calculated without any hedging strategy, the portfolio returns the largest VaR. The first group of hedging strategies (2-5) examined includes 100% hedges, based on technical relationships, for each input price variable. Forward contracting all input requirements (2) returns a VaR of zero, since all prices have been fixed. Implementing the risk minimizing hedge for each input in futures (3), options (4), and 50% futures-50% options (5) return similar VaR statistics. The important relationship to note is that the hedging strategy utilizing only futures contracts returns a lower VaR than either strategy containing options, and the strategy utilizing only options has the largest VaR of the three.

This relationship, where futures contract strategies tend to yield lower VaR statistics than options strategies, is caused by several factors. First, options are not held to maturity so the standard payoff to an option at maturity does not accurately represent the change in an option premium value over the 1-month time horizon for which VaR is calculated. Options also experience time decay, in that options lose a portion of their extrinsic value as maturity nears. This means that even if futures prices remain constant over the next time period, the value of the options will decrease. The rate of time decay increases as maturity nears, and the use of options expiring in 3 months results in a greater rate of decay than had options with a longer time to maturity been used.

Table 10. Case I: Value at Risk Statistics and Hedging Instrument Positions

| Hedging Strategy | Strategy | Value at Risk | Position Taken in Hedging Instruments ¹ | | | | | | | | | | | | |
|---------------------------------------|----------|---------------|--|----|----|----|----|---------|----|----|----|----|----|--|--|
| | | | Futures | | | | | Options | | | | | | | |
| | | | MW | NG | HO | SO | CC | MW | NG | HO | SO | CC | | | |
| No-Hedge | 1 | \$121,771 | | | | | | | | | | | | | |
| All Inputs Hedged | | | | | | | | | | | | | | | |
| Forward contracts | 2 | \$0 | | | | | | | | | | | | | |
| Futures contracts | 3 | \$102,128 | 47 | 4 | 4 | 5 | -7 | | | | | | | | |
| Options contracts | 4 | \$107,248 | | | | | | | 47 | 4 | 4 | 5 | *7 | | |
| 50% futures-50% options | 5 | \$102,494 | 24 | 2 | 2 | 3 | -4 | | 23 | 2 | 2 | 2 | *3 | | |
| Flour Only Hedged | | | | | | | | | | | | | | | |
| Forward contracts | 6 | \$56,743 | | | | | | | | | | | | | |
| Futures contracts | 7 | \$101,842 | 47 | | | | | | | | | | | | |
| Options contracts | 8 | \$110,823 | | | | | | | 47 | | | | | | |
| Flour & Natural Gas Hedged | | | | | | | | | | | | | | | |
| Forward contracts | 9 | \$40,633 | | | | | | | | | | | | | |
| Futures contracts | 10 | \$101,365 | 47 | 4 | | | | | | | | | | | |
| Options contracts | 11 | \$108,632 | | | | | | | 47 | 4 | | | | | |
| All Non-Flour Inputs Hedged | | | | | | | | | | | | | | | |
| Forward contracts | 12 | \$101,081 | | | | | | | | | | | | | |
| Futures contracts | 13 | \$121,589 | | 4 | 4 | 5 | -7 | | | | | | | | |
| Options contracts | 14 | \$117,660 | | | | | | | | 4 | 4 | 5 | *7 | | |
| Regression - Total Portfolio | | | | | | | | | | | | | | | |
| MGE wheat futures | 15 | \$101,770 | 48 | | | | | | | | | | | | |
| MGE wheat options | 16 | \$110,623 | | | | | | | 48 | | | | | | |
| MGE wheat & nat gas futures | 17 | \$101,164 | 49 | 4 | | | | | | | | | | | |
| MGE wheat & nat gas options | 18 | \$108,519 | | | | | | | 49 | 4 | | | | | |

¹MW = MGE hard red spring wheat futures; NG = NYMEX Henry Hub natural gas futures; HO = NYMEX heating oil futures; SO = CBOT soybean oil futures; CC = CBOT corn futures.

*Denotes position in put options, all other options position are in call options.

The third factor causing options hedging strategies to return higher VaR statistics than futures strategies is due to delta. The at-the-money options used in this analysis have deltas approximately equal to 0.5, indicating that for every 1-unit change in the futures price, the option value will change by 0.5 units. Therefore, if cash and futures prices were perfectly correlated, and cash prices moved 1 unit against a firm's position, a futures strategy would exactly offset the incurred losses. The equivalent options strategy would only move 0.5-units and would not provide as much hedging effectiveness as a futures hedge. Using deep in-the-money or out-of-the-money options with drastically different values for delta may have a significant affect on the VaR of options strategies, however, this was not explored.

The fourth reason for this relationship between futures and options strategies has to do with the variability of prices used in the analyses. In times of high prices volatility, these three characteristics of options are more than offset by the benefits of an option's truncated payoffs. In the base case scenarios, however, volatilities are low enough that even the largest simulated losses do not move prices to a great enough extent that the truncated payoffs of options are realized. This becomes more evident later when we examine the effects of increases in the variances of price changes.

This illustrates the impact that the length of historical data and statistical distribution choices can have on the VaR results. While the flexibility of distribution and parameter selection in Monte Carlo simulation allows the user to choose any distribution and parameters that he feels adequately represents the future price movement possibilities. This freedom, however, also allows the user to make poor choices that inaccurately estimate future movements. This is referred to as model risk and could affect the ranking of strategies in all three cases.

The second group (6-8) of strategies focuses on hedging only the flour portion of the portfolio with different combinations of instruments. Forward contracting the flour requirements (6) results in the third lowest VaR for this case study. This is because flour is the most prominent component of the procurement division's portfolio, making up over half of the portfolio value. The same relationship exists within this group, as forward contracts (6) returns the lowest VaR, followed by the futures strategy (7). When hedging only flour requirements, an options hedge (8) again provides the highest VaR, or the least hedging effectiveness.

Since natural gas was the second most prominent input, in terms of absolute value of the requirements, a group of hedging strategies was examined that considered hedging only flour and natural gas (9-11). These strategies provided some of the lowest VaR statistics; however, the same pattern held, in that forward contracting (9) reduced risk the most, and the options strategy (11) provided the least effective hedge.

The hedging strategies involving all inputs except flour (12-14) provide an interesting illustration of the effects of correlation between each cash input variable and the instrument used to hedge the associated price risk. In this group of strategies, the VaR ranking of the strategies does not follow the same pattern as observed in the other groups. Here, forward contracting (12) results in the lowest VaR, but the highest VaR in the group is returned for the futures hedge (13).

This can be explained by observing the correlations in Table 3 on page 17. The correlation between Minneapolis flour and MGE wheat futures of 0.718 is the highest correlation coefficient observed between a cash input and its associated futures contract. Hedging strategies where MGE wheat futures are used return lower VaR statistics than the options strategies. While the correlation between heating oil futures and #2 diesel fuel is only slightly lower, at 0.674, the correlations between cash beet sugar, Decatur, soy oil, natural gas, mill feeds, and the futures contracts used to hedge each of these price risk variables respectively, range from 0.442 to 0.200. As shown in Table 10, when the flour component is not hedged the implication of these lower correlations on the VaR statistics becomes much more prominent. When correlations between cash and futures positions are high, futures hedging strategies return lower VaR statistics than options strategies. As correlations decline between cash and futures, hedging effectiveness decreases to the point where, as observed in the all non-flour inputs hedge (12-14), the truncated payoffs offered by an options position (14) return a lower VaR statistic, and provide greater hedging effectiveness than futures contracts (13).

The final group of hedging strategies (15-18) contains futures and options positions calculated differently than those in the previous strategies. The first step was to value the current portfolio, had it been held at each historical monthly time period, and observe the total change in portfolio value for each period. The change in price of individual futures contracts, as well as different combinations of multiple futures contracts, was regressed against the total change in portfolio value. All possible contract combinations were evaluated, and those that were significant at the 5% confidence level were considered. This method was used to calculate the minimum variance hedge ratio for the entire portfolio, instead of calculating the ratio for each input individually. By calculating the minimum variance hedge ratios for the entire portfolio, the benefits of diversification that naturally occur in multiple asset portfolios were taken into account.

Although the hedge ratios changed only slightly from those used in strategies 7-8 and 10-11 where the same combinations of hedging instruments were used, the strategies in this group (15-18) yielded lower VaR statistics. The level of risk reduction observed in the VaR statistics shows that this method of hedge ratio calculation provides superior hedging effectiveness compared to calculating the ratio for each factor independently. As described earlier, the use of futures (15, 17) results in lower VaR statistics than the equivalent options strategies (16,18).

When evaluating the scenarios analyzed for Case I, the four lowest VaR statistics were observed for the strategies utilizing forward contracts. With forward contracts, the firm eliminates both futures risk, and basis risk, which provides 100% reduction of price risk for the inputs hedged in this manner. VaR does not lead the user to an optimal portfolio, however. This application of VaR addresses only price risk, and since managers must consider numerous other sources of risk, as well as expected return, VaR is not sufficient for portfolio selection. For instance, forward contracts are typically less liquid, and lifting the hedge if desired may be difficult. The firm might also be uncertain of the exact quantity of inputs needed, and if inputs were forward contracted, the firm would have much less flexibility. A forward contract also specifies a supplier, which prohibits the firm from changing suppliers before the actual input purchase is made.

The two futures strategies (15, 17) where hedge ratios were found through regression with the change in total portfolio value, and the futures strategies hedging only flour (7), and flour and natural gas (10), were similar. Aside from the forward contracting strategies, these four futures contract strategies provide the highest level of hedging effectiveness. With only one exception, hedging strategies utilizing options consistently offered the least hedging protection; however all strategies resulted in VaR statistics at least marginally lower than the VaR observed for the unhedged position (1).

Thus far, every hedging strategy examined calls for a 100% hedge ratio to be used. Figure 2 illustrates the effect that scaling this hedge ratio from 0-100%, in increments of 10%, would have on the VaR statistic when hedging only flour requirements. The figure shows that any level of VaR between \$121,771, and \$101,081 for the case of futures, or \$110,823 for the case of options, can be achieved by varying the size of the hedge position. The difference between the futures and options series is due to the indivisibility of futures and options contracts. For example, a 20% hedge calls for exactly 9.37 contracts; however, futures and options contracts are only available in integer units and had to be rounded.

The distribution of changes in portfolio values for strategy 7 in Case I is shown in Figure 3. This figure is a histogram reporting the number of occurrences, out of 10,000, found in each histogram bin, when values for change in portfolio value are divided into 25 bins with a range of \$25,000 each. This illustrates that the focus of VaR is on the far left hand tail of the distribution. The 95% confidence interval implies that, when strategy 7 is used, one out of every twenty periods will experience losses greater than \$101,842.

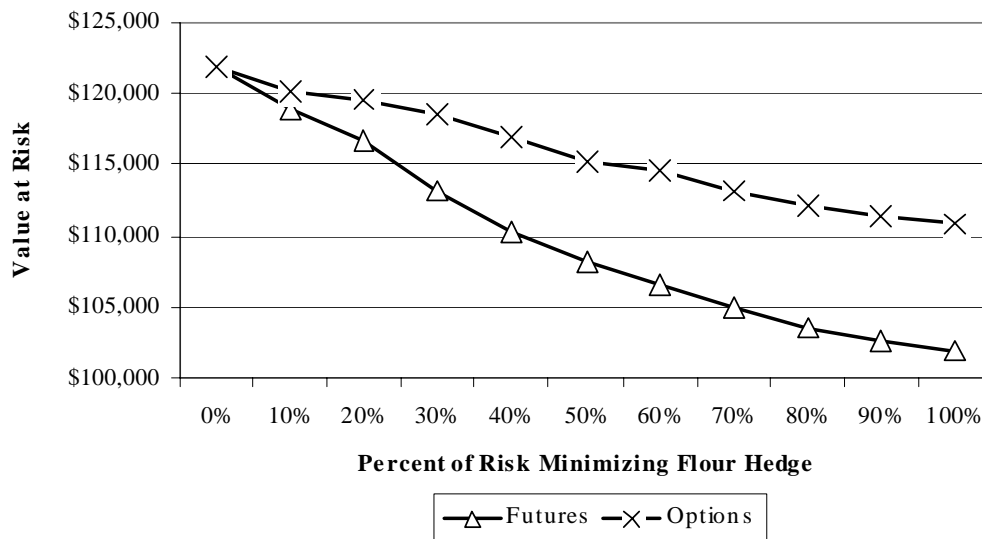


Figure 2. Case I: Value at Risk Statistics for Varying Percentages of the Risk Minimizing Hedge Ratio for Strategies 7 and 8

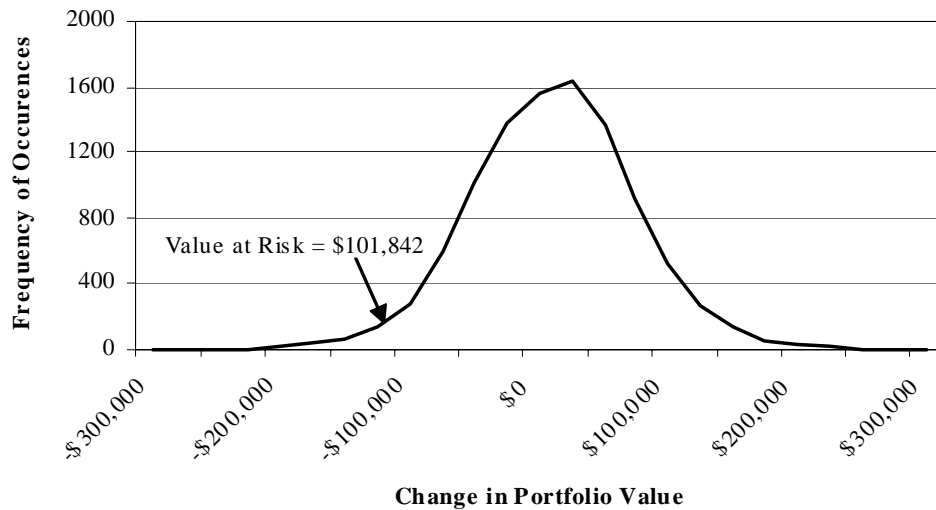


Figure 3. Case I: Distribution of 1-Month Changes in Portfolio Value when Hedging the Flour Position with Futures Contracts in Strategy 7

Confidence Interval

The 95% confidence interval (C.I.) was used for VaR statistics in Cases I, II and III. This section shows the impact of C.I. choice for Case I strategies, by calculating VaR at the 90%, 95%, and 99% C.I. The most obvious observation is that the absolute magnitude of VaR increases as the confidence interval increases, for every strategy. The primary relationship found throughout this study is that forward contracts yield the lowest VaR, options yield the highest VaR, and futures strategies rank between them, except when cash/futures correlations are low or price variability is high. This relationship holds for each of the confidence intervals shown in Table 11.

One interesting result of calculating VaR at these different confidence intervals is that at both that 90% and 99% C.I., the unhedged strategy (1) does not return the highest VaR. The highest VaR is instead found for strategy 13 for both confidence intervals. While the ordinal rankings of strategies at the 90% and 95% confidence intervals are very similar, the rankings for the 99% C.I. show some interesting differences. First, the rank of forward contracting all non-flour inputs (12) falls, while the rank of hedging flour and natural gas with futures contracts (10) increases. The largest increase in rank was observed for hedging all inputs with futures contracts (3).

While the ordinal rankings varied depending on which C.I. was used, most discrepancies were relatively minor and resulted from the distributions of changes in portfolio value observed for each individual hedging strategy. The important aspect to focus on, is that the same general conclusions would be drawn, no matter which C.I. was used when results were presented to the firm's management. The decision makers would see that forward contracting provides the greatest risk reduction by far, and that futures contracts where flour is hedged provide the next best series of strategies. They would also see that hedges not including flour provide relatively low levels of risk reduction.

Table 11. Case I: Value at Risk Statistics at Different Confidence Intervals

| Hedging Strategy | Strategy | 90% C.I. | | 95% C.I. | | 99% C.I. | |
|---------------------------------------|----------|---------------|------|---------------|------|---------------|------|
| | | Value at Risk | Rank | Value at Risk | Rank | Value at Risk | Rank |
| No-Hedge | 1 | \$92,492 | 17 | \$121,771 | 18 | \$181,037 | 17 |
| All Inputs Hedged | | | | | | | |
| Forward contracts | 2 | \$0 | 1 | \$0 | 1 | \$0 | 1 |
| Futures contracts | 3 | \$79,777 | 10 | \$102,128 | 9 | \$147,716 | 6 |
| Options contracts | 4 | \$82,644 | 11 | \$107,248 | 11 | \$157,432 | 11 |
| 50% futures-50% options | 5 | \$79,746 | 9 | \$102,494 | 10 | \$150,974 | 9 |
| Flour Only Hedged | | | | | | | |
| Forward contracts | 6 | \$42,999 | 3 | \$56,743 | 3 | \$82,641 | 3 |
| Futures contracts | 7 | \$79,446 | 7 | \$101,842 | 8 | \$148,980 | 8 |
| Options contracts | 8 | \$85,543 | 15 | \$110,823 | 15 | \$160,306 | 15 |
| Flour & Natural Gas Hedged | | | | | | | |
| Forward contracts | 9 | \$31,555 | 2 | \$40,633 | 2 | \$56,904 | 2 |
| Futures contracts | 10 | \$78,413 | 5 | \$101,365 | 6 | \$145,438 | 4 |
| Options contracts | 11 | \$83,005 | 13 | \$108,632 | 13 | \$158,427 | 13 |
| All Non-Flour Inputs Hedged | | | | | | | |
| Forward contracts | 12 | \$75,892 | 4 | \$101,081 | 4 | \$156,861 | 10 |
| Futures contracts | 13 | \$93,108 | 18 | \$121,589 | 17 | \$183,874 | 18 |
| Options contracts | 14 | \$89,272 | 16 | \$117,660 | 16 | \$179,210 | 16 |
| Regression - Total Portfolio | | | | | | | |
| MGE wheat futures | 15 | \$79,601 | 8 | \$101,770 | 7 | \$148,888 | 7 |
| MGE wheat options | 16 | \$85,295 | 14 | \$110,623 | 14 | \$159,851 | 14 |
| MGE wheat & nat gas futures | 17 | \$78,546 | 6 | \$101,164 | 5 | \$145,982 | 5 |
| MGE wheat & nat gas options | 18 | \$82,939 | 12 | \$108,519 | 12 | \$157,696 | 12 |

Stress Testing

Stress testing procedures were performed to evaluate the effects of unlikely economic events. Scenarios commonly evaluated in stress testing programs fit into two basic categories. They consist of economic events that have occurred in the past and events that are believed to be possible in the future. In order to demonstrate these two types of stress testing, four events (Table 12) were considered. The first two represent events that have not occurred in the past but are still reasonable. For the event titled maximum observed losses, the portfolio was valued assuming that the largest 1-month price increase, observed for each variable in the available historical data, was realized in the coming period. The second event assumes that all prices experience a four standard deviation price increase in the next period.

The last two scenarios are used to demonstrate the losses that the current portfolio would sustain for each of the hedging strategies if historical economic events were experienced. The two historical periods selected were September 2002, and May 1996. September 2002 was the largest 1-month loss that the current, unhedged portfolio would have sustained since April of 1994. Situations with price movements similar to those in this period were observed several times and resulted in four of the largest losses. Although May 1996 would have only resulted in the 5th largest loss to the current unhedged portfolio, it was examined due to the unique price movements observed. In this event, all three agriculture input prices increased, fuel prices decreased, and mill feed prices dropped dramatically, despite increases in other agricultural commodity prices.

Table 12. Case I: 1-Month Price Movements for Each Stress Event

| Financial Variables | Maximum | 4 Standard | Sep-02 | May-96 |
|-----------------------------------|-----------|------------|---------|----------|
| | Observed | Deviation | | |
| | Increases | Increases | | |
| Inputs | | | | |
| Mpls Spring Standard Patent Flour | \$1.93 | \$2.23 | \$1.91 | \$1.36 |
| Midwest Beet Sugar | \$2.90 | \$2.69 | \$0.98 | \$0.00 |
| Decatur, Soybean Oil | \$36.68 | \$14.08 | -\$0.10 | \$0.71 |
| Midwest On-Road #2 Diesel Fuel | \$0.17 | \$0.21 | \$0.09 | -\$0.01 |
| Natural Gas - Industrial | \$1.98 | \$1.19 | \$0.12 | -\$0.28 |
| Futures Contracts | | | | |
| MGE Hard Red Spring Wheat | \$0.93 | \$0.76 | \$0.88 | \$0.40 |
| CBOT Soybean Oil | \$8.67 | \$5.72 | -\$0.47 | \$0.81 |
| CBOT Corn | \$0.82 | \$0.63 | \$0.07 | \$0.36 |
| NYMEX Heating Oil | \$0.23 | \$0.19 | \$0.07 | -\$0.05 |
| NYMEX Henry Hub Natural Gas | \$2.91 | \$1.73 | \$0.48 | -\$0.03 |
| Other | | | | |
| Mpls, FOB Truck Mill Feeds | \$35.50 | \$39.80 | \$9.70 | -\$24.05 |
| U.S. 91-Day Treasury Bills | \$2.71 | \$2.48 | \$0.00 | \$0.03 |

The results of these four economic stress events are shown in Table 13, where the strategy, VaR, and rank columns are in the same format as Table 10. Each portfolio value in the columns representing stress events was calculated analytically,² assuming the price changes given in Table 12, instead of through simulation used to calculate VaR. When all the hedging strategies are observed, the maximum observed increases scenario results in the largest losses, followed by the four standard deviation increase scenario. The losses realized under the two historical events are, on average, much smaller and vary dramatically depending on the hedging strategy in question.

The results of these stress events allow the user of this information to draw some conclusions. The first of these is that the strategies involving forward contracting of the flour requirements provide some of the lowest losses for the stress events considered, with the exception being the case when the maximum observed losses are realized. Another is that, although forward contracting all non-flour inputs ranks fourth in terms of VaR, the losses realized by that portfolio under the four stress events make the strategy much less appealing than if VaR had been utilized alone. Finally, strategies using the hedge ratios found through regression for both MGE wheat and NYMEX natural gas futures contracts consistently rank in the top five, whether evaluated using VaR or the four stress events.

Variance Stressing

When @RiskTM was used to estimate the distribution parameters that best approximated the historical distribution of prices for each price risk variable, the entire sample set was included. This method of distribution estimation aggregates periods of price variability into one distribution ignoring that variability of prices fluctuates over time. For this reason, the conventional methods of stress testing were supplemented by analyzing the VaR of the current portfolio in periods of both increased and decreased price variability to illustrate the importance of accurately portraying the price variability likely to occur in the next time period.

Case I was used to illustrate how the portfolio VaR would act under three different situations. These situations include periods when the variability of changes in price decreased by half, doubled, and quadrupled. Transforming the variance of the variables with normal and logistic distributions was accomplished by modifying the *beta* parameter in the @RiskTM distribution function according to the variance function. Due to the complexity of the function for the variance of a log-logistic distribution, trial and error methods were used to approximate the *alpha* parameter in the @RiskTM functions which corresponded to the desired magnitude of the distribution's variance.

² Stress testing is done analytically since the portfolio is valued at a given set of prices. Simulation would result in numerous equivalent portfolio values, because no stochastic variables are used. This is in contrast to VaR, where thousands of portfolio values are calculated, ordered, and the 5th percentile is chosen.

Table 13. Case I: Portfolio Losses Realized Under Select Stress Events

| Hedging Strategy | Strategy | Value at Risk | Rank | Maximum Observed | | 4 Standard Deviation | | Sep-02 | Rank | May-96 | Rank |
|---------------------------------------|----------|---------------|------|------------------|------|----------------------|------|-----------|------|------------|------|
| | | | | Increases | Rank | Increases | Rank | | | | |
| No-Hedge | 1 | \$121,771 | 18 | \$560,867 | 18 | \$426,529 | 18 | \$246,920 | 18 | \$187,068 | 16 |
| All Inputs Hedged | | | | | | | | | | | |
| Forward contracts | 2 | \$0 | 1 | \$0 | 1 | \$0 | 1 | \$0 | 1 | \$0 | 1 |
| Futures contracts | 3 | \$102,128 | 9 | \$191,326 | 3 | \$151,779 | 3 | \$13,558 | 4 | -\$112,997 | 8 |
| Options contracts | 4 | \$107,248 | 11 | \$230,121 | 7 | \$194,685 | 8 | \$62,089 | 10 | \$136,620 | 14 |
| 50% futures 50% options | 5 | \$102,494 | 10 | \$211,747 | 4 | \$173,826 | 6 | \$37,548 | 8 | \$124,985 | 9 |
| Flour Only Hedged | | | | | | | | | | | |
| Forward contracts | 6 | \$56,743 | 3 | \$329,867 | 11 | \$159,159 | 4 | \$17,420 | 5 | \$24,468 | 2 |
| Futures contracts | 7 | \$101,842 | 8 | \$343,433 | 13 | \$248,016 | 12 | \$40,219 | 9 | \$92,833 | 6 |
| Options contracts | 8 | \$110,823 | 15 | \$386,020 | 15 | \$289,798 | 15 | \$81,741 | 14 | \$128,457 | 12 |
| Flour & Natural Gas Hedged | | | | | | | | | | | |
| Forward contracts | 9 | \$40,633 | 2 | \$99,989 | 2 | \$20,648 | 2 | \$3,488 | 2 | \$56,976 | 3 |
| Futures contracts | 10 | \$101,365 | 6 | \$227,009 | 6 | \$178,773 | 7 | \$21,110 | 6 | \$93,866 | 7 |
| Options contracts | 11 | \$108,632 | 13 | \$284,100 | 10 | \$233,917 | 10 | \$69,576 | 12 | \$128,996 | 13 |
| All Non-Flour Inputs Hedged | | | | | | | | | | | |
| Forward contracts | 12 | \$101,081 | 4 | \$231,000 | 8 | \$267,370 | 13 | \$229,500 | 17 | \$162,600 | 15 |
| Futures contracts | 13 | \$121,589 | 17 | \$408,759 | 17 | \$330,292 | 16 | \$220,259 | 15 | \$207,232 | 18 |
| Options contracts | 14 | \$117,660 | 16 | \$404,968 | 16 | \$331,415 | 17 | \$227,268 | 16 | \$195,232 | 17 |
| Regression - Total Portfolio | | | | | | | | | | | |
| MGE wheat futures | 15 | \$101,770 | 7 | \$338,807 | 12 | \$244,218 | 11 | \$35,821 | 7 | \$90,828 | 5 |
| MGE wheat options | 16 | \$110,623 | 14 | \$382,300 | 14 | \$286,889 | 14 | \$78,227 | 13 | \$127,210 | 11 |
| MGE wheat & nat gas fut | 17 | \$101,164 | 5 | \$217,757 | 5 | \$171,177 | 5 | \$12,315 | 3 | \$89,856 | 4 |
| MGE wheat & nat gas opt | 18 | \$108,519 | 12 | \$276,660 | 9 | \$228,099 | 9 | \$62,547 | 11 | \$126,502 | 10 |

The results of this analysis are in Table 14, where the first, third, and fourth VaR columns represent the cases of altered variances, and the second column is the VaR calculated in the base scenario. The conclusion is that the magnitude of the VaR increases as the variance of the relevant price change variables increases. It is also interesting that the group of hedging scenarios involving all non-flour inputs (12-14) is the only group in which the ordering of forward, futures, and options strategies hold across all four variance levels. Forward contracts (12) return the lowest VaR, followed by options (14), and finally futures (13).

For every other group of strategies, at least one altered variance scenario does not conform to the original observed relationships. For instance, the 50% futures 50% options strategy (5) for hedging all inputs returns a VaR between the futures (3) and options strategies (4) under the observed, historical variance scenario. However, as the variances are either increased or decreased, the VaR of the 50% futures 50% options strategy (5) declines below the futures strategy (3). The relationship observed between forward, futures, and options strategies breaks down again in the case of hedging flour (6-8), flour and natural gas (9-11), and the regression strategies (15-18) as well. In these groups, futures contract strategies return the least desirable VaR statistics only when variances are quadrupled. This result, described earlier in this section, demonstrates that as the variability of price changes increases, so does the hedging effectiveness of options relative to futures.

Case II: U.S. Bread Baking Company

The U.S. bread baking company case is an extension of Case I, the procurement operations example. All input quantities and prices are the same between the two, with the only difference being the addition of white pan bread price risk in Case II. While Case I considers only input price risk, Case II considers both input and output price risk simultaneously.

The current prices and cash portfolio positions listed in Table 9 are supplemented by adding the position in the output, white pan bread. The value of the current long position in 18,000,000 1-pound loaves of white pan bread, at \$1.02 per loaf, is then calculated to be \$18,288,000. The current cash portfolio value, \$15,911,453, is found by summing the value of the long bread position and the value of the procurement operations cash portfolio. Therefore, inclusion of the output, or revenue portion of the firm's budget, leads to the traditional positive portfolio value and allows changes in input prices and output prices to be considered simultaneously.³ The VaR statistics for Case II, as listed in Table 15, reveal that the observed relationships between the futures and options strategies are consistent with those found in Case I.

³ While the utility maximizing hedge ratio was expanded in this scenario to include the strategic component, this component reduces to zero because observed correlation between bread and all other prices were not statistically different than zero. Therefore, the utility maximizing hedge ratio and the risk minimizing hedge ratio are equivalent, and hedging strategies 1 through 14 were equivalent for Cases I and II.

Table 14. Case I: Value at Risk Statistics Under Periods of Increased and Decreased Price Variability

| Hedging Strategy | Strategy | Value at Risk | | Value at Risk | | Value at Risk | | Value at Risk | |
|---------------------------------------|----------|---------------|------|---------------------------|------|---------------|------|---------------|------|
| | | Variance / 2 | Rank | Obs ¹ Variance | Rank | Variance * 2 | Rank | Variance * 4 | Rank |
| No-Hedge | 1 | \$67,542 | 18 | \$121,771 | 18 | \$174,337 | 17 | \$209,175 | 18 |
| All Inputs Hedged | | | | | | | | | |
| Forward contracts | 2 | \$0 | 1 | \$0 | 1 | \$0 | 1 | \$0 | 1 |
| Futures contracts | 3 | \$62,725 | 10 | \$102,128 | 9 | \$147,689 | 10 | \$179,090 | 10 |
| Options contracts | 4 | \$64,405 | 12 | \$107,248 | 11 | \$150,259 | 11 | \$174,910 | 6 |
| 50% futures 50% options | 5 | \$61,959 | 6 | \$102,494 | 10 | \$147,112 | 9 | \$174,460 | 5 |
| Flour Only Hedged | | | | | | | | | |
| Forward contracts | 6 | \$39,724 | 3 | \$56,743 | 3 | \$85,648 | 3 | \$129,804 | 3 |
| Futures contracts | 7 | \$62,308 | 8 | \$101,842 | 8 | \$147,050 | 8 | \$183,571 | 14 |
| Options contracts | 8 | \$65,987 | 15 | \$110,823 | 15 | \$154,979 | 15 | \$183,165 | 13 |
| Flour & Natural Gas Hedged | | | | | | | | | |
| Forward contracts | 9 | \$28,113 | 2 | \$40,633 | 2 | \$60,545 | 2 | \$97,775 | 2 |
| Futures contracts | 10 | \$61,402 | 5 | \$101,365 | 6 | \$146,453 | 6 | \$179,725 | 9 |
| Options contracts | 11 | \$64,276 | 11 | \$108,632 | 13 | \$151,709 | 13 | \$179,662 | 7 |
| All Non-Flour Inputs Hedged | | | | | | | | | |
| Forward contracts | 12 | \$51,486 | 4 | \$101,081 | 4 | \$142,162 | 4 | \$142,176 | 4 |
| Futures contracts | 13 | \$66,999 | 17 | \$121,589 | 17 | \$177,030 | 18 | \$209,247 | 17 |
| Options contracts | 14 | \$65,884 | 14 | \$117,660 | 16 | \$169,077 | 16 | \$201,186 | 16 |
| Regression - Total Portfolio | | | | | | | | | |
| MGE wheat futures | 15 | \$62,623 | 9 | \$101,770 | 7 | \$146,992 | 7 | \$184,436 | 15 |
| MGE wheat options | 16 | \$66,188 | 16 | \$110,623 | 14 | \$154,579 | 14 | \$182,780 | 12 |
| MGE wheat & nat gas futures | 17 | \$62,015 | 7 | \$101,164 | 5 | \$145,953 | 5 | \$179,795 | 11 |
| MGE wheat & nat gas options | 18 | \$64,722 | 13 | \$108,519 | 12 | \$151,026 | 12 | \$179,008 | 8 |

¹ Obs Variance indicates that the observed, historical variance was used for each variable in calculating the VaR statistic.

Table 15. Case II: Value at Risk Statistics and Hedging Instrument Positions

| Hedging Strategy | Strategy | Value at Risk | Position Taken in Hedging Instruments ¹ | | | | | | | | | | | | |
|---------------------------------------|----------|---------------|--|----|----|----|----|---------|----|----|----|----|----|--|--|
| | | | Futures | | | | | Options | | | | | | | |
| | | | MW | NG | HO | SO | CC | MW | NG | HO | SO | CC | | | |
| No-Hedge | 1 | \$257,411 | | | | | | | | | | | | | |
| All Inputs Hedged | | | | | | | | | | | | | | | |
| Forward contracts | 2 | \$225,723 | | | | | | | | | | | | | |
| Futures contracts | 3 | \$248,615 | 47 | 4 | 4 | 5 | -7 | | | | | | | | |
| Options contracts | 4 | \$251,599 | | | | | | | 47 | 4 | 4 | 5 | *7 | | |
| 50% futures 50% options | 5 | \$250,182 | 24 | 2 | 2 | 3 | -4 | | 23 | 2 | 2 | 2 | *3 | | |
| Flour Only Hedged | | | | | | | | | | | | | | | |
| Forward contracts | 6 | \$232,113 | | | | | | | | | | | | | |
| Futures contracts | 7 | \$248,567 | 47 | | | | | | | | | | | | |
| Options contracts | 8 | \$255,154 | | | | | | | 47 | | | | | | |
| Flour & Natural Gas Hedged | | | | | | | | | | | | | | | |
| Forward contracts | 9 | \$227,681 | | | | | | | | | | | | | |
| Futures contracts | 10 | \$247,458 | 47 | 4 | | | | | | | | | | | |
| Options contracts | 11 | \$252,564 | | | | | | | 47 | 4 | | | | | |
| All Non-Flour Inputs Hedged | | | | | | | | | | | | | | | |
| Forward contracts | 12 | \$247,201 | | | | | | | | | | | | | |
| Futures contracts | 13 | \$253,573 | | 4 | 4 | 5 | -7 | | | | | | | | |
| Options contracts | 14 | \$251,454 | | | | | | | | 4 | 4 | 5 | *7 | | |
| Regression - Total Portfolio | | | | | | | | | | | | | | | |
| MGE wheat futures | 15 | \$247,360 | 52 | | | | | | | | | | | | |
| MGE wheat options | 16 | \$255,628 | | | | | | | 52 | | | | | | |

¹ MW = MGE hard red spring wheat futures; NG = NYMEX Henry Hub natural gas futures; HO = NYMEX heating oil futures; SO = CBOT soybean oil futures; CC = CBOT corn futures.

* Denotes put contracts, all other options position are in call options.

The forward contracting of inputs resulted in the lowest VaR levels, followed by futures contracts, and finally, options strategies. This relationship held when all inputs were hedged (2-5), only flour was hedged (6-8), and when flour and natural gas were hedged (9-11). As with Case I, this relationship broke down when all non-flour inputs were hedged (12-14) as the strategy utilizing options contracts returned a lower VaR statistic. This is due to the low correlations observed between the physical non-flour inputs and the instruments used to hedge them. The lowest VaR found for a hedging strategy utilizing futures contracts was again found by regression. This strategy calls for a slight increase in the number of wheat futures when compared to the utility maximizing futures position.

Inclusion of output price risk in this model results in observed VaR statistics more than double those in Case I. This is due to the fact that even small changes in the price of the \$18,288,000 bread portfolio have large impacts on the portfolio value. The effects of the hedging strategies are also muffled, since the price of bread exhibits no significant correlation to any of the cash or futures variables considered. Therefore, hedging strategies targeting input price risks have less of an effect because they are dominated by output price risk. It is also interesting that VaR could be reduced from \$121,771 to zero, in Case I. However, the largest VaR reduction observed for Case II is only \$31,660. All hedging strategies examined did result in VaR statistics lower than those observed for the unhedged portfolio, indicating at least minimal risk reduction. All hedging strategies in this section focused on reducing input price risk. While it may be possible for this firm to forward contract bread sales, this would result in output price risk falling below zero. This would mean that all price risk would be due to inputs and any strategy involving output forward contracting would result in a VaR equal to the equivalent Case I strategy.

Input/Output Correlation Effects

In the case of the U.S. bread baking company, no significant contemporaneous correlation was found between any of the firm's inputs and the output. While this relationship is not uncommon for a firm dealing in a consumer goods industry, firms producing intermediary goods tend to observe at least some level of correlation between inputs and outputs. When input/output correlations exist, they may impact the effectiveness of hedging strategies, and not accounting for these relationships can result in hedging strategies that actually increase the VaR of a portfolio. This result is extremely undesirable, and the following illustration outlines how a firm with correlated inputs and outputs could account for this relationship to achieve the desired result of hedging.

Before proceeding with this analysis, it is important to emphasize that significant positive correlations were not observed between bread and flour or bread and MGE wheat futures at any point over the time period examined. The imposed correlations between these factors are hypothetical, and the analysis is included only to illustrate how firms in industries where input/output correlations are present can account for them.

In order to demonstrate the correlation effects, the VaR statistics for only the flour and bread components of Case II are evaluated when unhedged and under three hedging strategies. The first strategy involves forward contracting the flour requirements; however, the output, bread, was not forward contracted. The other strategies hedged the flour exposure, and in some instances the bread exposure, with wheat futures and options contracts. The first assumption made was that the correlation between flour and bread would be equivalent to the correlation between wheat futures and bread when the correlations were varied between 0 and 0.6. This assumption was made solely to maintain the integrity of the correlation matrix given in Table 3, page 17, since varying either correlation individually resulted in an invalid correlation matrix. Correlations greater than 0.6 were not evaluated, because even with the assumption that flour/bread and wheat futures/bread correlations were equivalent, correlations of this magnitude invalidated the matrix.

For each correlation level examined, utility maximizing hedge ratios were calculated for positions in MGE wheat contracts. The strategic component of this hedge ratio was included and the changes resulting from this component of the ratio are shown in Table 16. When forward contracts are used, quantities exactly offsetting cash flour requirement are used. Positions in futures and options contracts are taken according to the utility maximizing hedge ratio for the specific correlation level in question.

Table 16. Value at Risk Statistics for Varying Levels of Input/Output Correlation for the Flour, Bread, and MGE Wheat Futures Components of Case II

| Only Flour Hedged | Correlation between MGE wheat/bread and flour/bread | | | | | | |
|----------------------|---|-----------|-----------|-----------|-----------|-----------|-----------|
| | 0.0 | 0.1 | 0.2 | 0.3 | 0.4 | 0.5 | 0.6 |
| No-Hedge | \$242,927 | \$232,536 | \$222,803 | \$214,280 | \$206,876 | \$196,017 | \$183,396 |
| Forward contracts | \$225,793 | \$225,793 | \$225,793 | \$225,793 | \$225,793 | \$225,793 | \$225,793 |
| Hedge Ratio | -1.953 | 0.454 | 2.861 | 5.267 | 7.674 | 10.081 | 12.487 |
| Futures contracts | \$236,153 | \$230,237 | \$223,427 | \$214,250 | \$203,391 | \$188,425 | \$166,615 |
| Options contracts | \$240,091 | \$231,741 | \$223,519 | \$214,675 | \$204,961 | \$193,935 | \$175,832 |

The VaR statistics for each correlation and hedging strategy are listed in Table 16. When comparing the unhedged portfolio to the forward contracting strategy, the effects of input/output correlation are apparent. If forward contracts are used, price risk associated with the input equals zero and the VaR is composed entirely of the output price risk, which is constant for all levels of correlation. The VaR of the unhedged portfolio is greater than the VaR of the portfolio when flour is forward contracted under correlations of zero and 0.1. However, the VaR of the unhedged position actually declines below that of the forward contracting strategy when the correlation rises above 0.2, and when the input/output correlation reaches 0.6, the VaR of the unhedged portfolio is nearly 20% lower than the VaR when forward contracts are used. This is because as the input/output correlation increases, flour and bread prices tend to offset each other. In all but the forward contracting case, VaR declined as correlation increased because price changes in correlated inputs and outputs offset each other to some extent. Therefore, hedging without regard for input/output correlation can increase a firm's risk exposure, emphasizing the importance of the input/output relationship.

By examining the hedge ratio row in Table 16, the effect that input/output correlation has on the utility maximizing hedge ratio can be seen. When the correlation is zero, the hedge ratio suggests a long position in nearly two bushels of wheat futures per hundredweight of flour that the firm is short. Even at a correlation of only 0.1, however, the sign on the hedge ratio has changed indicating a short position in futures contracts. In essence, this indicates that the strategic demand for short futures contracts, to hedge the output price risk, has more than offset the hedging demand for long futures contracts to offset the input price risk. As correlations increase to 0.6, the magnitude of this net short futures position grows, and the futures contracts essentially hedge the output price risk, supplementing the hedging effectiveness offered by input price fluctuations in an industry where input/output correlations exists.

Finally, it can be seen that both the futures and options hedging strategies offer lower VaR statistics than the unhedged position for each and every level of input/output correlation analyzed. It is also apparent that, although forward contracting provided the lowest VaR figure at a zero correlation, input/output correlations greater than 0.2 produce futures and options strategies lower than the forward contracting strategy. At the 0.6 correlation, the VaR for the futures strategy is more than 25% below that observed for the forward contracting strategy, and 9% less than the unhedged portfolio.

This example illustrates the importance of input/output correlations in the hedging strategy of a firm. Although the U.S. bread baking company developed in this study did not observe a significant contemporaneous input/output correlation, this scenario describes how this aspect of the hedging decision could be dealt with by a firm that does experience correlations between their input and output.

Case III: Mexican Flour Milling Company

The case of the Mexican flour milling company is used to demonstrate the application of VaR in the presence of foreign currency exchange risk. This risk component comes from the fact that the firm's input, wheat, is purchased in U.S. dollars, while the outputs, flour and mill feeds, are sold in Mexican pesos. The data used for this analysis is October 1, 2000, and the current prices for each of the relevant variables are given in Table 17. The current short cash wheat position is not listed in the position value Mexican peso (MP) column of the table. Instead, the value of this position is listed as a short position in U.S. dollars, since before purchasing the wheat, the firm's home currency must be converted to dollars, introducing the foreign currency exchange risk. The value of the position in U.S. dollars is then reported in Mexican pesos and summed with the value of the two output positions to obtain the current portfolio value of 30,718,300 Mexican pesos. As in Case II, the portfolio value consists of both cost and revenue items. The positive portfolio value is realized since output values exceed input values.

Monte Carlo simulation was used to estimate the risk minimizing hedge ratios for both KCBT wheat futures and CME Mexican peso futures for the portfolio as a whole, as opposed to calculating the hedge ratios for the wheat and U.S. dollar positions individually. Coincidentally, the risk minimizing hedge ratio for KCBT wheat futures was approximately -0.32, and the hedge ratio for CME Mexican peso futures was approximately 0.32. A positive hedge ratio was found

for CME Mexican peso futures due to the specifications of the contracts, calling for delivery of pesos, in exchange for dollars at maturity. Instead, the processor wants to exchange Mexican pesos for U.S. dollars, requiring a short position in the futures contracts to offset a short cash position, as opposed to the typical long futures position.

Table 17. Current Average Monthly Price as of October 1, 2000

| Financial Variables | Spot Position | Current Price | Units | Position Value MP¹ |
|--------------------------------------|--------------------------|--------------------------|--------------|--|
| Inputs | | | | |
| HRW 11% Protein, FOB U.S. Gulf Wheat | -1,569,750 | 3.40 | \$/bushel | |
| Futures Contracts | | | | |
| KCBT Hard Red Winter Wheat | | 3.04 | \$/bushel | |
| CME Mexican Peso | | 9.67 | MP/\$ | |
| Other | | | | |
| U.S. Dollars | -5,331,655 | 9.36 | MP/USD | -49,909,619 |
| U.S. 91-Day Treasury Bills | | 6.03 | % points | |
| Outputs | | | | |
| Flour Sold for Pesos | 31,022,727 | 2.17 | MP/kg | 67,226,250 |
| Mill Feeds Sold for Pesos | 11,987,182 | 1.12 | MP/kg | 13,401,669 |
| Cash Portfolio Value | | | | 30,718,300 |

¹ MP = Mexican pesos; indicates the positions are value in Mexican pesos.

Since hedge ratios were calculated for the portfolio as a whole, it cannot necessarily be stated that each futures contract was used to hedge a specific price risk variable. Since the presence of currency exchange risk makes the position taken in forward contracts more complex than in the previous two case studies, these positions are also listed when applicable. When forward contracting wheat requirements in this situation, forward contracting the exchange rate of Mexican pesos for U.S. dollars is required, since wheat forward contracts are made in U.S. dollars.

It is apparent from Table 18 that two of the forward contract strategies (2, 3) provide the best risk reduction. Hedging all inputs and outputs (2) results in a VaR of zero, since all prices, and the Mexican peso/U.S. dollar exchange rate. The second lowest VaR is observed when wheat, flour, and the exchange rate are forward contracted (3). This is because wheat and flour contribute an overwhelming majority of the value of this firm's portfolio. When only wheat and U.S. dollar requirements are forward contracted (4), or only the outputs of flour and mill feeds (5) are hedged, the VaR reduction is much less significant than the strategies where both wheat and flour are forward contracted simultaneously (2, 3).

Table 18. Case III: Value at Risk Statistics and Hedging Instrument Positions

| Hedging Strategy | Strategy | Value at Risk Mexican Pesos | Position Taken in Hedging Instruments ¹ | | | | | | | |
|-------------------------------------|----------|--------------------------------|--|-----|---------|-----------|-------------|-------------|-----------|-----|
| | | | Futures | | Options | | Forward | | | |
| | | | KW | CME | KW | CME | GW | FP | MLFP | USD |
| No-Hedge | 1 | 4,761,353 | | | | | | | | |
| Forward Contracts | | | | | | | | | | |
| Wheat, USD, flour & mill feeds | 2 | 0 | | | | 1,569,750 | -31,022,727 | -11,987,182 | 5,331,655 | |
| Wheat, USD & flour | 3 | 1,482,824 | | | | 1,569,750 | -31,022,727 | | 5,331,655 | |
| Wheat & USD (input) | 4 | 4,263,750 | | | | 1,569,750 | | | 5,331,655 | |
| Flour & mill feeds (outputs) | 5 | 4,471,146 | | | | | -31,022,727 | -11,987,182 | | |
| KCBT Wheat | | | | | | | | | | |
| Futures contracts | 6 | 4,247,508 | 100 | | | | | | | |
| Options contracts | 7 | 4,359,921 | | | 100 | | | | | |
| Basis contracts | 8 | 4,168,524 | -100 | | | 1,569,750 | | | 5,331,655 | |
| CME US \$/Mexican peso | | | | | | | | | | |
| Futures contracts | 9 | 4,546,492 | | -32 | | | | | | |
| Options contracts | 10 | 4,615,281 | | | | | | | *32 | |
| KCBT Wheat & CME pesos | | | | | | | | | | |
| Futures contracts | 11 | 3,979,698 | 100 | -32 | | | | | | |
| Options contracts | 12 | 4,226,655 | | | 100 | | | | *32 | |
| 50% futures - 50% options | 13 | 4,090,587 | 50 | -16 | 50 | | | | *16 | |
| Regression - Total Portfolio | | | | | | | | | | |
| KCBT wheat futures | 14 | 4,271,410 | 151 | | | | | | | |
| KCBT wheat options | 15 | 4,235,956 | | | 151 | | | | | |

¹ KW = KCBT hard red winter wheat futures; CME = CME US \$/Mexican pesos futures; GW = HRW 11% protein, FOB U.S. Gulf wheat; FP = Flour sold for Mexican pesos; MLFP = Mill feeds sold for Mexican pesos.

* Denotes put contracts, all other options position are in call options.

Three of the next lowest VaR statistics are observed for strategies utilizing KCBT wheat and CME peso futures (11-13). Both futures contracts have relatively high correlations to the cash position that they primarily offset, and KCBT wheat has a significant positive correlation to flour produced by the mill as well. These three strategies follow the same pattern observed in the previous case studies, where futures (11) return the lowest VaR, options (12) provide the highest VaR, and the half futures half options strategy (13) ranks between the futures and the options scenarios.

A strategy involving the use of basis contracts (8) was also evaluated and was found to rank fifth when compared to the other scenarios considered. Since a basis contract for a purchaser of an input would essentially substitute a short futures position for the short cash position, basis contracts are modeled in this manner. Basis contracts were relatively effective due to the positive correlation between cash wheat, KCBT wheat futures, and flour. As the correlation between input and output increases, the VaR reduction observed with basis contracts will increase as well.

When regression was used to determine the risk minimizing position in KCBT wheat futures, it was found to be half as great as when found through simulation. The most interesting aspect of these two scenarios (14, 15), is that the options strategy results in a lower VaR than when the equivalent number of futures contracts were used. Although the difference between the two VaR statistics is relatively small, it is likely due to the multiple sources of risk encountered when dealing in more than one currency.

CONCLUSIONS

Despite its increasing popularity, the adoption of VaR in the agricultural sector has lagged behind other sectors of the economy (Manfredo and Leuthold, 2001a). Those agricultural firms that do use VaR tend to be the larger, more diversified corporations. The benefits of VaR in the agricultural industry are not limited to large conglomerates; however, and this study provides empirical examples of how mid to large sized commodity end-users can use VaR to quantify price risk exposure. Agricultural processors can benefit from all three primary advantages VaR holds over traditional mean-variance analysis. By reporting price risk in terms of dollars as a single summary statistic, VaR provides a more intuitive measure of risk for decision makers, especially when the distribution of portfolio value changes is non-normal. VaR methodologies also separate the downside potential from the upside potential by focusing on the far left hand tail of a portfolio's distribution of returns. Although parametric VaR assumes normality of portfolio returns, both simulation methodologies allow for the non-linearity of return found for options and option-like instruments. Therefore, VaR simulation techniques allow returns to follow any distribution, and do not distort the risks of portfolios with significant options content.

The purpose of this paper is to demonstrate how VaR can be applied to the portfolio of a hypothetical U.S. bread baking company. Six of the bakery's most prominent commodity input components were considered. These included flour, bakery shortening, and sugar, while mill feed price risk was also included since it is commonly a component of flour pricing agreements. These commodities represent the input price risk components, and make up Case I, the procurement division of a U.S. bread baking company, which serves as the base case. This case

considers a portfolio of costs, and the risk of procurement cost changes is measured. Output price risk is considered in Case II by including white pan bread prices as a price risk variable. This portfolio contains both cost and revenue items, and the risk of payoff changes resulting from input and output price changes is considered. In Case III, a Mexican flour milling company, the only input considered is wheat, but multiple outputs, flour and mill feeds, are included. This results in a portfolio of cost and revenue items as well, measuring the risk of changes in portfolio value. The complicating factor is that foreign currency exchange risk is incorporated, since the input is purchased in U.S. dollars and both outputs are sold for Mexican pesos.

In each case, different hedging instruments were considered for use in various hedging strategies. Forward contracts were available for the precise input or output. A futures contract was also selected to hedge each input and output, as well as options on those futures contracts. Although each futures contract had a positive correlation to the physical asset it was used to hedge, the magnitude of these correlations varied greatly. Though VaR can be utilized by decision makers for numerous management decisions, in the cases analyzed in this study VaR estimates are used to quantify the price risk associated with different hedging strategies.

In Case I, flour and natural gas constitute the bulk of the procurement operations' price risk. Hedging other inputs with anything other than forward contracts actually increases risk, suggesting that hedging activities should be focused on flour and natural gas. Forward contracting only the flour or flour and natural gas requirements can eliminate a substantial amount of the risk exposure.

Considering both input and output price risk in Case II, suggests that output risk dominates input risk for the firm. While hedging all inputs in Case I resulted in a 100% reduction in price risk, the same hedge in Case II reduces risk exposure only 12%. Therefore, attempts to forward contract bread production would result in the largest risk reducing effects. It is also apparent that when output contracting is not available, hedging in only wheat futures reduces risk more than any of the other complex strategies utilizing multiple contracts.

In Case III, forward contracting only the input, or the outputs, is much less effective than strategies involving the combination of inputs and outputs. Strategies using KCBT wheat and CME pesos futures or options reduce price risk exposure more than utilizing either individually. Due to the correlations between wheat, wheat futures, and flour, basis contracts removing the basis component from the wheat price risk exposure provides significant risk reduction.

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