Cooperation and Cheating

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Abstract:
In this article, we extend the variable delivery claim framework (Cross, Buccola, and Thomann, 2006) to examine the option-to-cheat, that is, the option to shift production between contracts ex post. We use this framework to provide a solution to the age-old conflict between enforcement and the cooperative tradition of providing a “home” for member produce. We show that, in contrast to Nourse’s competitive yardstick hypothesis, the value of the cooperative-provided option increases as market competition intensifies. When the option-to-cheat is fairly-priced, it is Pareto improving, increasing grower returns, lowering cooperative per-unit costs and reducing contract shortfalls for investor-owned rivals at no additional per-unit cost. Our valuation framework is consistent with replication-based equilibria and is free from parametric specification of individual preference or firm cost structure.

Key words: fraud, path-dependent options, production contracts
Cooperation and Cheating

Agricultural cooperatives, under intense competition from investor-owned firms (IOF’s), struggle to control over-delivery and under-delivery by growers, that is, cheating. When crop yields are high at harvest time, some members shift excess produce from their IOF contracts to their more accommodating cooperative contracts, thereby receiving payment for produce that would otherwise be plowed under. Alternatively, when yields are low, some members shift produce away from their cooperative contracts to exploit higher IOF prices. An “extra row” can represent a few percent of contracted acreage and is difficult to discern amid harvest activity. Enforcement by managers can be costly and often conflicts with the cooperative philosophy of providing a “home” for member produce.

Nourse’s (1945) competitive yardstick hypothesis suggests that as market competition intensifies the incentive to cooperate wanes. We introduce a cooperative option-to-cheat and prove that its value is, in contrast to Nourse, strictly increasing as market competition intensifies. The cooperative role is transformed from IOF rival to beneficial partner.

Further, when the option-to-cheat is fairly priced it is a Pareto improving market mechanism. Many agricultural markets lack well-regulated, liquidly traded exchanges to fulfill contract shortfalls or dispose of excess production. High storage or transportation costs, geographic dispersion, and increasing differentiation all contribute to thin cash markets. Like giving each grower a small personal exchange, the shifting option provides cheating growers with a finite crop reserve in case of an IOF contract shortfall, or a
destination for a portion of excess production. Growers who cheat deliver more total produce and at better prices, thereby increasing net farm returns. With fewer contract shortfalls, IOF processors face lower delivery volatility with no increase in per-unit costs. Higher grower returns allow cooperatives to lower the per-unit forward price paid out to members.

Recent research effort has focused on detecting fraudulent crop insurance claims (Atwood, Robinson-Cox, and Shaik, 2006), designing compliance incentives for conservation programs (Giannakas and Kaplan, 2005) and pollution permit markets (Milak, 2006), and determining optimal tax enforcement for multinational firms (Peralta, et al., 2006). We extend the Variable Delivery Claim (VDC) framework (Cross, Buccola, and Thomann, 2006) to value the option-to-cheat and derive balanced forward contract prices. This framework is consistent with replication-based equilibria, thin markets, and heterogeneous contract terms, and provides estimates that are independent of parametric specification of individual preferences or firm cost structures.

**Economic and Contracting Environment**

To begin, we extend the cooperative economy of Cross and Buccola (2004), in which two processors, a cooperative and an IOF, purchase raw product from growers. The IOF processor has some monopoly power in its finished good market, whereas the cooperative supplies a separate and competitive output market. As an alternative buyer of raw product, the cooperative poses a credible threat to the IOF processor, enabling grower members to extract any IOF monopolistic rent.
There is no cash market for raw product. Therefore, growers forward contract half their acreage with the cooperative and half with the IOF. Denote these time $t = T$ payoffs $X_{1,T}$, $X_{2,T}$, respectively. The cooperative contract pays a relatively low, fixed forward price $s_{1,i}$ for all produce harvested. In contrast, the IOF pays a relatively high, fixed forward price $s_{2,i}$ for produce, but only up to a set threshold $k_i$ and nothing thereafter $s_{2,i} = 0$.

(figure 1 here)

Figure 1 above illustrates contract revenue per acre associated with two simplified contracts over a range of *ex post* agricultural yield levels $Y_T$, where $T$ is the terminal period. Here, for illustration, the cooperative contract pays $s_{1,1} = $1 per unit of produce harvested. The IOF contract pays $s_{2,1} = $2 per unit harvested up to a threshold of $k_1 = 5$ units, and nothing thereafter.

The grower’s combined revenue from the *portfolio* of two contracts (and two acres) is then simply the sum of the two contract payoffs $\sum_{i=1}^{2} X_{i,T}$, as illustrated in figure 2 below.

(figure 2 here)

Notice that the cooperative contract $X_{1,T}$ is an affine ($J = 0$) variable delivery claim and $X_{2,T}$ is a piece-wise linear variable delivery claim with one kink ($J = 1$). The combined portfolio payoff is also a $J = 1$ variable delivery claim.
Now consider a grower with the willingness and ability to shift some proportion, say $\delta$, of the harvest from one contract to the other \textit{ex post}. Denote the portfolio payoff
\[
\sum_{i=1}^{2} X_{i,T}(\delta), \text{ at time } T.
\]
Let the shift amount $\delta$ be a percentage of total agricultural yield $Y_T$, and let it take on values in the closed interval $[-\alpha, \alpha]$, where $\alpha$ is some positive constant $\alpha \in \mathbb{R}_+$. This interval may represent the limits of the grower’s inscrutability, a formal or tacit allowance to by the holder, or perhaps the cheating threshold below which the grower believes detection will not occur. The cheating grower then chooses the optimal proportion of produce to shift \textit{ex post} in order to maximize portfolio payoff
\[
\sum_{i=1}^{2} X_{i,T}(\delta).
\]

(figure 3 here)

The upper-most dashed-line in Figure 3 illustrates the payoff associated with the cheating portfolio. Notice that the cheating portfolio payoff has three kinks ($J = 3$), two of which are determined directly from the magnitude of the cheating parameter $\alpha$ and the original threshold value $k_1$.

Figure 3 also illustrates the revenue realized from each of the individual contracts $X_{1,T}(\delta)$ and $X_{2,T}(\delta)$. Notice that when terminal yield exactly matches the IOF delivery threshold $Y_T = k_1$, there is no incentive to cheat. This is because, a one-unit shift from contract one to contract two results in a loss of $1$ in return for $0$ gain, since contract two pays nothing for excess production. Alternatively, a one-unit shift from contract two to contract one results in a $2$ loss in return for a gain of $1$, the cooperative’s low price
per unit. Thus, no shifting takes place. When terminal yield coincides with the lower kink at $Y_T = k_i/(1 + \alpha)$, the grower will shift enough produce from contract one to fulfill contract two, which is the maximum allowable shift ($\delta = -\alpha$). This results in a reduced payoff for contract one and an elevated payoff for contract two, as indicated by the two dashed lines.

Finally, the lower most dashed line represents the net additional revenue from shifting product, $Z_T(\alpha)$, which is simply the difference between the portfolio payoff with and without cheating:

$$Z_T(\alpha) = \sum_{i=1}^{2} X_{i,T}(\delta) - \sum_{i=1}^{2} X_{i,T},$$

See the technical appendix for the general specification of $Z_T(\alpha)$. Notice that under this non-negative payoff the grower “cannot lose” by addition the cheating option. We will formalize this important concept later.

Though less central to our problem, both processors ensure that standard production practices are followed by specifying a set of fixed and variable inputs to be applied throughout production period at a cost of $C_{[0,T]}$. Because of the variable inputs, this value is path dependent.

**Valuation**

We will use the valuation method corresponding to the missing asset market of Björk (1998), in which the underlying yield process is observable, but not tradable. Valuation is based on the existence of a replicating portfolio, or arbitrage portfolio. We define an
arbitrage opportunity, following Harrison and Pliska (1981), as any self-financing
investment strategy $\phi$, with payoff $\pi$ at time $t$, fulfilling the weak arbitrage condition

(2) $\pi_0 = 0, \quad \pi_T \geq 0, \quad \mathbb{E}[\pi_T] > 0,$

where $\mathbb{E}$ is the conditional expectation operator. Initial and terminal periods are denoted 0
and $T$, respectively.

We model the underlying yield process $Y_t$ as the usual adapted Geometric
Brownian Motion (GBM), defined in the probability space $(\mathbb{P}^\rho, \Omega, \mathbb{F}),$ where $\mathbb{P}^\rho$ is the
probability measure with synthetic drift term $\rho$, $\Omega$ the sure event, and $\mathbb{F}$ the filtration.

The equation of motion for yield is then given by

(3) $dY_t = \rho Y_t dt + \sigma Y_t dW_t,$

$Y_t = Y_0 \exp\left\{\left(\rho - \frac{1}{2}\sigma^2\right)t + \sigma W_t\right\}, \quad t \in [0, T],$ with drift $\rho$, volatility $\sigma$, and initial value $Y_0$.

The discount process $\beta_t$ is the inverse of the risk-free bond $B_t$. The bond evolves
deterministically, according to

(4) $B_t = B_0 e^{-rt},$

where $B_0$ is a constant and $r$ the risk-free rate.

Two notions of “value” can be ascribed to the option-to-cheat: For the cheater,
the option leads to additional net revenue, given a fixed schedule of forward prices. For
the cooperative, a formalized option-to-cheat leads to a lower forward price paid to
growers, a *rebalanced* forward contract.
The payoff associated with the option-to-cheat $Z_T(\alpha)$ can be represented by a ($J = 3$) variable delivery claim,

$$
Z_T(\alpha) = \sum_{j=1}^{J-1} (s_{\alpha,j+1} - s_{\alpha,j})(Y_T - k_{\alpha,j})^+,
$$

where $(Y_T - k_{\alpha,j})^+ = \max(Y_T - k_{\alpha,j}, 0)$, and in vector notation

$$
s_\alpha = \begin{bmatrix}
0 \\
\alpha(s_{1,1} - s_{1,1}) \\
(s_{1,1} - s_{1,1}) \\
(s_{1,2} - s_{2,2}) \\
\alpha(s_{1,2} - s_{2,2})
\end{bmatrix},
$$

and

$$
k_\alpha = \begin{bmatrix}
\left(\frac{1}{1-\alpha}\right)k_0 \\
\left(\frac{1}{1+\alpha}\right)k_1 \\
k_1 \\
\left(\frac{1}{1-\alpha}\right)k_1
\end{bmatrix}.
$$

Cheating thus provides not a single option, but a sum of European-style “plain vanilla” call options with strike thresholds $k_\alpha$, weighted by a schedule of fixed prices $s_\alpha$. This is a special case\(^1\) of the general cheating option provided in the appendix.

Denote the *ex ante* value function $\mathbb{V}[\bullet]$. The value of $Z_T(\alpha)$ is then

$$
\mathbb{V}[Z_T(\alpha)] = \mathbb{E}^\rho[\beta_T Z_T(\alpha)],
$$
where $\mathbb{E}^\rho$ is the discounted conditional expectation operator with respect to the synthetic drift term $\rho$ and information up to time $t = 0$. As seen in figure 3, the payoff $Z_T(\alpha)$ is everywhere non-negative, leading to a strictly positive expected value and thus an arbitrage opportunity, as in (2).

It remains to determine by how much the cooperative must reduce forward price $s_{1,1}$ in order to eliminate the arbitrage opportunity embedded within $\sum_{i=1}^{2} X_{i,T}(\delta)$. By the definition of a forward contract

\begin{equation}
\mathbb{E}^\rho \left[ \sum_{i=1}^{2} X_{i,T}(\delta) \right] = 0.
\end{equation}

Solving equation (1) for $\sum_{i=1}^{2} X_{i,T}(\delta)$ and substituting into (9), we obtain

\begin{equation}
\mathbb{E}^\rho \left[ \sum_{i=1}^{2} X_{i,T} + Z_T(\alpha) \right] = 0.
\end{equation}

The general expression for the variable delivery claim $X_{i,T}$ is given by

\begin{equation}
X_{i,T} = \sum_{j=0}^{J} (s_{i,j+1} - s_{i,j})(Y_T - k_j)^+ - C_{[0,T]}, \quad i = 1, 2,
\end{equation}

where $s_0$ is zero, $k_0$ is the lower limit of $Y_T$, and $C_{[0,T]}$ is the path-dependent input cost rule. Holding all other forward prices fixed, we then solve (10) for the arbitrage-free first forward price $s_{1,1}^*$,

\begin{equation}
 s_{1,1}^* = \frac{\mathbb{E}^\rho \left[ NUM \right]}{\mathbb{E}^\rho \left[ 2(Y_T - k_1)^+ - 2Y_T + (1 + \alpha)(Y_T - \left( \frac{1}{1 + \alpha} \right) k_1)^+ \right]},
\end{equation}
where

\[
NUM = s_{1,2} (Y_r - k_1)^+ + \sum_{j=2}^{J} (s_{1,j} - s_{1,j-1}) (Y_r - k_j)^+ - C_{[0,T]}
\]

(13)

\[
+ \left( s_{1,2} - s_{2,2} \right) (Y_r - k_1)^+ + X_{2,r}
\]

\[
+ s_{2,1} \left[ (Y_r - k_1)^+ - Y_r - \left( 1 + \alpha \right) \left( Y_r - \left( \frac{1}{1 + \alpha} \right) k_1 \right)^+ \right].
\]

**Statics**

To test Nourse’s (1992) competitive yardstick hypothesis, we want to know how the cheating option value changes as market competition intensifies, that is, as the forward price offered by the IOF is driven up by either a rival cooperative, collective bargaining, or competing IOF’s. Because cheating exploits price differentials between contracts, forces that increase these differentials, such as IOF competition, enhance the value of cheating.

To formalize this, consider the non-trivial case, in which at least some cheating occurs and the IOF contract accepts some delivery, \( \alpha, k_1 > 0 \). Also, let the expected terminal yield is not zero, \( \mathbb{E}^\rho \left[ Y_r \right] > 0 \). Under these conditions, we provide our central claim:

**Claim:** The value of the cheating option increases as market competition intensifies:

(14) \[
\frac{\partial V}{\partial s_{2,1}} \left[ Z_r (\alpha) \right] > 0,
\]

where \( s_{2,1} \) is the IOF forward price, \( \alpha, k_1 > 0 \), and \( \mathbb{E}^\rho \left[ Y_r \right] > 0 \).
The proof is provided in the appendix and follows directly from the first order condition

\[
\frac{\partial \mathbb{V}[Z_T(\alpha)]}{\partial s_{2,1}} = \mathbb{E}^\nu \left[ \beta_r \left( \alpha Y_T + (Y_T - k_i)^+ - ((1 + \alpha) Y_T - k_i)^+ \right) \right].
\]

We illustrate the values of this condition over a range of cheating and threshold levels in the results section.

We are also interested in how two other economic forces affect the value of the cheating option: technical innovation and maturing capital markets. Technological innovation, such as GMO crops, improves agricultural productivity, leading to higher yields or reduced production volatility or both. Interestingly, neither the Black-Scholes (1973) nor the Björk (1998) valuation models are a function of the “native drift,” that is the rate of increase in yield over time. Both depend on the synthetic drift term to obtain arbitrage-free derivative values. Thus, yield-enhancing innovation does not affect the value of the option to cheat, \textit{ceteris paribus}. Technological innovation that reduces yield volatility, however, have an important impact on the value of yield-based derivative options. In the results section, we will examine derivative of the value of cheating with respect to yield volatility, as given by

\[
\frac{\partial \mathbb{V}[Z_T(\alpha)]}{\partial \sigma}.
\]

Finally, as capital markets mature, interest rates tend to fall and risk premiums narrow. The cooperative is uniquely suited to raise equity in thin capital markets, due to its ability to retain equity from member patronage (Cross and Buccola, 2004). We are
therefore interested in cheating option values as interest rates and risk premiums decrease. If the value of cheating also wanes as capital markets mature, the following first order condition will hold:

\[
(17) \quad \frac{\partial V[Z_T(\alpha)]}{\partial \rho} > 0.
\]

This condition is also illustrated in the results section.

**Estimation Methods**

Because no convenient analytical expression exists for the density of the arithmetic average of log-normal random variables (Kemna and Vorst, 1990), we employ standard Monte Carlo Integration (Boyle, Brodie, and Glasserman, 1997), which is consistent, asymptotically normal, and accurate to an arbitrary level depending on the number of draws \(m\) (Campbell, 1997).

**Data**

To illustrate the value of the cheating option, both from the grower and cooperative perspective, we begin with the arbitrage-free market of Cross, Buccola, and Thomann (2006). This market corresponds to the missing asset market, in which cooperative and IOF processors each contracted processing tomatoes under one of two standardized variable delivery claims. Tri Valley Growers, the large and now defunct cooperative processor, competed with a number of large investor-owned rivals, including Hunts, Heinze, Ragu, Campbell’s, and others. We consider the 20-year period from 1977 to
1996 when financial difficulties at Tri Valley became widely known, and utilize both raw
data and estimated model parameters from this study.

IOF forward prices are published by the California Tomato Growers Association
(1977-2001). These prices, along with Tri Valley Grower’s arbitrage-free forward price
estimates with and without cheating are illustrated in figure 5 in the results section. All
value terms are expressed in constant 2005 dollars adjusted using the Consumer Price
Index (CPI) as published by the Bureau of Labor Statistics.

Kern County yield data and delivery threshold estimates are illustrated below in
figure 4 over the 20-year study period. Yield volatility $\sigma$ was estimated to be 0.1849,
resulting in a market price of risk estimate of $0.32 per unit of volatility. This premium is
added to the risk free interest rate to obtain synthetic drift parameters. The 3-month U.S.
Treasury index is used for the risk-free rate. It averaged 7% and ranged from a high of
14% in 1981 to a low of 3% in 1993.

(figure 4 here)

Arbitrage-free farm production costs parameters are estimated from studies
published by the University of California Cooperative Extension Service (UCCES)

For illustration, we use a cheating magnitude of $\alpha = 0.10$, except where
otherwise stated. Interviews with cooperative managers suggest that actual shifting
varied widely across members and over time, based on changing market conditions and
cooperative enforcement policy.
Results

The introduction of the cheating option reduces the arbitrage-free cooperative forward price by 6.7% on average or $5.60 per ton, with a high of 8.5% in 1990 and a low of 3.8% in 1994. Figure 5 below illustrates cooperative prices with and without cheating along with IOF forward prices for comparison.

(figure 5 here)

Average deliveries and delivery volatilities are illustrated for the cheating and non-cheating case in figure 6 below. We note two key implications: First, average deliveries increase under cheating for both the cooperative and IOF contracts. Deliveries to the IOF processor increase because of fewer contract shortfalls. Cooperative deliveries increase because growers shift produce from the IOF contract to the cooperative in years of surplus. Second, cheating increases delivery volatility for the cooperative, shifting volatility away from the IOF processor. A close look at the two contracts with no cheating suggests why the additional liquidity provided by a formal shifting mechanism may be Pareto improving. The basic design of the IOF contract, with no cheating, already leads to more stable, targeted deliveries at higher per-unit costs (forward prices). This is consistent with the expectations of the more stable, higher-value, branded output markets served by Hunts, Heinz, Ragu, Campbell’s, and other IOF’s. By contrast, the cooperative contract, with no cheating, leads to higher volatility and lower per-unit costs (forward prices), consistent with the needs of the lower-value, commodity-style markets served by Tri Valley Growers. Cheating strengthens the relative differences between the two contracts, reducing shortfalls for the IOF processor and providing a home for
cooperative member produce. These improvements cause no additional per-unit costs to the IOF and lower per-unit costs to the cooperative.

(figure 6 here)

The extent to which competitive pressure enhances the value of cheating is expressed by first order condition (14). The higher this value, the greater the enhancement. This condition is clearly dependent on two parameters of interest: the potential degree of cheating $\alpha$ and the IOF delivery threshold $k_i$. As illustrated in figure 7 below, the first order condition is everywhere positive and increasing in both variables.

(figure 7 here)

This raises the question of why the IOF processor would not enter the commodity-style output market, thereby internalizing the shifting mechanism and eliminating a competitor. One factor could be return-on-equity. Publicly-traded IOF’s achieved significantly higher and more stable net returns than did Tri Valley Growers during the study period (Cross and Buccola, 2004). Returns in the commodity-style markets may appear unattractive to IOF’s. Tri Valley Growers attempted at one point to enter the higher-value branded market, investing heavily in an unsuccessful branded product line (Hariyoga, 2004). To the extent that crossing competitive lines is either unattractive or costly, the shifting mechanism remains a cooperative-provided service to the market.

Our last graph, figure 8, shows that maturing capital markets depress the value of cheating. Moving along the x-axis labeled $\rho$ in figure 8, as interest rates fall the value of cheating decreases for all levels of volatility. The impact of volatility-reducing
technological innovation, however, is interest rate dependent. For high levels of $\rho$, innovation enhances the value of cheating. For lower values of $\rho$, by contrast, cheating values decline as volatility ebbs. This result is reasonable, given the fact that higher returns are required when interest rates and risk-premiums are high.

(figure 8 here)

**Conclusion**

This article extends the VDC framework (Cross, Buccola, Thomann, 2006) to the problem of crop shifting, an age-old problem, encouraged by cooperative social norms, discouraged by fair-minded managers, lenders, and retired members. We found the fairly-priced shifting provision to be Pareto-improving, transforming the cooperative from IOF-adversary to beneficial partner. The cheating-option increases grower net returns, lowers cooperative per-unit costs, and avoids IOF production shortfalls at no additional per-unit cost to the IOF. Because equity returns may be lower for member-owned firms than for investor-owed firms, traditional and new-generation cooperatives are equally well-suited to provide this option. Finally, we showed that as market competition proliferates, the value of cheating option increases, as shifters exploit widening forward price differentials.
References


Appendix to Cooperation and Cheating

General formula for the option to cheat

The cheating payoff $Z_T(\alpha)$ from equation (5) can be expressed as functions of the cheating limit $\alpha \in \mathbb{R}_+$ and terminal yield $Y_T$, as follows:

(A.1) $Z_T(\alpha) = F_1(\alpha) + F_2(\alpha) + F_3(\alpha),$

where

(A.2) $F_1(\alpha) = \sum_{j=1}^{J+1} \alpha \left| s_{i,j} - s_{2,j} \right| Y_T 1_{[Y_T \in D_{j,1}]}$

(A.3)

\[
F_2(\alpha) = \sum_{j=1}^{J+1} \max \left\{ \left( s_{i,j} - s_{2,j} \right) \left( k_j - Y_T \right)^+ + (1 + \alpha) \left( s_{i,j+1} - s_{2,j} \right)^+ \left( Y_T - \frac{1}{1+\alpha} k_j \right) \right\} 1_{[Y_T \in D_{j,2}]}
\]

(A.4)

\[
F_3(\alpha) = \sum_{j=1}^{J+1} \max \left\{ \left( s_{i,j+1} - s_{2,j+1} \right) \left( Y_T - k_j \right)^+ + (1 - \alpha) \left( s_{i,j} - s_{2,j} \right)^+ \left( \frac{1}{1-\alpha} k_j - Y_T \right) \right\} 1_{[Y_T \in D_{j,3}]}
\]

Here, the cheating payoff takes on values over three region-types $D_{j,1}, D_{j,2},$ and $D_{j,3}$, $j = 1, \ldots, J$, corresponding to intervals of the terminal yield domain, as follows:
$$D_{j,1} = \left[ \left( \frac{1}{1-\alpha} \right)^{k_{j-1}}, \left( \frac{1}{1+\alpha} \right)^{k_j} \right]$$

(A.5) $$D_{j,2} = \left[ \left( \frac{1}{1+\alpha} \right)^{k_j}, k_j \right]$$

$$D_{j,3} = \left[ k_j, \left( \frac{1}{1-\alpha} \right)^{k_j} \right]$$

Figure 3 illustrates each of the three region types. Regions $D_{j,1}$, $j = 1, \ldots, J$, correspond to terminal yield values that lie between contract thresholds, such that no thresholds $k_j$ lie within the interval $(1+\delta)Y_T$ for any possible value of $\delta \in [-\alpha, \alpha]$. Regions $D_{j,2}$ correspond to terminal yield values just below the thresholds $k_j$, such that $(1+\alpha)Y_T$ reaches or exceeds the threshold. Finally, regions $D_{j,3}$ correspond to terminal yield values just above the thresholds $k_j$, such that $(1-\alpha)Y_T$ reaches or exceeds the threshold.

Proof of the claim

From the identity in (15), the claim is true, if the following inequality holds:

$$\mathbb{E}^\rho \left[ \beta_T \left( \alpha Y_T + (Y_T - k_1)^+ - ((1+\alpha)Y_T - k_1)^+ \right) \right] > 0.$$  

(A.7)  

Passing the expectation through, we will show directly that

$$\alpha \mathbb{E}^\rho \left[ Y_T \right] + \mathbb{E}^\rho \left[ (Y_T - k_1)^+ - ((1+\alpha)Y_T - k_1)^+ \right] > 0.$$  

(A.8)  

The relationship is equality if either $\alpha$ or $k_1$ is zero. Therefore, let $\alpha, k_1 > 0$. Expressing the conditional expectation operator as an integral, the LHS of (A.8) is
\[(A.9) \quad \alpha \int_{-\infty}^{\infty} Y_T(z) \varphi(z) \, dz + \int_{-\infty}^{\infty} \left( Y_T(z) - k_i \right)^* \varphi(z) \, dz - \int_{-\infty}^{\infty} \left( (1 + \alpha) Y_T(z) - k_i \right)^* \varphi(z) \, dz, \]

where \( Y_T(z) = Y_0 e^{\tilde{\rho} + \sigma z}, \) \( \varphi(z) \) is the standard normal density, \( Y_0 \) is a positive constant, and \( \tilde{\rho} = \rho - \frac{1}{2} \sigma^2. \) Changing the limits of integration, we obtain

\[(A.10) \quad \alpha \int_{-\infty}^{\infty} Y_T(z) \varphi(z) \, dz + \int_{z_0}^{\infty} \left( Y_T(z) - k_i \right) \varphi(z) \, dz - \int_{z_a}^{\infty} \left( (1 + \alpha) Y_T(z) - k_i \right) \varphi(z) \, dz, \]

where \( z_0 = \frac{\ln(k_i / Y_0) - \tilde{\rho}}{\sigma}, \) and \( z_a = \frac{\ln(k_i / (1 + \alpha) Y_0) - \tilde{\rho}}{\sigma}. \) Changing the limits of integration again and canceling terms, we can rewrite our derivative

\[(A.11) \quad \alpha \int_{-\infty}^{\infty} Y_T(z) \varphi(z) \, dz - \int_{z_a}^{z_0} \left( Y_T(z) - k_i \right) \varphi(z) \, dz \equiv F(\alpha). \]

Note that for \( z_a \leq z \leq z_0, \) \( k_i > Y_0 e^{\tilde{\rho} + \sigma z}. \) So, \( F(\alpha) > 0, \) verifying our claim.
The contracts of interest are defined by convex contract payoffs $s_{1,1} \leq s_{1,2}, s_{2,1} \leq s_{2,2}$ and
the relative price relationships illustrated in figure 1, specifically, $s_{1,1} \leq s_{2,1}, s_{1,2} \geq s_{2,2}$ and
$s_{1,1} \geq s_{2,2}, s_{1,2} \leq s_{2,1}$.
Figure 1: Simplified cooperative and IOF contract payoffs over a range of terminal yield values
Figure 2: Payoff from the sum of a cooperative and IOF contract over a range of terminal yield values
Figure 3: Payoffs for the cheating portfolio, the cooperative and IOF contracts, and the option to cheat
Figure 4. Kern County tomato yields and contract thresholds ($k_i$), 1977-1996
Figure 5. Investor-owned forward prices vs. cooperative forward prices – with and without cheating, 1977-1996
Figure 6. Average delivered tons and delivery volatility from 1977 to 1996 by type of forward contract
Figure 7: Condition values over a range of cheating and threshold levels
Figure 8: Cheating option values over a range of volatility and drift levels