Productivity and Efficiency Analysis for Livestock Grazing under Grazing Pressure using Directional Distance Function

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Abstract

With the use of first hand field survey data from 193 yak grazing households combined with remote sensing Net Primary Productivity data on the Qinghai-Tibetan Plateau, a directional output-orientation distance function is developed with four inputs, grassland area, labor, capital and initial livestock stocking, and two outputs, good output of livestock grazing revenue and undesirable output of grazing pressure. The average technical efficiency is estimated to be 0.82, and shadow price of grazing pressure to livestock revenue is estimated to be -1.8. According to Morishima elasticity of substitution between inputs, there is significant complementary relationship between grassland area, labor and capital. Elasticity of substitution between grassland and initial livestock stocking is estimated to be 0.50. Treating grazing pressure as an undesirable output of livestock grazing in the directional distance function is a new step in the general direction of better accounting for natural resource depletion in efficiency and production analysis.

Keywords: directional distance function, grazing pressure, technical efficiency, shadow price, Morishima elasticity of substitution
1. Introduction

The concerns about environmental problems caused by economic development in developing countries have received a lot of attention in recent years. Grassland is one of the main land use types on earth and is essential for livestock grazing and grassland ecosystem service supply; the method of finding a balance between environment and livestock grazing has been a major focus in research (De Haan et al., 1997; White, et al., 2000; McDowell, 2008). However, demand for livestock products is being accelerated by population growth, economic growth and expanding urbanization, especially in developing countries or regions. High demand for livestock products has resulted in increasing grazing pressure, which is associated with overgrazing.

Overgrazing appears when the stocking rate exceeds the proper grassland carrying capacity, and is accompanied with increasing grazing pressure, which threatening both economically and ecologically the sustainable use of grassland, e.g. to result in grassland degradation. Three quarters of the world’s grazing lands are so degraded that they have lost more than 25% of their capacity to support animals (White, et al., 2000; UNEP, 2005), as well as that on The Qinghai-Tibetan Plateau is a perfect example of a region heavily affected by advancing grassland degradation over wide areas and overgrazing is assumed to be one of the main courses (Akiyama, et al., 2006; Zhou, et al., 2006; Zhang, 2008; Harris, 2010).

Grassland in the Qinghai Province of China, one of the largest grasslands in China, was found to have a high overgrazing status by comparing actual and proper livestock carrying capacity (Fan et al., 2011; Zhang et al., 2014). Fan et al. (2011) studied the temporal-spatial dynamics of grazing pressure during the period from 1988 to 2005. Over grazing was considered as one of the main factors resulting in grassland ecosystem degradation, although the grazing pressure was steadily reduced. In recent research on the overgrazing status in the Sanjiangyuan region, overgrazing still remained a problem in 2010 (Zhang, Zhang and Liu et al., 2014). Combining the data from the field survey, the overgrazing ratio is more than 300% in sample counties (Table 1), overgrazing remains a serious problem and the overgrazing status is highly correlated with grazing pressure. The strong effect of the overgrazing status on grassland degradation makes us interested in researching the performance of livestock grazing under the control of grazing pressure, by adopting the grazing pressure as an undesirable output from livestock grazing using the directional distance function.
The directional distance function was probably first proposed by Chung et al. (1997) and Chambers et al. (1998) with a similar introduction previously from the Luenberger index. It has surged in popularity over the last 10 years (Färe, et al., 2013; Feng and Serletis, 2014). Rather advantageously, the directional distance function loses the assumption that radial changes of inputs or outputs take place simultaneously. In reality, the pursuit of more undesirable outputs (“bad outputs”) is not encouraged proportional in producing good outputs. It prevails to use directional distance analysis for polluting technology, which produces pollution as a byproduct, such as electric utilities producing electricity and air pollution (Atkinson and Dorfman, 2005; Färe et al., 2005; Cuesta et al., 2009; Coelli et al., 2013; Wang et al., 2013; Yao et al., 2015) and dairy farming (Reinhard 1999; Reinhard et al. 2000; Fernández et al., 2002; Reinhard 2002; Sauer and Latacz-Lohmann, 2014; Njuki and Bravo-Ureta, 2015).

Before the development of the directional distance function, the productivity of the decision making unit when some outputs are undesirable has been studied. Pittman was perhaps the first to develop an index of productivity change which takes environmental effects into account. He modeled pollution as an input in the production function because of the relation between an environmentally detrimental variable and output (Pittman 1981; Pittman 1983). Undesirable outputs in pulp mills industry sector have been heavily researched; such topics have included the environmental effects of undesirable outputs in the Finnish pulp and paper industry (Hetemäki, 1996), Swedish pulp and paper industry (Brannlund et al. 1998) and the paper recycling industry in Vietnam (Van Ha et al., 2008). Färe et al. (1986; 1989; 1993) modeled environmental effects as undesirable outputs with U.S. electricity generation data by using econometric models. The directional distance function was then applied later by other researchers (Cropper and Oates,1992; Yaisawarng and Klein, 1994).

To evaluate the environmental goods - sometimes called nonmarket goods - such as air pollution emissions, soil pollution or ecological diversity loss from human economic activity, relative shadow prices of nonmarket goods are derived based on the distance function as well as elasticities of complementary or substitutionary relationships among inputs or outputs (Blackorby and Russel, 1989; Färe et al.,1993; 2005; Morrison Paul et al., 2000; 2005; Hailu, 2000; Cuesta, 2009; Rahman, 2010; Serra et al., 2011). In this paper, we incorporate grazing pressure as an undesirable output from livestock grazing to determine the environmental-grazing relationship.
We extend the contribution of the directional distance function by incorporating the grazing pressure as an undesirable output and deriving the shadow prices of grazing pressure to grazing revenue, elasticity of directional distance with respective to inputs and outputs, and elasticity of complementary or substitutionary relationships among inputs. The directional distance function with MLE estimation procedure is developed by using 193 households’ level data on livestock grazing. We would like to stress a deeper understanding of the performance of extensive livestock grazing on the Qinghai-Tibetan Plateau by taking into account the grazing pressure.

The structure of the paper is as follows. Section 2 presents the theoretical framework, methodology. Section 3 presents the empirical model specification and data description. The empirical analysis results are presented in section 4, followed by section 5 which concludes with discussion.

2. Theoretical framework and methodology

A multi-input multi-output directional distance function incorporating grazing pressure as the bad output is developed in order to measure the production performance of grassland grazing under the framework of environmental efficiency. As grassland grazing on the Tibetan Plateau still adopts the traditional half-nomadic pastoral system (Davies and Hatfield, 2007; Harris, 2010), this might be advantageous for the distance function, not considering the exact price of inputs and outputs. We adopt the distance function approach instead of the deterministic approach because of the advantage of the stochastic approach to separate the random noise from the technical inefficiency term.

2.1 Conceptual framework

The directional distance function is applied to all properties of the distance function introduced by Shephard (1970) with defining inputs $x = (x_1, \cdots, x_K) \in \mathbb{R}^K$, outputs $y = (y_1, \cdots, y_M) \in \mathbb{R}^M$, and the output possibility set $P(x) = \{y : x \text{ can produce } y\}$, which is assumed to satisfy the set of axioms depicted in Färe et al. (2000). The directional distance function measures the distance from the production unit to the efficiency boundary along with a directional vector, given the directional vector $g = (-g_x, g_y)$ with $g_x \in \mathbb{R}_+^N$ and $g_y \in \mathbb{R}_+^M$, determined by which of the inputs would be contracted and which outputs would be expanded, as described in Figure 1, when firms adjust the production behavior along the vector from producing point A. Then, the directional distance function is given by
\[ \bar{D}(x, y; g_x, g_y) = \sup\{\vartheta: (x - \vartheta g_x, y + \vartheta g_y) \in P \} \]  

where \( \bar{D}(x, y; g_x, g_y) \geq 0, \vartheta \in \mathbb{R} \), which inherits all properties of the directional distance function described in Chambers et al. (1998) and Färe et al. (2005). The directional distance function can take the quadratic form as:

\[
\bar{D}(x, y; g_x, g_y) = \sum_{k=1}^{N} \alpha_k x_k + \sum_{j=1}^{M} \beta_j y_j + \sum_{k=1}^{N} \sum_{l=1}^{N} \alpha_{kl} x_k x_l + \sum_{j=1}^{M} \sum_{h=1}^{M} \beta_{jh} y_j y_h \\
+ \sum_{k=1}^{N} \sum_{j=1}^{M} \gamma_{kj} x_k y_j
\]

The translation property of the directional distance function allows for an empirical use:

\[ \bar{D}(x - \vartheta g_x, y + \vartheta g_y; g_x, g_y) = \bar{D}(x, y; g_x, g_y) - \vartheta \]  

This property indicates the producer decreases the distance to efficiency boundary by scalar \( \vartheta \), while output is improved by \( \vartheta g_y \) and input is reduced by \( \vartheta g_x \) simultaneously, given the technology is available. It is a radial input distance function if \( g_y = 0 \), as the firm close to the efficient frontier from point A to point B; and it is a radial output distance function if \( g_x = 0 \), when the firm is moving the producing point from point A to point C, therefore, the radial distance function is – in special cases - the directional distance function (Färe and Grosskopf, 2000). Based on the directional distance function framework, the output oriented directional distance function and the input oriented directional distance function can be derived. We use the output oriented directional distance function for grassland grazing in this paper.

[Figure 1. Directional distance function]

Specifically, the producer is efficient in the director \((-g_x, g_y)\) if

\[ 0 = \bar{D}(x, y; g_x, g_y) + \varepsilon_i \]  

where \( \varepsilon_i = v_i - u_i, v_i \sim \text{i.i.d. } N(0, \sigma_v^2) \) and \( u_i \sim N(\mu_i, \sigma_u^2)^+, i = 1, 2, \ldots, N. \)

Hence, by substituting (4-4) into (4-3), we get

\[
-\vartheta_i = \bar{D}(x - \vartheta g_x, y + \vartheta g_y; g_x, g_y) + v_i - u_i
\]  

5
The translation property given above requires the restrictions as discussed in Färe et al. (2005), which will be explained in more detail in next section. The mean \( \mu_i \) is defined for technical inefficiency \( u_i \), where the technical inefficiency model is written in equation (6).

\[
\mu_i = \sum_{c=1}^{c} \tau_c \times Z_{ci}
\]

(6)

where \( Z_{ci} \) is a vector of household characteristic variables associated with the technical inefficiency effects and \( \tau_i \) is a vector of unknown parameters to be estimated (Battese and Coelli, 1988, 1995, 1997; Coelli and Battese, 1996). Maximum Likelihood Estimation (MLE) could be used to estimate the parameters (Aigner, Lovell and Schmidt, 1977).

Given the production frontier specified as

\[
Y_i = f(X_i, \beta). \exp(v_i - u_i)
\]

(7)

Technical efficiency (TE) is defined as the ratio of the observed output to the corresponding potential output, which is written as

\[
TE_i = \frac{f(X_i, \beta). \exp(v_i - u_i)}{f(X_i, \beta). \exp(v_i)} = \exp(-u_i)
\]

(8)

2.2 Relative shadow prices and Morishima elasticity of substitution

Thanks to the duality theory between the distance function and cost function or revenue function (input distance function for cost minimization function, output distance function for revenue maximization function), shadow prices for non-market goods can be derived (Shephard, 1970; Färe and Primont, 1996). Assuming that the directional input (output) distance function and the cost (revenue) functions are differentiable, application of Shepherd's dual lemma would lead to the shadow price formulas:

\[
\nabla_x \tilde{D}(x, y) = r^*(x, y)
\]

(9)

where \( r^*(x, y) \) is the cost minimizing inputs prices vector.

Because the input prices are not available and optimal cost of production cannot be accurately estimated in this paper, then relative shadow prices can be derived from the following formula:

\[
R_{kl} = \frac{r_k^*}{r_l^*} = \frac{\partial \tilde{D}(x, y)/\partial x_k}{\partial \tilde{D}(x, y)/\partial x_l}
\]

(10)
where \( r_k^* \) and \( r_l^* \) are the shadow prices of the inputs \( x_k \) and \( x_l \), respectively. This ratio is the relative shadow price of input \( x_k \) with respect to input \( x_l \). These shadow prices reflect the trade-off between different inputs (Färe et al., 1993; Hailu and Veeman, 2000).

Because of the duality between the input distance function and the cost function, the degree of substitutability along the surface frontier, such as frontier curvature, can be calculated. The indirect Morishima elasticity of substitution (MES) from a directional input distance function can be computed as formula (11) (Blackorby and Russell, 1989).

\[
MES_{kl} = -\frac{\partial (\bar{D}_k/\bar{D}_l)}{\partial (x_k/x_l)}
\]

(11)

where the subscripts on the distance functions refer to partial derivatives with respect to inputs. This represents the change in relative marginal products and input prices required to affect substitution under cost minimization. High values reflect low substitutability and low values reflect relative ease of substitution between the inputs (Morrison-Paul et al., 2000). The MES can be simplified as follows:

\[
MES_{kl} = \varepsilon_{kl} - \varepsilon_{kk}
\]

(12)

where \( \varepsilon_{kl} \) and \( \varepsilon_{kk} \) are the constant output cross and own elasticity of shadow prices with respect to input quantities. The first term provides information on whether pairs of inputs are net substitutes or net complements, and the second term is the own price elasticity of demand for the inputs. It should be noted that these elasticities are indirect elasticities. Therefore, for \( \varepsilon_{kl} > 0 \) net complements are indicated, and for \( \varepsilon_{kl} < 0 \) net substitutes are indicated.

The shadow price elasticities with respect to input quantities are given by:

\[
\varepsilon_{kl} = (\alpha_{kl} + S_k S_l)/S_k \text{ if } k \neq l
\]

(13)

\[
\varepsilon_{kk} = [\alpha_{kk} + S_k (S_k - 1)]/S_k \text{ if } k = l
\]

(14)

where \( S_k \) is the first order derivatives of the distance function with respect to input \( x_k \), that is

\[
S_k = \partial \bar{D}/\partial x_k
\]

(15)
3. Empirical model specification and data descriptive statistics

3.1 Empirical model specification and estimation measurement

Following Chambers et al. (1998, 2002) and Färe et al. (2005), we first build the output oriented directional distance function in equation (16). The advantage of the output oriented directional distance function is that it allows us to expand the good output while contracting the bad output while leaving inputs unchanged, as showed in Figure 2. Assuming point A is the production point of a household, then the household improves the production along the directional vector \( g = (g_y, -g_b) \), that is to add \( \vartheta g_y \) to good output \( y \) while subtracting \( \vartheta g_b \) from the bad output \( b \).

\[
D_0^\rightarrow (x, y, b; g_y, -g_b) = \sup \{ \vartheta : (y + \vartheta g_y, b - \vartheta g_b) \in P \}
\]  \( (16) \)

While satisfying the translation property, equation (16) can be denoted as equation (17).

\[
D_0^\rightarrow (x, y + \vartheta g_y, b - \vartheta g_b; g_y, -g_b) = D_0^\rightarrow (x, y, b; g_y, -g_b) - \vartheta
\]  \( (17) \)

We parametrically estimate the directional distance using stochastic estimation methods following Kumbhakar and Lovell (2000), then the empirical stochastic specification form is written in equation (18).

\[
-\vartheta_i = D_0^\rightarrow (x, y + \vartheta g_y, b - \vartheta g_b; g_y, -g_b) + v_i - u_i
\]  \( (18) \)

Assuming \( g = (g_y, -g_b) = (1, -1) \), the quadratic form for our case, 4 inputs and 2 outputs (1 good output \( y \) and 1 bad output \( b \)), is denoted by equation (19).

\[
D_0^\rightarrow (x, y, b; 1, -1) = \alpha_0 + \sum_{k=1}^{4} \alpha_k x_k + \beta_1 y + \beta_2 b + \frac{1}{2} \sum_{k=1}^{4} \sum_{l=1}^{4} \alpha_{kl} x_k x_l + \frac{1}{2} \beta_{11} y^2 + \frac{1}{2} \beta_{22} (b)^2 + \sum_{k=1}^{4} \gamma_{k1} x_k y + \sum_{k=1}^{4} \gamma_{k2} x_k b + \delta y b
\]  \( (19) \)

To hold the translation property, the required restrictions are

\[
\beta_1 - \beta_2 = -1, \beta_{11} = \beta_{22} = \delta, \gamma_{k1} = \gamma_{k2}, k=1,2,3,4.
\]

Additional, symmetry conditions require: \( \alpha_{kl} = \alpha_{lk}, k = l = 1,2,3,4. \)
In our case, we choose \( \theta = b \), then the quadratic form of the empirical specification for grassland grazing is

\[
-\vartheta_i = \overrightarrow{D}_o(x, y + b, 0) + v_i - u_i
\]

where \( y^* = y + b \). \( y \) describes the good output of grassland grazing, denoted by the revenue of livestock meat and milk produced in the year; \( b \) denotes the bad output, grazing pressure, defined as ratio of livestock live weight divided by grassland forage biomass. \( X \) is the vector of inputs with \( x_1 \) = grassland area size, \( x_2 \) = labor, \( x_3 \) = household productive capital and \( x_4 \) = initial yak stock. \( v_i \) is a random error term, intended to capture events beyond the control of the herdsman and \( u_i \) is a non-negative random error term, intended to capture technical inefficiency in production. In order to compare different effects of the directional vector, we use \( g = (g_y, -g_b) = (1, -1) \) and \( g = (g_y, -g_b) = (1, 0) \) in empirical analysis, the setting with \( g = (1, 0) \) means ignoring the bad output in the production process.

The technical inefficiency model referred to equation (6) in this chapter is written as

\[
\mu_i = \tau_0 + \sum_{c=1}^{8} \tau_c * Z_{ci}
\]

where \( Z \) is a vector of explanatory variables associated with the technical inefficiency effects including total NPP change of each household \( (z_1) \), household size \( (z_2) \), distance from fixed home to summer pasture \( (z_3) \), grazing experience \( (z_4) \), summer pasture area \( (z_5) \) and winter pasture area size \( (z_6) \), dummy variable of pasture plot \( (z_7) \) and dummy variable of whether there is leased-in grassland from other households \( (z_8) \).

3.2 Data and descriptive statistics

The social-economic data used in this paper was drawn from field survey data in the Sanjiangyuan region in Qinghai province conducted by the Center for Chinese Agricultural Policy (CCAP) of the Chinese Academy of Sciences in August and October, 2012. The Net Primary Production data is from the MODIS GPP/NPP Project. The Sanjiangyuan region in
China, known as the Three-River Headwaters in English, is located in the northeast of the Qinghai-Tibetan Plateau, where more than 90% of the local people are of Tibetan Ethnic Minority. The average elevation of the Sanjiangyuan region is between 3500 and 4800 m. Like other parts of the Tibetan plateau, a cold season from approximately November to the following May and a warm season from June to October can be identified. The annual mean temperature is about 1 to 2 degrees Celsius, and the annual precipitation ranges from 600 mm to 800 mm. The stratified random sampling method was used to select observations and 144 of them are available for this paper.

Classic inputs are aggregated into four categories (grassland area, labor, capital and initial yak) and outputs are aggregated into two categories (y as good output of revenue from grassland grazing and bad output b, grazing pressure). There are two kinds of pastures on the Qinghai-Tibetan Plateau: Summer/autumn pasture and winter/spring pasture, where grassland area \( x_1 \) is the sum of summer pasture area and winter pasture area for each household. Labor \( x_2 \) consists of family labor, measured by person. Capital \( x_3 \) consists of productive machinery (irrigation machine, transportation machine and so on). It is calculated by sum aggregation of each item obtained from the questionnaires. Initial yak stock \( x_4 \) means the initial yak input at the beginning of the year and is calculated by multiplying the average weight of a yak by the yak number per household. Good output y denotes the revenue of yak meat produced in the year and revenue of the other outputs, including the revenue from Tibetan sheep meat, output of milk, yak hide, Tibetan sheep wool and so on. Bad output b denotes the grazing pressure of livestock grazing on the Qinghai-Tibetan Plateau; detailed information of bad output b, grazing pressure, is given in the next paragraph.

The important variable that we are focusing on is the bad output, grazing pressure. Grazing pressure is international terminology which forms the relationship between animal live weight and forage mass per unit of grassland on the grazed land given a specific time (Allen et al., 2011). Grazing pressure is highly positively correlated to the over grazing ratio: The more the livestock stocking rate surpasses the proper carrying capacity, the higher the grazing pressure. As grazing pressure is the relationship of animal-to-forage ratio, we calculate grazing pressure as the ratio between livestock live weight and total grassland Net Primary Productivity (NPP), where total grassland NPP is used as representative of grassland biomass. Grassland total NPP is computed with unit NPP multiplying the total grassland area, while unit NPP is computed with daily MODIS land cover, FPAR/LAI and global GMAO surface meteorology at 1km for the global vegetated land surface (Zhao and Running, 2010). These variables provide the
initial calculation for growing season and carbon cycle analysis, and are used for agriculture, range and forest production estimates. After matching the rough boundary of summer pasture and winter pasture to the 1km NPP raster data file, and getting samples of NPP for each pasture according to the pasture area, we summarize the NPP of pastures for each grazing household.

For the technical inefficiency model, operational and farm-specific variables were considered including the total NPP change of each household’s pasture \( (z_1) \), household size \( (z_2) \), distance from fixed home to summer pasture \( (z_3) \), grazing experience \( (z_4) \), summer pasture area \( (z_5) \) and winter pasture area size \( (z_6) \), dummy variable of pasture plot \( (z_7) \) and dummy variable of whether there is leased-in grassland from other households \( (z_8) \). Total NPP change of each household \( (z_1) \) is calculated by total NPP in the year 2012 subtracted by total NPP in the year 2011. Household size \( (z_2) \) is the population in one family. Distance from fixed home to summer pasture \( (z_3) \) measures the geographic distance from the fixed home to the summer pasture. Grazing experience \( (z_4) \) denotes how many years of grazing experience each household head has. Summer pasture area \( (z_5) \) and winter pasture area \( (z_6) \) are pasture area size in summer and winter respectively, while dummy variable of pasture plot \( (z_7) \) means whether the summer pasture and winter pasture are located in the same plot or adjacent plots. Dummy variable of lease-in grassland \( (z_8) \) measures whether there is leased-in grassland from other households, which equals 1 if the household grazes on leased-in grassland, and 0 for otherwise. A statistic description of variables in the directional distance function and technical inefficiency model is shown in Table 2.

\[ \text{Table 2. Statistic descriptive of variables} \]

4. Results

Before presenting the performance of inputs and outputs in the directional distance function, we compared the directional distance functions with different assumptions of directional vectors \( g = (g_y, -g_b) = (1, -1) \) and \( g = (g_y, -g_b) = (1, 0) \) (Table 3). We assigned model 1 with directional vector \( g = (1, -1) \), which means to expand good output while subtracting bad output of the grazing pressure. In model 2, \( g = (1, 0) \) is assumed, denoting the expansion of good output without subtracting the bad output grazing pressure. A Hausman test is used to compare the two models with a null hypothesis that there are no systematic differences between the two. From the Hausman test below Table 3, we can see the null hypothesis is strongly rejected, indicating that there are systematic differences between the estimates of the
two models. Taking into account the distributional signals and monotonicity conditions, model 1 with directional vector \( g = (1, -1) \) is preferred. Hereafter, all analysis and estimates will be based on the directional vector of \( g = (g_y, -g_b) = (1, -1) \) in model 1. The \( \delta_u \) is estimated to be 0.586, which means relatively unignorably inefficiency term \( u_i \), which supports us in setting the directional distance function combined with technical inefficiency model, as showed Table 3, where the likelihood value is -3.554 with degree of freedom 31. The likelihood value of model 1 is -36.824 with degree of freedom of 23. According to the likelihood ratio test, \( LR \, Chi^2(8) = 66.54 \), which is significantly larger than the criteria value of \( x_{0.005}^2(8) = 21.955 \); this implies that the setting of the technical inefficiency model would definitely improve the model specification. We took likelihood ratio tests for the technical inefficiency model setting (Appendix Table 1). According to all the likelihood ratio tests and one-sided inefficiency random components, we choose the final model setting in Table 4, and consequently calculate the first order conditions, shadow price and Morishima elasticity of substitutions between inputs.

**Table 3. Directional distance function with different directional vector**

**Table 4. Estimates of directional distance function and technical inefficiency model**

### 4.1 Parameter estimates of directional distance functions

The one-step approach for both the directional distance function and technical inefficiency model using maximum likelihood is presented in Table 4, with all variables divided by mean. Most coefficients are estimated to be significant; especially the first order coefficient and second order coefficient of good output \( y \), and cross interacting terms between input and good output \( y \). The output distance function is concave in outputs, thus, \( \partial^2(D_o(x, y, b; 1, -1))/\partial y\partial y = \beta_{11} \leq 0 \), and according to the translation property, \( \partial^2(D_o(x, y, b; 1, -1))/\partial b \partial b = \partial^2(D_o(x, y, b; 1, -1))/\partial y \partial b = \beta_{11}, \beta_{11} \) is estimated to be -0.263, significant at the 1% statistical level.

Based on the estimates from the directional distance function, the elasticities of the directional distance function with respect to inputs and outputs are calculated to get a full understanding of the performance of inputs and outputs in the grassland grazing process, elasticities of the sample mean are presented (Table 5). A T Test is used to test whether the elasticities are different from zero at the 10% statistical level. The monotonicity conditions of the directional distance function require \( \partial^2(D_o(x, y, b; 1, -1))/\partial x \geq 0 \). With the exception of input \( x_4 \), the
initial yak at the beginning of the year, elasticity of distance with respect to inputs grassland area size, labor and capital have expected positive signs, implying that increasing the input of any of these inputs will increase production potential substantially. The largest elasticity of the directional distance with respect to inputs comes from the grassland area, which is estimated to be 0.64, implying a 1% increase of grassland area would enhance production potential by 0.64%. The monotonicity conditions of the directional distance function for outputs require \( \partial (D_o(x, y, b; 1, -1))/\partial y \leq 0 \) and \( \partial^2 (D_o(x, y, b; 1, -1))/\partial b \geq 0 \). The elasticity of distance with respect to good output \( \varepsilon_y \) is -0.36 and elasticity of bad output grazing pressure \( \varepsilon_b \) is estimated to be 0.64; both are significant at the 1% statistical level. A 1% increase in good output would reduce the distance by 0.35%, while a 1% increase in bad output, grazing pressure, would expand the distance by 0.64%.

[Table 5. Elasticity of distance with respect to inputs and outputs]

4.2 Shadow price of grazing pressure

As grazing pressure cannot be traded in the market directly, the relative shadow prices of grazing pressure to revenue of livestock grazing are calculated for a better understanding of their relationship with each other. The relative shadow price of grazing pressure is estimated to be -1.8 at the sample mean, which means the “price” coming from grazing pressure is higher than production of one unit of good output. As there is no reason to interpret shadow price for the observations that violate the monotonicity conditions (Färe et al., 2005), we summarize the relative shadow price for a partial sample which meets the monotonicity conditions in the third column of Table 6. Thus, we can see that the relative shadow price of grazing pressure is -3.99, which means that the higher cost household should pay for one unit production of good output, again confirming that grazing pressure is a bad output from livestock grazing. In previous literature on environmental efficiency analysis, most of the studies on shadow prices of environmental outputs were assumed to be negative (Reinhard, 1999; Färe et al., 1993, 2005; Hailu and Veeman, 2000), which mean that these environmental outputs are “bad outputs”. The value of \( M_{by} \) is estimated to be -0.55 for all samples, and -0.73 for partial samples which meet the monotonicity condition. A more negative \( M_{by} \) indicates a greater change of relative shadow price of grazing pressure to good output livestock revenue, thus resulting in a greater cost to reduce the grazing pressure.

[Table 6. Relative shadow price of outputs and elasticity of transformation]
4.3 Morishima elasticity of substitution between inputs

Morishima elasticity of substitution (MES) can be used to measure changes in relative output and input quantities as a consequence of changes in relative prices (Färe et al., 2005; Sauer et al., 2012). Using equations (11) to (15), we calculate MES for substitution or complementarity relations among inputs based on estimates of the directional output distance function (Table 7). A positive MES indicates a complementary relationship between two inputs and negative MES indicates a substitutionary relationship between inputs; in terms of absolute value of MES, high values reflect a low degree of complementarity or substitutability and low values reflect a high degree of complementarity or substitution between the inputs (Blackorby and Russell 1989; Morrison-Paul et al., 2000; Rahman, 2010). Most of the elasticities are positive and are significantly different from zero by the T-test. An exception is the substitution elasticity between the grassland area size and the initial livestock stock which is equal to 0.50; there are further complementary relationships among other combinations.

[Table 7. Morishima elasticity of substitution between inputs]

4.4 Estimates for technical inefficiency model and technical efficiency

The general-to-specific modeling method (Hendry, 2000) was used in variable selection for deciding on the technical inefficiency model specifications. We first estimate a model including all control variables (Appendix Table 2), and then we drop the least significant variables according to a likelihood ratio test and estimate the model again. This procedure is repeated until only variables that are significant enough to pass the likelihood ratio test at the 10% level remain. The final determinants for the variation of a grazing households’ technical inefficiency are estimated in the technical inefficiency model (right part of Table 3). Because technical inefficiency is the dependent variable in the technical inefficiency model, a negative parameter coefficient for the variables indicates a negative effect on technical inefficiency and conversely, meaning a positive effect on technical efficiency.

Total grassland NPP change ($z_1$) is estimated to be negative in relation to the technical inefficiency, -0.265 significant at the 1% statistical level. This indicates that the more the total NPP on grassland changes, the greater the efficiency of the household will be. However, as the total NPP change might be assumed to be consumed by livestock, which would let us be aware of the more the total NPP change, the higher trend to be overgrazed. Household size ($z_2$) is estimated to be positively related to technical inefficiency, which can be explained as larger household sizes would distract the household head’s attention away from grazing, thus
resulting in higher inefficiency. There is no significant effect of distance from the fixed home to the summer pasture \((z_3)\) on inefficiency, but it is suggested that this variable be kept in the model by a general-to-specific process. More grazing experience would increase the technical efficiency, which was obtained from estimates of grazing experience \((z_4)\), -1.058. We treat summer pasture area \((z_5)\), winter pasture area size \((z_6)\) and the dummy variable of pasture plot \((z_7)\) as a variable block, and we can see winter pasture area size is positively related to technical inefficiency, with 1.033 significance at the 1% statistical level. The dummy variable of lease-in grassland \((z_8)\) has a highly positive affect on technical inefficiency, which means to lease in grassland from other households would increase the technical efficiency.

We calculate each household’s technical efficiency after estimation of the stochastic distance function and technical inefficiency model. The average estimated technical efficiency is 0.82 (Table 8), which indicates that on average, grazing households can improve technical efficiency by 18% in terms of expanding livestock revenue and reducing grazing pressure given unchanged inputs. The distribution of technical efficiencies seems satisfactory from the histogram graph (Figure 3), and we can see that about 13% of the households have a technical efficiency smaller than 0.70, whereas 12% of households have efficiency greater than or equal to 0.70 and less than 0.80; 39% of households have efficiency greater than or equal to 0.80 and less than 0.90, and 35% households operate with a technical efficiency larger than 0.90 (Table 8).

\[\text{Figure 3. Histogram graph of technical efficiency}\]

\[\text{Table 8. Summary of technical efficiency}\]

5. Conclusion and discussion

Incorporating grazing pressure as the undesirable output from livestock grazing using the directional distance function is a new step toward environmental efficiency analysis under the field of productivity and efficiency analysis. The environmental variable, grazing pressure, as the undesirable output from livestock grazing, plays a significant role in the directional distance function and technical inefficiency model. The average technical efficiency is estimated to be 0.82, implying that the grassland production potential can be increased by 18% with directional adjustment of reduction of the grazing pressure.

With reference to Figure 4, there is a reverse U-shape relationship between cumulative grazing pressure and livestock production per unit area grassland. The livestock production per unit area increases according to cumulative grazing pressure until point A, and then
begins to decline at the critical cumulative grazing pressure. However, the ecological risk is monotonously positively increasing as cumulative grazing pressure increasing, which is strongly associated with the overgrazing status. The higher probability of ecological risk would result in a higher probability of grassland degradation (McDowell, 2008). As from the findings of our study, livestock grazing is probably operating on the line from point A to point B, according to estimates of the directional distance function, which means we would be better to leave constant or increase the production potential of livestock grazing in the Sanjiangyuan region without increasing the grazing pressure given the amount grassland size, labor, and capital.

Livestock grazing can have negative impacts on the environment if it is not controlled within acceptable limits. An efficiency livestock grazing monitor approach is suggested to ensure a proper livestock stocking rate. The tradeoff between traditional livestock grazing production and ecological and environmental protection of grassland calls for more scientific research on how to improve production potential under the sustainable grassland use. Findings of how environmental variables and grazing pressure affect the production potential and technical inefficiency of livestock grazing in this study would be helpful for the development of scientific strategies and programs for local economic development and environmental protection, as well as for the effectiveness of ecological protection projects.

There are a few limitations in this paper, for example, there is an assumption that the quality of livestock meat is homogenous for different livestock age groups. In terms of the approximate pasture boundary matching from a long time schedule and large scale level of remote sensing data to minor scale household level data, there is inevitable measurement error to some extent. For the grazing pressure measurement, we have not considered the grazing pressure from wild stock (Fisher, 2004). The consideration of the impact of both domestic stock and wild stock for analysis of sustainable livestock grazing could be considered in future work when wild stock data is available.
Reference


Figure 1. Directional distance function

\[ g = (-g_x, g_y) \]
Figure 2. Output orientation directional distance function

\[ g = (-g_b, g_y) \]
Figure 3. Histogram graph of technical efficiency
Figure 4. Relationship between cumulative grazing pressure, ecological risk and livestock production

Figure source: McDowell, 2008, P.136.
<table>
<thead>
<tr>
<th>Grazing status relate variable</th>
<th>County</th>
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<tbody>
<tr>
<td>Proper carrying capacity (SU/km²) (Zhang, Zhang, Liu et al., 2014)</td>
<td>Tongde</td>
</tr>
<tr>
<td></td>
<td>127.07</td>
</tr>
<tr>
<td></td>
<td>90.58</td>
</tr>
<tr>
<td></td>
<td>81.34</td>
</tr>
<tr>
<td>Overgrazing ratio of 2010 (%) (Zhang, Zhang, Liu et al., 2014)</td>
<td>Zeku</td>
</tr>
<tr>
<td></td>
<td>112.25</td>
</tr>
<tr>
<td></td>
<td>323.5</td>
</tr>
<tr>
<td></td>
<td>47.6</td>
</tr>
<tr>
<td>Overgrazing ratio from 1988 to 2005 (%) (Fan et al., 2010)</td>
<td>Maqin</td>
</tr>
<tr>
<td></td>
<td>600</td>
</tr>
<tr>
<td></td>
<td>500</td>
</tr>
<tr>
<td></td>
<td>300</td>
</tr>
<tr>
<td>Overgrazing ratio from our field survey (%)</td>
<td></td>
</tr>
<tr>
<td></td>
<td>347</td>
</tr>
<tr>
<td></td>
<td>490</td>
</tr>
<tr>
<td></td>
<td>568</td>
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</tbody>
</table>

Note: The proper carry capacity is referred to Zhang, Zhang, Liu et al., 2014
Table 2. Statistic descriptive of variables

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<tr>
<th>Variable Description</th>
<th>Symbol</th>
<th>Measurement</th>
<th>Unit</th>
<th>Mean</th>
<th>Std. Dev.</th>
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<td><strong>Inputs variables</strong></td>
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<td></td>
<td></td>
<td></td>
<td></td>
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<tr>
<td>Grassland area size</td>
<td>( x_1 )</td>
<td>mu</td>
<td></td>
<td>937.38</td>
<td>1413.14</td>
</tr>
<tr>
<td>Labor</td>
<td>( x_2 )</td>
<td>person</td>
<td></td>
<td>2.31</td>
<td>1.27</td>
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<tr>
<td>Productive capital</td>
<td>( x_3 )</td>
<td>1000yuan</td>
<td></td>
<td>129.21</td>
<td>186.87</td>
</tr>
<tr>
<td>Initial yak at the beginning of 2011</td>
<td>( x_4 )</td>
<td>1000kg</td>
<td></td>
<td>7.17</td>
<td>8.15</td>
</tr>
<tr>
<td><strong>Outputs variables</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Good output: revenue from livestock grazing</td>
<td>( y )</td>
<td>1000yuan</td>
<td></td>
<td>105.26</td>
<td>112.02</td>
</tr>
<tr>
<td>Bad output: grazing pressure</td>
<td>( b )</td>
<td>-</td>
<td></td>
<td>0.21</td>
<td>0.28</td>
</tr>
<tr>
<td><strong>Household characteristics variables</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Total NPP change in 2011</td>
<td>( z_1 )</td>
<td>1000kgC</td>
<td></td>
<td>4.94</td>
<td>13.57</td>
</tr>
<tr>
<td>Household size</td>
<td>( z_2 )</td>
<td>head</td>
<td></td>
<td>4.72</td>
<td>1.67</td>
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<tr>
<td>Distance from fixed home to summer pasture</td>
<td>( z_3 )</td>
<td>km</td>
<td></td>
<td>15.24</td>
<td>19.68</td>
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<tr>
<td>Grazing experience</td>
<td>( z_4 )</td>
<td>year</td>
<td></td>
<td>29.87</td>
<td>11.97</td>
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<tr>
<td>Summer pasture area</td>
<td>( z_5 )</td>
<td>1000mu</td>
<td></td>
<td>542.98</td>
<td>911.04</td>
</tr>
<tr>
<td>Winter pasture area</td>
<td>( z_6 )</td>
<td>1000mu</td>
<td></td>
<td>395.65</td>
<td>649.29</td>
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<tr>
<td>Dummy variable pasture plot (1 = the winter pasture and summer pasture are different plots; 0 = other)</td>
<td>( z_7 )</td>
<td></td>
<td></td>
<td>44</td>
<td>149</td>
</tr>
<tr>
<td>Dummy variable of whether there is leased-in grassland (1=yes; 0 = no)</td>
<td>( z_8 )</td>
<td></td>
<td></td>
<td>43</td>
<td>150</td>
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</table>

No. of dummy = 1 No. of dummy = 0
Table 3. Directional distance function with different directional vector

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<td><strong>dep. var.: -θ</strong></td>
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<tr>
<td>Constant</td>
<td>0.050</td>
<td>0.125</td>
<td>-0.165**</td>
<td>0.074</td>
</tr>
<tr>
<td>$x_1$</td>
<td>0.350**</td>
<td>0.143</td>
<td>0.251***</td>
<td>0.044</td>
</tr>
<tr>
<td>$x_2$</td>
<td>-0.464***</td>
<td>0.173</td>
<td>0.097</td>
<td>0.079</td>
</tr>
<tr>
<td>$x_3$</td>
<td>0.172***</td>
<td>0.065</td>
<td>-0.048</td>
<td>0.032</td>
</tr>
<tr>
<td>$x_4$</td>
<td>-0.232</td>
<td>0.170</td>
<td>0.361***</td>
<td>0.066</td>
</tr>
<tr>
<td>$y^*$</td>
<td>-0.263***</td>
<td>0.074</td>
<td>-0.678***</td>
<td>0.034</td>
</tr>
<tr>
<td>$0.5x_1^2$</td>
<td>-0.029**</td>
<td>0.014</td>
<td>-0.031***</td>
<td>0.006</td>
</tr>
<tr>
<td>$0.5x_2^2$</td>
<td>0.555***</td>
<td>0.178</td>
<td>-0.027</td>
<td>0.065</td>
</tr>
<tr>
<td>$0.5x_3^2$</td>
<td>0.027</td>
<td>0.031</td>
<td>0.014</td>
<td>0.012</td>
</tr>
<tr>
<td>$0.5x_4^2$</td>
<td>0.087</td>
<td>0.067</td>
<td>-0.121***</td>
<td>0.035</td>
</tr>
<tr>
<td>$0.5y^2$</td>
<td>-0.125***</td>
<td>0.023</td>
<td>0.011</td>
<td>0.02</td>
</tr>
<tr>
<td>$0.5b^2$</td>
<td>a</td>
<td></td>
<td>-0.11**</td>
<td>0.045</td>
</tr>
<tr>
<td>$x_1y$</td>
<td>0.310***</td>
<td>0.029</td>
<td>0.056**</td>
<td>0.028</td>
</tr>
<tr>
<td>$x_2y$</td>
<td>-0.166***</td>
<td>0.056</td>
<td>0.271***</td>
<td>0.035</td>
</tr>
<tr>
<td>$x_3y$</td>
<td>-0.003</td>
<td>0.018</td>
<td>0.017</td>
<td>0.012</td>
</tr>
<tr>
<td>$x_4y$</td>
<td>0.057*</td>
<td>0.030</td>
<td>0.074***</td>
<td>0.019</td>
</tr>
<tr>
<td>$x_1b$</td>
<td>-1.29***</td>
<td>0.029</td>
<td>-0.017</td>
<td>0.012</td>
</tr>
<tr>
<td>$x_2b$</td>
<td>-0.365***</td>
<td>0.041</td>
<td>-0.014</td>
<td>0.013</td>
</tr>
<tr>
<td>$x_3b$</td>
<td>0.007</td>
<td>0.013</td>
<td>0.012</td>
<td>0.012</td>
</tr>
<tr>
<td>$x_4b$</td>
<td>-0.077***</td>
<td>0.022</td>
<td>-0.017</td>
<td>0.013</td>
</tr>
<tr>
<td>$x_1x_2$</td>
<td>-0.200**</td>
<td>0.085</td>
<td>-0.073**</td>
<td>0.032</td>
</tr>
<tr>
<td>$x_1x_3$</td>
<td>0.067***</td>
<td>0.013</td>
<td>-0.001</td>
<td>0.006</td>
</tr>
<tr>
<td>$x_1x_4$</td>
<td>-0.393***</td>
<td>0.038</td>
<td>-0.039</td>
<td>0.03</td>
</tr>
<tr>
<td>$x_2x_3$</td>
<td>-0.112**</td>
<td>0.056</td>
<td>0.008</td>
<td>0.021</td>
</tr>
<tr>
<td>$x_2x_4$</td>
<td>0.439***</td>
<td>0.120</td>
<td>-0.202***</td>
<td>0.042</td>
</tr>
<tr>
<td>$x_3x_4$</td>
<td>-0.065</td>
<td>0.041</td>
<td>-0.014</td>
<td>0.013</td>
</tr>
<tr>
<td>$y^b$</td>
<td>0.017</td>
<td>0.031</td>
<td></td>
<td></td>
</tr>
<tr>
<td>ln(σ^2_v)</td>
<td>-20.355*</td>
<td>10.815</td>
<td>-4.360***</td>
<td>0.102</td>
</tr>
<tr>
<td>ln(σ^2_u)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Constant</td>
<td>-1.070***</td>
<td>0.102</td>
<td>-12.967</td>
<td>92.348</td>
</tr>
<tr>
<td>$δ_v$</td>
<td>0.000</td>
<td>0.000</td>
<td>0.113</td>
<td>0.006</td>
</tr>
<tr>
<td>$δ_u$</td>
<td>0.586</td>
<td>0.030</td>
<td>0.002</td>
<td>0.071</td>
</tr>
<tr>
<td>$δ^2$</td>
<td>0.343</td>
<td>0.035</td>
<td>0.013</td>
<td>0.001</td>
</tr>
<tr>
<td>Log likelihood</td>
<td>-36.824</td>
<td>146.841</td>
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</tbody>
</table>

Hausman test:

$$\text{Chi}^2(20) = \frac{(b-B)'[(V_b-V_B)^{-1}](b-B)}{698.38}$$

Prob>chi^2 = 0.0000

Notes: 1. a, according to restrictions, they can be calculated. 2. Coefficient of parameter b in Model1 can be calculated according to restrictions, and parameter b is transformed to left side variable in Model2. 3. *Significant at 10% level (P < 0.10), **Significant at 5% level (P < 0.05), ***Significant at 1% level (P < 0.01).
Table 4. Estimates of directional distance function and technical inefficiency model

<table>
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</thead>
<tbody>
<tr>
<td><strong>Stoc. frontier normal/half-normal model</strong></td>
<td></td>
<td></td>
<td><strong>Technical inefficiency model</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Dependent variable: $- \theta$</td>
<td></td>
<td></td>
<td>Dependent variable: $\ln\sigma^2_u$</td>
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<tr>
<td>Constant</td>
<td>0.084</td>
<td>0.098</td>
<td>Constant</td>
<td>-4.358***</td>
<td>1.662</td>
</tr>
<tr>
<td>$x_1$</td>
<td>0.197**</td>
<td>0.096</td>
<td>$z_1$</td>
<td>-0.265***</td>
<td>0.081</td>
</tr>
<tr>
<td>$x_2$</td>
<td>-0.300**</td>
<td>0.135</td>
<td>$z_2$</td>
<td>1.176**</td>
<td>0.548</td>
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<tr>
<td>$x_3$</td>
<td>0.097</td>
<td>0.059</td>
<td>$z_3$</td>
<td>-0.348</td>
<td>0.212</td>
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<td>$x_4$</td>
<td>-0.063</td>
<td>0.113</td>
<td>$z_4$</td>
<td>-1.058*</td>
<td>0.595</td>
</tr>
<tr>
<td>$y^*$</td>
<td>-0.639***</td>
<td>0.059</td>
<td>$z_5$</td>
<td>-0.016</td>
<td>0.174</td>
</tr>
<tr>
<td>$0.5 \cdot x_1^2$</td>
<td>-0.069***</td>
<td>0.016</td>
<td>$z_6$</td>
<td>1.033***</td>
<td>0.208</td>
</tr>
<tr>
<td>$0.5 \cdot x_2^2$</td>
<td>0.263**</td>
<td>0.110</td>
<td>$z_7$</td>
<td>1.781</td>
<td>1.258</td>
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<tr>
<td>$0.5 \cdot x_3^2$</td>
<td>-0.105***</td>
<td>0.020</td>
<td>$z_8$</td>
<td>-1.947***</td>
<td>0.543</td>
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<tr>
<td>$0.5 \cdot x_4^2$</td>
<td>-0.152***</td>
<td>0.049</td>
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<tr>
<td>$0.5 \cdot y^2$</td>
<td>-0.071***</td>
<td>0.013</td>
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</tr>
<tr>
<td>$x_1^\prime y$</td>
<td>0.485***</td>
<td>0.031</td>
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<tr>
<td>$x_2^\prime y$</td>
<td>-0.076**</td>
<td>0.036</td>
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<tr>
<td>$x_3^\prime y$</td>
<td>-0.092***</td>
<td>0.014</td>
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<td>$x_4^\prime y$</td>
<td>0.105***</td>
<td>0.020</td>
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<td>$x_1^\prime x_2$</td>
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<tr>
<td>$x_1^\prime x_3$</td>
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<td>0.022</td>
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<tr>
<td>$x_1^\prime x_4$</td>
<td>-0.409***</td>
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<td>$x_2^\prime x_4$</td>
<td>0.214***</td>
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<td>$x_3^\prime x_4$</td>
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<tr>
<td>$\ln\sigma^2_v$</td>
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<td>Log likelihood=-3.554</td>
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<tr>
<td>Constant</td>
<td>-3.681***</td>
<td>0.231</td>
<td>Number of observation =193</td>
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<td>Wald Chi^2(20) = 5807.730</td>
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<td>Prob.&gt;Chi^2=0.000</td>
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Notes: *Significant at 10% level (P < 0.10), **Significant at 5% level (P < 0.05), ***Significant at 1% level (P < 0.01).
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<td>$\varepsilon_{x1}$</td>
<td>0.64***</td>
<td>0.68</td>
<td>-0.73</td>
<td>3.93</td>
</tr>
<tr>
<td>$\varepsilon_{x2}$</td>
<td>0.08***</td>
<td>0.33</td>
<td>-1.02</td>
<td>1.67</td>
</tr>
<tr>
<td>$\varepsilon_{x3}$</td>
<td>0.12***</td>
<td>0.18</td>
<td>-0.71</td>
<td>0.57</td>
</tr>
<tr>
<td>$\varepsilon_{x4}$</td>
<td>-0.04</td>
<td>0.64</td>
<td>-6.08</td>
<td>1.64</td>
</tr>
<tr>
<td>Outputs elasticity</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\varepsilon_{y}$</td>
<td>-0.36***</td>
<td>0.73</td>
<td>-1.51</td>
<td>6.58</td>
</tr>
<tr>
<td>$\varepsilon_{b}$</td>
<td>0.64***</td>
<td>0.73</td>
<td>0.51</td>
<td>7.58</td>
</tr>
</tbody>
</table>

Notes: T-Test for elasticity different from 0, *Significant at 10% level (P < 0.10), **Significant at 5% level (P < 0.05), ***Significant at 1% level (P < 0.01).
Table 6. Relative shadow price of outputs and elasticity of transformation

<table>
<thead>
<tr>
<th>Variable</th>
<th>Full sample (Obs. = 193)</th>
<th>Partial sample (Obs. = 173)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Relative shadow price: $\frac{\partial \tilde{D}(x,y,b)}{\partial y}$</td>
<td>-1.80</td>
<td>-3.99</td>
</tr>
<tr>
<td>$\frac{\partial \tilde{D}(x,y,b)}{\partial b}$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$M_{by}$: Morishima elasticity substitution of $b$ to $y$</td>
<td>-0.55</td>
<td>-0.73</td>
</tr>
</tbody>
</table>
Table 7. Morishima elasticity of substitution between inputs

<table>
<thead>
<tr>
<th>MES(row, column)</th>
<th>$x_1$ (grassland area)</th>
<th>$x_2$ (labor)</th>
<th>$x_3$ (capital)</th>
<th>$x_4$ (initial yaks)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$x_1$ grassland area</td>
<td>-</td>
<td>0.43***</td>
<td>0.71***</td>
<td>-0.50***</td>
</tr>
<tr>
<td>$x_2$ labor</td>
<td>0.75</td>
<td>-</td>
<td>0.73***</td>
<td>0.75***</td>
</tr>
<tr>
<td>$x_3$ capital</td>
<td>2.07***</td>
<td>1.99**</td>
<td>-</td>
<td>1.96**</td>
</tr>
<tr>
<td>$x_4$ initial yaks</td>
<td>1.02</td>
<td>2.07</td>
<td>1.96</td>
<td>-</td>
</tr>
</tbody>
</table>

Notes: T-Test for elasticity different from 0, *Significant at 10% level (P < 0.10), **Significant at 5% level (P < 0.05), ***Significant at 1% level (P < 0.01).
Table 8. Summary of technical efficiency

<table>
<thead>
<tr>
<th>Variable</th>
<th>Obs.</th>
<th>Percentage</th>
<th>Mean</th>
<th>Std. Dev.</th>
<th>Min.</th>
<th>Max.</th>
</tr>
</thead>
<tbody>
<tr>
<td>TE &lt; 0.70</td>
<td>25</td>
<td>13%</td>
<td>0.42</td>
<td>0.24</td>
<td>0.02</td>
<td>0.70</td>
</tr>
<tr>
<td>0.70 ≤ TE &lt; 0.80</td>
<td>24</td>
<td>12%</td>
<td>0.76</td>
<td>0.03</td>
<td>0.70</td>
<td>0.80</td>
</tr>
<tr>
<td>0.80 ≤ TE &lt; 0.90</td>
<td>76</td>
<td>39%</td>
<td>0.86</td>
<td>0.03</td>
<td>0.80</td>
<td>0.90</td>
</tr>
<tr>
<td>TE ≥ 0.90</td>
<td>68</td>
<td>35%</td>
<td>0.93</td>
<td>0.02</td>
<td>0.90</td>
<td>0.97</td>
</tr>
<tr>
<td>Technical efficiency (TE)</td>
<td>193</td>
<td>100%</td>
<td>0.82</td>
<td>0.19</td>
<td>0.02</td>
<td>0.97</td>
</tr>
</tbody>
</table>
### Appendix

#### Appendix Table 1. Hypothesis test for model selection

<table>
<thead>
<tr>
<th>Model</th>
<th>Description</th>
<th>Likelihood value</th>
<th>Degree of freedom</th>
</tr>
</thead>
<tbody>
<tr>
<td>H₀</td>
<td>Final model presented in paper</td>
<td>-3.554</td>
<td>31</td>
</tr>
<tr>
<td>Hₙ</td>
<td>Directional distance function without setting technical inefficiency mode: ( \tau_0 = \tau_1 = \tau_2 = \tau_3 = \tau_4 = \tau_5 = \tau_6 = \tau_7 = \tau_8 = 0 )</td>
<td>-36.823</td>
<td>23</td>
</tr>
<tr>
<td>H₂</td>
<td>Full model setting</td>
<td>-1.985</td>
<td>35</td>
</tr>
<tr>
<td>H₃</td>
<td>( \tau_1 = 0 )</td>
<td>-8.678</td>
<td>30</td>
</tr>
<tr>
<td>H₄</td>
<td>( \tau_2 = 0 )</td>
<td>-6.888</td>
<td>30</td>
</tr>
<tr>
<td>H₅</td>
<td>( \tau_3 = 0 )</td>
<td>-5.016</td>
<td>30</td>
</tr>
<tr>
<td>H₆</td>
<td>( \tau_4 = 0 )</td>
<td>-5.964</td>
<td>30</td>
</tr>
<tr>
<td>H₇</td>
<td>( \tau_5 = \tau_6 = \tau_7 = 0 )</td>
<td>-18.008</td>
<td>27</td>
</tr>
<tr>
<td>H₈</td>
<td>( \tau_8 = 0 )</td>
<td>-11.782</td>
<td>30</td>
</tr>
</tbody>
</table>
### Appendix Table 2. Model setting with all reasonable variables in technical inefficiency model

<table>
<thead>
<tr>
<th></th>
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<th></th>
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<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Stoc. frontier normal/half-normal model</strong></td>
<td></td>
<td></td>
<td><strong>Technical inefficiency model</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Dependent variable: $-\theta$</td>
<td></td>
<td></td>
<td>Dependent variable: $\ln\sigma_2 u$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Constant</td>
<td>0.074</td>
<td>0.095</td>
<td>Constant</td>
<td>-5.404***</td>
<td>2.014</td>
</tr>
<tr>
<td>$x_1$</td>
<td>0.202**</td>
<td>0.094</td>
<td>$z_1$</td>
<td>-0.246***</td>
<td>0.079</td>
</tr>
<tr>
<td>$x_2$</td>
<td>-0.292***</td>
<td>0.133</td>
<td>$z_2$</td>
<td>1.131**</td>
<td>0.54</td>
</tr>
<tr>
<td>$x_3$</td>
<td>0.097</td>
<td>0.059</td>
<td>$z_3$</td>
<td>-0.234</td>
<td>0.173</td>
</tr>
<tr>
<td>$x_4$</td>
<td>-0.053</td>
<td>0.112</td>
<td>$z_4$</td>
<td>-1.031*</td>
<td>0.566</td>
</tr>
<tr>
<td>$y^*$</td>
<td>-0.631***</td>
<td>0.064</td>
<td>$z_5$</td>
<td>-0.015</td>
<td>0.18</td>
</tr>
<tr>
<td>$0.5 x_1^2$</td>
<td>-0.068***</td>
<td>0.015</td>
<td>$z_6$</td>
<td>0.959***</td>
<td>0.202</td>
</tr>
<tr>
<td>$0.5 x_1^2$</td>
<td>0.267**</td>
<td>0.111</td>
<td>$z_7$</td>
<td>1.764</td>
<td>1.295</td>
</tr>
<tr>
<td>$0.5 x_2^2$</td>
<td>-0.107***</td>
<td>0.02</td>
<td>$z_8$</td>
<td>-1.662***</td>
<td>0.513</td>
</tr>
<tr>
<td>$0.5 x_3^2$</td>
<td>-0.139***</td>
<td>0.049</td>
<td>$z_9$</td>
<td>-0.049</td>
<td>0.077</td>
</tr>
<tr>
<td>$0.5 y^2$</td>
<td>-0.071****</td>
<td>0.013</td>
<td>$z_{10}$</td>
<td>0.587</td>
<td>0.663</td>
</tr>
<tr>
<td>$x_1 y$</td>
<td>0.483***</td>
<td>0.031</td>
<td>$z_{11}$</td>
<td>0.559</td>
<td>0.421</td>
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<tr>
<td>$x_2 y$</td>
<td>-0.080***</td>
<td>0.038</td>
<td>$z_{12}$</td>
<td>-0.082</td>
<td>0.398</td>
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<tr>
<td>$x_3 y$</td>
<td>-0.090***</td>
<td>0.014</td>
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<tr>
<td>$x_4 y$</td>
<td>0.100***</td>
<td>0.02</td>
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<tr>
<td>$x_1 x_2$</td>
<td>-0.087</td>
<td>0.07</td>
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<tr>
<td>$x_1 x_3$</td>
<td>0.025</td>
<td>0.022</td>
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<tr>
<td>$x_1 x_4$</td>
<td>-0.411***</td>
<td>0.057</td>
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<tr>
<td>$x_2 x_3$</td>
<td>0.140***</td>
<td>0.042</td>
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<tr>
<td>$x_2 x_4$</td>
<td>0.214***</td>
<td>0.073</td>
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<tr>
<td>$x_3 x_4$</td>
<td>0.154***</td>
<td>0.026</td>
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<tr>
<td>$\ln \sigma_2 u$</td>
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<tr>
<td><strong>Log likelihood</strong></td>
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<tr>
<td></td>
<td>-1.985</td>
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<td></td>
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</tr>
<tr>
<td><strong>Number of observation</strong></td>
<td></td>
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<td></td>
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</tr>
<tr>
<td></td>
<td>193</td>
<td></td>
<td></td>
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</tr>
<tr>
<td><strong>Wald chi2(20)</strong></td>
<td></td>
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<tr>
<td></td>
<td>5530.070</td>
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<tr>
<td><strong>Prob&gt;chi2=0.000</strong></td>
<td></td>
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<td></td>
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</tr>
</tbody>
</table>

Notes: *Significant at 10% level (P < 0.10), **Significant at 5% level (P < 0.05), ***Significant at 1% level (P < 0.01).
Appendix Table 3. Descriptive statistics for additional variables in appendix table 4.3

<table>
<thead>
<tr>
<th>Variable Description</th>
<th>Symbol</th>
<th>Unit</th>
<th>Mean</th>
<th>Std. Dev.</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Continuous variables</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Total direct subsidy from government</td>
<td>$z_9$</td>
<td>1000yuan</td>
<td>9.52</td>
<td>21.40</td>
</tr>
<tr>
<td>grazed month of summer pasture</td>
<td>$z_{10}$</td>
<td>month</td>
<td>5.52</td>
<td>1.38</td>
</tr>
<tr>
<td>duration of getting the use right of pasture</td>
<td>$z_{11}$</td>
<td>year</td>
<td>19.56</td>
<td>7.59</td>
</tr>
<tr>
<td><strong>Dummy variables</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Dummy variable of education (1=has been</td>
<td>$z_{12}$</td>
<td>No. of dummy = 1</td>
<td>58</td>
<td>135</td>
</tr>
<tr>
<td>education; 0 = no education)</td>
<td></td>
<td>No. of dummy = 0</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>