Additive versus Proportional Pest Damage Functions: Why Ecology Matters

Paul D. Mitchell

May 8, 2001

Copyright 2001 by Paul D. Mitchell. All rights reserved. Readers may make verbatim copies of this document for non-commercial purposes by any means, provide that this copyright notice appears on all such copies.
Additive versus Proportional Pest Damage Functions: 
Why Ecology Matters

Abstract

Economic analyses of pests typically assume damage is either additively separable from 
pest free yield or proportional to it. This paper describes the ecological assumptions 
required for additive and proportional damage functions to demonstrate that both 
specifications are reasonable. Ecological research supports a proportional damage 
function for competitive pests such as weeds, while for insect pests the appropriate 
damage function depends on the level of pest free yield. Theoretical analysis identifies 
differences between additive and proportional damage functions in terms of the impact of 
pest control on output variance and the concavity of output in the pest control input.

Keywords: Pest Economics, Damage Function, Damage Control, Risk Reducing Input, 
Increasing Returns, Functional Response.
Most economic analyses of pests assume that pest damages are either additively separable from potential (damage free) output or proportional to it. Additive damage models are of the general form \( q = y - L \), where \( q \) is realized output, \( y \) is potential output, and \( L \) is damage. Proportional damage models assume \( L = y \phi \), where \( \phi \) is the proportion of output lost to damage, so that \( q = y(1 - \phi) \). The key difference is that with additive models, damage does not depend on potential output while damage does depend on potential output with proportional models.

Since the two function types imply a different correlation structure between damage and realized output, they can imply different impacts of damage control inputs on output variance. As a result, when the effects of uncertainty are included, the assumed functional structure can significantly impact the estimated value of damage control and the optimal use of damage control inputs. In addition, these two damage functions differ in terms of requirements for output to be concave in the damage control inputs, and hence have different ranges of input and output prices with discontinuous input demand.

Lichtenberg and Zilberman demonstrate that damage control inputs should not be treated as standard inputs in a production function. Rather a two-stage process is needed that first models damage abatement as a function of damage control inputs, then uses damage abatement as the productive input. Lichtenberg and Zilberman do not explicitly preclude additive damage functions (p. 263-264), but discussion and analysis following the general specification in their paper assume only proportional damages.

Subsequent research extending and refining the Lichtenberg-Zilberman model has maintained this proportional damage assumption. Chambers and Lichtenberg extended the model to include multiple pest control inputs, while Babcock, Lichtenberg and
Zilberman extended the model to include multiple pests. Carrasco-Tauber and Moffit explored the sensitivity to abatement function specifications. Blackwell and Pagoulatos developed a general dynamic pest model to argue that the Lichtenberg-Zilberman model omits state variables, and that the correct model uses the proportion of pests surviving, not the proportion of pests abated. Saha, Shumway and Havenar explored specification issues including interaction between pest control and direct inputs, separability between pest control and direct inputs in damage abatement, and alternative stochastic specifications. Carpentier and Weaver also explored separability issues and developed a method to address heterogeneity bias when estimating pesticide productivity with panel data. Fox and Weersink pointed out the possibility of increasing returns to scale for damage control inputs and Hennessy developed a simple empirical test for concavity violations and associated increasing returns to scale.

Other noteworthy papers in pest economics assume proportional damages. Harper and Zilberman incorporate secondary pest impacts as an externality and Underwood and Caputo analyze the impact of pesticide taxes and information subsidies on adoption of information-based pest control strategies. Marsh, Huffaker, and Long develop a model for management of a vector-borne virus pathogen in a crop system. Sunding and Zivin analyze the regulation of pesticide use to reduce harvest worker poisoning. Zivin, Heuth and Zilberman also use a proportional damage function in their wildlife management model. After Cousens’ work concerning yield loss due to weeds, economic analyses of weed management assume a proportional damage function (Pannell; Archer and Shogren; Swinton and King).
Additive pest damage models have also been widely used in pest economics. Additive damage functions were used in models deriving action thresholds for optimal timing of pesticide application (Headley), optimal timing and dose with single and multiple applications (Hall and Norgaard; Talpaz and Borosh), and optimal timing and dose in the presence of a pollution externality and a common property resource (Regev, Gutierrez and Feder). Shoemaker determined optimal pest and predator populations in a dynamic context with chemical control, while Feder and Regev derived optimal taxes/subsidies to implement socially optimal pest and predator populations in the presence of a pesticide pollution externality. Regev, Shalit and Gutierrez compared socially and individually optimal pesticide use when the pest develops resistance to chemical control. Feder studied optimal pesticide use with uncertainty and risk aversion in a static context. Moffit, Hall, and Osteen and Marra and Carlson developed a threshold approach in the presence of uncertainty. Rollins and Briggs assume an additive damage function in their principal-agent model of wildlife crop damage compensation. Saphores applies real option theory to develop a pest treatment threshold in a stochastic process model.

This review indicates that both types of damage functions are well represented in the pest economics literature. However, this dichotomy in assumed damage function structure and its economic implications seems to have gone unexamined in a comprehensive manner. Horowitz and Lichtenberg come closest to such an analysis.

Horowitz and Lichtenberg develop a general model with multiple sources of uncertainty to clarify the conditions under which pesticides reduce or increase output variability. Though they do not state the issue in terms of additive versus proportional
damage functions, but focus on the sources of uncertainty, they realize that a special case of their Case 1 encompasses what here is termed an additive damage function. They note that pesticides are risk reducing when output and damage are uncorrelated, but because they assume a proportional damage function, they conclude that this only occurs when uncertainty about pest free yield is minimal, such as for irrigated agriculture in the western United States. As shown below, ecological theory indicates that this need not be the case—additive loss can be independent of pest free yield, regardless of the level of uncertainty in pest free yield. Horowitz and Lichtenberg provide reasonable examples and appeal to ecological principles to illustrate the applicability of their three cases, but do not utilize specific theoretical or empirical research from the ecological literature as support, since such justification was not the purpose of their paper. Rather they developed a model to demonstrate reasonable cases in which pesticides could be risk increasing, as opposed to the conventional view that pesticides must be risk reducing. In addition, since Fox and Weersink had yet to publish their paper, Horowitz and Lichtenberg did not address concavity and increasing returns.

This paper has two purposes. First it describes the ecological assumptions required for additive and proportional damage functions in order to demonstrate that both specifications are reasonable. For competitive pests such as weeds, ecological theory and empirical work support the use of a proportional damage function. However, for insect pests, ecological theory and empirical work indicates that the structure of the damage function depends on the level of pest free yield. Pest free yields below some critical level imply a proportional damage function, while above this critical level, an additive damage function is implied.
Secondly, this paper identifies economic differences between the two damage functions in terms of the impact of pest control on output variance and the concavity of output in the pest control input. With complete pest control or eradication, in general an additive damage function must satisfy a less restrictive condition for output variance to decrease with pest control. With incomplete pest control, the damage functions must satisfy different conditions for pest control to decrease output variance, but which is more restrictive cannot be determined except in a few special cases. Results concerning the possibility of increasing returns to scale with each damage function are similar—which damage function is more restrictive cannot be determined except for special cases.

Results indicate that economic difference exist between the damage specifications and ignoring these differences can lead to biases in economic analysis of a wide variety of agricultural pest issues, including the value of transgenic crops for pest control, the cost of restricting the use of pesticides, the value of pest eradication programs, the cost of pest invasions into new areas, and the impact of crop insurance on pesticide use.

**Ecological Foundation for Additive and Proportional Damage Functions**

Competitive and predator-prey systems are probably the most studied interspecific relationships in population ecology (Begon, Mortimer, and Thompson; Roughgarden; Gotelli). The other ecological relationships, commensalism, amensalism, and mutualism, are important, but not as widely studied. Predation is used broadly to also include herbivore-plant, host-parasite, host-parasitoid, and host-pathogen relationships. To apply this classification to pest-crop systems, the pest and crop are either competitors, or the pest is the predator and the crop its prey. The pest population
measures pest abundance as a species, harvested biomass or yield measures crop productivity as a species, and pest damage as a function of the pest population reduces harvested yield. This section describes the necessary assumptions for competitive and predatory pest-crop systems to exhibit additive and proportional damage functions.

**Competitive Systems**

The original Lotka-Volterra model of interspecific competition modified single species population models by using a constant proportionality factor to convert a competing species’ population to an equivalent population of the other species. Begon, Mortimer, and Thompson review papers demonstrating the empirical validity of the approach for a wide variety of competing species and discuss refinements developed for modeling competition among plant species. The original Lotka-Volterra model assumes that total productivity loss is a constant proportion of the product of both species’ populations, which in a pest-crop system implies a proportional pest damage function. Later refinements allowed this proportionality factor to change as a function of the competing species population. These refined models imply a general damage function, which in a pest-crop system means that damage is neither proportional to nor additively separable from productivity without competition.

Probably the most common example of a competitive relationship in agriculture is the weed-crop interaction. Cousens motivates his meta-analysis of yield loss due to weeds by noting the largely arbitrary nature of the models chosen for estimating yield loss and the general lack of use of even simple biological theory to guide model choice. After providing ecological justification for his derivation of a general yield loss model,
Cousens performs extensive statistical testing of numerous functional forms with several data sets to find that the hyperbolic proportional model best fits these data. This result implies that a proportional damage function is correct for the weed-crop system, so that

\[ L = y\phi(p), \]

where \( y \) is weed free yield, \( p \) is some measure of weed density, and \( \phi(p) \) is Cousens’ hyperbolic function. Cousens’ analysis has established the hyperbolic proportional model as the standard model in weed science and weed economics (Lindquist et al.; Swinton et al.; Pannell).

**Predator-Prey Systems**

Common predator-prey examples in agriculture include insect pests of crops, livestock grazing systems (both predator attacks on livestock and livestock harvesting of forage), and humans as predators harvesting populations such as fish or forest products. This paper focuses solely on the pest-crop system and leaves extensions to these other systems unexplored.

The original Lotka-Volterra predator-prey model assumed the average number of prey captured by each predator was a constant proportion of the total number of prey. In a series of papers, Holling (1959, 1965, 1966) refined this original model to include the effects of predator satiation, time for handling prey and similar requirements. The assumption is that as prey become more available, other factors limit the predation rate so that eventually it reaches some maximum. Holling used the term “functional response” to denote the function determining how the predation rate (loss per pest) increases to this maximum as a function of pest availability (pest free yield) and described three types.
For a Type 1 functional response, the predation rate increases linearly with the availability of prey until it reaches the maximum. For a Type 2 functional response, the predation rate asymptotically approaches the maximum, increasing at a decreasing rate. For a Type 3 functional response, the predation rate follows a sigmoid curve as prey become more available, first rising at an increasing rate, then asymptotically approaching the maximum at a decreasing rate of increase. Figure 1 illustrates each functional response and Begon, Mortimer, and Thompson review empirical examples of each.

Crop loss due to pest damage can be either proportional or additive for these functional responses, depending on pest-free yield. When pest free yield is sufficiently low so that loss per pest is below the plateau, crop loss per pest is proportional to pest free yield. Some proportion \( \phi(p) \) of pest free yield is lost, where \( \phi(p) \) is the product of the pest population and the slope of the functional response curve at the given pest free yield. When pest free yield is sufficiently high so that loss per pest reaches the plateau, crop loss per pest is some constant independent of pest free yield. Crop loss is \( \alpha(p) \), where \( \alpha(p) \) is the product of the pest population and the loss per pest at the functional response curve’s plateau. The first case implies a proportional damage function \( L = y\phi(p) \) while the second case implies an additive damage function \( L = \alpha(p) \).

Agricultural systems are typically managed for the crop to be highly productive. Furthermore, the common practice of planting monocultures of genetically similar and phenologically synchronized plants can create a habitat favorable for pests. As a result, in some pest-crop systems it seems possible for pest free yield to be sufficiently high and the conditions right for each individual pest to cause the maximum amount of damage. In these situations, an additive damage function is correct, otherwise a proportional damage
function is correct. When pest free yield is stochastic, the correct damage function specification depends on the realized value of the pest free yield. The issue is further complicated because the functional response depends on environmental factors, so that the critical pest free yield changes (Begon, Mortimer, and Thompson).

Additive and proportional damage functions are appropriate for different situations. If the predominate pest or pest of concern is a weed, assuming a proportional damage function can be justified by appeal to the ecological theory for interspecific competition. For insect pests, ecological theory does not provide a definitive damage function specification, but does indicate an appropriate function to estimate to guide model choice, i.e. the functional response. The correct pest damage function is an empirical issue specific to each pest-crop system, depending on the pest free yield relative to the threshold defined by the functional response.

**Economic Model**

The analysis here focuses on the implications of additive and proportional damage functions in terms of the impact of pest control on output variance and the concavity of output in the pest control input. Output variance changes determine the risk benefits or costs of pest control, while output that is locally non-concave in the pest control input implies locally increasing returns to scale and so discontinuities for input demand. The analysis seeks to identify conditions that indicate whether pest control reduces output variance and whether output is concave in the pest control input, then to examine how these conditions differ for additive and proportional damage functions.
For simplicity, assume a single output \( q \), a single pest control input \( x \), and that all other inputs are at optimal levels and so can be ignored. Two sources of uncertainty exist—pest free yield \( y \) and the pest population \( p \). Pest free yield is stochastic since it depends on a random variable \( \Theta \), where for example \( \Theta \) measures climatic factors that increase crop yield, and the pest control input \( x \) may affect pest free yield. As such, 
\[
y = f(x, \Theta),
\]
where \( f(\bullet) \) is a differentiable function, \( y_\Theta = f_\Theta > 0 \), and single (and double) subscripts denote first (and second) derivatives. The pest control input reduces the pest population, but the pest population is also stochastic since it depends on the random variable \( \Omega \) where for example \( \Omega \) is some measure of weather factors such as degree-days that benefit the pest population. As such, \( p = g(x, \Omega) \), where \( g(\bullet) \) is a differentiable function. Assume \( p_\Omega = g_\Omega(x, \Omega) > 0 \) and \( p_x = g_x(x, \Omega) < 0 \).

In many cases, the same random weather factors affecting crop growth also affect the pest population, implying that \( \Theta \) and \( \Omega \) (and hence \( y \) and \( p \)) are correlated. To model this correlation, assume \( \Theta = z(\Omega) \), so that \( z_\Omega \) determines the sign of the \( \text{Cov}[\Theta, \Omega] \).

However, to reduce model notation and complexity, use \( y_\Omega \) to denote the complete derivative of \( y \) with respect to \( \Omega \), i.e. \( y_\Omega = f_\Theta z_\Omega \). Since \( p_\Omega > 0 \), the sign of \( y_\Omega \) determines the sign of \( \text{Cov}[y, p] \). If \( z_\Omega > 0 \), then \( \Theta \) and \( \Omega \) are positively correlated and \( y_\Omega > 0 \), so that \( \text{Cov}[y, p] > 0 \). The reverse is true if \( z_\Omega < 0 \). If \( z_\Omega = 0 \), then \( \Theta \) and \( \Omega \) are uncorrelated, so that \( y_\Omega = 0 \) and \( \text{Cov}[y, p] = 0 \).

Output is \( q = y - L \), where \( L \) is either additive and \( L = \alpha(p) \) or proportional and \( L = y\phi(p) \). Assume \( L \) strictly increases in the pest population, whether the damage
function is additive or proportional, and that no loss occurs when \( p = 0 \), i.e. \( \alpha_p(p) > 0 \), \( \alpha(0) = 0 \), \( \phi_p(p) > 0 \), and \( \phi(0) = 0 \).

**Risk Management Impact of Pest Control**

*Complete Pest Control or Pest Eradication*

Complete pest control eliminates the pest from the crop before damage occurs or in some manner prevents all pest damage, but control is required each season since fields are potentially re-infested. Pest eradication eliminates the pest from the region so that control is no longer needed. Examples of complete pest control include Bt corn active against European and Southwestern corn borers and the Roundup Ready and Liberty Link herbicide resistant crops. Klassen (1989) and Myers, Savoie, and van Randen (1998) review several examples of past and current eradication programs for pests such as screwworm, boll weevil, gypsy moth, Mediterranean fruit fly, codling moth, and imported fire ant.

This special case ignores all use of the pest control input \( x \), and hence concavity issues, and focuses solely on the impact of complete pest control or eradication on output variance. As such, pest control becomes a binary choice. Before pest control \( q = y - L \), while with pest control \( q = y \). With additive damage, total loss is \( L = \alpha(p) \) so that \( L \) and \( y \) are only correlated when \( y \) and \( p \) are correlated (i.e. both functions of \( \omega \)). With proportional damage, total loss is \( L = y\phi(p) \) so that \( L \) and \( y \) must be correlated, even if \( y \) and \( p \) are not correlated.

**Proposition 1:** Complete pest control or pest eradication changes output variance by \( \Delta V = -V[L] + 2Cov[y, L] \), which is negative only if \( V[L] > 2Cov[y, L] \).
Proof. Output variance after complete control or eradication is \( V[y] \), the variance of pest free yield. Output variance before control is \( V[y - L] = V[y] + V[L] - 2\text{Cov}[y, L] \). The effect of pest control on output variance is \( \Delta V = V[y] - V[y - L] \). This simplifies to \( \Delta V = -V[L] + 2\text{Cov}[y, L] \), which is only negative if \( V[L] > 2\text{Cov}[y, L] \).

With additive pest damage, the sign of \( \text{Cov}[y, L] \) is the same as the sign of \( \text{Cov}[y, p] \), since \( L = \alpha(p) \) is a positive monotonic transformation of \( p \). If \( y \) and \( p \) are uncorrelated, then \( \text{Cov}[y, L] = 0 \) and complete pest control or pest eradication must reduce output variance. If \( y \) and \( p \) are negatively correlated, then \( \text{Cov}[y, L] < 0 \) and again complete pest control or pest eradication must reduce output variance. If \( y \) and \( p \) are positively correlated, then \( \text{Cov}[y, L] > 0 \) and complete pest control or pest eradication has an ambiguous effect on output variance. Unlike the case of additive damage, with a proportional damage function, \( \Delta V \) has an ambiguous sign regardless of the correlation between \( y \) and \( p \), implying that complete pest control or eradication has an ambiguous effect on output variance.

The primary implication of Proposition 1 is that in the case of complete pest control or eradication, an additive damage function is more apt to decrease output variance. This is clearly the case when \( y \) and \( p \) are uncorrelated. For the additive case \( \text{Cov}[y, L] = 0 \) so that \( \Delta V = -V[L] < 0 \). For the proportional case \( \text{Cov}[y, L] = \text{Cov}[y, y\phi] = V[y]E[\phi] > 0 \), so that \( \Delta V = -V[L] + 2\text{Cov}[y, L] \), which must exceed \(-V[L]\). As a result, the decrease in yield variance must be smaller for proportional damage than for additive damage.

Assuming or imposing an additive damage function when the true damage function is proportional creates an upward bias on estimates of the output variance.
reduction occurring with complete pest control or eradication. Similarly, assuming or imposing a proportional damage function when the true damage function is additive has the opposite effect—output variance reduction occurring with complete pest control or eradication is underestimated. Care must be taken when selecting or assuming a damage function if changes in output variance matter for the analysis of complete pest control or eradication, as for example when including risk effects in the evaluation of pest eradication programs, the invasion of pest species to new areas, or the value of complete pest control using transgenic crop varieties.

**Incomplete Pest Control**

Incomplete pest control occurs when use of the pest control input \( x \) does not prevent all pest damage to the crop, for example because the pesticide does not eliminate every individual pest, or because new individual pests continually hatch, emerge, sprout, immigrate, etc. The analysis here assumes a single perfectly divisible pest control input and leaves extensions of the analysis to pest threshold models for future research.

Most analyses assume the pest control input is homothetically separable from the inputs determining pest free yield, i.e. that the pest control input does not affect pest free yield. Notable exceptions include Harper and Zilberman, Carpentier and Weaver, and Saha, Shumway, and Havenar. However, many pest control inputs affect potential crop yields, and crop inputs can affect pest populations. Some herbicides damage both crops and weeds, or have carry-over effects that reduce yields of crops that follow. Mechanical control of weeds can damage crop roots and reduce yields, while mechanical control of insects can reduce yields, or cause bruises and blemishes. Chemical control of one insect
pest can cause secondary pest outbreaks that reduce yields and/or further increase pest control expenditures. Tillage not only reduces insect, weed and plant pathogen problems, but also increases soil aeration and early spring soil temperatures which increases crop yields by allowing earlier planting and establishment of better crop stands. Fertilizer and irrigation water increase crop yield, but also supply nutrients and water to weeds and affect insect and plant pathogen populations or crop ability to compensate for pest damage. As such, the analysis here assumes a non-separable pest control input and addresses a separable pest control input as a special case.

Proposition 2 and its corollary express the condition for the pest control input to be risk reducing in terms of the relative curvature of the loss function. The relative curvature of a function normalizes the curvature (second derivative) by the marginal (first derivative) so that the resulting ratio is unit invariant.

**Proposition 2:** With an additive damage function the input $x$ is risk reducing if

$$\frac{L_{pp}}{L_p} > -\frac{p_{x0}}{p_x p_{w0}} + K_1,$$

where $K_1 = \frac{y_{w0}}{L_p p_x p_{w0}}$. With a proportional pest damage function the pest control input $x$ is risk reducing if

$$\frac{L_{pp}}{L_p} > -\frac{p_{x0}}{p_x p_{w0}} + (1-\phi)K_1 + K_2 + K_3,$$

where $K_1$ is as previously defined, $K_2 = -\frac{y_{w0}}{yp_{w0}}$, and $K_3 = -\frac{y_x}{yp_x}$.

**Proof:** The input $x$ is risk reducing if the marginal damage reduction is larger when pest damage is larger. For the model as specified, since both the pest population and damage are strictly increasing in $\omega$ this requires that the marginal product of $x$ be increasing in $\omega$—$q_{x0} > 0$. This sign requirement is opposite that typically required since here increasing the pest population through $\omega$ decreases, instead of increases, output.
For an additive damage function, $q = y - \alpha(p)$, so that $q_x = y_x - \alpha_p p_x$ and
\[
q_{x0} = y_{x0} - \alpha_p p_{x0} p_x - \alpha_p p_{x0} p_x.
\]
Divide both sides of the condition $q_{x0} > 0$ by $-p_{x0} p_x$,

which is positive since $p_{x0} > 0$ and $p_x < 0$. Rearrange to obtain
\[
\alpha_p > \frac{\alpha_p p_{x0}}{p_{x0} p_x}
\]

and divide both sides by $\alpha_p > 0$. To obtain the reported expression, note that since $L = \alpha(p)$, $L_p = \alpha_p$ and $L_{pp} = \alpha_{pp}$.

For a proportional damage function, $q = y - y\phi(p)$, so that $q_x = y_x - y_x \phi$
\[
- y\phi_p p_x
\]
and $q_{x0} = y_{x0} - y_{x0} \phi - y_x \phi_p p_{x0} - y_x \phi_p p_x - y_x \phi_p p_{x0} p_x - y_x \phi_p p_{x0} p_x$. Divide both sides of the condition $q_{x0} > 0$ by $-p_{x0} p_x$, which is positive since $p_{x0} > 0$ and $p_x < 0$.

Rearrange to obtain
\[
y\phi_{pp} > \frac{-y\phi_p p_{x0}}{p_{x0} p_x} + (1 - \phi) \frac{y_{x0}}{p_{x0} p_x} - \phi \left( \frac{y_x + y_{x0}}{p_x + p_{x0}} \right)
\]

and divide both sides by $y\phi_p > 0$. To complete the proof, note that since $L = y\phi(p)$, $L_p = y\phi_p$ and $L_{pp} = y\phi_{pp}$.

**Corollary 1.** If pest free yield is separable from the pest control input and pest free yield and the pest population are uncorrelated, then $K_1 = K_2 = K_3 = 0$ and no difference exists between an additive and proportional damage function. If pest free yield is separable from the pest control input, then $K_1 = K_3 = 0$. If pest free yield and the pest population are uncorrelated, then $K_1 = K_2 = 0$.

**Proof.** These are special cases of Proposition 2. If $x$ and $y$ are separable and
\[
Cov[y, p] = 0
\]
then $y_x = y_{x0} = y_{x0} = 0$ so that $K_1$, $K_2$, and $K_3$ are zero. If $x$ and $y$ are
separable, then \( y_x = y_{x0} = 0 \), so that \( K_1 \) and \( K_3 \) are zero. If \( \text{Cov}[y, p] = 0 \), then
\[ y_{x0} = y_{x00} = 0 \], so that \( K_1 \) and \( K_2 \) are zero.

Proposition 2 and Corollary 1 indicate that the condition for a pest control input to be risk reducing generally differs for additive and proportional damage functions because with proportional damages, loss also depend on pest free yield. Only in the special case when \( y \) is separable from \( x \) and \( y \) and \( p \) are uncorrelated does the condition not differ. As a result, just as with complete pest control, care must be taken when assuming or imposing the general form of the damage function since an incorrect specification can bias estimates of the variance effect resulting from changes in pest control.

Unfortunately the terms in the proposition do not lend themselves to intuitive interpretations. As a result, discussion begins with the more restrictive cases in Corollary 1 before addressing Proposition 2.

Proposition 2 and its corollary imply that, depending on the sign of the right hand side, for a pest control input to be risk reducing, the loss function must either be sufficiently convex, or not too concave. The curvature of the loss function is an empirical issue for each pest-crop system, but concavity seems more likely, since it implies the reasonable result that the marginal increase in damage due to each additional pest is decreasing.

When \( y \) is separable from \( x \) and \( y \) and \( p \) are uncorrelated, the condition for \( x \) to be risk reducing simplifies to
\[ \frac{L_{pp}}{L_p} > -\frac{p_{x0}}{p_x p_{x0}} \], whether the damage function is additive or proportional. Since \( p_x > 0 \) and \( p_{x0} < 0 \), \( p_{x0} \) determines the sign of \( \frac{p_{x0}}{p_x p_{x0}} \). The sign of
depends on how weather affects pest control, the pest control’s mode of action, and the pest’s biology. A negative $p_{wp}$ implies that weather conditions favorable for pest growth make pest control more effective—$w$ and $x$ are complements for pest control. A positive $p_{wp}$ implies the opposite. If $p_{wp} < 0$, a convex or linear loss function ensures that $x$ is a risk reducing input, while a concave loss function cannot be too concave. If $p_{wp} > 0$, a concave or linear loss function implies that the pest control input is risk increasing; for $x$ to be risk reducing requires that the loss function be sufficiently convex to satisfy the condition. The sign of $p_{wp}$ is an empirical issue for each pest–crop system. However, $p_{wp} < 0$ seems likely for most systems, though this need not be the case for all systems.

If $y$ and $p$ are correlated, but $y$ is separable from $x$, the condition for a proportional damage function to be risk reducing also includes the term $K_2$. Since both $y$ and $p_w$ are positive, $y_w$ determines the sign of $K_2$, and the sign of $y_w$ is the same as the sign of $Cov[y, p]$. If $Cov[y, p] > 0$, then $K_2 < 0$, which implies that the condition for a pest control input to be risk reducing is more restrictive for an additive damage function than for a proportional damage function. If $Cov[y, p] > 0$, then $K_2 > 0$, and a proportional damage function has a more restrictive condition for a pest control input to be risk reducing.

The sign and magnitude of the correlation between $y$ and $p$ is an empirical issue specific to each pest-crop system, but positive, negative and no correlation are observed. For example, European corn borer populations can be decimated during the brief adult mating period by dry weather (no rainfall and low relative humidity) and by wet weather...
at larval hatch (Mason et al.). Because corn yield depends on cumulative weather over
the season, these acute events during critical periods for the insect have little impact on
yield. As a result, no correlation exists between \( y \) and \( p \) for this system (Showers et al.).
However, populations of problematic grasshopper species generally rise during drought
conditions when crop yields are below average (Hein and Campbell; Patrick), which
implies that \( \text{Cov}[y, p] < 0 \). On the other hand, phytophageous insects are limited by
dietary nitrogen (White; Evans), so that populations of pests such as silverleaf whitefly,
corn earworm/cotton bollworm, and cotton aphids generally increase when crop hosts
have more nitrogen available (Bi et al.; Broadway and Duffey, Nevo and Coll). Since
crops are also nitrogen limited, pest free yields also increase with nitrogen availability so
that \( \text{Cov}[y, p] > 0 \).

If \( y \) is not separable from \( x \) and \( y \) and \( p \) are uncorrelated, the condition for a
proportional damage function to be risk reducing also includes the term \( K_3 \). Since \( y > 0 \)
and \( p_x < 0 \), \( y \) determines the sign of \( K_3 \). If the pest control input also reduces the pest
free yield, then \( K_3 < 0 \) so that the condition for \( x \) to be risk reducing is more restrictive for
an additive damage function. However, if the pest control input also increases the pest
free yield, or the yield augmenting input also reduces the pest population, then \( K_3 > 0 \) so
that the condition for \( x \) to be risk reducing is more restrictive for a proportional damage
function. The signs of \( y \) and \( K_3 \) depend on the specific pest control input and crop, and
as previously discussed, a variety of relationships can exist so that it is not possible a
priori to assume a sign for \( y \) and \( K_3 \).

Proposition 2 addresses the most general case when \( y \) is not separable from \( x \) and
\( y \) and \( p \) are correlated. Not only are the terms \( K_2 \) and \( K_3 \) present, with signs and
implications as discussed, but also the term $K_1$. $K_1$ arises because of the interaction between $x$ and $\omega$ in determining pest free yield and its sign depends on the cross partial derivative $y_{x\omega}$. Regardless of its sign, $K_1$ has the same effect for both additive and proportional damage functions, but with proportional damages it is reduced by the factor $(1-\phi)$.

Using the sign of $y_{x\omega}$ to determine whether $x$ and $\omega$ are substitutes or complements for the production of $y$ depends on the signs of $y_x$ and $y_{\omega}$. For example, if $y_x$ and $y_{\omega}$ are positive and $y_{x\omega}$ negative, then weather good for the crop decreases the productivity of $x$ for producing the crop so that $x$ and $\omega$ are substitutes for producing $y$. However, if again $y_{x\omega} < 0$ and $y_x > 0$, but now $y_{\omega}$ is negative, weather good for crop production increases the productivity of $x$ for producing the crop so that $x$ and $\omega$ are complements. The main point is that the sign of $y_{x\omega}$ cannot be interpreted in isolation, but must be placed in context of the whole pest crop system. If $y_{x\omega} > 0$, then $K_1 < 0$ so that the condition for $x$ to be risk reducing is more restrictive for a proportional damage function. The reverse is true if $y_{x\omega} < 0$ and $K_1 > 0$.

The sign of $K_1$ depends on the interaction between $x$ and $\omega$ in determining pest free yield (the sign of $y_{x\omega}$), the sign of $K_2$ depends on the correlation between $y$ and $p$ (the sign of $y_{\omega}$), and the sign of $K_3$ depends on whether $x$ increases or decreases pest free yield (the sign of $y_x$). As a result, for the general case addressed by Proposition 2, a variety of relationships are possible in which various positive and negative effects offset one another. As such, whether an additive or proportional damage function is less
restrictive in terms of risk reducing or risk increasing effects of the pest control input is in
general ambiguous. Only in two cases can the difference between an additive and
proportional damage function be clearly identified.

If \( y \) and \( p \) are negatively correlated (\( y_o < 0, K_2 > 0 \)), \( x \) increases pest free yield
\((y_x > 0, K_3 > 0)\), and weather bad for the crop (but good for the pest) increases the
productivity of \( x \) for pest free yield \((y_{x0} > 0, K_1 < 0)\), then a proportional damage
function must satisfy a more restrictive condition for the pest control input to be risk
reducing. If \( y \) and \( p \) are positively correlated (\( y_o > 0, K_2 < 0 \)), \( x \) reduces pest free yield
\((y_x < 0, K_3 < 0)\), and weather good for the crop (and the pest) makes \( x \) even more
damaging to pest free yield \((y_{x0} < 0, K_1 > 0)\), then an additive damage function must
satisfy a more restrictive condition for the pest control input to be risk reducing. All
other combinations of \( y_x, y_o \) and \( y_{x0} \) create an ambiguous difference between an
additive and proportional damage function in terms of the risk reducing/increasing effects
of the pest control input.

In summary, Proposition 2 and Corollary 1 express as a restriction on the relative
curvature of the loss function the condition necessary for the pest control input to be risk
reducing when the damage function is either additive or proportional. In only the most
restrictive case (\( y \) and \( p \) uncorrelated and \( x \) separable from \( y \)) is the condition the same for
both an additive and a proportional damage function. However, it is not possible except
for a few special cases to determine whether an additive or proportional damage function
is more restrictive in terms of the condition that must be satisfied for the pest control
input to be risk reducing. These results indicate that the assumed general form of the
damage function has impacts on the risk management benefits of pest control and that assuming an incorrect general form can bias estimates of these risk benefits in an unpredictable manner. As such, specification testing is in order before imposing the form of the damage function for estimation.

**Returns to Scale and Pest Control**

As demonstrated by Fox and Weersink and Hennessy, the concavity of output in the pest control input becomes more difficult to ensure because of the damage and pest control functions. A lack of concavity implies the possibility of increasing returns to scale and a discontinuity in the demand for the pest control input over some range of input and output prices. Following Hennessy, a condition ensuring the concavity of output in the pest control input \( x \) is expressed in terms of the relationship between the relative curvatures of the loss function and the indirect control function. The indirect control function \( h(\bullet) \) is the inverse of the pest control function \( p = g(x, \phi) \). Since \( g(\bullet) \) is strictly decreasing in \( x \), it can be inverted to obtain \( x = h(p, \phi) \).

**Proposition 3:** With an additive damage function, output is concave in the pest control input if
\[
\frac{L_{pp}}{L_p} > \frac{h_{pp}}{h_p} + J_1, \text{ where } J_1 = \frac{y_{xx} h_p^2}{L_p}.
\]

With a proportional damage function, output is concave in the pest control input if
\[
\frac{L_{pp}}{L_p} > \frac{h_{pp}}{h_p} + (1 - \phi) J_1 + J_2, \text{ where } J_1 \text{ is as previously defined and } J_2 = -\frac{2 y_x h_p}{y}.
\]

**Proof:** The proof follows the method used by Hennessy. Express output in its parametric form, i.e. as a function of the pest population \( p \), then use the rules for
differentiating a function in its parametric form to obtain the first and second derivatives of output in the pest population. Because \( p = g(x, \omega) \) and \( x = h(p, \omega) \) are inverses, \( p_x = 1/x_p \) and \( g_x = 1/h_p \). For an additive damage function rearrange \( q_p = y_x h_p - \alpha_p \) to obtain \( y_x = \frac{q_p + \alpha_p}{h_p} \). Substitute these into \( q_x = y_x - \alpha_p g_x \) and simplify to obtain

\[
q_x = \frac{q_p}{h_p}.
\]

Using the quotient rule, making substitutions, and simplifying gives

\[
q_{xx} = \frac{q_{pp} - q_p h_{pp}/h_p}{h_p^2}.
\]

The numerator determines the sign of \( q_{xx} \). Substitute

\[
q_p = y_x h_p - \alpha_p \quad \text{and} \quad q_{pp} = y_x h_p^2 + y_x h_{pp} - \alpha_{pp}
\]

into the numerator, then rearrange to obtain the reported expression, noting that \( L_p = \alpha_p \) and \( L_{pp} = \alpha_{pp} \). Repeat the process for a proportional damage to again obtain

\[
q_{xx} = \frac{q_{pp} - q_p h_{pp}/h_p}{h_p^2},
\]

but now

\[
q_p = y_x h_p - y_x h_p \phi - y \phi_p \quad \text{and} \quad q_{pp} = y_x h_p^2 (1 - \phi) + y_x h_{pp} (1 - \phi) - 2 y_x h_p \phi_p - y \phi_{pp}.
\]

Substitute these into the numerator and rearrange to obtain the reported expression, noting that \( L_p = y \phi_p \) and \( L_{pp} = y \phi_{pp} \) to complete the proof.

**Corollary 2:** If pest free yield is separable from the pest control input, output is concave in the pest control input if \( \frac{L_{pp}}{L_p} > \frac{h_{pp}}{h_p} \) whether the damage function is additive or proportional.

**Proof:** This is a special case of Proposition 3. If \( y \) is separable from \( x \), then

\[
y_x = y_{xx} = 0 \quad \text{so that} \quad J_1 = 0 \quad \text{and} \quad J_2 = 0,
\]

whether damage is additive or proportional.
Proposition 3 indicates that in general the condition for ensuring the concavity of output in the pest control input differs for additive and proportional damage functions. Only in the special case when $y$ is separable from $x$ does the condition not differ, as reported by Corollary 2. In this special case, the concavity requirement is that the loss function be relatively more convex or less concave than the indirect control function. This requirement has been previously reported—Corollary 2 is equivalent to the proposition developed by Hennessy. However, Proposition 3 extends Hennessy’s proposition to address the more general case in which $x$ affects pest free yield. Proposition 3 finds that the concavity condition must be adjusted to account for the impact of $x$ on $y$ and that this adjustment differs for additive and proportional damage functions since $y$ also appears in the proportional damage function.

Concavity of output in the pest control input is important since it defines the range of input and output prices over which demand for the pest control input is continuous. The limits of continuous input demand impact the use of taxes or subsidies for addressing pest control externalities. Discontinuities can also create difference between the efficiency of taxes and standards for addressing pest control externalities. Because of the difference between additive and proportional damage functions, imposing an additive damage function when the true damage function is proportional, or vice versa, implies that the range of continuous input demand will be incorrectly estimated. Errors of this sort imply potential errors when developing policies to address pest control externalities. As such, depending on the goal of the analysis, care must be taken when assuming or imposing the general form of the damage function.
Whether or not the concavity condition is satisfied is an empirical question for each pest-crop system. As such, discussion here does not address this issue, but rather focuses on identifying the requirements that indicate whether the concavity condition is more restrictive for an additive or proportional damage function. Given the results in Proposition 3, this requires determining the sign of \(-\phi J_1 + J_2\), since the concavity condition for additive and proportional damage functions differ only by this expression.

If \(-\phi J_1 + J_2 > 0\), then the condition is more restrictive for a proportional damage function, since the loss function for a proportional damage function must be more convex or less concave than is required for an additive damage function. Similarly, if \(-\phi J_1 + J_2 < 0\), then the condition is more restrictive for an additive damage function.

The signs of \(y_x\) and \(y_{xx}\) determine whether the requirement is satisfied for the concavity condition to be more restrictive for a proportional damage function. To see this, use the definitions of \(J_1\) and \(J_2\), substitute in \(\phi = \frac{L}{y}\), and rearrange the condition \(-\phi J_1 + J_2 > 0\) to obtain

\[
- y_{xx} L > \frac{2y_x L_p}{h_p}.
\]

If \(y_x > 0\) and \(y_{xx} < 0\), then (1) becomes \(L > \frac{-2y_x L_p}{y_{xx} h_p}\). Then as long as \(L > 0\), a proportional damage function has a more restrictive condition. This is the standard case for a productive input—that it have a positive and diminishing marginal product. Thus any typical input that also has pest reduction properties will satisfy this condition. For
example if application of nitrogen fertilizer as anhydrous ammonia also reduces corn rootworm larval populations or tillage also reduces weed populations.

If \( y_x < 0 \) and \( y_{xx} > 0 \), then (1) becomes \( L < \frac{-2 y_x L_p}{y_{xx} h_p} \), which implies that \( L < 0 \) is needed for a proportional damage function to have a more restrictive condition. Thus as long as \( L > 0 \), an additive damage function must satisfy a more restrictive condition. This case implies a pest control input that damages the crop, with the marginal damage decreasing as use of the input increases.

If \( y_x > 0 \) and \( y_{xx} > 0 \), then (1) implies that the concavity condition is more restrictive for a proportional damage function if \( L < \frac{-2 y_x L_p}{y_{xx} h_p} \). This puts an upper bound on \( L \) since the right hand side is positive. Thus losses below this critical point imply that a proportional damage function has a more restrictive condition to satisfy, but losses above this critical value imply the opposite. This case seems unlikely, since it requires that the input \( x \) not only reduce the pest population, but also increases pest free yield at an increasing rate.

If \( y_x < 0 \) and \( y_{xx} < 0 \), then (1) implies that the concavity condition is more restrictive for a proportional damage function if \( L > \frac{-2 y_x L_p}{y_{xx} h_p} \). Thus losses above this critical level imply that a proportional damage function has a more restrictive condition, but losses below this critical value imply the opposite. This is the case of a pest control input that damages the crop, with the marginal damage increasing with use of the input.

In summary, if \( y_x \) and \( y_{xx} \) have opposite signs, then whether the concavity condition is more restrictive for an additive or proportional damage function can clearly
be determined. If $y_x$ and $y_{xx}$ have the same sign, then whether the concavity condition is more restrictive for an additive or proportional damage function depends on whether loss $L$ is above or below a critical level. The lack of concavity and associated increasing returns only become a concern if policy changes or other factors imply moving prices into the range of input demand discontinuities. As such, ignoring concavity problems can cause unexpected outcomes for policies meant to alleviate pesticide problems.

Conclusion

A review of the literature in pest economics indicated that most analyses assume either an additive or a proportional damage function. However, this dichotomy in assumed damage function structure and the associated economic implications has generally gone unexamined in a comprehensive manner. This paper described the ecological assumptions required for additive and proportional damage functions in order to demonstrate that both specifications are reasonable. For competitive pests such as weeds, ecological theory and empirical work support the use of a proportional damage function, but for insect pests, the level of pest free yield determines the appropriate structure of the damage function. A proportional damage function is appropriate when pest free yield is below some critical level, while pest free yield above this critical level implies an additive damage function.

In three propositions and two corollaries, this paper identified economic differences between the two damage functions in terms of the impact of pest control on output variance and the concavity of output in the pest control input. When complete pest control or eradication is possible, an additive damage function must in general
satisfy a less restrictive condition for output variance to decrease with pest control. When pest control is incomplete, the conditions for pest control to decrease output variance differ for each damage function structure and which is more restrictive cannot be determined analytically except in a few special cases. Results are similar concerning the concavity of output in the pest control input—which damage function is more restrictive cannot be analytically determined except for special cases.

These theoretical results indicate that differences exist between the damage function structures and that ignoring these differences can lead to biases in economic analysis of a wide variety of agricultural pest issues, including the value of transgenic crops for pest control, the cost of restricting the use of pesticides, the value of pest eradication programs, the cost of pest invasions into new areas, and the impact of crop insurance on pesticide use. Empirical analysis is needed to determine the magnitude of the biases that result from imposing an incorrect damage function structure—these biases may remain theoretical possibilities with little empirical importance, or may be quite substantial. Also, empirical analysis can indicate which pests of which crops exhibit additive or proportional damages so that one or the other damage function can be eliminated as empirically unlikely for some pest crop systems.

In addition to empirical applications, other areas remain unexplored. Hennessy has developed a concavity test for the case of multiple pest control inputs and it is likely that his method can be extended to develop a concavity test that allows interaction between pest control inputs and pest-free yield and that indicates differences between additive and proportional damage functions. Furthermore, the impacts of using data that aggregate across multiple control inputs and/or pests when some pests cause additive
damage and some cause proportional damage is not clear. It may be that the damage function should include both an additive and a proportional component. Also, optimal use of a pest control input may differ for additive and proportional damage functions, even after accounting for differences due to output variance impacts and concavity. Similarly, optimal thresholds may differ for additive and proportional damage functions.

Another interesting issue not pursued here is additive pest survival functions. Most pest analyses assume that the pesticide kill or survival function is proportional to the pest population. The ecological research of DeWitt and Yoshimura implies that additive kill/survival can occur and that additive and proportional kill/survival functions imply differences in terms of species evolution to adapt to environmental changes. For agricultural pests, this implies differences between additive and proportional kill/survival functions in terms of the development of pesticide resistance. Possible impacts on optimal pesticide use, output variance, or concavity remain to be explored.
Figure 1. Plots illustrating the general shape of the three types of functional response curves describing the loss per pest ($L/p$) as a function of the pest free yield ($y$).
References


