Modeling and Hedging Rain Risk

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Abstract

In this article we price a precipitation option based on empirical weather data from Germany using different pricing methods, among them Burn Analysis, Index Value Simulation and Daily Simulation. For that purpose we develop a daily precipitation model. Moreover, a de-correlation analysis is proposed to assess the spatial basis risk that is inherent to rainfall derivatives. The models are applied to precipitation data in Brandenburg, Germany. Based on simplifying assumptions of the production function, we quantify and compare the risk exposure of grain producers with and without rainfall insurance. It turns out that a considerable risk remains with producers who are remotely located from the weather station. Another finding is that significant differences may occur between the pricing methods. We identify the strengths and weaknesses of the pricing methods and give some recommendations for their applications. Our results are relevant for producers as well as for potential sellers of weather derivatives.

Keywords: *weather risk, weather derivatives, precipitation model, basis risk*

JEL Classification: *C8, Q14, Q54*

It is well known that weather is an important production factor in agriculture. Unfortunately, this production factor can hardly be controlled. In fact, weather risks are a major source of uncertainty in crop production. Traditionally, producers try to compensate for the negative economic consequences of bad weather events by buying insurance. In the mid-nineties a new class of instruments has emerged, namely weather derivatives. Weather derivatives are financial instruments that allow to trade weather related risks. These instruments include futures, options, and swaps. Common underlyings are derived

from temperature, rainfall or wind. Weather derivatives have the advantage that they are not affected by moral hazard or adverse selection, which may be a serious problem for insurance companies. However, a considerable risk may remain with the producer when using weather derivatives, because individual yield variations in general are not completely correlated with the relevant weather variable. Until now it has not been clear if weather derivatives would permeate in agriculture. Actually, literature increasingly deals with the question if weather derivatives can also play a role as risk management tools in the agribusiness. (Richards, Manfredo, and Sanders 2004; Turvey 2001; van Asseldonk and Oude Lansink 2003).

From a statistical viewpoint there are two alternatives with regard to the modeling of weather risk. On the one hand, the distribution of the weather event (e.g. a weather index) can be estimated directly, either parametrically or non-parametrically. On the other hand, a daily model of the underlying weather variable (temperature or rain) can be developed, from which the relevant weather index is then derived (Diebold and Campbell 2003). This procedure is more complex initially yet potentially favorable for two reasons: Firstly, the ways in which daily models can be used are very flexible, because practically all yield relevant events like the sums of precipitation or temperature for different accumulation periods, dry spells or extreme precipitation can be determined for any periods of time (Srikanthan and McMahon 2001). In contrast, a direct estimate of the distribution of the weather index is usually only valid for a particular index. Secondly – and this seems even more important than the higher flexibility – the accuracy of daily based models is higher due to a considerably larger number of observed values (Brix, Jewson, and Ziehmann 2002). For this reason, the pricing and the analysis of the effectiveness of temperature related derivatives mainly result from daily temperature models. Despite of the po-

tential advantages of a daily modeling the available studies on rainfall based derivatives in the agribusiness resort to a direct estimation of the precipitation distribution (Berg et al. 2005; Turvey 1999; Stoppa and Hess 2003; Skees et al. 2001).

The aim of this article is twofold. The first objective is the development and the estimation of a daily precipitation model which can be used for the assessment of rainfall based weather derivatives or insurances. The results are compared with those from other pricing models. This comparison shall answer the question whether the superiority of a daily model also applies in the context of rainfall modeling. The second objective is to analyze the basis risk of weather derivatives. Here we define basis risk as the uninsurable risk resulting from the difference of the weather index at the derivative's reference point and the location of agricultural production. While this aspect is less important concerning temperature related derivatives, it cannot be neglected when the effectiveness of precipitation derivatives is analyzed because of the high spatial variability of precipitation. To our knowledge the basis risk of rainfall related weather derivatives has not yet been quantified.

The article is structured as follows: In Section 2 a statistical model for the estimation of daily precipitation is presented. Furthermore, the statistical relationship between precipitation at different places is described with a de-correlation analysis. In Section 3 an empirical application of these concepts follows. Using precipitation data of the region of Brandenburg/Germany, a put option and a call option on the cumulated rainfall for different accumulation periods are priced and the effect on the risk exposure of farms is examined. Simple pricing procedures like the burn analysis and the index value simulation serve as a benchmark. The article ends with conclusions on the proposed pricing method-

ology and the efficiency of weather derivatives with respect to hedging against precipitation risks.

A precipitation model

A precipitation model should be able to capture the following characteristics of daily rainfall:

- The probability of rainfall occurrence obeys a seasonal pattern. Rainfall in Europe for example, is more likely in winter than in summer.

- The sequence of wet and dry days follows an autoregressive process. This means, the probability of a rainy day is higher if the previous day was wet.

- The amount of precipitation on a wet day varies with the season. Rainfall in Europe is more intensive in summer than in winter.

- The volatility of the amount of rainfall also changes seasonally. In Europe it is higher in summer than in winter.

In the following, a daily precipitation model is described which can depict these characteristics. According to Moreno (2002) and Cao, Li, and Wei (2004), the stochastic process of daily precipitation can be decomposed into a stochastic process for the binary event "rainfall" and "dryness" respectively, and a distribution for the amount of precipitation is given such that it describes a rainy day. Consider the random variable X_t:

$$(1) \quad X_t = \begin{cases} 0, \text{if day } t \text{ is dry} \\ 1, \text{ if day } t \text{ is rainy} \end{cases}$$

It is assumed that X_t follows a first order Markov chain. The probability p_t, that it will rain on day t can be calculated as:

(2) $p_t = p_{t-1} \cdot q_t^{11} + (1 - p_{t-1}) \cdot q_t^{01}$, $t = 1, 2, ..., n$

q_t^{01} describes the transition probability of rain on day t and dryness on the previous day $t - 1$. Analogously, q_t^{11} stands for the transition probability between two successive rainy days. Note that the transition probabilities q_t^{01} and q_t^{11} vary with time.

The precipitation amount y_t is modeled as a sequence of continuous random variables with independent distributions. In the literature, various distributions with a non-negative domain are discussed, among others the exponential distribution and the gamma distribution (Woolhiser and Roldan 1982). The mixed exponential distribution has proven to be especially flexible (Wilks and Wilby 1999). The density function is:

(3) $f(y_t | X_t = 1) = \dfrac{\alpha_t}{\beta_t} \cdot \exp\left(\dfrac{-y_t}{\beta_t}\right) + \dfrac{1 - \alpha_t}{\gamma_t} \cdot \exp\left(\dfrac{-y_t}{\gamma_t}\right)$, with $0 \le \alpha_t \le 1$ and $0 < \beta_t < \gamma_t$

The parameters of the mixed exponential distribution α_t, β_t and γ_t are also time varying. Thereby the seasonality of precipitation is taken into account. In this form however, the model is not estimable. In order to reduce the number of the parameters to be estimated, each of the time varying parameters is developed by a finite Fourier series:

(4) $\theta_{tj} = a_{j0} + \displaystyle\sum_{k=1}^{m_j} \left[a_{jk} \cdot \sin\left(\dfrac{t \cdot k}{N} + b_{jk}\right) \right]$,

where $\theta_{t1} = q_t^{01}$, $\theta_{t2} = q_t^{11}$, $\theta_{t3} = \alpha_t$, $\theta_{t4} = \beta_t$, $\theta_{t5} = \gamma_t$ and $N = \dfrac{365}{2 \cdot \pi}$

a_{jk} and b_{jk} denote the Fourier coefficients, and m_j is the maximum number of harmonics needed to specify the seasonal cycles. The Fourier coefficients for α_t, β_t and γ_t as well as for q_t^{01} and q_t^{11} are estimated simultaneously by maximizing the following log-likelihood functions (Woolhiser and Pegram 1979):

$$(5)\ \ln L_1 = \sum_{t=1}^{365}\left[c_t^{00}\cdot\ln\!\left(1-q_t^{01}\right)+c_t^{01}\cdot\ln\!\left(q_t^{01}\right)+c_t^{10}\cdot\ln\!\left(1-q_t^{11}\right)+c_t^{11}\cdot\ln\!\left(q_t^{11}\right)\right]$$

$$(6)\ \ln L_2 = \sum_{t=1}^{T}\ln\!\left[f\!\left(y_t\,\middle|\,X_t=1\right)\right]$$

In (5) c_t^{ij} is the observed number of transitions from state i at day $t-1$ to state j at day t.

Spatial Correlation of Rainfall

A weather derivative always refers to a particular weather station. The fact that the farm site is usually not located at this weather station results in a basis risk. One can expect the risk to increase with the distance to the weather station. This basis risk can be quantified by means of a de-correlation analysis. Rubel (1996) proposes the following nonlinear de-correlation function for the modeling of the spatial relationship of precipitation in Europe:

$$(7)\ \rho(d)=e_1\cdot\exp\!\left(-e_2\cdot d^{e_3}\right)$$

Herein ρ denotes the correlation coefficient between the precipitation at different places and d the distance between the weather station and the farmer's production site. e_1, e_2, and e_3 are parameters to be estimated. In spite of the de-correlation analysis being a popular instrument in meteorology, two points should be considered critically. Firstly, the de-correlation function is invariant of the direction. Thus, topographical differences potentially influencing the precipitation are neglected. Secondly, Embrechts, McNeil, and Straumann (1999) point out problems of using correlation coefficients when the underlying distributions are not elliptical. In spite of these weaknesses the concept of the de-correlation analysis is used in this study.

Valuation of rainfall options for grain producers in north-eastern Germany

Background, methods and data

Grain production in north-eastern Germany, Brandenburg in particular, is highly affected by rainfall risk. During the relevant months of April and May, Brandenburg has had between 4.5 mm and 136.3 mm of precipitation (with a mean of 80.2 mm) in the last 20 years, and the grain yields have fluctuated similarly. The high correlation between rainfall and yields results from the sandy soil having little water storing capacity and the lack of irrigation. Currently there is no possibility to insure against yield losses caused by low rainfall. In view of the extreme crop failures in the years of drought 2000 and 2003, in which the government had to provide disaster relief in order to save farmers from becoming insolvent, there is a pronounced interest to introduce some kind of rainfall insurance. By purchasing a put option on a rainfall index, a grain producer is (partially) insured against revenue losses due to little precipitation in the growing season. Moreover we consider a call option on cumulated rainfall during the harvesting period which compensates grain producers for quality losses and timeliness costs. In the following, let us assume that both options are available in the OTC market. The reference point is the weather station Berlin-Tempelhof which is central for Brandenburg. The contract specifications for the derivatives considered here are summarized in Table 1.

Table 1 about here

The estimation of the precipitation model is based on rainfall data measured in Berlin-Tempelhof from 1 January 1948 to 31 August 2004 ($T = 20\,683$). Records from 23 weather stations in Brandenburg from 1 January 1983 to 31 December 2003 are available

for the calculation of the de-correlation function. The weather stations are evenly distributed in the Brandenburg region within a radius of 100 km from Berlin. Pricing of the options is carried out by a burn analysis, an index value simulation and a daily simulation. These procedures are briefly described below. A more detailed description and critical discussion is found in Zeng (2000) and in Cao and Wei (2000).

Burn Analysis (Historical Simulation). In a non-parametric burn analysis, an empirical distribution of the rainfall index is derived from the historical rainfall data which consist of 56 observations here. Based on the empirical distribution, hypothetical payoffs of the option are determined for each of the 56 years and discounted with the risk-free interest rate *r*, which is assumed to be 5%. The price of the option is the mean of the discounted payoffs.

Index Value Simulation. An index value simulation requires determining a parametric distribution for the rainfall index. We use the EXCEL add-in BEST-FIT to test for the most appropriate distribution. Only distributions with a non-negative domain were considered. According to the Chi Square test, the Kolmogorov Smirnov test and the Anderson Darling test, the Weibull distribution in the case of the accumulation period „April/May", and the Erlang distribution in the case of the accumulation period „1st half of July" show the best fit to the 56 empirical observations. From these distributions, values for the precipitation index are randomly drawn 50 000 times, and the discounted payout of the option is determined. As before the option price is the average discounted payoff.

Daily Simulation. In this approach the daily precipitation is simulated instead of the rainfall index using the model described in previous section. The rainfall index is then derived from the simulated sample paths by summing up daily precipitation in the relevant accumulation period. The subsequent steps are identical to the index value simulation.

Note that all three pricing procedures suffer from a theoretical shortcoming in that there is no guarantee for the calculated option prices being arbitrage free. A well-known problem with pricing weather derivatives is that weather cannot be traded and hence it is impossible to construct a risk-free hedge portfolio. However, this is a prerequisite for the application of no-arbitrage models like the Black Scholes pricing formula. Different approaches to treat this problem are proposed in the literature (cf. Cao and Wei 1999; Alaton, Djehiche, and Stillberger 2002; Richards, Manfredo, and Sanders 2004). All methods suggested require the market price for weather risk to be quantified. The market price for weather risk could be derived implicitly from price quotations, if a market already existed for these weather derivatives. As this is not the case in the situation considered, the only resort is to parameterize this unknown value. We refrain from doing so here, as such an analysis has already been carried out in the papers mentioned above. Moreover, the no-arbitrage trait of derivative prices is not that relevant in the assumed situation of an OTC trade, as trading usually does not occur during the contract term. Still, when interpreting the results, one should take into account, that the option prices depicted, i.e. the costs of coverage from the farmer's point of view, constitute lower limits to the actual costs that are to be expected, as the sellers (insurance companies, banks) will charge risk premiums.

Estimation of the daily precipitation model

In order to estimate the parameters of the daily precipitation model (1) to (4), the likelihood functions (5) and (6) are maximized with a genetic algorithm. The parameters m_j

are determined using the Akaike Information Criterion (AIC). Figure 1 shows the actual and the estimated daily rainfall in the course of a year. Figure A1 in the appendix depicts the related transition probabilities q_t^{01} and q_t^{11}. Obviously the model does not only fit the yearly average but also the seasonality of the rainfall amounts well. From Figure A2 it can be seen that the standard deviation of the estimated daily precipitation is actually higher in summer than in winter. Hence, the model reflects the aforementioned characteristics of daily rainfall.

Figure 1 about here

A pitfall of the daily precipitation model is the underestimation of the variance of cumulated rainfall over a period of several weeks. This underestimation of the variance has already been observed in a different context and has been termed „low frequency variability bias" (Dubrovsky, Buchtele, and Zalud 2004). For example, the sample variance of the precipitation for the period of April/May is 1436, whereas the daily precipitation model only shows a value of 882. One can expect that the daily simulation will also result in biased option prices because options prices are sensitive towards volatility. Hansen and Mavromatis (2001) discuss various methods to reduce the low frequency variability bias. In this study we take several measures: Firstly, the transition probabilities q_t^{01} and q_t^{11} are estimated by their empirical sample counterparts, which show a clearly higher variability than those based on the Fourier series (see Figure A1). Moreover, the parameters of the mixed exponential distribution α_t, β_t and γ_t are determined in such a way that the resulting variances exactly fit the sample variance of the daily rainfall amounts shown

in Figure A2b in the appendix. Secondly, following Dubrovsky, Buchtele, and Zalud (2004), a second order Markov process instead of a first order one is estimated. Thus, longer sequences of consecutive rainy and dry days respectively may occur, which leads to a higher variance of the cumulated precipitation. In what follows the original and the modified daily precipitation model are called „daily simulation I" and „daily simulation II", respectively[1].

The calculation of the spatial basis risk is exemplified for the rainfall sum in April and May (Option 1). First, the correlation coefficients of the precipitation index between pairwise weather stations are determined. Next, the distances between the stations are quantified, which serve as an explanatory variable in the non-linear regression function (7). The parameter estimates for the de-correlation function are: $e_1 = 0.9331$, $e_2 = 0.0009$ and $e_3 = 1.2183$. Figure 2 shows the graph of the de-correlation function which has the expected negative slope. While the correlation of the total precipitation is approximately 0.9 between Berlin-Tempelhof and a station at a distance of 25 km, this value decreases to about 0.5 at a distance of 200 km. An R^2 of 0.66 proves that the estimated de-correlation function can be considered a good approximation to the empirical correlations. Nevertheless, it should be noted that the topographical situation in Brandenburg matches the assumption of a direction-independent relationship, and that this assumption may apply less, for example, to mountainous regions. Moreover, the scatter plot in Figure 2 shows heteroscedasticity, i.e. the relationship between distance and correlation becomes more imprecise with increasing distance.

Figure 2 about here

Valuation of the rainfall options

Table 2 presents the option prices obtained by applying the burn analysis, the index value simulation and both variants of the daily simulation. The burn analysis and the index value simulation lead to similar prices for both options. The similarity of the results implies that the Weibull distribution for the accumulation period „April/May" and the Erlang distribution for the accumulation period „1st half of July" approximate the respective empirical distribution of the precipitation index well. In contrast, the prices calculated with the daily simulation are lower compared to those obtained with the burn analysis. This difference is significant for the put option in the case of the daily simulation I. The reason is the aforementioned underestimation of the volatility. This effect can be mitigated, yet not eliminated, by the modification of the daily precipitation model described above. Furthermore, the differences of the pricing methods depend on the derivative to be valuated. Thus, all four methods result in similar prices for the call option referring to a shorter accumulation period.

In order to assess the reliability of the four models table 2 also displays the standard errors of the estimates[2]. Actually the daily simulation shows smaller standard errors than the burn analysis and the index value simulation (except for daily simulation II in the case of the call), but the gain in accuracy is not that pronounced as reported by Brix, Jewson, and Ziehmann (2002) for a temperature index. Interestingly the estimates of the burn analysis have smaller standard errors than those of the index value simulation while the former approach is frequently criticized in the literature for producing unstable results (see e.g. Cao and Wei 2000; Zeng 2000).

Table 2 about here

Analysis of the hedging effectiveness

Finally, we investigate how the availability of a precipitation derivative affects the risk exposure of grain producers located at different distances from the weather station by using the put option as an example. As we are mainly interested in the geographic basis risk here, we will simplify matters by neglecting other production risks. In addition, it is assumed that the relation between the cumulated rainfall in April and May and the grain yield is linear and that both variables are perfectly correlated. Due to these simplifying assumptions, it is possible to interpret the rainfall distributions directly as profit distributions. The distributions for the rainfall index are generated with the Index Value Simulation using the Weibull distribution and the de-correlation function estimated before. Table 3 and Figure 3 show that a producer located in the immediate vicinity of the weather station can eliminate the downside risk by acquiring a put option. The variance of the returns can be reduced by about 65 percent. The expected value of the profit distributions does not change, since the option price is defined as the expected value of the payouts according to the actual probabilities (as opposed to the risk-neutral ones). As mentioned before, this price constitutes a lower limit; due to risk premiums and transaction costs, the actual prices will be higher, which leads to a downward shift of the distributions with an option. Furthermore, it becomes clear that the risk reduction potential of the weather derivatives diminishes with increasing distance. When interpreting the results the simplifying assumptions regarding the „production function" should be kept in mind. In reality, yields are not perfectly correlated with the rainfall index resulting in a further reduction of the hedging effect (cf. Berg et al. 2005).

Figure 3 and Table 3 about here

Conclusions

An economic analysis of weather derivatives from the viewpoint of a potential buyer requires the solution of three interrelated problem areas: firstly, the statistical modeling of the relevant weather variables; secondly, the quantification of the relationship between the weather variables and the production; and thirdly, the development of a theoretically consistent pricing model. This paper focuses on the first aspect. A daily precipitation model is specified, from which indices can be derived that determine the payoff of the derivative. Based on this model "fair prices" can be calculated, which constitute a lower bound for the value of a derivative. A comparison with other, simpler procedures reveals clear differences in the pricing of the weather derivatives. First of all this finding underscores the importance of the model choice. However, it is difficult to draw an unequivocal conclusion regarding the superiority of one of the valuation approaches. On the one hand, application of the daily simulation has the advantage of yielding smaller confidence intervals for the resulting indices and prices compared with the non-parametric burn analysis and the index value simulation. On the other hand, the danger of a rather sophisticated daily precipitation model being wrongly specified is relatively high; such a risk is precluded when the precipitation index is estimated directly. In the present application it turned out that a daily simulation model tends to underestimate the volatility of monthly rainfall. This pitfall may be of minor importance in the context of meteorological or hydrological applications, but it is severe when the model is used for risk assessment and derivative pricing. Some measures to reduce this bias have been discussed and successfully implemented in this paper. Nevertheless the problem deserves further attention. Another shortcoming of the presented daily rainfall model is the ignorance of long term (interannual) variability of the parameters. That means, trends or an increase of rainfall vola-

tility due to climatic changes are not captured by this model. In principle, however, it is also possible to incorporate interannual variability into daily precipitation models (cf. Wilks and Wilby 1999). We conclude that the preferential statistical approach to option pricing depends, among other things, on the availability of weather data and on the context of its application. A systematic model validation on the basis of quasi-ex ante forecasts is suggested as a subject of further research.

Regardless of the issue of the appropriate pricing method, the following practical conclusions can be drawn: The risk reducing effect of precipitation derivatives is much more regionally confined than it is the case with temperature related derivatives. In the example of Brandenburg considered here, the correlation between the precipitation index of the weather station Berlin-Tempelhof and a remote farm site decreases to a value of 0.75 at a distance of 100 km. If one additionally takes into account the stochastic relation between precipitation on the one hand and production or returns on the other hand - which has been ignored in this study - the use of rainfall derivatives as risk management tools in agriculture appears questionable. It follows that potential suppliers of rainfall insurances should introduce a dense network of weather stations as reference points for the rainfall index in order to increase the attractiveness of this type of insurance, although this may lead to a fragmented demand. Moreover, the specification of adequate weather indices also requires further studies. The cumulated rainfall, on which we focus here, may not be specific enough from the viewpoint of many producers. For example, not only the rainfall sum is relevant for the yields but also the rainfall distribution over time. The precipitation model presented in this paper can be utilized to explore the effectiveness of any kind of rainfall based index.

References

Alaton, P., B. Djehiche, and D. Stillberger. 2002. "On Modelling and Pricing Weather Derivatives". *Applied Mathematical Finance* 9(1):1-20.

Asseldonk, M.A.P.M. van, and A.G.J.M. Oude Lansink. 2003. "Weather Based Index Insurance to Hedge Temperature Exposure of Greenhouse Horticultural Farms". In Wesseler, J., H.P. Weikard, and R.D. Weaver, eds. *Risk and Uncertainty in Environmental and Natural Ressource Economics*. Cheltanham, Edward Elgar, pp. 235-249.

Berg, E., B. Schmitz, M. Starp, and H. Trenkel. 2005. "Wetterderivate: Ein Instrument im Risikomanagement für die Landwirtschaft?" *German Journal of Agricultural Economics* 54(2):158-170.

Brix, A., S. Jewson, and C. Ziehmann. 2002. "Weather Derivative Modelling and Valuation: A Statistical Perspective". In Dischel, R.S., ed. *Climate Risk and the Weather Market*. London, Risk Books, pp. 127-150.

Cao, M., A. Li, and J. Wei. 2004. "Precipitation Modeling and Contract Valuation: A Frontier in Weather Derivatives". *The Journal of Alternative Investments*. Fall 2004:93-99.

Cao, M., and J. Wei. 2000. "Pricing the Weather". *Risk* 13:67-70.

__. 1999. "Pricing Weather Derivatives: an Equilibrium Approach". Working Paper, Queen's University Kingston, Ontario.

Diebold, F.X., and S.D. Campbell. 2003. "Weather Forecasting for Weather Derivatives". Working Paper 10141, NBER Working Paper Series, Cambridge.

Dubrovsky, M., J. Buchtele, and Z. Zalud. 2004. "High-Frequency and Low-Frequency Variability in Stochastic Daily Weather Generator and its Effect on Agricultural and Hydrologic Modelling". *Climatic Change* 63:145-179.

Embrechts, P., A. McNeil, and D. Straumann. 1999. "Correlation: Pitfalls and Alternatives". *Risk* 12(5):69-71.

Hansen, J.W., and T. Mavromatis. 2001. "Correcting Low-frequency Variability Bias in Stochastic Weather Generators". *Agricultural and Forest Meteorology* 109:297-310.

Moreno, M. 2002. "Rain Risk". Research Paper. Speedwell Weather Derivatives, London.

Richards, T.J., M.R. Manfredo, and D.R. Sanders. 2004. "Pricing Weather Derivatives". *American Journal of Agricultural Economics* 86(4):1005-1017.

Rubel, F. 1996. "Scale Dependent Statistical Precipitation Analysis". *Proceedings of the International Conference on Water Resource and Environment Research* 1, pp. 317-324.

Srikanthan, R., and T.A. McMahon. 2001. "Stochastic Generation of Annual, Monthly and Daily Climate Data: A Review". *Hydrology and Earth System Sciences* 5(4):653-670.

Skees, J., S. Gober, P.Varangis, R. Lester, and V. Kalavakonda. 2001. "Developing Rainfall-Based Index Insurance in Morocco". Policy Research Working Paper 2577, The World Bank, Washington D.C.

Stoppa, A., and U. Hess. 2003. "Design and Use of Weather Derivatives in Agricultural Policies: the Case of Rainfall Index Insurance in Morocco". Paper presented at the International Conference "Agricultural Policy Reform and the WTO: Where are we heading", Capri (Italy), June 23-26, 2003.

Turvey, C.G. 1999. "The Essentials of Rainfall Derivatives and Insurance". Working Paper WP99/06, Department of Agricultural Economics and Business, University of Guelph, Ontario.

__. 2001. "Weather Derivatives for Specific Event Risks in Agriculture". *Review of Agri-cultural Economics* 23(2):333-351.

Wilks, D.S., and R.L. Wilby. 1999. "The Weather Generation Game: A Review of Sto-chastic Weather Models". *Progress in Physical Geography* 23(3):329-357.

Woolhiser, D.A., and G.S. Pegram. 1979. "Maximum Likelihood Estimation of Fourier Coefficients to Describe Seasonal Variations of Parameters in Stochastic Daily Pre-cipitation Models". *Journal of Applied Meteorology* 18(1):34-42.

Woolhiser, D.A., and J. Roldan. 1982. "Stochastic Daily Precipitation Models: 2. A Comparison of Distributions of Amounts". *Water Resources Research* 18(5):1461-1468.

Zeng, L. 2000. "Pricing Weather Derivatives". *Journal of Risk Finance* 1(3):72-78.

Footnotes

[1] Beside these two models other model versions have been specified and estimated. For example, the mixed exponential distribution has been replaced by a kernel density estima-tor. The results, however, didn't change significantly.

[2] Standard errors are calculated with the bootstrap method with 200 iterations.

Table 1:   Specification of Rainfall Options

	Option 1	Option 2
Type	Put	Call
Payoff	$V \cdot \max(K - I_t, 0)$	$V \cdot \max(I_t - K, 0)$
Index I_t	$\displaystyle\sum_{t=01.04.}^{31.05.} y_t$	$\displaystyle\sum_{t=01.07.}^{15.07.} y_t$
Strike price K	91 mm	30 mm
Tick size V	1 € per 1 index points	1 € per 1 index points
Discount rate r	5 %	5 %
Time to expiration τ	9 months	9 months

Table 2: **Moments of the Rainfall Index and Option Prices for Different Valuation Models***

| | Burn Analysis | | | Index Value Simulation | | | Daily Simulation I | | | Daily Simulation II | | |
| | Rainfall Index | | Option Price | Rainfall Index | | Option Price | Rainfall Index | | Option Price | Rainfall Index | | Option Price |
	Mean	Standard Deviation		Mean	Standard Deviation		Mean	Standard Deviation		Mean	Standard Deviation	
Put	90.80	37.86	14.28	90.58	38.47	15.17	92.49	29.85	10.58	91.14	31.56	11.91
	(4.92)	(3.52)	(2.56)	(5.44)	(4.10)	(2.95)	(3.67)	(2.63)	(1.96)	(4.01)	(3.35)	(2.07)
Call	29.49	20.23	7.68	29.88	21.13	8.05	29.76	18.16	7.05	29.47	19.96	7.52
	(2.40)	(2.16)	(1.66)	(2.76)	(2.88)	(1.99)	(2.28)	(2.01)	(1.50)	(2.62)	(2.78)	(1.88)

* Standard errors in parentheses

Table 3: **Parameters of the Profit Distributions with and without Hedging at Different Locations**

		without option	with option			
Distance (Correlation coefficient)		0 km (1.00)	0 km (1.00)	25 km (0.89)	100 km (0.73)	200 km (0.53)
Mean		90.53	90.53	90.54	90.53	90.53
Variance		1468.91	555.75	689.36	906.15	1182.39
Quantiles	5 %	31.53	75.26	55.79	44.80	36.95
	10 %	41.90	75.26	61.83	54.30	47.01
	15 %	49.73	75.26	66.03	60.32	54.60
	50 %	88.24	75.26	85.84	87.87	88.89
	90 %	141.83	126.09	126.27	130.46	135.83
	95 %	157.36	141.62	141.63	144.08	149.92
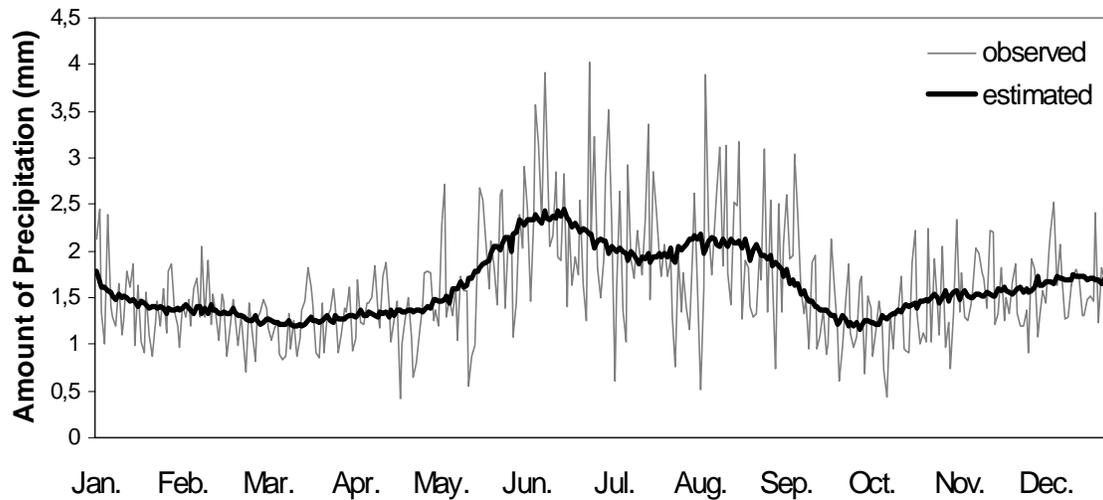
Figure 1:   **Observed and estimated average daily precipitation**

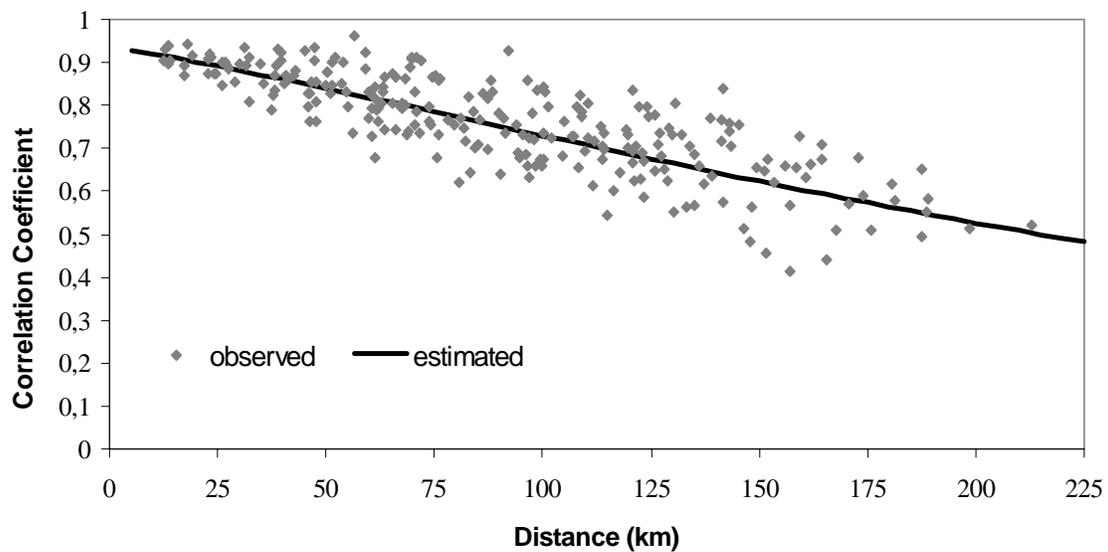**(weather station Berlin-Tempelhof)**

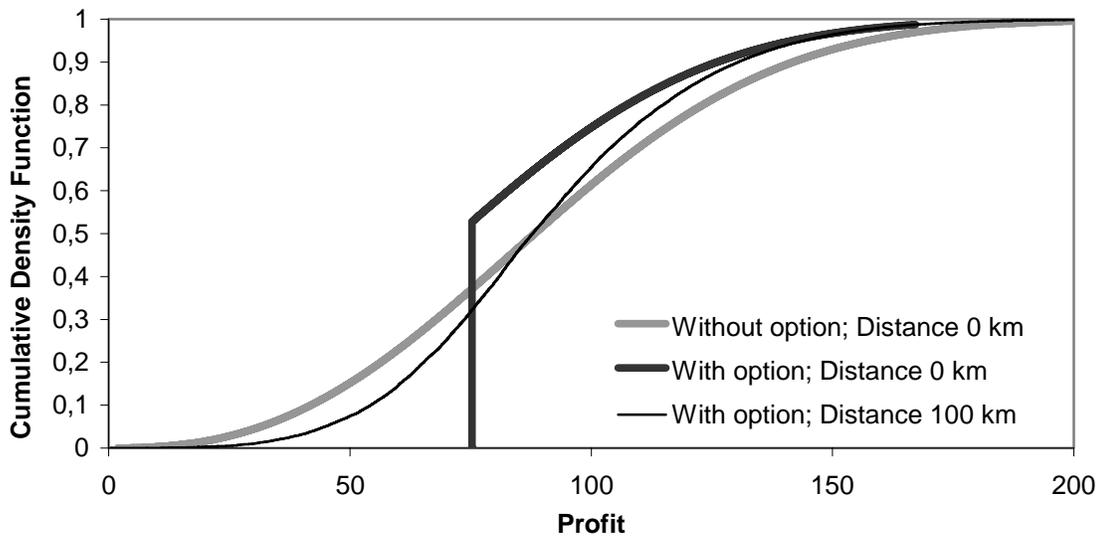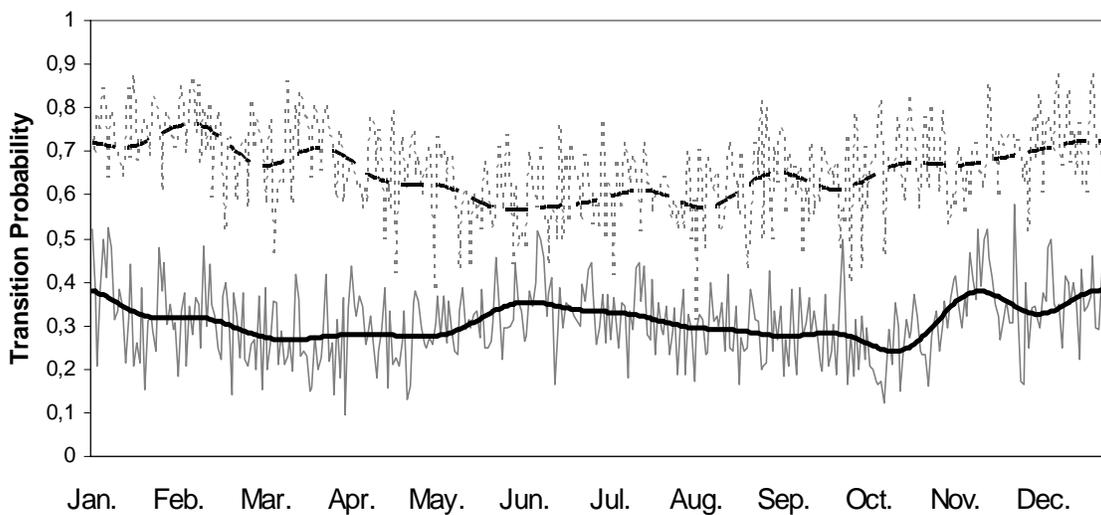**Figure 2:**   **De-correlation function for cumulated precipitation in Branden-**
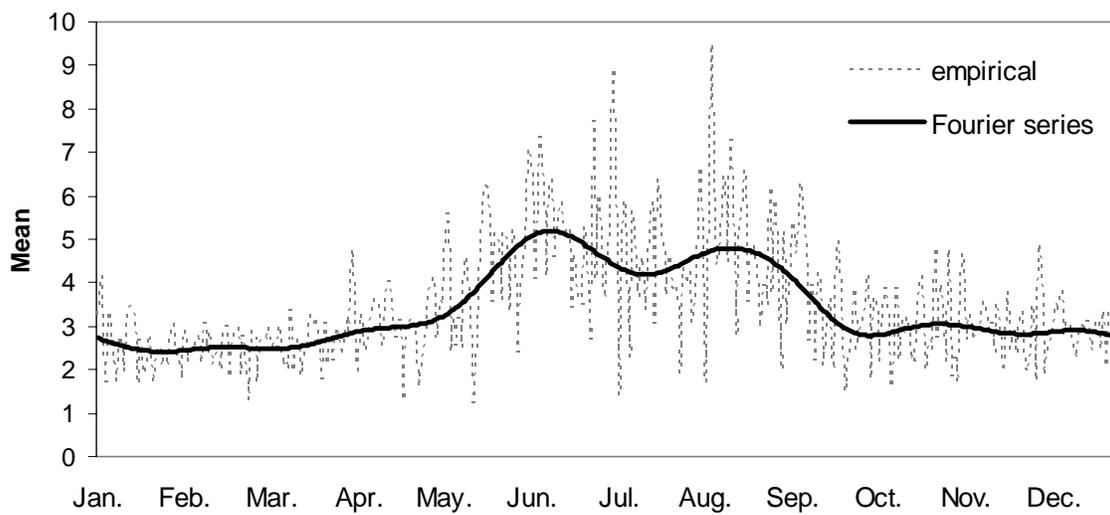
burg, Germany

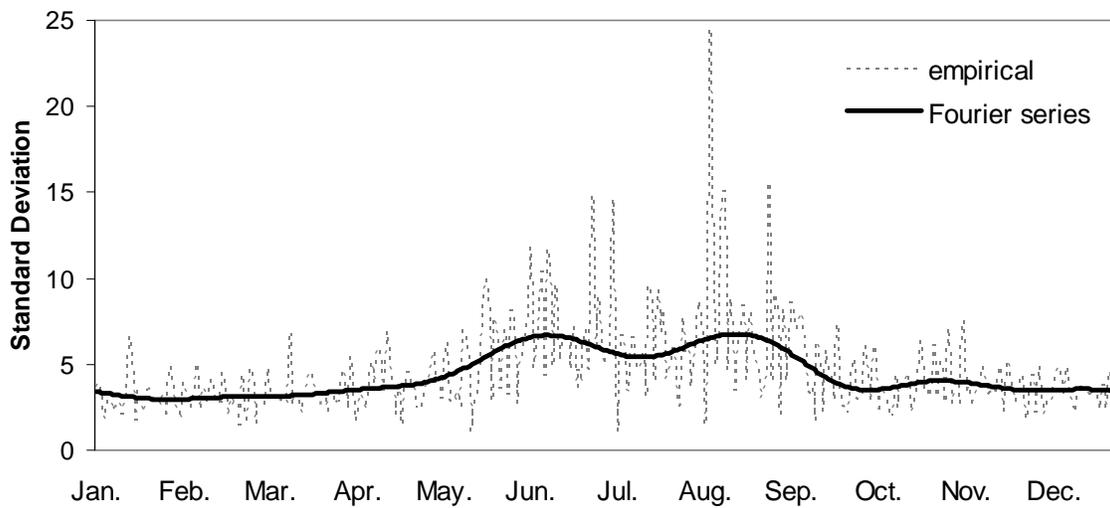**Figure 3:** **Profit distributions with and without hedging**

Appendix

——— q_t^{01} empirical ━━━ q_t^{01} Fourier series ⋯⋯ q_t^{11} empirical ━ ━ q_t^{11} Fourier series

Figure A1: **Conditional transition probabilities**

a) Mean

b) Standard deviation

Figure A2: **Conditional mean and standard deviation of daily precipitation**