On the usefulness of the directional distance function in analyzing environmental policy on manure management

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Selected Paper prepared for presentation at the American Agricultural Economics Association Annual Meeting, Long Beach, California, July 23-26, 2006

Preliminary draft, May 2006.

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In many European countries, water quality problems associated with the use of synthetic fertilizers and the disposal of animal waste have become a major environmental policy issue. Nitrates in drinking water supplies and eutrophication of inland and coastal waters are of particular concern. Increasing concentrations of nitrates in groundwater - the primary source of drinking water in many regions - have been observed, notably in France. Agriculture is not the only source of nitrates in ground and surface waters but it is one of those that give most cause for concern. There is widespread interest in implementing policies that will be more effective in protecting water quality without causing undue economic harm to agricultural producers.

The two farming practices that most concern policymakers are the use of large amounts of fertilizer for crop growth and the disposal of livestock manure. Both materials are sources of nitrogen, which is transformed into nitrate once in the soil. Nitrate that is not used by plants or transformed back into atmospheric nitrogen leaches through the soil or runs off into water supplies. Intensive livestock production is an important source of pollution, due to an insufficient area of land available to farmers for spreading manure. This is particularly relevant for pig production. The direct impact of pig production on the environment is in some areas really severe. Along with the expansion of production, there have also been significant structural changes in the pig sector. Pig farming has become more intensive, with fewer farms producing a larger number of pigs, often with very little land, and more specialized, with feed obtained from off-farm sources. Developments in production technologies have allowed significant productivity gains, particularly for large-scale producers. Pig farming has become more regionally concentrated. A major factor encouraging the development and uptake of productivity-enhancing technologies has been the intense competition in the meat market and the long-run decline in real prices received by farmers, which in turn is driven by productivity improvements.

In response to high nitrate levels in water supplies, the European Union passed its Nitrate Directive in 1991. Its objective is to limit the amount of nitrogen remaining in the soil as a residual after uptake by crops. The Directive limits the spreading of organic nitrogen per farm to 170 kilograms per hectare in nitrate vulnerable zones. Each European member state has organized the implementation of this Directive, defining a set of constraints relevant for their own country on the use of nitrogen fertilizer, the numbers of livestock, and the storage and disposal of manure. In France, this implementation has been in effect since 1993.

The purpose of this paper is to model the manure management policy implemented in the European Union, and more specifically the limit imposed on the spreading of organic nitrogen. A theoretical model is defined in such a way that a number of specificities concerning livestock production can be introduced. Manure, the by-product of livestock production, is not in fact directly a polluting output. It can be used on farms as a fertilizer for productive land and only actually becomes a pollutant when it is disposed of in the environment without taking account of the uptake capacity of the soil or crops. Up to a certain level therefore, it can be used as an input in the crop production process. Beyond this level, it
is considered as an undesirable output that has to be disposed of. The European regulation on the maximum amount of organic nitrogen that can be spread per farm is applicable to the by-product of livestock production, i.e. directly to the manure itself, rather than to the actual polluting output, which is the nitrogen surplus. This particular feature, which is introduced into the theoretical framework, can be used to assess the impact of this regulatory measure on the production choices made by livestock farmers.

As a result of the specialization of pig production, farms have very little of their own land available for disposing of their undesirable output. They have to look for land outside their own farm perimeter, but within a restricted geographical area limited by the cost of transporting the manure. The theoretical framework is used to investigate how the land can be shared out optimally between the non-productive purpose of spreading manure in a manner compliant with the environmental regulation and the productive function of providing crops.

In the second part of this paper, we define an empirical model derived from the previous theoretical model, using the directional distance function. This function allows a relatively flexible representation of the production technology and environmental regulation. It also provides a framework for deriving shadow prices of pollutant, of productive and non productive use of land and of the constraint on organic manure involved by the European environmental regulation. The directional distance function also enables us to take any inefficiency present into account and therefore to measure any indirect benefits resulting from the environmental regulation.

The final part of this paper provides an illustration of the analysis framework, developed from a sample of farms in Brittany, France, some specializing in the intensive production of pigs in 1996 and others not. The illustration gives a value to the non-productive function of land, in other words to the disposal of nitrogen excess through spreading. This paper provides an analysis of the impact the European environmental regulation on manure management has on the production choices made by livestock farmers, and also an assessment of certain indirect costs and/or benefits borne by producers.

**Theoretical model**

This section presents a theoretical model incorporating undesirable output in agricultural production and more specifically considering the regulation concerning manure management in the European Union.

We consider the pig sector and assume that farms produce two consumption goods: livestock (A) and crops (C) using two distinct farming technologies. As a first step, we assume that there is two separate production function. We do not consider the link between crop and livestock production processes. All crops are assumed to be sold on the crops market. These two production functions are \( f_A(L_A, k_A) \) and \( f_C(L_C, k_C) \), respectively, with \( L_A \) the land area allocated to the disposal of manure from rearing activities and \( L_C \) the land area allocated to crop production. At this step, we assume that the total land available is \( L = L_A + L_C \) and \( L_C \subseteq L \). \( L_A \) is the land area where manure can not be spread for best agronomic practices, like lands located near rivers, drinking sources or in sensible areas. \( L_A \) and \( L_C \) are variable input while \( L \) is fixed in the short run. The main difference in the production function is the given technology and the sector specific capital \( k_A \) and \( k_C \). We assume that these production functions are differentiable and concave.
Simultaneously with production of marketable goods, we assume that an unmarketable good is produced by each farm. As a first step, we focus on undesirable output or bad. However, this model can be easily extended to unmarketable desirable goods as landscape. Thus, we assume the production of a bad output \( b \) causing environmental deterioration:

\[
b = \alpha_A f_A(L_A, k_A) + \alpha_C f_C(L_C, k_C)
\]

with \( \alpha_A \) and \( \alpha_C \) non negative coefficients.

The environmentally undesirable output related to pig and crops production is nitrogen surplus, i.e., manure that cannot be spread on farm’s land due to the crop production process chosen by the farmer related to the good agricultural practices involved.

In environmental economics, a first best optimal solution can be derived by setting up a social planner’s utility maximization problem. The bad output causing a nuisance is considered as an argument of the social planner’s utility function as it has an impact on the society’s well-being. This utility function is \( U(A, C, b) \). We assume this function to be differentiable, strictly quasi-concave, increasing in desirable outputs and decreasing in undesirable output (\( U_A, U_C > 0 \) and \( U_b < 0 \)).

The optimality conditions of this economy are derived from the following program:

\[
\begin{align*}
\max U(A, C, b) \\
\text{s.t.} \\
f_A(L_A, k_A) &= A \\
f_C(L_C, k_C) &= C \\
b &= \alpha_A f_A(L_A, k_A) + \alpha_C f_C(L_C, k_C) \\
\bar{L} &= L_A + L_C \\
L_C &\leq \bar{L}
\end{align*}
\]

For the maximization problem of the social planner, we have the following Lagrangian:

\[
\ell = U(A, C, b) + \lambda_b (f_A(L_A, k_A) - A) + \lambda_C (f_C(L_C, k_C) - C) \\
+ \lambda_3 (b - \alpha_A f_A(L_A, k_A) - \alpha_C f_C(L_C, k_C)) + \lambda_4 (\bar{L} - L_A + L_C) + \lambda_5 (L_C - \bar{L})
\]

and the first conditions are

\[
\begin{align*}
(3) \quad \frac{\partial \ell}{\partial A} &= U_A - \lambda_1 = 0 \\
(4) \quad \frac{\partial \ell}{\partial C} &= U_C - \lambda_2 = 0 \\
(5) \quad \frac{\partial \ell}{\partial b} &= U_b + \lambda_3 = 0 \\
(6) \quad \frac{\partial \ell}{\partial L_A} &= \lambda_b f_A^{L_A} - \lambda_2 \alpha_A f_A^{L_A} - \lambda_4 = 0 \\
(7) \quad \frac{\partial \ell}{\partial L_C} &= \lambda_C f_C^{L_C} - \lambda_3 \alpha_C f_C^{L_C} - \lambda_5 = 0
\end{align*}
\]

From (3), (5) and (6), we have:

\[
(7) \quad U_A = \lambda_2 \alpha_A + \frac{\lambda_4}{f_A^{L_A}} \Rightarrow U_A + \alpha_A U_b = \frac{\lambda_4}{f_A^{L_A}}
\]
and from (4), (5) and (7), we have:

\[(8) \quad U_C = \lambda_1 a_C + \frac{\lambda_5}{f_L} \Rightarrow U_C + \alpha_C U_b = \frac{\lambda_5}{f_L} \]

Equations (7) and (8) are the optimality conditions that guarantee an efficient input use in livestock and crop farming processes. In particular, the environmental deterioration is optimal up to the point where the marginal utility value of the marketable goods plus the marginal disutility value of the unmarketable good equals the private cost of production of the desirable goods. Consequently, each technology (livestock and crops) should be optimally implemented when the following equation hold:

\[(9) \quad \frac{U_A + \alpha_A U_b}{U_C + \alpha_C U_b} = \frac{\lambda_A f_L^A}{\lambda_C f_L^C} \]

Assume that competitive farms are producing both products at market prices \( p_A \) and \( p_C \), respectively. Farms are assumed to determine the level of desirable goods on the basis of a profit maximisation objective function. In the specific case of pig production, there exists an environmental standard concerning the spreading of organic manure from rearing activities. The regulation states the maximum amount of organic manure spread such as:

\[(10) \quad \alpha_A f_A \leq \bar{b}_A L_A \]

where \( \bar{b}_A = 170 \text{ kg/ha} \) of organic manure spread on fields. Thus, we have:

\[(11) \quad \pi_A = p_A f_A - r_A L_A - q_A (\bar{b}_A L_A - \alpha_A f_A) \]
and
\[(12) \quad \pi_C = p_C f_C - r_C L_C \]

where \( r_A \) and \( r_C \) are land rent and \( q_A \) the shadow price or Lagrangian multipliers related to the organic manure constraint. The optimality conditions imply:

\[(13) \quad p_A f_{L_A}^A - r_A + q_A \alpha_A f_{L_A}^A - q_A \bar{b}_A = 0 \]
and
\[(14) \quad p_C f_{L_C}^C - r_C = 0 \]

Thus, we can write that:

\[(15) \quad \frac{p_A + q_A (\alpha_A - \frac{\bar{b}_A}{f_{L_A}^A})}{p_C} \quad \frac{f_{L_A}^C}{r_A f_{L_A}^A} \quad \frac{f_{L_C}^C}{r_C f_{L_A}^A} \]

and that:
The above optimal conditions imply that regulation setting a constraint $\bar{b}_d$ on the spreading of organic manure from rearing activities correspond to a specific relation where the level of the restriction and the shadow price related to organic manure. If the negative environmental impact of these activities is not introduced, then the usual measures of economic efficiency are biased. The shadow price \( q_A \) provides an indication on how producers value the constraints on organic manure induced by the regulation on organic manure.

Furthermore, the above model provides some additional information concerning the value of agricultural land in two different functions. The rent value \( r_d \) characterises the value of land in its unproductive function that is the disposal of manure while the rent value \( r_c \) is the value of land in its productive function.

**Empirical model**

In this section, we provide an empirical model for the previous theoretical framework. We based our analysis on developments from Färe et al. (2001, 2005) and Färe and Grosskopf (2004). These authors provide a generalisation of the Shephard output distance function that allows one to consider non-proportional changes in outputs. The directional output distance function allows one output to be expanded while another is contracted. This property is particularly suitable when we study polluting outputs since most of the regulation concerning these undesirable goods is interested in pollution reduction. The directional output distance function is also a measure of efficiency. Since it simultaneously accounts for pollution reduction and good improvements, it is a combined environmental and technical efficiency measures.

Let \( y = (y_1, \ldots, y_M) \in R^M_+ \) and \( b = (b_1, \ldots, b_S) \in R^S_+ \) be vectors of goods and bad outputs, respectively. Let \( x = (x_1, \ldots, x_N) \in R^N_+ \) be a vector of inputs. The reference technology can be described as an output possibilities set \( P(x) \) that for a given vector of inputs denotes all technically feasible output vectors. This output set is assumed to be convex and compact with \( P(0) = \{0,0\} \). Furthermore, inputs and good outputs are assumed to be freely disposable and bad outputs only weakly disposable. Livestock good and organic manure good are assumed to be null-joint. This means that livestock cannot be produced without organic manure. The directional output distance function is defined on \( P(x) \) as:

\[
(17) \quad \bar{D}_o(x, y, b; g) = \max \left\{ \beta : (y + \beta g_y, b - \beta g_b) \in P(x), g_y \in R^M_+, g_b \in R^S_+ \right\}
\]

The function in (17) inherits its properties from \( P(x) \). If \( \bar{D}_o(x, y, b; g_y, -g_b) = 0 \), the farm is efficient in the \((g_y, -g_b)\) direction. Feasible but inefficient farms will take a value greater that zero, reflecting the additional good output and reduction in pollution that this particular productive unit could achieve if it were on the best practice frontier. The higher the value the more inefficient is the productive unit.

This approach can be used to derive shadow prices based on the duality between the directional output distance function and the revenue function. Following Färe et al. (2005), let
\( D_o(x,y,b;1,-1) = 0 \) be the selected output distance function. This function completely characterises the technology. Let \( p = (p_1, \ldots, p_M) \in R^M_+ \) and \( q = (q_1, \ldots, q_S) \in R^S_+ \) represent absolute prices of good and bad outputs, respectively. In our specific case, \( p \) is desirable good prices (livestock and crops) while \( q \) is the price of undesirable outputs (pollution). The revenue function can be specify as:

\[
R(x, p, q) = \max \left\{ py - qb : D_o(x, y, b;1,-1) \geq 0 \right\}
\]

Chambers et al. (1998) have shown that the lagrangian multiplier for the above problem is:

\[
\lambda = pg_y - qg_b = p.1 - q.1 \text{ for } g = (1,-1).
\]

Therefore, we can write the revenue function as:

\[
R(x, p, q) = \max \left\{ py - qb + (p.1 + q.1)D_o(x, y, b;1,-1) \right\}
\]

Equation (19) is an unconstrained maximisation problem. From the first-order conditions, we get:

\[
(1 + q.1)\nabla_y D_o(x, y, b;1,-1) = -p
\]

\[
(1 + q.1)\nabla_b D_o(x, y, b;1,-1) = q
\]

and

\[
-\frac{p}{q} = \frac{\nabla_y D_o(x, y, b;1,-1)}{\nabla_b D_o(x, y, b;1,-1)}
\]

Furthermore, if we assume that one price is known, for example, one of the market prices of the desirable outputs, then we may compute the absolute price for the undesirable output as argued in Färe et al. (2005). Here, we assume that \( p_1 \), the absolute price for the first output is known and is equalled to its shadow price \( (p_1 = q_1) \). Then, we may compute the price of the bad output \( b \) as:

\[
q = -p_1 \left( \frac{\nabla_y D_o(x, y, b;1,-1)}{\nabla_b D_o(x, y, b;1,-1)} \right)
\]

**Empirical illustration**

Let \( J \) observations \( (j=1, \ldots, J) \). The program in (17) can be rewritten in a DEA (Data Envelopment analysis) framework as:

\[
D_o(L_A, L_C, k_A, k_C, C, A, b;1,1,-1) = \max \beta
\]
\[
\begin{align*}
\sum_{j=1}^{J} z_j C_j & \leq C_j \\
\sum_{j=1}^{J} z_j A_j & \leq A_j \\
\sum_{j=1}^{J} z_j b_j & = b_j \\
\sum_{j=1}^{J} z_j L_{A,j} & \geq L_{A,j} \\
\sum_{j=1}^{J} z_j L_{C,j} & \geq L_{C,j} \\
\sum_{j=1}^{J} z_j k_{A,j} & \geq k_{A,j} \\
\sum_{j=1}^{J} z_j k_{C,j} & \geq k_{C,j}
\end{align*}
\]

The dual values for each equation can be evaluated at the optimum of these linear programs. Thus, if we assume that \( p_c \), the absolute price for crops output is known and is equalled to its shadow price (\( p_c = q_c \)). we have:

\[(24) \quad q_b = -p_c \left( \frac{\nabla_b \bar{D}_p(L_A, L_C, k_A, k_C, C, A, b; l, 1, -1)}{\nabla_c D_p(L_A, L_C, k_A, k_C, C, A, b; l, 1, -1)} \right) \]

with \( q_b \) the shadow price of the polluting good, i.e., the nitrogen surplus.

Based on the same reasoning, it is possible to derive the price of land in its two distinct function: the productive one and the disposal one.

\[(25) \quad r_A = -p_c \left( \frac{\nabla_{L_A} \bar{D}_p(L_A, L_C, k_A, k_C, C, A, b; l, 1, -1)}{\nabla_c D_p(L_A, L_C, k_A, k_C, C, A, b; l, 1, -1)} \right) \]

In order to evaluate the shadow price of the constraint on organic manure, we have to develop the link between nitrogen surplus and organic manure as follows:

\[(26) \quad b = \alpha_A f_A(L_A, k_A) + \alpha_C f_C(L_C, k_C) \text{ and } \alpha_A f_A(L_A, k_A) \leq \bar{b}_A L_A \]

In the case of nitrate management, the equation (26) can be rewritten as:

\[(27) \quad b = N_{org} + N_{min} - N_{exp} \text{ and } N_{org} \leq 170 \cdot L_{Norg} \]

with \( N_{org} \) the level of organic manure produced by rearing activities on farm, \( N_{min} \) the level of mineral fertilizers used in crops production, \( N_{exp} \) the level of nitrogen exported by crops
during their growth process and $L_{\text{org}}$ the land acreage on which organic manure can be spread. When, we introduce the above equations, we can derive the shadow prices for the nitrogen surplus and for the constraint on the spreading of organic manure.

**Data**

This analytical framework is illustrated on a sample of farm data from Brittany pig farms that participate in the Farm Accountancy Data Network in 1996. The selective sample consists on 188 farms that are specialised in pig production and have a nitrogen surplus. Some of them do not respect the constraint imposed by the European regulation on organic manure.

The directional output distance function is estimated on the following input and output variables. The good outputs are total livestock products and the total crops products in euros. The bad output is nitrogen surplus. It is an estimation based on the nitrogen balance of each farm. Inputs are an aggregate in euros describing the capital structure of the farm with capital, machinery and buildings, labour in annual worker units, arable area in hectare and an aggregate in euros of other variable inputs. These variables describe the technology of this productive sector and are used to derive the shadow prices of the polluting good, of the restrictive constraint on organic manure spreading and of the productive and non-productive land use.

**Results and Discussion**

Forthcoming

**References**


Färe R. and S. Grosskopf (1998), Shadow Pricing of Good and Bad Commodities, American Journal of Agricultural Economics 80, 584-590.

