The Interaction of Working and Speculative Commodity Stocks

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Abstract

This paper models the interaction of working (also called pipeline) and speculative commodity stocks. We model working inventories (i.e., raw material inventories carried by processors) based on Ramey's (1989) model of inventories as factors of production, which allows us to represent storage under inter-temporal price backwardation, observed in commodity markets. We incorporate both speculative and working stocks in a simple model to analyze the interaction and to simulate the relationship between inter-temporal commodity price spreads and stocks. Our model replicates common price patters found in commodity markets.

The Interaction of Working and Speculative Commodity Stocks

I. Introduction

While the economic role of competitive (also known in the literature as speculative) storage has been extensively studied (Wright and Williams, 1982, and Williams and Wright, 1991), the role of processors' "working stocks" has received only marginal attention in the theoretical literature. However, empirical work has emphasized the effect of working stocks on the backwardation portion of the "supply of storage curve" (see Working, 1949; Brennan, 1958; Telser, 1958; Miranda and Glauber, 1993; Miranda and Rui, 1996). Two notable exceptions in the literature are the book by Weymar (1968), on the world cocoa market, and Lowry (1988), who theoretically modeled the interaction between stocks carried by merchants and speculators.

Recent econometric models of commodity prices (see Deaton and Laroque, 1992, 1995, 1996 and Miranda and Rui, 1996) have shown that the interaction between working and speculative stocks is important in terms of explaining the actual distribution of commodity prices. Therefore, an understanding of the interaction is important for the assessment of commodity policies, such as the effect of government intervention to stabilize commodity prices or to foster private storage, or measuring the effect of external shocks on commodity markets.

A problem associated with the modeling of working stocks is the lack of a suitable theory explaining the presence of storage under backwardation (which is a phenomenon frequently observed in commodity markets, as shown in table 1), without appealing to the controversial "convenience yield" concept (see Kaldor, 1939, Working, 1949 and Brennan, Williams and Wright, 1997). Weymar and Lowry use the "convenience yield" explanation without clarifying its microfoundations. Weymar models processors and speculators similarly. Lowry, following Brennan (1958), models merchants (instead of processors) as the ones who carry stocks under backwardation. However, the distinction between merchants and speculators is not clear in the Lowry model since they both buy and sell raw material and they are both risk neutral.

Table 1: Price Spread and Off Farms U.S. Stocks for Selected Commodities 1968-97 [1]

Year	Wheat		Corn		Soybeans	
	Spread [2] ¢/bushel	Stocks Mill. bushels	Spread [2] ¢/bushel	Stocks Mill. bushels	Spread [3] ¢/bushel	Stocks Mill. bushels
1968	6.5	649.0	6.7	809.9	-5.3	301.2
1969	3.1	740.7	-2.1	818.2	-26.9	452.3
1970	-5.5	679.3	-5.6	767.8	-6.1	525.0
1971	-5.3	685.0	-2.9	670.2	-9.9	369.1
1972	-8.4	611.3	3.3	898.1	-26.8	333.7
1973	-11.6	366.4	-6.3	955.0	-140.9	358.4
1974	-21.0	388.0	-20.3	849.8	-37.5	405.9
1975	-9.3	594.2	-29.3	704.9	-12.8	323.3
1976	12.5	878.5	-4.3	913.0	22.5	456.3
1977	14.8	887.8	12.0	1,159.3	-192.5	390.2
1978	4.8	596.2	0.8	1,360.2	-49.8	455.4
1979	-8.8	655.5	12.8	1,322.5	-59.3	467.6
1980	26.3	790.1	29.0	1,416.3	60.0	580.3
1981	19.5	809.0	16.8	1,346.5	54.8	493.9
1982	20.3	991.2	19.0	1,506.4	14.8	460.1
1983	16.0	986.9	-12.8	1,956.3	43.8	504.5
1984	-16.5	953.7	-55.5	1,317.6	-54.5	415.8
1985	-17.5	1,330.6	-14.3	1,131.7	5.5	408.6
1986	-51.8	1,456.4	-29.3	1,965.1	-15.8	n.a.
1987	-10.5	1,175.5	14.3	3,224.2	-5.5	749.9
1988	14.5	764.7	14.8	3,214.6	18.8	594.6
1989	-8.5	567.1	-3.3	2,182.9	-7.5	475.2
1990	-11.3	864.7	-5.3	1,901.9	24.5	519.7
1991	16.5	614.4	7.3	1,724.5	32.0	634.6
1992	-10.3	670.1	-1.8	1,950.8	24.5	672.3
1993	-35.0	664.8	15.0	2,048.6	14.8	638.7
1994	-7.3	633.8	-15.3	1,785.5	-35.5	595.9
1995	0.5	602.9	11.5	2,089.7	24.5	734.9
1996	-31.0	501.1	-93.0	1,799.3	-10.5	678.4
1997	-0.3		-21.3	1,624.1	-180.5	541.9

Source: Chicago Board of Trade; New York Sugar, Coffee and Cocoa Exchange, and USDA, Agri cultural Statistics, various yearbooks.

Notes:

n.a. indicates not available.

[1] Price spreads and commodity stocks correspond to the first working day of April. Since 1986 stocks correspond to March 1st instead of April 1st due to a change in USDA methodology. Off farms stocks correspond to stocks at mills, elevators, warehouses, terminals, processors and those owned by the Commodity Credit Corporation which are in bins and other storage faci lities under CCC control.

[2] Measured as the September futures price minus the preceding May futures price on April 1st.

[3] Measured as the November futures price minus the preceding May futures price on April 1st.

We model processors' inventories using a model originally developed for manufacturing inventories. Specifically, we use Ramey's model (1989), which models inventories as factors of production. This approach allows us to develop a microeconomic framework to understand the demand for inventories by processing firms (working stocks). We show that the "convenience yield" concept is unnecessary to explain the presence of working stocks under backwardation, since their existence is explained by the processors' willingness to pay for a factor of production (inventories). Furthermore, we show that under certain conditions the "supply of storage" equation used in empirical work by Brennan (1958), and Miranda and Rui (1996) represent only those inventories carried by processing firms. Therefore, these models may not account for the effect of speculative storage when the price spread (i.e., the futures price minus the spot price) is highly positive.

The results of simulating our model support Lowry's point of view that the quantity of working stocks changes with inter-temporal price spreads, but does not support his finding that when price spreads are high, all stocks in the market are speculative. When inter-temporal price spreads are high, speculative stocks increase, driving the extraordinary profits in the storage business to zero. This reduces the cost of carrying working stocks for processing firms, causing working stocks to increase up to the storage capacity of processing firms or to reach a maximum in accordance with conditions in the processed good market. This relationship can only be observed in a model that allows both speculators and processors to interact in the market. In addition, our results allow us to simulate the Working curve (i.e., the empirical curve that relates commodity price spreads with stocks, as drawn by Holbrook Working in 1933 which was the basis for Working's supply of storage theory, see Working, 1949).

II. Modeling the Interaction of Working and Speculative Stocks

The purpose of this section is threefold: first, to derive a demand for working inventories carried by processing firms, based on Ramey's approach (1989) of treating inventories as factors of production.[1] Second, to show that under certain conditions the "supply of storage equation" used in the empirical literature may be interpreted as the demand for inventories carried by processors. Third, to incorporate both components of the demand for inventories (i.e., from processors and speculators) into a commodity market model.

II.1 Derivation of Processor's Demand for Inventories

Let us assume that the output of a processed good Q_t of a competitive processing industry is represented by a quasi-fixed proportions production function (i.e., $Q_t = \min\{\frac{I_t}{\lambda}, f(K_t)\}$)[2] where λ is a parameter of the production function (i.e., the

[1] As pointed out by Ramey, 1989, the economics literature on inventories is focused on inventories of final goods. Besides Ramey's model, the other alternative model for raw material inventories available in the literature is Williams and Wright, 1991, chapter 10. We decided to use Ramey's model because it allows us to build a simpler model for the commodity market, and it captures the empirical stylized facts (see Abramovitz, 1950, chapters 9 and 10, and for more microeconomic evidence, see United States Senate, 1954). The use of the Williams and Wright model would have required us to build two interconnected dynamic models, one for each type of stockholder, and this would be far more difficult to solve numerically.

[2] The derivation does not require a quasi-fixed proportion production function. We have chosen it because of its algebraic tractability and because it is bounded when the price of the factor of production is equal to zero. Also, with the aim of clarifying this approach,

turnover parameter), K_t is a composite index of the "other" factors of production, I_t is the raw material inventories carried by the industry, and f (.) is an increasing function that relates other factors of production to output. [3]

It is important to note that the production function used represents the value added portion of the gross output of the industry (see Arrow, 1974, Jorgenson, 1990). Thus, during time period t, it is the utilization of raw materials by the processing industry and not inventories that are transformed to create the new product. In the approach proposed by Ramey (1989), inventories - similar to capital and labor - contribute to the creation of value added. The corresponding production function for the gross output (i.e., H_t which includes value added and raw materials), would be, following Arrow (1974) and under the separability assumption, equal to $H_t = H[V(I_t, K_t), RM_t]$, where $V(I_t,K_t)$ is the value added function (i.e., assumed in our case equal to $\min\{\frac{I_t}{\lambda}, f(K_t)\}$) and RM_t is the utilization of raw materials at time period t (assuming a technical coefficient equal to 1).

The key assumption in this model is that raw material inventories carried by processors are considered factors of production, because they provide a service to the production of the processed good. Abramovitz (1950) pointed out that the incentive for keeping stocks reflects in part goods passing through production stages antecedent to actual fabrication, and it also reflects in part the need to provide a reserve stock to cover raw material requirements for a reasonable number of weeks. Another motive for keeping stocks, mentioned by Abramovitz, is for precautionary reasons, since the commodity

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we derive the processor's demand for inventories using the primal approach instead of following Ramey (1989) who uses the dual approach.

[3] See Abramovitz, 1950, p. 238, for the analysis of the relation of output and raw material inventories for processing firms.

reserve safeguards production against interruptions in the flow of materials due to production difficulties encountered by suppliers, strikes, transportation delays, etc. These motives have also been pointed out by Williams (1987, p. 1002) and Ramey (1989, p. 340-41). On the other hand, Timms (1962) in his study of business production functions, explains the rationale for carrying inventories by manufacturing firms:

> "Inventories serve several purposes aside from the fact that they must exist wherever the time dimension is involved. Perhaps the most important role of inventories is decoupling or disengaging successive stages of production. For instance, conversion processes are disengaged from purchasing operations by the existence of an inventory of raw materials. […] The use of [raw material] inventories to disengage successive stages provides freedom to operate each stage most efficiently. Conversely, the operation of a particular stage is not compromised by the demands of preceding and succeding stages. Thus each stage may be scheduled most efficiently and costs thereby lowered." (Timms, 1962, p. 404-5).

Under the above assumptions, risk neutral processors maximize expected profits ($E[\pi]$) at period t :

$$(1) \qquad \underset{I_t, K_t}{\text{Max}} \; E[\pi] = E[P_t \min(\frac{I_t}{\lambda}, f(K_t)) - m_t I_t - w_t K_t]$$

Where P_t is the price of the processed product minus the price of the material (i.e., $P_t^p - p_{c,t}$, where P_t^p is the price of the processed good at time period t and $p_{c,t}$ is the commodity price at period t), m_t is the rental price of raw materials inventories (defined slightly different than in Ramey, 1989, as $m_t = (1+r)(p_{c,t} + ko) - p_{c,t+1}$ under the assumption that storage costs are paid in advance).[4] $p_{c,t+1}$ is raw material price of the beginning at the next period, r is the interest rate, ko is the storage cost (both the interest

[4] For an analysis of different alternatives for pricing inventories, see Giganti, 1990.

rate and the storage cost are assumed fixed per unit),[5] and w_t is the rental price of the composite factors of production, assumed exogenous to the industry. Cost minimization implies that $Q_t = \frac{I_t}{\lambda}$ and $Q_t = f(K_t)$. Then, the expected cost function is given by:

$$(2) \qquad E[C(m_t, w_t, Q_t)] = E[m_t]\lambda Q_t + w_t f^{-1}(Q_t)$$

Let us assume (for the purposes of developing the supply of storage function below) that $f^{-1}(Q_t) = Q_t(\ln(Q_t) - 1)$. Thus, substituting the expected cost function (2) into the profit function (1) and maximizing with respect to output, we obtain expressions for the output (Q_t) and processors' raw material inventories (I_t).

$$(3) \qquad \begin{aligned} Q_t &= \exp\left\{\frac{P_t - \lambda\{(1+r)(p_{ct} + ko) - E[p_{ct+1}]\}}{w_t}\right\} \\ I_t &= \lambda \exp\left\{\frac{P_t - \lambda\{(1+r)(p_{ct} + ko) - E[p_{ct+1}]\}}{w_t}\right\} \end{aligned}$$

Equation (3) show that an increase in the value added, increases the level of activity and therefore the level of inventories carried by processing firms. Also, it is useful to analyze the numerator in the inventories equation in (3). If we assume that $E\lfloor p_{c,t+1} \rfloor$ represents the futures price of the commodity at period t for delivery in period t+1, then under backwardation $E\lfloor p_{c,t+1} \rfloor$ would be less than $p_{c,t}$ and this means that processing firms face an expected depreciation in the value of their commodity stocks. Even under this situation, firms are willing to carry some stocks since the demand for inventories depends on the expression $P_t - \lambda\{(1+r)(p_{ct} + ko) - E[p_{ct+1}]\}$. In other words, the cost of carrying inventories is paid from the processed product price (net of the cost

[5] The fixed marginal storage cost seems to be a good approximation of what is observed

of the raw materials used), as is the case with wages or the rental cost of the capital. This result captures Holbrook Working's idea that:

> "The owners of large storage facilities are mostly engaged either in merchandising or in processing, and maintain storage facilities largely as a necessary adjunct to their merchandising or processing business. And not only are the facilities an adjunct; the exercise of the storing function itself is a necessary adjunct to the merchandising or processing business. Consequently, the direct costs of storing over some specified period as well as the indirect costs may be charged against the associated business which remains profitable, an so also may what appear as direct losses on the storage operation itself ". (Working, 1949, p. 1260).

II.2 Supply of Storage Equation

The origins of the supply of storage equation can be traced to Working (1949), Brennan (1958) and Telser (1958). Working sketched the function without choosing any specific functional form but with the empirical support of his 1933 paper (see Working, 1933 figure A.1, and Working's supply of storage in figure A.2 in the appendix). Brennan (1958) and Telser (1958) derived a supply of storage function, incorporating the concept of "convenience yield" from Kaldor (1939) and Working (1949), to explain storage under backwardation.

The modern commodity storage model (Miranda and Glauber, 1993; Miranda and Rui, 1996) adopts the supply of storage function from Brennan and introduces it into a market model, disregarding Brennan's demand for storage. This modern approach is clearly expressed in Helmberger and Chavas (1996):

> "It is sometimes asserted in agricultural economics textbooks that the quantity and the price of stocks is determined by the supply and demand for storage, where the price is defined as the difference between the expected price (or futures price) and the current price. This assertion rests on a flawed theory that does not provide an adequate explanation of expected prices. As we have seen, the level of stocks is determined by competition among buyers of a commodity for consumption

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in the storage business, on this topic see Paul, 1970.

(exports, processing, seeds, etc.) and for storage, with the supply of commodity being predetermined" (Helmberger and Chavas, 1996, p. 173).

A controversial component of this modern approach is the assumption that "convenience yield" exists, a concept criticized for being ad-hoc and with their microfundations not clarified (see Deaton and Laroque, 1995; and Brennan, Williams and Wright, 1997). The introduction of inventories into the processors' production function (say as working capital), allows us to model inventories held by processors without the requirement of a "convenience yield" function. It also allows us to show that the stocks represented by the supply of storage equation in the modern literature, may only correspond to the stocks carried by processing firms. Let us consider equation (4), which is the supply of storage equation used by Miranda and Rui, 1996.[6]

$$(4) \qquad \frac{E\left[p_{c,t+1}\right]}{(1+r)} - p_{c,t} = \theta_0 + \theta_1 \ln(I_t)$$

Equation (4) is a reduced form equation for particular parameters of the processors' demand for inventories (I_t) shown above, and for the case of a quasi fixed proportions production function. To show this, let us write the processors' demand for inventories (3) as in (5), and let us define $P_t^* = \frac{P_t}{\lambda}$, where P_t is the price of the processed good minus the price of the raw material used:

$$(5) \qquad I_t = \lambda \exp\left[\frac{\lambda}{w_t} \left[P_t^* - \left\{ (1+r)(p_{c,t} + ko) - E\left[p_{c,t+1}\right] \right\} \right] \right]$$

Let us simplify expression (5) by introducing parameters $ß_0$ and $ß_1$.

[6] Miranda and Glauber (1993) use a similar equation but they express the price spread in relative terms.

$$
(6) \qquad
\begin{aligned}
&\beta_0 = \lambda \\
&\beta_1 = \frac{\lambda}{w_t} \\
&I_t = \beta_0 \exp\left\{\beta_1\left[P_t^* - \left\{(1+r)(p_{c,t} + ko) - E[p_{c,t+1}]\right\}\right]\right\}
\end{aligned}
$$

Taking the natural logarithms of both sides and factoring terms we obtain (7)

$$
(7) \qquad \frac{E[p_{c,t+1}]}{(1+r)} - p_{c,t} = ko - \frac{P_t^*}{(1+r)} - \frac{1}{(1+r)\beta_1}\ln(\beta_0) + \frac{1}{(1+r)\beta_1}\ln(I_t)
$$

Re-writing (7) in terms of the parameters θ_0 and θ_1 we obtain the storage cost function (i.e., expression (4)) that has been used in the previous literature. The values for the parameters θ_0 and θ_1 are given in (8):

$$
(8) \qquad
\begin{aligned}
&\theta_0 \equiv ko - \frac{P_t^*}{(1+r)} - \frac{1}{(1+r)\beta_1}\ln(\beta_0) \equiv ko - \theta_1\left(\frac{P_t}{w_t} + \ln(\beta_0)\right) \\
&\theta_1 \equiv \frac{1}{(1+r)\beta_1}
\end{aligned}
$$

Therefore, the commodity model presented by Miranda and Glauber, 1993, and Miranda and Rui, 1996, only incorporates processors inventories, excluding speculative stockholding. Furthermore, what they call the arbitrage equation may be understood as a demand for inventories, unlike the arbitrage condition in Williams and Wright (1982).

In addition, the requirements for treating θ_0 and θ_1 as parameters are: (1) the value added in the processed good relative to the price of the other factors of production (i.e., $\frac{P_t}{w_t}$) must be constant, (2) the price of other factors of production (i.e., w_t) must be constant, and (3) the period considered in the model has to be relatively short to preclude technological change.

The derivation of the parameters of the supply of storage function also allows us to predict changes in the slope and intercept of the supply of storage function. Changes in both, slope and intercept, of the supply of storage were noted by Working (1953). Working's graph is reproduced in figure A.3 in the appendix. The data used in A.3 are the same data used in his original graph (Working, 1933, reproduced in A.1). It should be noted that the curves correspond to a period with little government intervention, so changes in the supply of storage cannot be explained by stocks held by the government's Commodity Credit Corporation.

II.3 The Model

The next step is to add speculators to the model. They are incorporated through an arbitrage condition (9), as in, Williams and Wright, 1991.

$$
\begin{aligned}
&\text{If } S_t \geq 0 \quad \frac{1}{1+r} E(p_{ct+1}) - P_c(A_t - S_t - I_t) = ko \\
(9)\qquad & \\
&\text{If } S_t = 0 \quad \frac{1}{1+r} E(p_{ct+1}) - P_c(A_t - S_t - I_t) < ko
\end{aligned}
$$

In equation (9) A_t corresponds to the commodity availability (i.e., current production plus carryover from the previous period), S_t are speculators' carryover, I_t is processors' carryover and $P_c(\ )$ is the inverse consumption function. The arbitrage condition (9) states that speculators will carry inventories only if they expect asset appreciation. Furthermore, under extraordinary profits, free entry to the storage business will eliminate those profits, resulting in speculators covering storage costs only. To complete the model, we include the demand for the processed good, which we assume is non-stochastic, and we assume expectations are formed rationally as in Muth (1961). Thus, (10) presents our entire commodity market model.

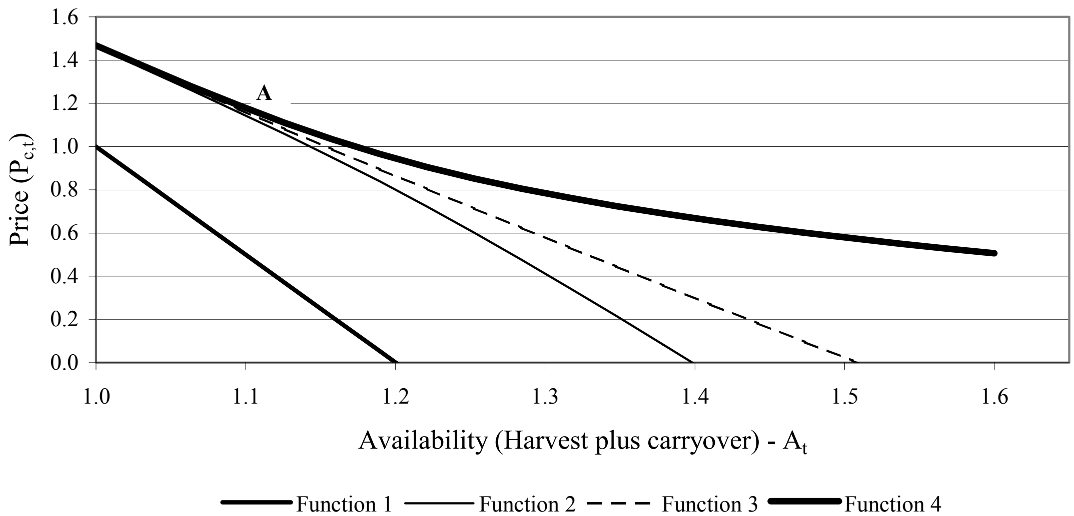
$$
\begin{aligned}
&P_t = P(Q_t) \\
&Q_t = \exp\left\{\beta_1\left[P_t^* - \left\{(1+r)(p_{c,t} + ko) - E\left[p_{c,t+1}\right]\right\}\right]\right\} \\
(10)\qquad &I_t = \beta_0 Q_t \\
&\text{If } S_t \geq 0 \quad \frac{1}{1+r}E(p_{c,t+1}) - P_c(A_t - S_t - I_t) = ko \\
&\text{If } S_t = 0 \quad \frac{1}{1+r}E(p_{c,t+1}) - P_c(A_t - S_t - I_t) < ko
\end{aligned}
$$

III. Model results

A model like (10) has to be solved numerically. To do so, we have used the polynomial approximation technique (see Williams and Wright, 1991; and Judd, 1998). The method consists of replacing $E\left[p_{c,t+1}\right]$ by a low order polynomial, which is a function of the total stocks carried into the market, and which is a state variable that captures the dynamics of the system. Since our interest is the study of the interaction of working and speculative stocks, we only consider the case of a price inelastic stochastic supply, which is driven by multiplicative disturbances (the elastic supply case can easily be extended following Williams and Wright, 1991). The supply shocks are assumed to be approximately normal with mean 1 and standard deviation 0.01.

The solution to the model (i.e., the policy function, or the aggregate demand function) is presented in figure 1. We have assumed in the simulation that the inverse consumption function (i.e., $P_c(\ )$) is linear. It should be noted that the linearity of this function does not imply linearity of the aggregate market demand. In figure 1, we have represented four different demand functions, the first considers consumption only (i.e., without carryover); the second considers consumption plus processors' stocks,[7] the third corresponds to the demand function when there are no speculators in the market, and the fourth considers consumption plus total stocks (i.e. the model's solution).

Figure 1: Market Demand Functions



Notes:
Function 1 = Price without carryover
Function 2 = Price considering only processors' stocks (but from a solution including speculators).
Function 3 = Price when only processors carry stocks.
Function 4 = Price when processors and speculators carry stocks.

When the expected profits from carrying stocks are high, because the supply of the commodity is high (beyond the quantity implied by point A in figure 1), then speculators enter the storage business, driving extraordinary profits to zero. In this case the difference between the discounted expected price and the current price is given by the cost of storing the commodity, exactly satisfying the arbitrage condition. On the other hand, it should be noted that the exact satisfaction of the arbitrage condition implies that the rental price of the inventories is zero. Therefore, for the case of our specific

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[7] Even if we are computing the price function with processors' stocks only, those stocks were computed from a model that also includes speculators.

production function, the amount stored is given by the equilibrium in the market of the processed good, and the output and inventories would be determined by the solution of the system (11).

$$
(11) \qquad \begin{aligned} P_t^P &= P\left(\frac{I_t}{\beta_0}\right) \\ I_t &= \beta_0 \exp\left\{ \beta_1 \left[\frac{P_t^P - p_{c,t}}{\beta_0} \right] \right\} \end{aligned}
$$

The previous result contrasts with Lowry's (1988) results, where under extraordinary profits all of the inventories carried are speculative stocks (Lowry, 1988, p. 313), a feature that seems counter-intutive. As shown here, processors have an incentive to increase their inventories as well. In addition, it seems implausible that they would sell their inventories, stop production, and simply start speculating with the raw material. This effect can only be captured if we consider inventories carried by processors for their own business separately from inventories carried for speculative purposes. This is not possible with a supply of storage model, since the same optimization function is used by both agents.

A central feature of the model is related to the Working curve (i.e., the empirical curve drawn by Holbrook Working in 1933, that relates inter-temporal price spreads to inventories). Figure 2a represents the Working curve based on the original information (see Working, 1933; see also figure A.1 in the appendix that reproduces the original Working curve). In figure 2a, we have excluded farm stocks from total stocks, considering only what Working called "commercial stocks." This was done in order to present the curve as representing a more homogeneous category of stocks. However, the form of the curve is not modified with the exclusion of farm stocks.

We have over-imposed a cubic regression line, instead of Working's original functional form (probably quadratic or logarithmic, unfortunately not reported by him) to show that the data does not fit a curve that has an increasing slope for the portion where the price spread is positive (as implied by equation (4)). It would probably be better to consider a flat or a decreasing line as a more accurate representation (as in Working (1953), reproduced in graph A.3 in the appendix). This pattern has also been noted in the commodity literature (for wheat, see Gray and Peck (1981) reproduced in figure A.4 in the appendix; where they use interest rate adjusted spreads instead of price spreads used by Working (1933, 1953); and for soybeans see Gardner and Lopez (1996) whose graph is reproduced in figure A.5 in the appendix).[8] This type of pattern cannot be explained with only processors storage, it requires the presence of speculators to drive the extraordinary profits in the storage business to the point where the difference between the interest rate-adjusted expected price and the current price is equal to the marginal storage cost (i.e. ko), which seem to be stable (see Paul, 1970).

[8] The figure presented by Gardner and Lopez (1996), see figure A.5 in the appendix, is not strictly comparable to the previous figures since they adjust the futures price by the interest rate, which moves the price spread downwards. The effect of considering stocks as a proportion of the available supply makes the interpretation more difficult, but since their graph resembles the pattern observed for wheat, it seems consistent with our results.

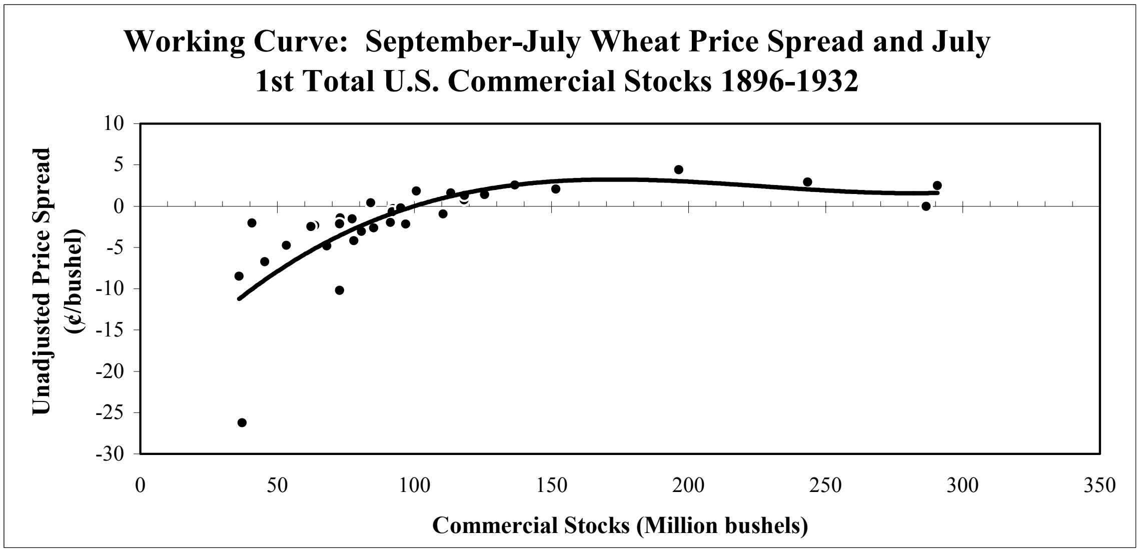

Figure 2.a

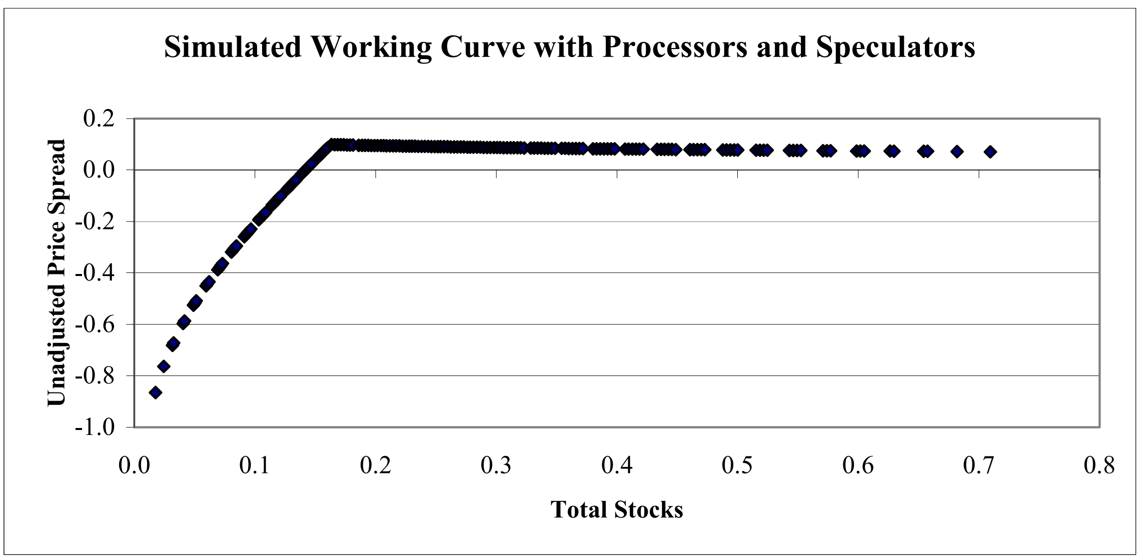

Figure 2.b

Note : Figure 2.b was computed using the model presented in the appendix.
Figure 2.a is based on Table VI, Working, 1933. The regression line was generated by fitting a cubic polynomial.

Furthermore, it should be noted that there exists the possibility of working stocks being so large compared to the supply, that they eliminate the possibility of meaningful speculative storage. Therefore, the specific form of the Working curve depends on the characteristics of the market studied. Finally, figure 2b has been constructed based on model (10), solving the model for the assumed parameters and computing the equilibrium spot price, the next period is expected spot price and stocks, and plotting the unadjusted spread (i.e., the expected price not adjusted by the interest rate) against equilibrium stocks. The curve seems to fit the main characteristics of the Working curve presented in figure 2a and is very similar to figure A.3 in the appendix.

IV. Final remarks

This paper has two main objectives. The first was to derive an equation for processors' storage. Our model, based on Ramey (1989) allows us to explain stocks held under backwardation without requiring an ad-hoc assumption (i.e., convenience yield).

The second objective was to analyze the interaction between working stocks and speculative stocks. For processors, the entrance of speculators into the market implies that the rental price of carrying their inventories is zero, so their demand for inventories depends exclusively on the conditions in the processed good market, or on their own storage capacity. Therefore, higher inter-temporal price spreads are only a temporary situation, and processors do not cease carrying inventories.

Using our model, we have simulated the Working curve. The results seem to fit the patterns observed in commodity markets. Of course, measurement of actual working and speculative stocks would require econometric estimation, which is beyond the scope of the paper. However, recent advances in maximum likelihood methods available in the literature (i.e., see Deaton and Laroque, 1995 and Miranda and Rui, 1996) are useful starting points for estimating a model that incorporates both types of stockholders.

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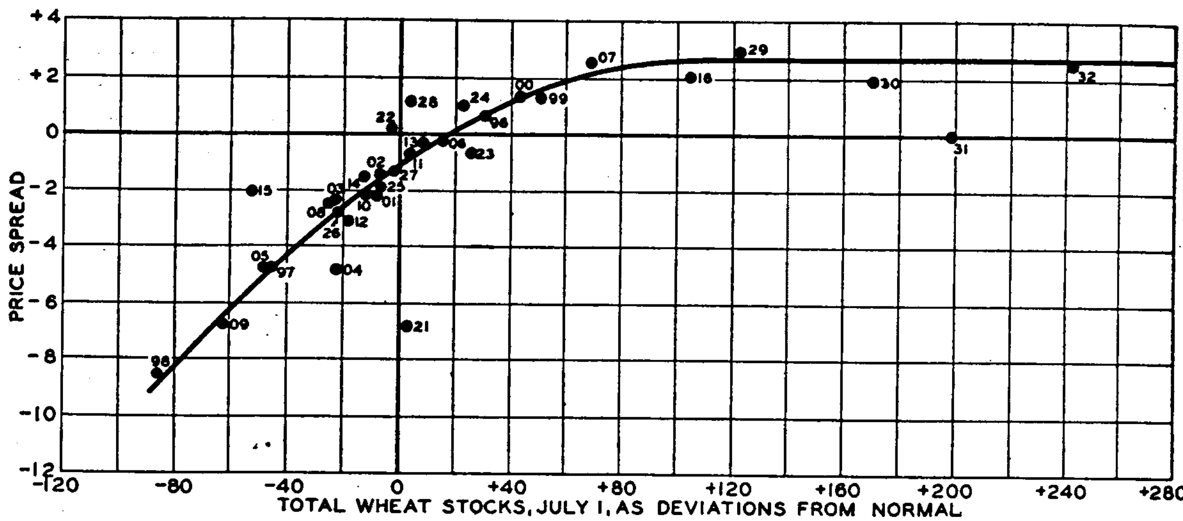
Appendix

Figure A.1
The Supply of Storage (1933)

CHART 7.—RELATION BETWEEN CHICAGO JULY–SEPTEMBER SPREAD IN JUNE AND TOTAL UNITED STATES WHEAT STOCKS, JULY 1*

(Cents per bushel; million bushels)



* Spreads and stocks from Appendix Table VI, but with spreads (column B) for 1921–30 reduced one-third to adjust for the markedly higher price and cost level in this period; stocks expressed as deviations from the "normals" represented by the trend line in Chart 6. The position of each dot reflects both the price spread and the total wheat stocks in one year, the dots being identified by the last two digits of the number of the year.

As here shown, and with the adjustments described above, the statistics of July–September price spread in June and total United States wheat stocks present a relationship only incompletely discernible in the separate curves of Chart 6. With the exception of eight years, one of which (1917) is so exceptional that the point representing it would lie far below the bottom of the chart, the relationship is extraordinarily close. The explanation of the unusual discrepancy in each of the eight abnormal years is given in Section III below. Usually the discrepancy results from some technical market situation, including corners and "squeezes."

Source: reproduced from Working (1933)

Figure A.2
The Supply of Storage (1949)

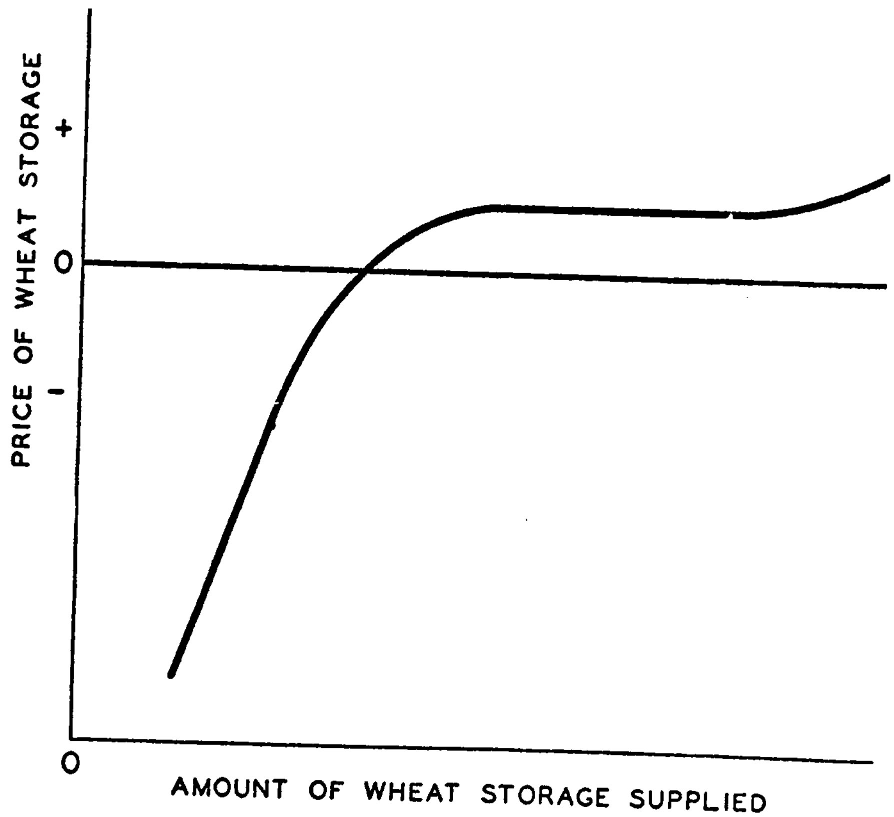
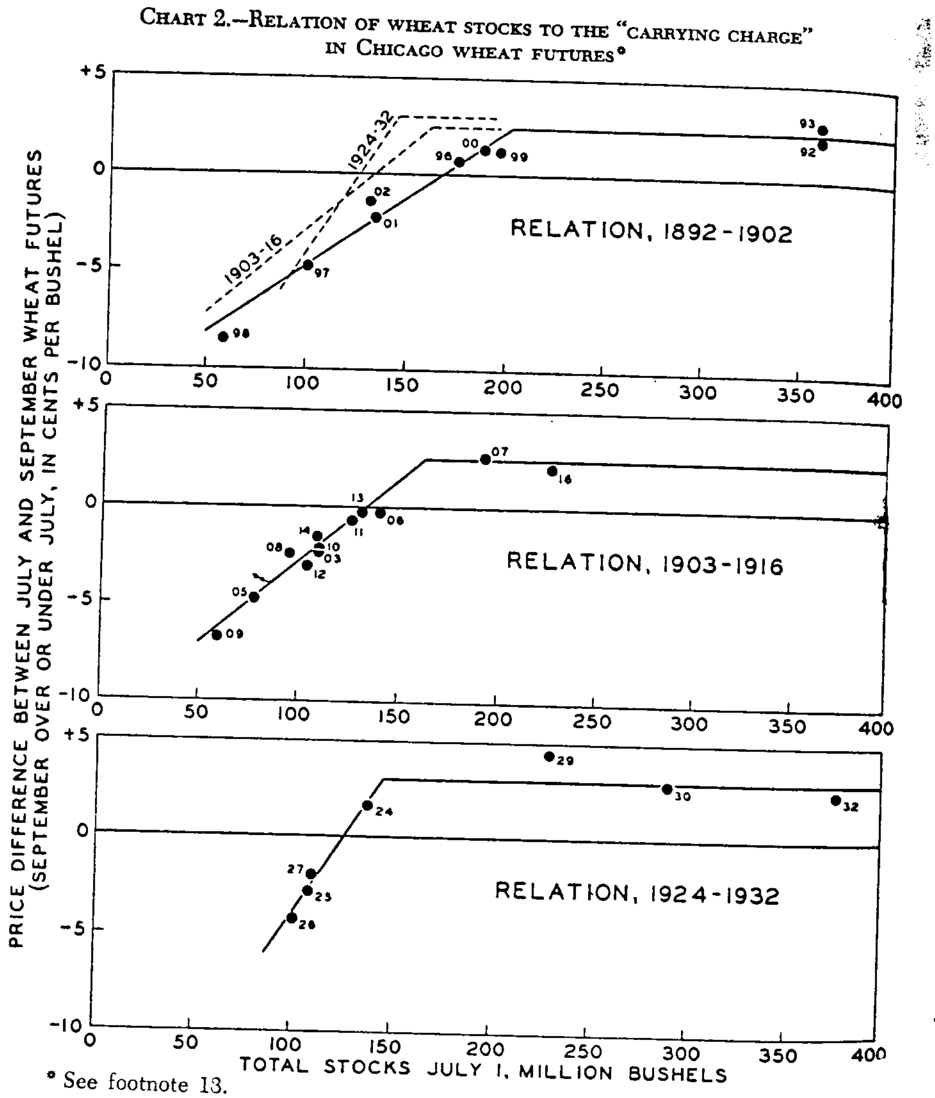


FIGURE 2. COMPLETE STORAGE SUPPLY CURVE

Source: reproduced from Working (1949).

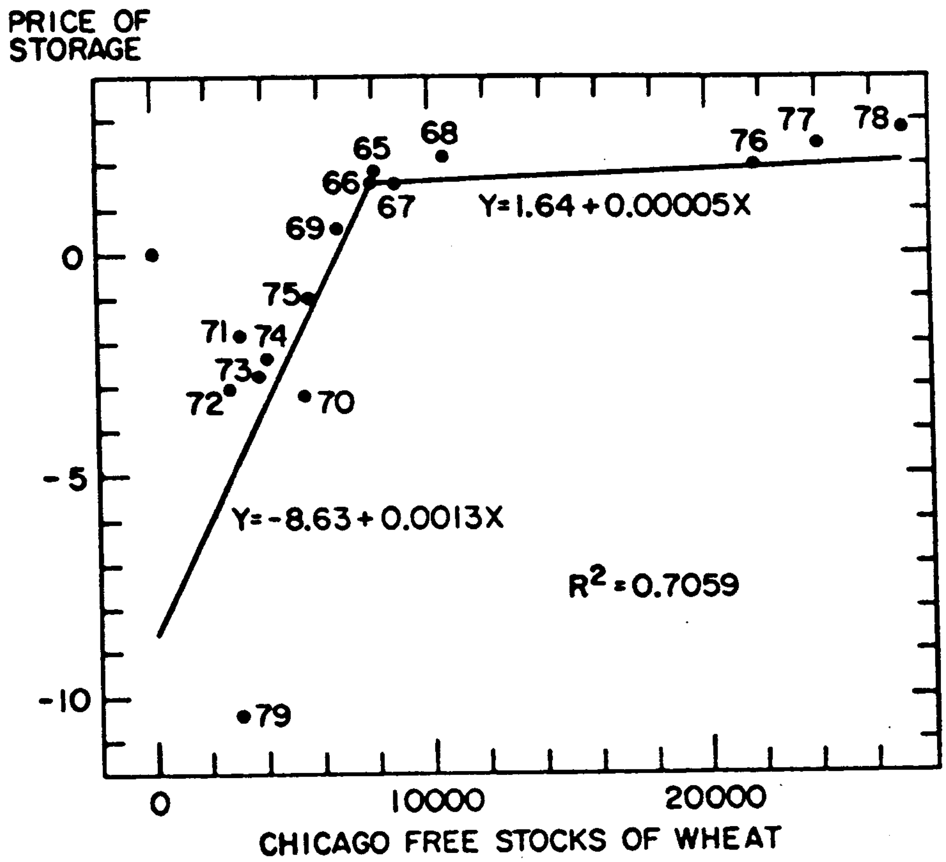
Figure A.3
The Supply of Storage (1953)



Source: reproduced from: Working (1953).

Figure A.4
The Supply of Storage (1981)

Chart 8. Price of storage, May-March, and Chicago stocks on March 15, 1965-79*



*Based on data from *Statistical Annuals*, Chicago Board of Trade, various years. Price of storage measured as the interest-rate adjusted difference between the May and March futures, as a percent of the March price. Chicago free stocks of wheat, including deliverable supplies in Toledo for the years 1974-79, are measured (in 1,000 bushels) on the second Friday in March. R^2 is the overall fit of the storage curve shown, constrained to be quasi-continuous.

Source: reproduced from Gray and Peck (1981)

Figure A.5
The Supply of Storage (1996)

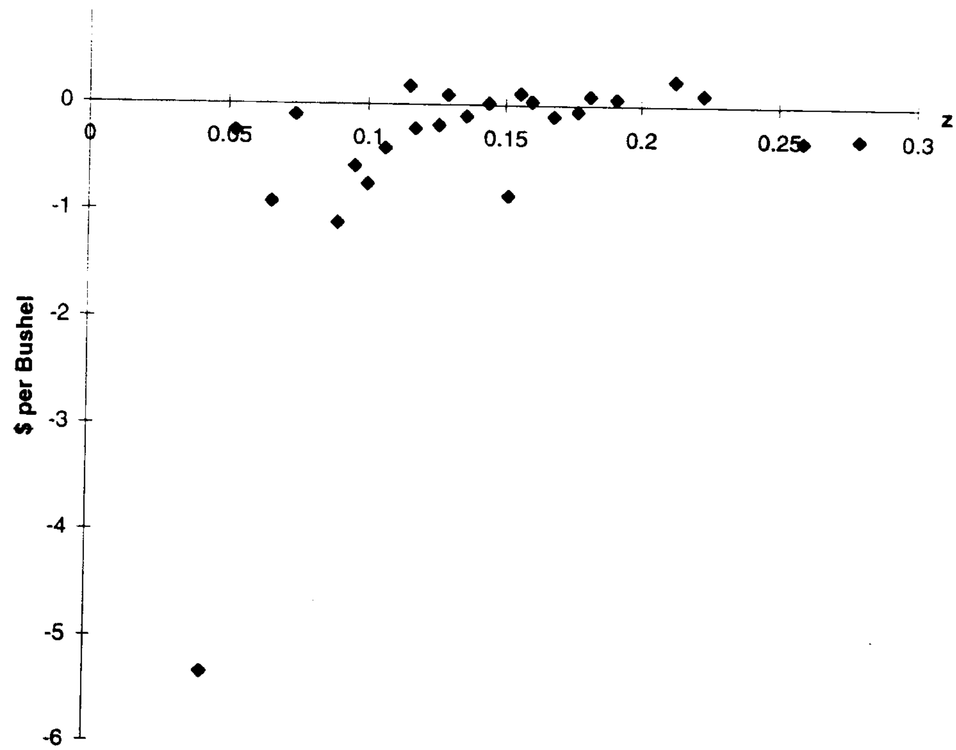


Figure 4. Adjusted soybean futures spreads

Source: reproduced from Gardner and Lopez (1996).

Note: The horizontal axis (z) represents the ratio of stocks to available supply. The vertical axis represents the adjusted price spread (i.e. futures price adjusted by the interest rate, minus the current price).

Simulation Model

The specific model used for simulation is:

$$
\begin{aligned}
&P_{p,t} = 2 - 0.5\,Q_t \\
&Q_t = \exp\left\{0.31\left[P_t^* - \left\{(1+0.05)(p_{c,t} + 0.05) - E[p_{c,t+1}]\right\}\right]\right\} \\
&I_t = 0.09\,Q_t \\
&\begin{cases} \dfrac{1}{1+r}E(p_{c,t+1}) - 6 + 5(A_t - S_t - I_t) = 0.05, \text{ if } S_t \geq 0 \\ \dfrac{1}{1+r}E(p_{c,t+1}) - 6 + 5(A_t - S_t - I_t) < 0.05, \text{ if } S_t = 0 \end{cases} \\
&A_t = h_t + S_{t-1} + I_{t-1}
\end{aligned}
$$

The shocks were approximated by 9 points using a Gaussian quadrature.