Grain emergency reserve cooperation – A theoretical analysis of benefits from a common emergency reserve

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Abstract

This paper presents a two country model with private stockholders and producers featuring rational expectations which is used to evaluate emergency reserve, private storage and trade related policies to stabilize grain prices. Contrary to existing works, this paper looks at extreme events besides price volatility, both representing political concerns. Findings illustrate the benefits from trade and that private storage, even if subsidized, hardly manages to avoid extreme price spikes although it is very efficient in reducing price volatility. In contrast, a (common) public emergency reserve allows compensating large supply shortages at a reasonable level of fiscal costs while leaving the lower quantiles of the price distribution largely unaffected. A private storage subsidy significantly impacts trade whereas a reserve hardly does. Policy makers looking for stabilization mechanisms may consider either option or a combination thereof. Free trade is beneficial if stocking policies match while otherwise a free-rider problem is created.
1 Introduction

High and volatile food prices are a major concern for governments in developing countries as they have serious impacts on the poor. Therefore, many developing countries actively use trade and storage policies to stabilize local prices and keep them at a low level. When food prices have spiked in the 2007-2008 world food crisis, many countries including Argentina, Ethiopia, and India restricted exports in order to prevent local prices from increasing to global price levels [Headey, 2010]. These measures have been very effective; in India, for example, the rice price increased by only 7.9%, whereas the world rice price increased by 160% between June 2007 and June 2008 [World Bank, 2010]. It has even been shown that similar restrictive trade policies are part of an optimal strategy to stabilize prices for a small open economy [Gouel and Jean, 2015]. However, these actions come at the cost of the other countries on the world market since export restrictions imposed by a net exporter will lead to a further increase of prices on the world market. During the world food crisis, these restrictions played an important role in setting the world price for maize, wheat, rice and soybeans [Headey, 2010]. Furthermore, while consumers typically benefit from these policies, producers and traders do not. In the long run, this may lead to suboptimal levels of production and result in even higher prices on the world market. Therefore, the need to improve international grain markets, i.e. by reducing trade restrictions, has often been emphasized [Bouët and Laborde Debucquet, 2012]. However, the different countries face very different incentives. Even though it might be desirable to limit export and import restrictions from a world-wide perspective, countries in need of local price stability may continue to use them as long as they are not bound by an international agreement. But they are unlikely to commit to such an agreement if they do not face incentives to do so. As a result, this situation seems to represent a collective action problem: If countries act individualistically rational and uncoordinated, the outcome for everyone involved will probably be worse than if all countries cooperate and choose a common strategy [Bouët and Laborde Debucquet, 2012].

Apart from trade, many other possible coping mechanisms to deal with supply shortages have been discussed. One of them which drew more interest in recent years is the setup of emergency grain reserves, usually considered as a common reserve for several countries. If shared between countries, such a reserve may work as an insurance mechanism. This means that in normal times all countries help to fill up the reserve while whenever a food shortage arises in one country, it may take some grains from the reserve. This risk sharing mechanism works if supply shortages are unlikely to coincide in the participating
countries. But even if they do, such a reserve will still help to dampen the effects and allow more time to arrange for other measures such as trade to compensate the shortage. In the last years, calls for measures of this kind such as international grain reserves or public emergency reserves [ICTSD, 2011, Rashid, 2013, von Braun and Torero, 2009] have increased and feasibility studies have been conducted [ECOWAS Commission, 2012].

Gustafson was the first person who analyzed private storage models with rational expectations [Gustafson, 1958], even before rational expectations were formally introduced [Muth, 1961]. Later, many works have been provided on models which analyze different price band and price peg policies of government stocks [Miranda and Helmberger, 1988, Wright and Williams, 1982, 1988, Miranda and Glauber, 1993, Gouel, 2013c]. In this work, we use a price peg policy, i.e. the minimum price and the maximum price for buying and selling operations for the reserve are the same. Significant progress was made when the first models combined private storage with trade [Williams and Wright, 1991, Miranda and Glauber, 1995]. Recently, many authors have investigated optimal food price stabilization policies in different settings: in a closed economy [Gouel, 2013a], in a small open economy [Gouel and Jean, 2015], in a poor grain importing country [Romero-Aguilar and Miranda, 2014], or far a large country calibrated for wheat in India [Gouel, 2014]. In a work which probably comes the most closest to ours, reserves and private storage subsidies are compared for wheat in the Middle East [Larson et al., 2014]. This is also the only work to the knowledge of the authors, which considered extreme events by calibrating the trigger price at the 90th percentile. Without including private storage, different papers have analyzed the costs (and benefits) of (non-) cooperative trade policies [Bouët and Laborde Debucquet, 2012, Gouel, 2015]. In the context of the WTO trade negotiations, these policies have been discussed extensively (compare e.g. [Bouët and Laborde, 2010, Laborde and Martin, 2012, Jean et al., 2010]).

Despite a large amount of literature on optimal grain reserves in a single country, there is currently a research gap in modeling the benefits from cooperation between different countries with respect to emergency reserves while including private storage (For an analysis of the sustainability of regional food reserves without private storage and trade, see [Romero-Aguilar and Miranda, 2015]). Furthermore, with the exception of the work of Larson et al. [2014], the role of extreme events has been largely ignored. This work combines emergency reserves with private storage in a two country setting where both countries may have different trade or storage policies and provides a focus on extreme events which makes it unique in several ways. The presented theoretical analysis shows
that an emergency reserve may be very effective in preventing very high prices in times of extreme supply shortfalls, i.e. when production shortfalls are combined with minor imports and low private carry-over stocks. Such a reserve has almost no impacts on trade and very little on private storage. However, a storage subsidy is found to be more efficient for minor supply shortfalls, i.e. it can reduce the overall volatility at lower costs than the emergency reserve.

The following sections illustrate how the effects of a private storage subsidy and an emergency reserve differ in their effects on prices. The subsidy reduces the standard deviation of the price distribution along with the lower percentiles of the upper half of the distribution (e.g. the 90th percentile) but at the costs of a higher skewness and kurtosis which implies that it has very little effects on extreme events, i.e. the highest percentiles. In contrast, the reserve reduces the skewness and kurtosis and therefore the prices of the highest percentiles but has little affect an the standard deviation and the lower percentiles of the upper half of the distribution. To understand these effects it is useful to revise how such distributions look like. Figure 1 shows 100,000 random draws from a normal distribution in blue bars along with the fitted normal distribution as a red

Figure 1: Normal distribution and percentiles: The blue bars show the occurrences of 100,000 random numbers taken from a normal distribution with mean 0 and standard deviation of 1. The red line is the fitted normal distribution function and the black bars indicate different percentiles from the 0.01st to the 99.99th percentile.
In addition, different percentiles are indicated. Reducing the standard deviation while leaving the highest percentiles largely unaffected would mean to condense the inner part of the distribution (e.g. from the 10th to the 90th percentile) while leaving the outer parts largely unaffected. The price volatility is expected to decrease in such a case. In contrast, reducing the prices at the highest percentiles means that only the utmost parts – the extreme events – are shifted towards the mean of the distribution while the big majority of the distribution which is between, say, the 10th and 90th percentile remains largely unaffected. As a result, the standard deviations hardly change while the kurtosis (and maybe skewness) does.

2 Theoretical model

The model is an extension the first trade-storage models which were developed [Williams and Wright, 1991; Miranda and Glauber, 1995]. Its specification closely follows the approach from [Gouel, 2011] and [Gouel and Jean, 2015]. However, it differs in several ways: (1) It explicitly includes two countries, (2) both of them have a public reserve following simplified rules as well as private storage, (3) it includes flexible trade policies, and (4) it has an explicit focus on extreme price events. These are the result of large supply shortfalls which arise if production, carry-over stocks, and imports combined are well below their expected level.

In our model, there are two countries, A and B, indexed by \( i \in \{A, B\} \). If one country \( i \) is chosen, the other country is indexed by \( -i = \{A, B\} \setminus \{i\} \). A homogeneous food product is produced, consumed, and stored in both countries and can be traded between them. The only other good is the numeraire. This partial equilibrium model has annual time steps and combines trade, private stockholders, and public storage.

2.1 Private stockholders

It is often argued that public storage is likely to crowd out private storage. This could imply additional efficiency losses due to distorted market allocations. In order to evaluate to which extent this holds and to compare the other scenarios with a private storage subsidy, there are risk-neutral profit maximizing stockholders incorporated into the model. One representative stockholder exists in each country who acts competitively according to the competitive storage model [Williams and Wright, 1991]. At time \( t \) the quantity \( S_{i,t} \) is bought for price \( P_{i,t} \) in country \( i \) and in time period \( t + 1 \) this quantity is sold for price
Storage losses $\delta_i$ and constant marginal storing costs $k_i$, which are considered to be equal in both countries, apply but may be (partly) compensated by the constant marginal storage subsidy $m_i$. As a result, the stockholder’s profit maximization problem can be written as

\[
V_i^S(S_{i,t-1}, P_{i,t}) = \max_{S_{i,t} \geq 0} \sum_{j=0}^{\infty} \beta^j \left[ \delta_i P_{i,t+j} S_{i,t+j-1} - (P_{i,t+j} + k_i - m_i) S_{i,t+j} \right]
\]

where $V_i^S$ is the stockholder’s value function which includes the sum over all buying and selling operations and therefore depends on the stock levels, the market prices, the storage costs and the storage subsidy. There are two discount factors in this equation, $\beta_i = 1/(1 + r_i)$ (with $r$ representing the interest rate) is the monetary discount factor whereas $\delta_i$ represents the discount factor originating from the storage losses. $E_t[\cdot]$ is the rational expectations operator. Representing this equation in a recursive form allows to rewrite the problem as the following Bellman equation:

\[
V_i^S(S_{i,t-1}, P_{i,t}) = \max_{S_{i,t} \geq 0} \left( \delta_i P_{i,t} S_{i,t-1} - (P_{i,t} + k_i - m_i) S_{i,t} + \beta_i E_t \left[ V_i^S(S_{i,t}, P_{i,t+1}) \right] \right).
\]

This equation can be rewritten as a complementarity problem using the first-order condition on the stocks, the envelope theorem, and the non-negativity constraint on the stocks [Gouel, 2011]. The resulting complementarity problem reads as

\[
S_{i,t} \geq 0 \perp P_{i,t} + k_i - m_i - \delta_i \beta_i E_t(P_{i,t+1}) \geq 0.
\]

Here, the $\perp$ symbol represents the orthogonality of the mixed complementarity problem. In general, a mixed complementarity problem $x^{min} \leq x \leq x^{max} \perp F(x)$ consists of a continuously differentiable function $F : \mathbb{R}^n \to \mathbb{R}^n$ and $x^{min} \in (\mathbb{R} \cup -\infty)^n$, $x^{max} \in (\mathbb{R} \cup +\infty)^n$. 

\footnote{To be precise, one would need to consider different costs for placing into the stock, releasing from the stock, storing itself, and rotating the crop as well as for keeping the storage capacity. Furthermore, all these parameters would depend on the actual stock levels. However, this would massively increase the complexity without any expected additional insights which is why only constant marginal storage costs which cover all these processes were considered.}
as lower and upper bounds, respectively, such that for each $i \in \{1, \ldots, n\}$ one out of the following conditions holds:

\begin{align*}
F_i(x) = 0 \text{ and } x_{i}^{\min} & \leq x_i \leq x_{i}^{\max} \text{ or } \quad (4) \\
F_i(x) > 0 \text{ and } x_{i}^{\min} &= x_i \quad \text{or} \quad (5) \\
F_i(x) < 0 \text{ and } x_i &= x_{i}^{\max} \quad . \quad (6)
\end{align*}

If $x_{i}^{\max} = \infty$ (or $x_{i}^{\min} = -\infty$), then $F(x) \geq 0$ ($\leq 0$) $\forall$ $x$, as it is the case for the private storage problem above. If $x_{i}^{\max} = \infty$ and $x_{i}^{\min} = -\infty$, then $F(x) = 0$ is a “traditional” function.

2.2 Public emergency reserve

Both countries have a public emergency reserve. These follow simple rules which makes the results more understandable and transferable to real-world situations. Only two parameters are used to operate the reserve, the reserve capacity, $c_i$, and the trigger price, $T_i$. As long as the observed price is below the trigger price, the reserve is filled up to its capacity whereas its stocks released to prevent any price increase above the trigger price. This can be expressed as the following complementarity problem:

\begin{align*}
P_{i,t} - T_i \perp 0 & \leq R_{i,t} \leq c_i \quad . \quad (7)
\end{align*}

where $R_{i,t}$ represents the level to which the reserve in $i$ is filled at time $t$. If both countries have a reserve, we called it reserve cooperation because they share the costs but also the benefits from a public reserve. In contrast, if only one country has a reserve, depending on the trade policy the benefits may be shared while the costs never are.

2.3 Production

Production in $i$ depends on the price expectations $E_t[P_{i,t+1}]$ about the future prices at $t + 1$ in time period $t$ with the knowledge available then. Therefore, the representative and risk-neutral producer in each country makes his planting decision $H_{i,t}$ at time period $t$ while the crop is only harvested one period later. Additionally, there are random, normally distributed yield shocks $e_{i,t}$ with mean 1 and variance $\sigma_i$ so that the final production can be written as $H_{i,t}e_{i,t+1}$. As the evidence against normally distributed harvest shocks
remains weak [Just and Wenninger, 1999], this is a reasonable assumption. The resulting profit-maximizing production decision of the producers reads as

$$\max_{\{H_{i,t+j} \geq 0\}} \mathbb{E}_t \left( \sum_{j=0}^{\infty} \beta^j \left[ \delta_i P_{i,t+j} H_{i,t+j-1} + e_{i,t+j} - \Psi_i(H_{i,t+j}) \right] \right).$$

(8)

Here, $\Psi_i(H_{i,t+j})$ represent the production costs for producing $H_{i,t}$. As before, this problem can be rewritten in recurse form providing

$$\beta_i \mathbb{E}_t (P_{i,t+1} + e_{i,t+1}) = \Psi'_i(H_{i,t}).$$

(9)

This result can be interpreted as follows: The marginal cost of production is equal to the expected, discounted marginal profit from one unit of planned production. Following economic theory, the first derivative of the production cost function needs to be strictly increasing which can be fulfilled by choosing a convex, isoelastic function of the form

$$\Psi_i(H_{i,t}) = h_i \frac{H_{i,t}^{1+\mu_i}}{1+\mu_i}.$$  

(10)

with scale parameter $h_i$ and $\mu_i \geq 0$ as the inverse supply elasticity.

2.4 Trade

All possibilities for spatial arbitrage are used by the representative trader who is trading competitively between the two countries. Trade is instantaneous with per unit trading costs of $\Theta_i$ for exports from $i$ to $-i$, i.e. the other country. In addition, a country may impose an export tariff $\phi_{i,t}$. As trade happens instantaneously, instant profits rather than expected profits are maximized. Expressed as a complementarity problem, the trader’s behavior can be described as

$$P_{i,t} - P_{-i,t} + \Theta_i + \phi_{i,t} \perp 0 \leq X_{i,t} \leq X_{i,t}^{\max}.$$ 

(11)

Here, $P_{-i,t}$ is the price at $t$ in the country which is not $i$, and $X_{i,t}$ are the exports from country $i$ to the other country. A direct result from this equation is that there are never simultaneously exports to and imports from the same country, i.e. $X_{i,t} \geq 0 \perp X_{-i,t} \geq 0$. Governments may set a limit to the maximum trade which is allowed; this limited is represented by $X_{i,t}^{\max}$. Furthermore, governments may decide that exports are only allowed
as long as their reserve remains untouched. For the numerical implementation, this case can be represented by adjusting the export tariff if the reserve is used. The following complementarity condition describes this behavior

$$T_i - P_{i,t} + R_{i,t} - c_i \perp 0 \leq \phi_{i,t} \leq \phi_{i,t}^{\text{max}}$$

where $\phi_{i,t}^{\text{max}}$ describes the maximum export tariff. This equation sets $\phi_{i,t} = 0$ as long as the price is below the trigger (which implies that the reserve is filled completely), it sets $\phi_{i,t} = \phi_{i,t}^{\text{max}}$ if the reserve is not filled up to its capacity, and in the remaining cases it adjusts $\phi_{i,t}$ such that exports only occur up to the point where the reserve would be touched.

### 2.5 Consumption

The consumers in both countries have an isoelastic consumption function

$$D_i(P_{i,t}) = \gamma_i P_{i,t}^{\alpha_i}$$

where $\gamma_i$ is a normalization parameter and $-1 \neq \alpha_i < 0$ is the price elasticity. This implies that consumers have a constant income and do not save and, as a result, not insure themselves. If they did, there would be another maximization problem for the consumer which needed to be solved and this would increase the complexity while hardly allowing any new insights.

### 2.6 Fiscal costs

Fiscal costs only arise if a government intervenes into a market which can be done by paying a subsidy to private stockholders, by having a public emergency reserve, or by limiting trade. For a constant marginal private storage subsidy $m_i$, the storage costs $k_i$ are shared between the government who pays $m_i$ and the private stockholder who pays $k_i - m_i$. The subsidy $m_i$ has an upper bound, $m_i \leq k_i + \bar{P}_i (1 - \delta_i \beta_i)$, because otherwise storage would always become profitable and therefore stock levels would diverge. However, there may also be a tax ($m_i < 0$) on storage resulting in $k_i - m_i > k_i$ and therefore very low
private stock levels. For any level of private subsidy, the fiscal costs can be calculated as 
$S_{i,t}m_i$ resulting in mean fiscal costs of

$$\bar{S}_{i,t}m_i = \frac{1}{n} \sum_{t=1}^{n} S_{i,t}m_i. \quad (14)$$

For a public emergency reserve the government has to pay the full storage costs which we assume to be equal to the gross marginal storage costs $k_i$ for private stockholders. Therefore, the per-period fiscal costs for storing the amount in the reserve is $R_i,t k_i$. In addition, fiscal revenues arise as the reserve is filled up when prices are below the trigger price and stock out take place for the trigger price or even higher prices if the reserves is replenished completely. This produces the per-period revenues of $(R_i,t - R_{i,t-1})P_i,t$. As a result, the total fiscal costs for the reserve are

$$\frac{1}{n} \sum_{t=1}^{n} \left[ (R_i,t - R_{i,t-1})P_i,t + R_i,t k_i \right]. \quad (15)$$

If trade is limited by a variable export tariff $\phi_{i,t}$, then the government can collect fiscal revenues from exports. If trade is not limited by a variable export tariff but by a fixed quota which dictates the maximum level of exports, then the government can still collect the profits from the traders (e.g. by selling the quota in auctions). In both cases, the revenues can be calculated as

$$\frac{1}{n} \sum_{t=1}^{n} (P_{-i,t} - P_{i,t} - \Theta_i)X_{i,t}. \quad (16)$$

All the fiscal costs and revenues are summed up and expressed as share of the agricultural GDP. For this, their sum needs to be multiplied by

$$\left[ \frac{1}{n} \sum_{t=1}^{n} (P_{i,t}H_{i,t}e_{i,t}) \right]^{-1} \approx 1. \quad (17)$$

In the results section, mean the fiscal costs are provided as percentage share of the agricultural GDP.
2.7 Market equilibrium

To limit the number of state variables, the private carry-over stocks, the emergency reserve, and the harvest can be combined to one state variable per country, availability $A_{i,t}$, which results in the following transition equation

$$A_{i,t} = (1 - \delta_i)(S_{i,t-1} + R_{i,t-1}) + H_{i,t-1}e_{i,t}. \quad (18)$$

As the shocks are considered at the beginning of each period, the knowledge of the availability in both countries fully determines the state of the model. Then, the market equilibrium condition reads as

$$A_{i,t} - X_{i,t} + X_{-i,t} = D_{i,t}(P_{i,t}) + S_{i,t} + R_{i,t}. \quad (19)$$

When the model is solved, a recursive equilibrium is calculated by evaluating how the response variables change dependent on the state variables. This means the following functions are calculated by using the aforementioned behavioral equations for the different agents: $S_{i,t}(A_{i,t}, A_{-i,t})$, $R_{i,t}(A_{i,t}, A_{-i,t})$, $H_{i,t}(A_{i,t}, A_{-i,t})$, $X_{i,t}(A_{i,t}, A_{-i,t})$, $\phi_{i,t}(A_{i,t}, A_{-i,t})$, and $P_{i,t}(A_{i,t}, A_{-i,t})$.

For simplicity, we assume that storage costs are the same in both countries and storage losses are zero. Changing this assumption does slightly affect the specific result but it does not influence the general behavior of the model and therefore the conclusion remain valid even if these assumptions were relaxed.

3 Calibration

The default values for the solving the RE-MCP problem and simulating the scenarios are illustrated in table 1. For each configuration, that is each set of parameters, the model is solved on a 50x50 grid of the state variables. This selection is justified and explained in the appendix. Typical values, which were found in the literature and in other models, were used for all parameters. However, these values are theoretical values which only have a relative interpretation as the model is not calibrated for two specific countries. In most scenarios, trade was not restricted. In these cases, the maximum is set to 100,000 which is far beyond any level which was ever achieved. Whenever no export tariffs shall apply, the maximum export tariff was set to zero. For the reserve, the characteristics of
<table>
<thead>
<tr>
<th>Parameter</th>
<th>Symbol</th>
<th>Default value</th>
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</thead>
<tbody>
<tr>
<td>Reserve price trigger in A</td>
<td>$T_A$</td>
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</tr>
<tr>
<td>Reserve price trigger in B</td>
<td>$T_B$</td>
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</tr>
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<td>Reserve capacity in A</td>
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<td>Reserve capacity in B</td>
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<td>Mean of supply shock in B</td>
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<tr>
<td>SD of supply shock correlation A to B</td>
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<tr>
<td>SD of supply shock correlation B to A</td>
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<tr>
<td>Marginal per-unit private storage costs in A</td>
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<tr>
<td>Marginal per-unit private storage costs in B</td>
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<td>Supply elasticity in A</td>
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<tr>
<td>Supply elasticity in B</td>
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<tr>
<td>(Demand) Price elasticity in A</td>
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<tr>
<td>(Demand) Price elasticity in B</td>
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<tr>
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<td>Interest rate in B</td>
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<td>Trade costs from B to A</td>
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<td>Relative country size of B</td>
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<tr>
<td>Maximum export tariff B to A</td>
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**Simulation parameters**

<table>
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<tr>
<th>Parameter</th>
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<td>Max. grid point</td>
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<td>Shock nodes</td>
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<tr>
<td>Simulation method between grid points</td>
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<td>Interpolation</td>
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Table 1: The default values for the simulations. Unless specified differently, the above specifications were used for solving and simulating the model.
<table>
<thead>
<tr>
<th>Scenario</th>
<th>Description</th>
<th>Variables differing from default</th>
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<tbody>
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<td>1</td>
<td>No trade, no reserve</td>
<td>$\Theta_A = \Theta_B = \infty; c_A = c_B = 0$</td>
</tr>
<tr>
<td>2</td>
<td>No reserve</td>
<td>$c_A = c_B = 0$</td>
</tr>
<tr>
<td>3</td>
<td>No trade</td>
<td>$\Theta_A = \Theta_B = \infty$</td>
</tr>
<tr>
<td>4</td>
<td>Reserve only in country A</td>
<td>$c_B = \infty$</td>
</tr>
<tr>
<td>5</td>
<td>Reserve only in country B</td>
<td>$c_A = 0$</td>
</tr>
<tr>
<td>6</td>
<td>Trade only if reserves are untouched</td>
<td>$\phi_A = \phi_B = \infty$</td>
</tr>
<tr>
<td>7</td>
<td>Trade only up to capacity of reserve</td>
<td>$X_A^{\max} = X_B^{\max} = c_A = c_B$</td>
</tr>
<tr>
<td>8</td>
<td>Common reserve</td>
<td></td>
</tr>
<tr>
<td>9</td>
<td>Tiny common reserve with low trigger</td>
<td>$c_A = c_B = 0.02; T_A = T_B = 1.3$</td>
</tr>
<tr>
<td>10</td>
<td>Small priv. storage subsidy, no reserve</td>
<td>$m_A = m_B = 0.03; c_A = c_B = 0$</td>
</tr>
<tr>
<td>11</td>
<td>Large priv. storage subsidy, no reserve</td>
<td>$m_A = m_B = 0.06; c_A = c_B = 0$</td>
</tr>
<tr>
<td>12</td>
<td>Common reserve and small subsidy</td>
<td>$m_A = m_B = 0.03$</td>
</tr>
<tr>
<td>13</td>
<td>Tiny low trigger reserve &amp; tiny subsidy</td>
<td>$m_A = m_B = 0.02; Rest as in scenario 9 $</td>
</tr>
</tbody>
</table>

Table 2: The different scenarios which are considered in the calculations

The response variables have been calculated for a reserve size between 0.5% and 10% of the mean harvest and trigger prices from 1.1 to 1.6 (with the expected price being almost equal to one). For private storage, different subsidies have been considered ranging from zero effective storage costs to a storage tax of 0.04. The other simulated scenarios are summarized in table 2. The models are solved and simulated in MATLAB using the RECS solver [Gouel, 2013b] and the CompEcon toolbox [Packler and Miranda, 2011].

4 Results

The presented model has been solved and simulated for a number of different scenarios. At first, the 13 different scenarios which are depicted in table 2 were simulated. This allows to investigate the benefits from trade, from a reserve, and from a storage subsidy. Afterwards, the storage subsidy was varied from 0.06 to -0.04 resulting in effective storage costs between 0 and 0.1. Negative subsidies are interpreted as a storage tax. The results are depicted in figure 3. Next, different reserve configurations, i.e. reserves with different capacities and trigger prices were investigated and the effects on prices are presented in figure 4. Finally, the effectiveness of the subsidy is compared to the effectiveness of a reserve given a level of fiscal costs.

The 13 cases depicted in figure 2 include the scenarios with different trade, private storage, and public reserve policies. For all these scenarios, some statistical properties of all the response variables are presented. The price characteristics are shown in table 3.
Table 3: Price characteristics in country A for the simulation of the different scenarios. The columns show the mean, standard deviation, skewness, kurtosis, different percentiles, and finally the fiscal costs for the respective scenario.

<table>
<thead>
<tr>
<th>Prices</th>
<th>Costs</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mean</td>
<td>SD</td>
</tr>
<tr>
<td>1</td>
<td>1.016</td>
</tr>
<tr>
<td>2</td>
<td>1.009</td>
</tr>
<tr>
<td>3</td>
<td>1.014</td>
</tr>
<tr>
<td>4</td>
<td>1.008</td>
</tr>
<tr>
<td>5</td>
<td>1.008</td>
</tr>
<tr>
<td>6</td>
<td>1.008</td>
</tr>
<tr>
<td>7</td>
<td>1.010</td>
</tr>
<tr>
<td>8</td>
<td>1.009</td>
</tr>
<tr>
<td>9</td>
<td>1.009</td>
</tr>
<tr>
<td>10</td>
<td>1.007</td>
</tr>
<tr>
<td>11</td>
<td>1.004</td>
</tr>
<tr>
<td>12</td>
<td>1.006</td>
</tr>
<tr>
<td>13</td>
<td>1.007</td>
</tr>
</tbody>
</table>

The reserve and availability characteristics are depicted in Table 4, and the characteristics of private storage, production, and exports are illustrated in Table 5. As graphs are more intuitive than tables, some of the statistical properties of the prices are also shown in Figure 2. As all scenarios have the same impacts on the response variables in both countries, only the properties of the variables in country A are shown in all tables and figures. The only non-symmetric case is if the reserve exists only in one country so that this case was split up in two scenarios with a reserve once only in country A and once only in country B. Overall, the following observations can be made from comparing the statistical properties of the response variables for the 13 different scenarios:

- Scenario 1, no trade and no reserve: A huge price range, in particular on the upper end, is observed and comes with relatively high mean prices and a high standard deviation, skewness and kurtosis. In extreme events, prices may double or more. The mean private storage levels are relatively large. The supply response is the strongest along with the no trade scenario which is indicated by the high standard deviation.

\( ^2 \) To avoid rounding problems, a maximum deviation of 0.1% was allowed for the frequencies above zero which implies that, for example, a reserve level of above 0.034965 at a reserve capacity of 0.035 was still considered to be full. For frequencies of zero (empty reserve, no trade), the absolute maximum allowed deviation was 0.001.
Figure 2: Price characteristics and fiscal costs of different scenarios: For the simulated scenarios the box-plots show the 1st, 10th, 50th, 90th, and 99th percentile of the price distribution (left axis). The red crosses illustrate the fiscal costs expressed in % of agricultural GDP which are shown on the right axis.

- **Scenario 2, no reserve**: Trade is a no-cost policy which is very effective in preventing high prices. Trade manages to reduce all moments of the prices and massively decreases the highest percentiles of the prices. However, trade also strongly reduces private stocks and the supply response in all parts of the distribution. The mean private stocks are almost halved (the difference in the mean private stocks can be regarded as a crowding out factor). Once trade is allowed, it is hardly affected by the different scenarios except if trade is only allowed when the reserve remains untouched, if it is limited by the capacity of the reserve, or if there is a large private storage subsidy.

- **Scenario 3, no trade**: The introduction of an emergency reserve reduces all moments of the price distribution and decreases the prices of the higher percentiles massively. The reserve does not affect the minimum prices and hardly affects the prices below the mean because it is usually filled up to its capacity. When compared to introducing trade, the reserve seems less attractive though: Trade reduces the prices of the highest
percentiles even more and does not produce any fiscal costs which clearly underlines the benefits from trade. However, this result depends on the reserve’s capacity and trigger price; for high capacities the reserve can reduce the highest percentiles of the prices even more than trade can but it does so only at very high fiscal costs. In contrast, private storage is influenced by the introduction of a reserve but much less than by allowing trade. Therefore, such a reserve presents itself as a stabilization tool with very little impacts on private storage. In this scenario, the frequency of the reserve being empty or non-full is the highest except for the scenarios with the tiny reserve of size 0.02. While the reserve only affects the highest percentiles of the supply response, trade mostly affects the lowest percentiles (not shown in the tables).

- Scenario 4 and 5, reserve only in country A or B, respectively: If one country has a reserve, both countries directly benefit from it. The mean, SD, skewness, and kurtosis are all reduced and there is a huge decline of the prices in the higher percentiles. The benefits are almost completely shared, i.e. the benefits largely “leak” into the other country, so that one country is paying the cost to stabilize both of them while almost having no benefits over the other country. Yet, the effects of one reserve alone are already very significant as the standard deviation and the
<table>
<thead>
<tr>
<th></th>
<th>Private storage</th>
<th>Production</th>
<th>Exports</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Mean</td>
<td>SD</td>
<td>50%</td>
</tr>
<tr>
<td>1</td>
<td>0.029</td>
<td>0.033</td>
<td>0.018</td>
</tr>
<tr>
<td>2</td>
<td>0.015</td>
<td>0.026</td>
<td>0.000</td>
</tr>
<tr>
<td>3</td>
<td>0.026</td>
<td>0.032</td>
<td>0.012</td>
</tr>
<tr>
<td>4</td>
<td>0.015</td>
<td>0.026</td>
<td>0.000</td>
</tr>
<tr>
<td>5</td>
<td>0.015</td>
<td>0.026</td>
<td>0.000</td>
</tr>
<tr>
<td>6</td>
<td>0.015</td>
<td>0.026</td>
<td>0.000</td>
</tr>
<tr>
<td>7</td>
<td>0.017</td>
<td>0.025</td>
<td>0.001</td>
</tr>
<tr>
<td>8</td>
<td>0.015</td>
<td>0.026</td>
<td>0.000</td>
</tr>
<tr>
<td>9</td>
<td>0.014</td>
<td>0.025</td>
<td>0.000</td>
</tr>
<tr>
<td>10</td>
<td>0.024</td>
<td>0.034</td>
<td>0.004</td>
</tr>
<tr>
<td>11</td>
<td>0.044</td>
<td>0.050</td>
<td>0.028</td>
</tr>
<tr>
<td>12</td>
<td>0.023</td>
<td>0.034</td>
<td>0.002</td>
</tr>
<tr>
<td>13</td>
<td>0.019</td>
<td>0.030</td>
<td>0.000</td>
</tr>
</tbody>
</table>

Table 5: Private storage, production, and export characteristics in country A for the simulation of the different scenarios. The frequency of exports being larger than zero is shown in the last column.

prices of the highest percentiles (when the reserve is touched) are much lower than in the scenarios without trade or without reserve. Private storage in either country remains basically unaffected from the reserve when compared to scenario 2. Due to trade as second stabilization mechanism, the reserve remains more filled than in scenario 3 without trade. The supply response is comparable to the scenario without trade and exports hardly change compared to the scenario 2 without reserve. Since the benefits from the reserve are largely shared, this scenario also shows that if is possible to share a reserve which is – for logistical or other reasons – located in one country only while the costs are shared.

- Scenario 6, trade only occurs whenever the reserves remain untouched: In this scenario, the frequency of an empty or non-full reserve is the lowest along with scenario 12. Compared to the aforementioned scenarios with trade, the frequency of exports is slightly reduced while private storage remains unchanged. Compared to the scenario with only one reserve in both countries (or to scenario 8, the common reserve), the 99th percentile slightly decreases while the 99.9th percentile increases. This illustrates the mechanism of such a restrictive trade policy: While prices in the “normal times” are a bit more stable due to the anticipated prevention of letting the other country induce a crisis, prices during extreme events in both country are
higher in the country which is more affected because of the lack of cooperation – i.e. sharing of the burden of high prices – in this case. In addition, now both countries have a reserve and therefore both need to pay the fiscal costs which means they are more than doubled compared to scenario 4 or 5. For the same fiscal costs but with unlimited trade, the highest percentiles (99.9% and above) of the prices can be reduced significantly as can be seen in scenario 8.

- Scenario 7, trade is limited by the capacity of the reserve: The frequency of trade is the highest in this scenario while the amounts traded are among the lowest. Basically, this implies that part of the trading is not instantaneous any more but delayed by one or more years, depending on the trading limit and supply shock. However, the trade quota generates revenues which significantly reduces the fiscal costs of this policy. The frequency of having an empty or non-full reserve is the highest among all scenarios which have trade and the default reserve size. Compared to scenario 8 where trade is unlimited, there is only a very little increase in private stocks. Interestingly, the mean price, the standard deviation and the highest percentiles are all higher than in scenario 6 or 8. Therefore, limiting the per-period amount of trade seems to have a more devastating impact on the price stability than limiting trade to periods where the reserve remains untouched. As a result, if only one country builds up a reserve and wants to protect itself from paying the costs to stabilize prices in other country, a trade policy based on whether the reserve is touched is a better option than introducing time-independent quotas.

- Scenario 8, the common reserve (or two identical reserves and unlimited free trade): While most of the price characteristics can be compared to the previous scenarios, a significant decline of the prices in the highest percentiles can be observed. Except for scenario 12 which includes an additional subsidy, the price in the 99.9th percentile is by far the lowest. Therefore, such a common reserve is the best mechanism to compensate extreme supply shortfalls. Interestingly, for smaller supply shortfalls prices can be reduced slightly more if trade is limited to times when the reserve is untouched. This might be a result of the decrease in private stocks (in particular in the highest percentiles) if trade is not limited as well as a result of the slight reduction in trade which produces less trade costs. This leads to lower prices as the trade costs are included in the prices. In addition, the frequencies of an empty or non-full reserve are also higher than in the scenario where trade is limited to times
when the reserve is full. However, compared to the other scenarios where trade is allowed, private storage remains almost unaffected. The reserve is only touched in around 10% of the cases and is replenished with a probability of around 0.7% implying that prices will only surpass the trigger price with a probability of around 1.4%. The production and trade are very similar to other scenarios which include unlimited trade and a reserve.

- Scenario 9, the tiny common reserve with a low trigger: This scenarios almost halves the fiscal cost while also lowering the price of the 95th percentile to 1.29 because of the lower trigger price. This is lower than in all the previous scenarios; however, the price of the 99th and 99.9th percentile are comparable to scenarios 4, 5, 6, and 7. The lower trigger price helps to keep the price in the high but not very high percentiles low but only at the costs of higher prices in the very highest percentiles. Due to the small capacity and low trigger price the reserve is used more often than in any other scenario but a frequency of usage of 20% and of replenishment of 3.5% is still not high. Interestingly, private storage levels are slightly reduced because the lower trigger price outweighs the lower capacity. Trade and production are remain unaffected when compared to scenario 8.

- Scenarios 10 and 11, no reserve but a small or large private storage subsidy: Compared to scenario 2 without reserve and private storage subsidy, the standard deviation and the mean price as well as the prices in the high percentiles and the mean price decrease significantly while the minimum price increases. However, when compared to the reserve, the prices of the highest percentiles do not change much which leads to an increase in the skewness and kurtosis. Therefore, no matter how large the subsidy, private storage fails to compensate for large supply shortfalls in a way which is similar to what could be achieved by a reserve. Only when the supply is relatively high, private storage occurs at all which in particular prevents private storage from compensating supply shortages if several of these occur in a row. Depending on the reserve characteristics, private stocks are more efficient in reducing prices up to a certain percentile but for the highest percentiles a reserve is far more efficient. The mean private storage levels in scenario 10 are much higher than in scenario 8 or 9, in scenario 11 they are even higher than the reserve capacity and the mean private storage of scenario 9 combined and yet the prices of the highest percentiles remain slightly higher (because the private storage is already mostly
used to compensate for small harvest failures) while at the same time producing much higher fiscal costs. Trade is heavily reduced by the storage subsidy and is so in different ways; the frequency, the mean exports, and the higher percentiles are reduced. Policymakers therefore need to be aware of these large impacts of a storage subsidy on trade. However, the private storage subsidy is the most efficient policy to reduce the standard deviation of prices and therefore also the expected volatility in normal times.

- Scenario 12 and 13, a small private storage subsidy combined with a reserve: This scenario brings a combination of the effects, i.e. the minimum and the mean price are mostly determined by the size of the subsidy whereas the prices of the highest percentiles are mostly determined by the characteristics of the reserve. Private storage increases while the reserve only needs to be touched in case of bigger supply shortfalls. Trade is slightly reduced through the subsidy. Scenario 13 produces relatively small fiscal costs while achieving a high degree of price stabilization for almost all kinds of supply shortfalls.

Having compared these different distinct scenarios, it seems reasonable to analyze the impact of a storage subsidy and an emergency reserve on a more continuous scale. This is presented in the following parts. For all these scenarios, it is assumed that trade is unlimited and both countries have the same subsidy or the same reserve. Figure 3 shows how the effective private per-unit storage costs \((k_i - m_i)\) influence different percentiles of the prices (left axis) and which fiscal costs it produces for the government (right axis). High subsidies significantly reduce the highest percentiles of the prices. However, this exponential decrease is accompanied by an exponential increase in fiscal costs reaching as high as 0.26% of the agricultural GDP when private storage is effectively free. In addition, the prices of the lower percentiles (the 25th percentile is indicated in cyan) do increase so the standard deviation is reduced because prices from both ends of the distribution are shifted towards the mean.

For the public emergency reserve, there are two parameters which can be varied, the capacity and the trigger price. Figure 4 shows the 99.9th percentile, the 99th percentile, the 90th percentile, the fiscal costs as well as the frequency of an empty or non-full reserve as colors in separate plots with the reserve capacity on the x-axis and the trigger price on the y-axis. This allows evaluating how these variables change if the reserve capacity and/or the trigger price are changed. In the graphs with the different price percentiles it can be seen that if the trigger price is very low, the reserve might not be able to keep the
Figure 3: Price distribution (left y-axis) of the 99.9th percentile (blue), the 99th percentile (green), the 90th percentile (red), and the 25th percentile (cyan) as well as fiscal costs expressed as % of agricultural GDP (dashed black, right y-axis) for different private storage subsidies (x-axis). As the default per-unit private storage costs are calibrated to 0.06, higher values on the x-axis represent a storage tax which brings revenues, i.e. negative fiscal costs.

price of the respective percentile below it. If the reserve is supposed to affect prices at the 90th percentile already, it is necessary to set the trigger price below 1.3. However, these low trigger prices are likely to fail in compensating large supply shortages unless they come with a big reserve and therefore high costs. To give a numerical example, a reserve capacity of 0.025 combined with a trigger price of 1.2 would allow to keep the 90th percentile below the trigger price while the 99th percentile reaches around 1.4 and the costs of this policy would be around 0.077% of the agricultural GDP.

A common concern with public stockholding is the crowding out of private storage. Research has shown that crowding out effects can be observed [Kozicka et al., 2015]. Therefore, figure 5 depicts the crowding out of private storage for the different setups of a public reserve. The private stocks at the 99.9th percentile, the 99th and the 90th percentile, and, most importantly, the mean private stocks are shown. Clearly, any reserve which is supposed to impact the price distribution will also impact private storage. However, it can
Figure 4: Characteristics of the reserve dependent on its capacity and trigger price: The colors of the six plots show the price at the 99.9th percentile, at the 99th percentile, at the 90th percentile, the costs (in % of agricultural GDP), the frequency of an empty reserve and the frequency of a non-full reserve, respectively. On the x-axis is always the capacity of the reserve, on the y-axis the trigger price. All graphs are for country A only and show the scenario for the common reserve (unlimited trade).
Figure 5: Private stocks for different reserve scenarios: The colors of the six plots show the private stocks at the 99.9th percentile, at the 99th percentile, at the 90th percentile, and the mean. On the x-axis is always the capacity of the reserve, on the y-axis the trigger price. All graphs are for country A only and show the scenario for the common reserve (unlimited trade).

It can be seen that if the reserve’s capacity is not too high and the trigger price is not too low, the impact of the reserve on private storage can be minimized. In the numerical example above with a reserve capacity of 0.025 and a trigger price of 1.2, the mean private stocks would be reduced from roughly 1.5% to 1.24% of the production. A price trigger of 1.275 would already prevent the mean private stocks from going below 1.3% of the production, independent of the capacity. Overall, the impact on private storage seems to be small.
compared to the other scenarios before. Prohibiting or limiting trade for example has a much bigger impact on private storage.

All these graphs in figures 4 and 5 allow policy makers to decide for a reserve which is optimal for their risk preferences. This means that policy makers would need to decide about their risk preferences first (e.g. which is the maximum acceptable frequency of the reserve being empty and non-full or how high is the maximum acceptable price in two specific percentiles) and then they can use these graphs to find the combination of trigger price and capacity which ensures this expectation. Alternatively, it is also possible to come from a costs minimization perspective, say by defining a budget for the maintenance of the reserve and then a second parameter as before. In general, two parameters have to be specified to obtain a unique solution.

Finally, the results of the different private storage subsidies shall be compared to the different reserve scenarios. A policy maker would be interested in comparing the effectiveness of these policies given a level of fiscal costs. This is illustrated in figure 6 where different percentiles are shown in different colors for the specific scenarios. As explained before, the reserve’s parameters are only uniquely defined when two parameters are chosen to be optimized. Therefore, choosing a level of fiscal costs is not yet enough. Instead, two different scenarios are chosen: The dotted curves show the case where the reserve is chosen to minimize the price at the 99th percentile given the level of fiscal costs at the x-axis while for the dashed curves the price at the 90th percentile has been minimized. The solid lines represent the storage subsidy as comparison. As before, it can be seen that the reserves reduced the prices at the highest percentiles much more for any level of fiscal costs. However, the prices at the 75th percentile are increased while they are decreased if a subsidy is paid to private stockholders. This subsidy nevertheless increases the prices of the lowest percentiles (here the 10th percentile in magenta) which are unaffected by the reserve. Therefore, the differences of these policies are rather distinct. The dashed lines show that if the reserve is optimized to minimize the price at the 90th percentile, it still manages to reduce the prices at this percentile to levels below what can be reached through a subsidy. But even if the reserve is optimized for the 90th percentile, it is able to reduce the price in the higher percentiles significantly more than a private storage subsidy could for the same costs.

It should be kept in mind that the grid size for the reserve calculations is limited – on the x-axis the capacity was varied from 0.005 to 0.1 in steps of 0.005, on the y-axis the trigger price was changed from 1.1 to 1.6 in steps of 0.025. Therefore, some fluctuations
Figure 6: Fiscal costs for stabilization through a reserve or a subsidy: The different lines show prices of the 99.9th percentile (blue), the 99th percentile (green), the 90th percentile (red), the 75th percentile (black), the 50th percentile (cyan), and the 10th percentile (magenta) on the y-axis dependent on the fiscal costs as percentage of the agricultural GDP on the x-axis. The solid lines represent the storage subsidy. The emergency reserve is represented by the dotted lines when optimized for a minimal price at the 99th percentile, and by the dashed lines when optimized for a minimal price at the 90th percentile. The small fluctuations of prices for the reserve are a result of the limited density of the grid on which the reserve calculations were performed.

are visible in figure 6 and the lines for the reserve configurations could even be slightly lower if the grid density were increased.

5 Conclusion

The theoretical two country model with private stockholders and producers with rational expectations provides a number of insights about how governments can protect their population from extreme prices. Unsurprisingly, free trade turned out to be a highly efficient and free of costs way to compensate harvest failures. A private storage subsidy
may be an additional tool to stabilize prices but while it is very efficient in reducing the
standard deviation of prices, it is likely to fail at compensating extreme events, i.e. massive
supply shortages. Such shortages are a result of production, private stocks, and imports
combined being significantly below the sum of their expected values. A public reserve
following very simple rules – storing up to some capacity limit if prices are below a trigger
price while releasing if they are above – turns out to be a much more efficient way to reduce
the highest percentiles of prices and therefore help in extreme events. Such a reserve
can be set up in a way that it hardly affects private storage and only produces minimal
fiscal costs. Already for 0.08% of the agricultural GDP, a decent level of insurance against
extreme events can be reached. Another consideration for a policy maker is that a private
storage subsidy may heavily impact trade while a reserve hardly does. While it could be
useful to limit the need for trade if infrastructure is bad, it also implies that in case of large
supply shortfalls there may be fewer companies ready to start importing. However, any
such measures are much less helpful if the policies of the countries are not aligned. If
trade is not limited and only one country has a reserve, the benefits of this reserve will leak
into the other country while the costs do not. Both countries would then benefit from the
insurance mechanism which is maintained and paid for by one country only. Nevertheless,
if for logistical or other reasons the maintenance of a reserve is easier in one country,
the other could pay a compensation as both countries are almost equally protected from
supply shortages.

6 Acknowledgments

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eration and Development of Germany (BMZ) within the research project “Volatility in
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Christophe Gouel for providing the RECS solver and giving advice on its usage and David
Laborde for helpful comments and discussions.
Figure 7: Testing the grid density: Relative deviation for the availability, public stocks, private stocks, planned production, price and exports in/from country A. The yellow bars show the maximal relative deviation, the red bars the mean relative deviation of the respective grid size compared to the reference case with a grid size of 120x120. The numbers above the graphs are the mean absolute deviation divided by 1000. The range of the respective response variables is indicated in the headlines after the variable name.
7 Appendix

7.1 Precision of the solution of the rational expectations mixed complementarity problem

The results depend not only on the model parameters but also on the parameters which were chosen to solve and simulate the model, i.e. the parameters in the lower half of table 1. In order to test for the precision of the results, different values were tested. The highest and lowest grid points need to be chosen such that the simulated realizations never exceed these values. In order to find the perfect foresight solution, a time horizon of 5 periods before convergence to the steady state is expected turned out to be sufficient for all cases. With the solution methods detailed in table 1 all models could be solved. In order to evaluate the necessary grid points (and therefore grid density), the grid points for each dimension were varied from 10 to 120. Figure 7 shows the absolute and relative deviation of the response variable for the different grid sizes from 10x10 to 80x80 with 120x120 as reference case for comparison. While a low grid density leads to less precision, high grid densities require a lot of calculation time. To compute these results, 900,000 realizations of the shock variable were used in order to guarantee a minimal bias from the simulations. The yellow bars show the maximal deviations, the red bars the mean deviations which are, divided by 1000, also indicated by the numbers above the bars. To ensure that the differences are not the result of different shocks, the same realizations of the shocks were used for all scenarios. It can be seen that even with very low grid sizes, the mean deviation is very small. However, the maximal deviation remains significant for some response variables if the grid densities are too low. We decided for a grid size of 50x50 which offers a high and sufficient precision while not requiring excessive computation times.

7.2 Precision of the estimated parameters

Imprecise results may not only be the outcome of a low grid size but also of using only few stochastic realizations of the shocks for estimating the moments, percentiles, and frequencies of the response variables. Hence, the deviations of the moments and percentiles depending on the simulated realizations are calculated and illustrated in figures 8 and 9 respectively. The first and second moments can already be estimated with a high precision when few realizations are used, whereas the skewness and in particular the kurtosis still differ significantly for many realizations. The percentiles appear to be rather
Figure 8: Dependency of the price moments on simulated realizations: The deviation of the mean (dark blue), the standard deviation (dark green), the skewness (red) and the kurtosis (light blue) of the price is shown for different amounts of simulated realization ranging from 20,000 to 580,000 with the simulation of 600,000 realizations as reference case. While the absolute deviation of the mean and standard deviation are always below 0.0006 if more than 100,000 realizations are used, the skewness and kurtosis may still deviate by up to 0.006 and 0.04, respectively.

precise if at least 100,000 realizations are used. Only the 99.9 percentile remains slightly imprecise but a deviation of less than 0.0035 is acceptable for all our analyses. Overall, we therefore conclude that simulating 120,000 realizations provides a sufficient level of precision. This number is split up in 600 cases starting from the steady state which are each time followed by 200 stochastic realizations.
Figure 9: Dependency of different price percentiles on simulated realizations: The deviation of different price percentiles (0.1, 1, 5, 10, 25, 50, 75, 90, 95, 99, 99.9) is shown for different amounts of simulated realization ranging from 20,000 to 580,000 with the simulation of 600,000 realizations as reference case. The absolute deviation of all percentiles is below 0.002 for more than 100,000 realizations except for the 99.9 percentile (light blue) whose deviation is below ±0.0035.

References


