The Dynamics of Reintroducing, Supplementing and Controlling Endangered Predator Populations

Daniel Rondeau

Department of Economics
Cornell University

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Correspondence should be addressed to
Daniel Rondeau, Department of Economics, 453 Uris Hall, Cornell University, Ithaca, NY, 14853, USA.
(607) 266-0015 (voice); (607) 255-2818 (fax); drr7@cornell.edu.

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A dynamic model is developed to analyze the reintroduction of endangered predators. Non-convexities and the conditions under which reintroduction is sub-optimal are studied. Following reintroduction, costly population control should be initiated before marginal animals impose net costs, providing an economic interpretation to changes in the sign of the shadow price.

Keywords: optimal control, non-convexity, shadow price, endangered species, wildlife management

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The reintroduction of locally extirpated species and the supplementation of threatened populations are increasingly relied upon in efforts to ensure the continued existence of animals in danger of extinction. While they provide nonconsumptive benefits, population recovery plans are controversial when they involve the return of predators. Strong opposition to the reintroduction of wolves to Yellowstone, the southwestern United States, the Adirondacks and Maine (Stevens, 1997); and of Grizzlies to Idaho (MacCracken et al., 1994) are testimony to the fears of local populations, and the danger posed by predators. Coyotes, cougars and bobcats cause losses of livestock estimated at $65 million per year (GAO, 1995). Alligator, coyotes and mountain lions occasionally attack and kill humans and the number of such attacks is on the rise (Kenworthy, 1997). In Florida, authorities receive 15,000 calls annually from residents requesting the removal of Alligators (Kenworthy, 1997).

While society may benefit from the protection of endangered predators, reestablishing self-sustaining populations implies the potential for population growth and increased conflict with humans. Yet, the intertemporal analysis of costs and benefits from wildlife with the potential to harm humans is notably absent from the literature.

In this paper, an optimal control model of wildlife management is constructed to analyze the characteristics of efficient population recovery and control programs. The model is designed to accommodate management strategies ranging from reintroduction to costly population control of a species for which additional animals may be either desirable or undesirable. After a presentation of the model, I explore the characteristics of the dynamical system. Because of the predator's potential to harm as well as benefit society, the shadow price of the population is allowed to be either positive or negative, creating multiple equilibria which are studied in order to determine the optimal solution to the model. The conditions under which species reintroduction is uneconomical and the properties of optimal wildlife management plans are then discussed. Concluding remarks summarize the findings.

1. Model Essentials

The objective of a benevolent wildlife manager is to choose a sequence of harvesting rates to maximize the discounted flow of net benefits from wildlife:

$$\begin{aligned} &\underset{\{Y\}}{\textit{Maximize}} \int_0^\infty e^{-\delta t}[V(X)+N(Y)-D(X)]\,dt \\ &\textit{Subject to} \quad \dot{X}=F(X)-Y \\ &\qquad\qquad\qquad Y_{\min}\le Y \le Y_{\max} \\ &\qquad\qquad X\ge 0 \qquad X(0)=X_0 \ \textit{given} \end{aligned} \tag{P}$$

where the time index, t, has been suppressed from X and Y for convenience. δ is the discount rate, X is the population of a predator species expressed as a proportion of carrying capacity, and Y is the rate of harvesting where $Y_{min} \le Y(t) \le Y_{max}$; $Y_{min} < 0$, $Y_{max} > 0$. A negative harvest rate indicates supplementation of the population. Throughout, it is assumed that Y_{max} is sufficiently large to be economically irrelevant. *V(X)* and *D(X)* are increasing and respectively strictly concave and strictly convex functions measuring the rates of nonconsumptive benefits and damage to society when the population level is X. *N(Y)* is a strictly concave and single-peaked function indicating the net benefits from harvesting wildlife at rate Y. The maximum of this function is $\bar{Y}>0$ with $N(Y<0)<0$, $N(0)=0$, $N_y>0$ for $Y<\bar{Y}$, $N_y(\bar{Y})=0$, and $N_y<0$ for $Y>\bar{Y}$. For Y<0, *N(Y)* represents the cost of supplementation, which, given our assumptions, is increasing at an increasing rate (in negative Y). The taking of animals beyond $\overline{Y}$ would never be part of the solution in a standard harvesting model but must be entertained here since a large stock can have detrimental effects on welfare. Harvesting rates above $\overline{Y}$ will be interpreted as "pest" control. It is also convenient to define $\overline{X}=\{X{:}V_x(\overline{X})-D_x(\overline{X})=0\}$ as the (unique) population level at which the damage caused by an additional unit of the stock equals the nonconsumptive benefits received from it. The law of motion for the animal population is given by $\dot{X}=F(X)-Y$ where the natural rate of growth, F(X), has $F(0)=F(1)=0$ and $F(X)>0$ for $0<X<1$.

2. Necessary Conditions and Phase Space

We show elsewhere (Rondeau, 1997) that a solution to (P) exists. Forming the current value Hamiltonian $\mathcal{H}=V(X)-D(X)+N(Y)+\mu[F(X)-Y]$ and the Lagrangean $\mathcal{L}=\mathcal{H}+\lambda(Y-Y_{min})$ the necessary conditions for an optimal trajectory are given by

$$N_y-\mu+\lambda=0 \tag{1}$$

$$\dot{\mu}-\delta\mu=-[V_x-D_x+\mu f_x] \tag{2}$$

$$\dot{X}=F(X)-Y \tag{3}$$

$$\lim_{t\to\infty} e^{-\delta t}X(t)\mu(t)=0 \tag{4}$$

$$X(0)=X_0 \tag{5}$$

as well as by the slackness conditions $\lambda(Y-Y_{min})=0$, $\lambda\geq 0$. Since we allow the net benefits from a marginal unit of the stock and the marginal return on harvesting to be either positive or negative, the sign of the costate variable μ is not restricted. It is therefore possible for the Lagrangean to be non-concave and (1) to (5) are not sufficient for a maximum (Kamien and Schwartz, 1991). Combining (1) with the slackness condition, we obtain that $Y=Y_{min}$ if

$N_y(Y_{min})<\mu$ and $Y=Y^*$ if $N_y(Y^*)=\mu$ (1'). Note from 1' that harvesting beyond profitability will be optimal whenever the costate variable is negative.

The dynamical system of this problem in the X-Y space is given by equations (3) and (after manipulation of equations 1 and 2) $\dot{Y} = \left[-V_x + D_x + N_y(\delta - F_x) - \dot{\lambda}\right] / N_{yy}$.

Figure 1 presents topologies of this system for which an interior steady state solution exists.[1] The $\dot{X}=0$ isocline is represented by the dome shaped curve while all other dotted curves represent the $\dot{Y}=0$ isocline for different parameterizations of the system.

The number and variety of admissible representations is of limited relevance to policy design since many of the topologies share common types of steady states and stability properties. It is shown in Rondeau (1997) that if parameter values result in a unique interior steady state (panels D, F, G, and H), the long term equilibrium is a saddle point. In this context, the optimal policy is to follow a unique manifold leading to the steady state. On the other hand, when a diagram has three equilibria, a central unstable node or spiral is outflanked by two saddle points. These topologies are more challenging to analyze but richer in economic interpretation and we focus on them for the remainder of the paper.

3. Competing Trajectories and Multiple Equilibria: Choosing the Right Path

For a study of systems with multiple equilibria, it is useful to consider Figure 2 which is a complete diagram of Figure 1C ($\overline{X} > X_\delta$), as well as Figures 3 and 4 which elaborate on Figure 1A ($\overline{X}<X_\delta$). In all cases, the pair ($\overline{X},\overline{Y}$) is interior to F(X) indicating not only that a marginal animal can be undesirable, but also that the profit maximizing rate of harvesting

[1] All simulations use *$V(X) = p(1-e^{-aX})$; $D(X)=cX^g$; $N(Y) = bY-zY^2$*; and *$F(X)=rX(1-X)$*. Constants are positive. Systems simulated using Mathematica 3.01.

is smaller than the natural rate of growth of the predator population at $\overline{X}$ [i.e. $\overline{Y}<F(\overline{X})$].

3.1 Globally Optimal Solution When a Single Spiral Leads to a Saddle Point

Inspection of Figures 2 and 3 reveals that from any initial condition $X_0 \in [0,X_{sup}]$ it is possible to adopt harvesting plans leading to either X_1^{∞} or X_3^{∞}. Adapting some of the methods introduced by Davidson and Harris (1981) and Tahvonen and Salo (1996) we make use of the properties of the maximized Hamiltonian to identify the globally optimal solution. From our assumptions on V(X), D(X), N(Y), and F(X) and by (1') to (3), it holds piecewise continuously that:

$$-\frac{d}{dt}\left[e^{-\delta t}\mathcal{H}\right]=\left[e^{-\delta t}\ \frac{d\mathcal{H}}{dt}-\delta\mathcal{H}\right]=\delta\, e^{-\delta t}\left[V\left(X\right)-D\left(X\right)+N\left(Y\right)\right]-e^{-\delta t}\lambda\,\dot{Y}$$

The last term is equal to zero in all periods. Integrating both sides and dividing by δ yields

$$1/\delta\left[V\left(X_0\right)-D\left(X_0\right)+N\left(Y_0\right)\right]+\left[N_Y\left(Y_0\right)+\lambda_0\right]\left[F\left(X_0\right)-Y_0\right]=\int_0^{\infty}e^{-\delta t}\left[V\left(X\right)-D\left(X\right)+N\left(Y\right)\right]dt$$

This equation simply states that on trajectories leading to a steady state, the net present value of a wildlife management program is equal to receiving, at every instant and forever, the value given by the Hamiltonian for the initial stock and harvesting levels.

Define, $M(X_0,Y_0)=(1/\delta)\{V(X_0)-D(X_0)+N(Y_0)+[N_Y(Y_0)+\lambda][F(X_0)-Y_0]\}$, set $\lambda=0$ (for an interior path) and take the partial derivative with respect to Y_0. The result indicates the effect of increasing the initial rate of harvesting on the net present value of a program leading to a steady state. The direction of this effect is dependent on the value of Y relative to $F(X_0)$:

$$\partial M/\partial Y_0 = (1/\delta)N_{yy}[F(X_0)-Y_0]\begin{cases} <0 \text{ for } Y_0<F(X_0) \\ =0 \text{ for } Y_0=F(X_0) \\ >0 \text{ for } Y_0>F(X_0) \end{cases} \tag{6}$$

For a given initial stock level X_0, the value of an optimal management program is decreasing in Y if $Y_0<F(X_0)$ and increasing if $Y_0>F(X_0)$. This result establishes that when more than one adjustment paths can be chosen in systems such as those in Figures 2 and 3, the lower trajectories leading to X_3^∞ are globally optimal.

3.2 Globally Optimal Solution When Two Spirals Lead to Separate Saddle Points

In the topology of Figure 4, either saddle point can be reached from initial stock levels in the vicinity of (X_2^∞,Y_2^∞). In addition, on trajectories leading to either steady state multiple candidate initial harvesting levels define several alternative time paths. To find the globally optimal trajectory, define X_{sup} as the largest stock level on the path leading to (X_1^∞,Y_1^∞) (call it "path 1"). It can be shown that none of the candidates before X_{sup} on path 1 can be optimal initial harvesting levels. The argument relies first on (6), from which we establish that trajectories starting on the outer portion of any given spiral are superior to those starting in the interior; and second, on the fact that the change in the value of the maximized Hamiltonian resulting from a change in X_0 along a trajectory is given by $dM(X_0,Y_0)/dX_0 = N_y(Y_0)<0$. The sign of this derivative is negative since it can be shown that path 1 lies entirely above $\overline{Y}$. With $N_{yy}<0$, the value of a management program decreases at a slower rate as one moves the initial population and harvesting levels along the lower portion of path 1, than it increases on the portion of the path above F(X). As a result, any program beginning before (X_{sup}, Y_{sup}) on path 1 is dominated by the alternative sequence beginning after it. With a parallel argument, we find that any management plan beginning before (X_{inf}, Y_{inf}) is inferior to a program beginning after it.

Determining the globally optimal solution still requires choosing which of path 1 or 3 maximizes welfare for initial stocks between X_{inf} and X_{sup}. Suppose that the initial stock level is X_{inf}. By (6), we conclude that the path with the highest initial harvesting level is optimal. On the other hand, if the initial stock is X_{sup} the path with the smallest initial harvesting is preferred. Since from X_{inf} it is optimal to take the high path toward $(X_1^{\infty},Y_1^{\infty})$ and from X_{sup} it is optimal to take the lower path to $(X_3^{\infty},Y_3^{\infty})$, there exists a critical stock located between X_{inf} and X_{sup}, below which it is optimal to adopt path 1 and above which path 3 is preferred. The system represented by Figure 4 has therefore two basins of attraction.

4. Properties of Optimal Population Recovery and Control Programs

Reintroduction and Supplementation. Under certain conditions preventing the existence of interior steady states, it is not beneficial to reintroduce a predator to its former habitat. From the expressions defining the dynamical system, a steady state population must solve $[\delta - F_x(X)]=[V_x(X)-D_x(X)]/N'(F(X))$. Since $F(\bullet)$ is continuous and single peaked with $F(0)=F(1)=0$, the marginal benefits from harvesting can only take a limited range of values in the interval $[N_y(F(0)), N_y(F(X_{msy}))]$. If 1) $F(X_{msy})<\overline{Y}$; 2) the marginal net non-consumptive benefits of the first animal, $V_x(0)-D_x(0)$, are sufficiently small (or negative); and 3) the discount rate is greater than the marginal growth rate of the first unit of the stock $[\delta>F_x(0)]$ then, no interior steady state exists. Since the cost of reintroducing the first animal is always greater than the benefits from harvesting it, these conditions ensure that it will never be beneficial to reintroduce the species.[2] These conditions are naturally related to the conditions under which it is optimal to harvest a species to extinction in conventional harvesting models as found by Clark (1973), Cropper *et al.* (1979) and Cropper (1988).

Conservation, Harvesting and Pest Control. In the situations illustrated in Figures 2 to 4, the reintroduction and supplementation of a population of predators is optimal. The initial supplementation phase driven by high marginal non-consumptive benefits is followed by an instant of conservation "during" which the population is self-regulated. As the population increases naturally, the optimally managed population would be removed from the list of protected species and become the subject of harvesting, albeit at sufficiently low levels to allow continued population growth. From then on, society will benefit from the consumption of animal products as well as from and the amenity value of the stock. However, regardless of the steady state population the species is ultimately headed for, the fact that the equilibrium harvesting rate is above $\overline{Y}$ implies that pest control activities will become inevitable.

Costly population control would start before X reaches the level $\overline{X}$ at which an additional animal is considered a nuisance (see Rondeau for a proof). This result has intuitive appeal. As the stock increases, the net present value of future losses associated with a

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[2] By assumption, N(•) is continuous at Y=0 with $N_y(0)>0$ and $N_{yy}(0)<0$. This ensures that the marginal cost of reintroducing the first (pair of) animal(s) is larger then the marginal benefits from harvesting the first unit. Empirical observations support this observation. The average cost of a U.S. Fish and Wildlife Service's recovery plan exceeds $3 million (1994). The ten most expensive programs cost between $29 and $88 million. Species recovery also imposes indirect costs such as restrictions on forestry.

marginal animal grows and eventually causes the shadow price to go from positive to negative. By equation (1), this change in the sign μ corresponds precisely to the instant at which it becomes optimal to incur net harvesting costs to control the population's rate of growth. At that instant marginal animals would still yield positive *instantaneous* net benefits. Nonetheless, the prospect of their "offsprings" inflicting future damage makes it optimal to restrict growth, even at a cost to society. Once the shadow price has signaled the beginning of pest control, society will incur control costs forever in order to lower the growth rate and limit the damage inflicted by the population.

Earlier efforts to boost natural productivity may then be seen as foolish and unfair. But it should not be so. Additional inspection of the conditions maximizing the value of a program along the optimal path reveals that the instant when the shadow price changes from positive to negative coincides precisely to the time when the present value of the forward portion of the control program is maximized. That is, $dM(X_0,Y_0)/dX_0=N_y(Y_0)$, and setting this expression to zero and using 1' implies $N_y(\bar{Y})=0=\mu$.

Thus, the intergenerational distribution of costs and benefits is likely to be unequal and hinder the timely implementation of optimal management decisions. Current generations may not want to pay the high cost of reintroduction and supplementation, resulting in extinction or sub-optimal conservation efforts. If the species subsists, following generations enjoying the non-consumptive benefits of a recovering population may not wish to curb its growth, leaving more distant generations with a larger than optimal population that imposes high damage rates and requires costly control. Unfortunately, there is little room for errors given the unstable properties of the system and the absence of self-correcting mechanisms.

Final Remarks

The model of predator management developed in this paper differs substantially from conventional models of renewable resources. It allows species reintroduction and other enhancement programs, conflicts between animals and humans, and the possibility of profitable harvesting as well as costly pest control. Yet, typical solutions have emerged where a single optimal trajectory leads to a saddle point steady state. In addition, the conditions under which reintroduction should not take place are extensions of the conditions leading to optimal extinction in those conventional resource harvesting models.

Less orthodox solutions have also emerged where a change in the sign of the shadow price makes the Hamiltonian non-concave. In these cases, the optimal management plan is only one of several possible equilibrium paths. When the reintroduction and supplementation of endangered predators are beneficial, the optimal policy is one of controlled but rapid initial population increase. Notwithstanding this initial prescription, the model also calls for active population control well before the animal reaches its nuisance level. This policy is justified by the costs that large future populations of predators may impose. Its timing corresponds precisely to a change in the sign of the shadow price.

The analysis underscores the importance of long term planning for the optimal management of predators. The results likely extend to other species such as the white-tailed deer, bison, or fox that carry diseases or can cause injury to humans. The model offers a test-bench for case studies of the management of such populations. Nonetheless, the analysis leaves unresolved the daunting questions raised by distributional inequities and the political challenges they pose for the timely implementation of efficient management policies.

Figure 1. Topologies and Bifurcations

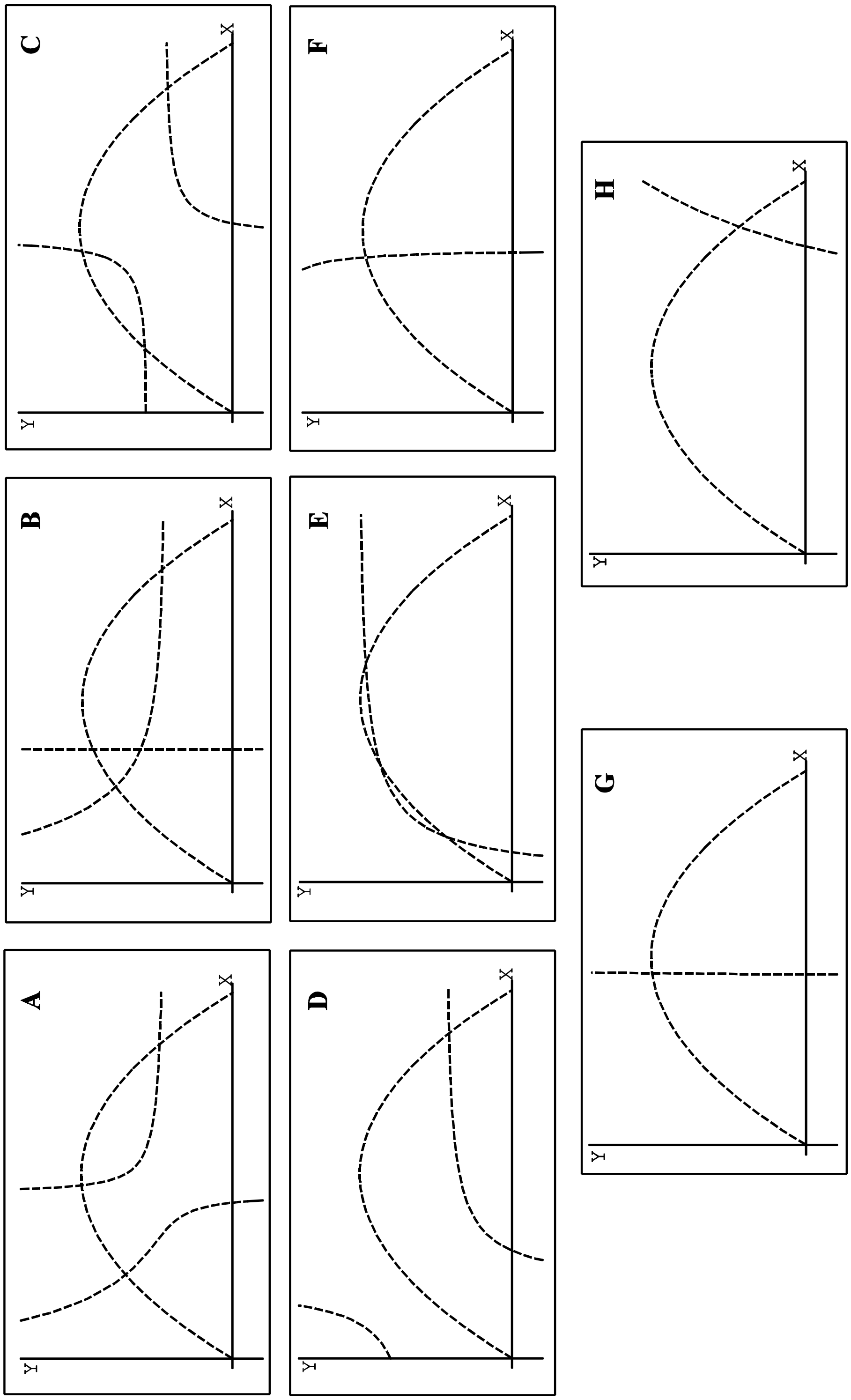


Figure 2

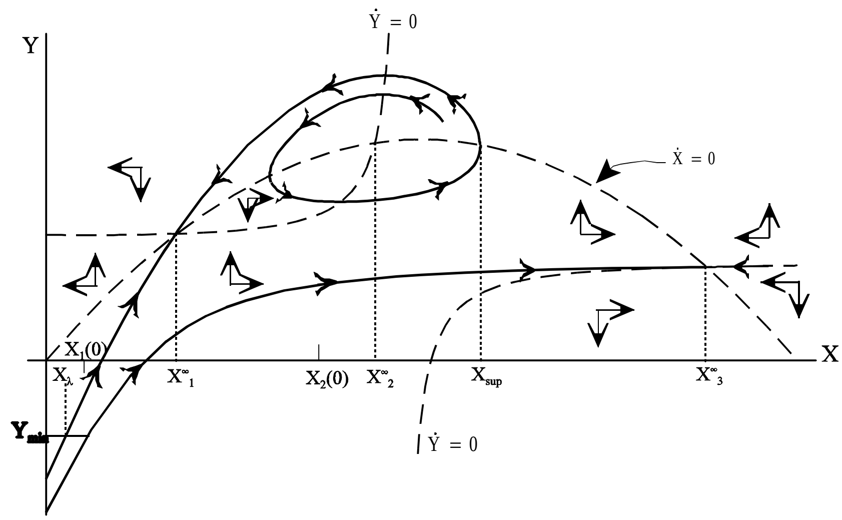


Figure 3

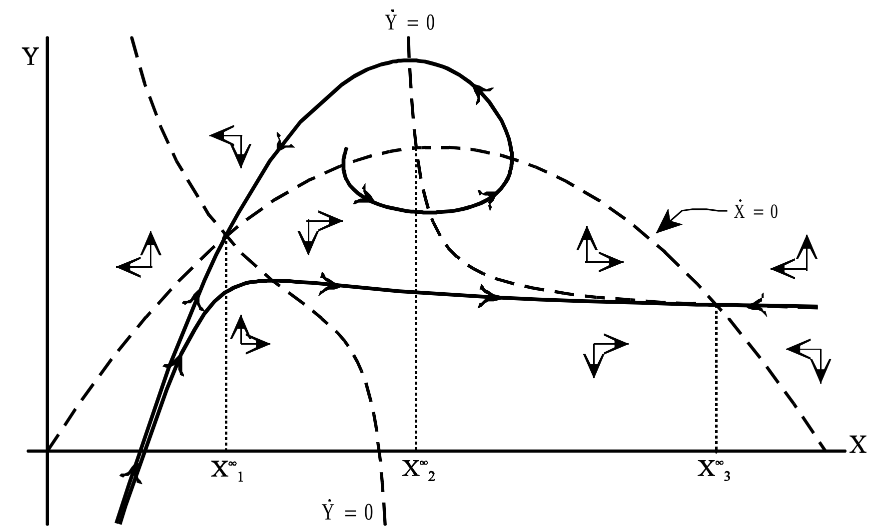
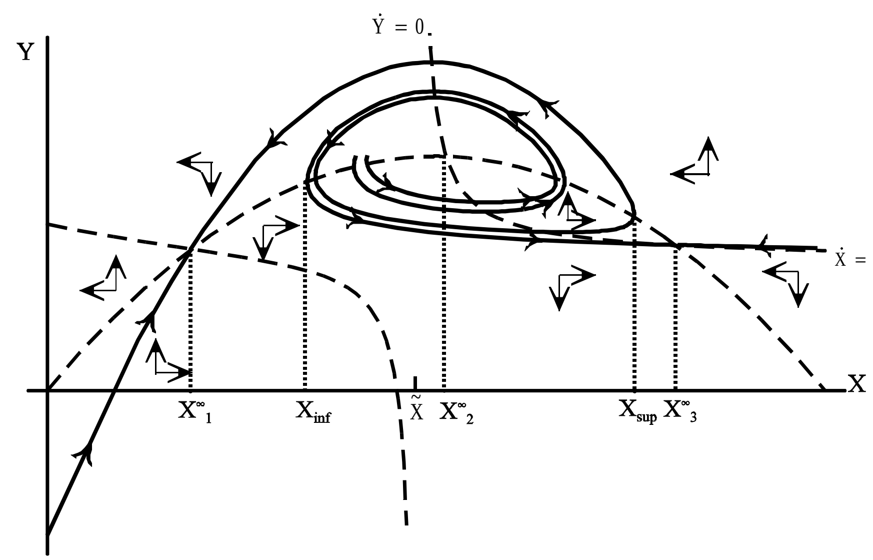


Figure 4



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