World Dairy Product Trade:
Analysis with a Mixed Complementarity Problem Formulation

Charles F. Nicholson

Department of Applied Economics and Management
Cornell University
Ithaca, NY 14853-7801
Email: cfn1@cornell.edu

and

Phillip M. Bishop

New Zealand Institute of Economic Research
P.O. Box 3479
Wellington, NEW ZEALAND
Email: phil.bishop@nzier.org.nz

Selected Paper Presented at the 2001 AAEA Annual Meetings,
Chicago, IL, August 5-8, 2001

Abstract:

World trade in dairy products is being transformed by trade policy liberalization and technological change in dairy processing. The mixed complementarity problem formulation to modeling product-specific dairy trade can overcome a number of the limitations of existing optimization models, and can facilitate desirable extensions to previous analyses.

Keywords: Dairy, trade, mixed complementarity problem

World Dairy Product Trade:
Analysis with a Mixed Complementarity Problem Formulation

Charles F. Nicholson
Cornell University

and

Phillip M. Bishop
New Zealand Institute of Economic Research

Introduction

Dairy has long been a highly regulated industry in the United States and in other developed countries. However, in the early 1990s the US dairy industry entered a period of domestic and trade policy reform. Three major policy events—the 1994 North American Free Trade Agreement (NAFTA), the 1995 Uruguay Round Agreement (URA), and the 1996 Federal Agriculture Improvement and Reform Act (FAIR)—represented a significant modification of previous policies by opening up markets and limiting government price support. Although the extent to which these efforts will be carried forward in future policies (*e.g.*, the next round of WTO negotiations and Farm Bill) is uncertain in the current political environment, these changes retain significant potential to influence world dairy trade.

In addition to trade and domestic policy reform, technological developments in the dairy and food processing industries will take on a greater importance in coming years. Current microfiltration technologies permit the fractionation of milk into its basic nutritive components: proteins, fats, lactose and minerals (Rizvi, 1987; Rizvi and Bhasker, 1995). These basic building blocks of milk are already being used to build customized products for industries as diverse as medicine and pharmaceuticals, health foods, and specialized food preparations and ingredients. Component separation is ubiquitous in the world dairy industry, and already the basic milk fractions are being further separated into various specialty products. Separation allows dairy processing companies to formulate products that can be transported more cheaply, stored for longer periods, and reformulated into a variety of customized food ingredients and value-added products. These developments will place tremendous pressure on policies aimed at pricing milk and protecting domestic producers. Technological change in dairy processing thus has the potential to markedly alter dairy trade patterns over the next two decades. The implications of future component separation technologies and product formulations for world and U.S. dairy markets has not previously been studied, so the potential impacts are unknown.

Many of the analytical models developed to date fail to account for important facets that determine prices, trade patterns, and competitiveness in the dairy industry today, at least for analyses of product-specific trade policies such as those likely to be negotiated under the next WTO round. The limitations of previous models are discussed in detail in Bishop *et al*. (1994). The characteristics of the world dairy industry that should be addressed in a model of dairy trade are summarized in Table 1. First, the characteristics of milk and dairy products make product-specific trade modeling a challenge. One characteristic is jointness in production. That is, milk is viewed by dairy processors as a combination of components (e.g., fat, proteins, and lactose) that can be (and are) separated and recombined in numerous product forms. This implies that economic models of dairy product trade must include sufficient disaggregation of dairy components, and explicit balancing constraints for each component.

Related to the need to account for component separation is the observation that much dairy trade is in "intermediate products." That is, dairy products processed from milk received at one location frequently are transported to another location and are used to make a different dairy product. An example is the use of nonfat dry milk in cheese manufacturing. Trade in intermediate products is a significant portion of total world trade, although an exact accounting is difficult to determine. In the mid-1990s, however, more than half of dairy product trade consisted of milk powders, butter and related products, and casein-type products, all of which have potential uses in manufacturing other dairy products. The importance of intermediate product trade nearly always implies that an explicit "processing" sector needs to be specified in dairy trade models. The political importance of the dairy sector in most countries also has resulted in a plethora of government interventions in dairy production, marketing and trade. Thus, any product-specific model of dairy trade must be able to account for a full range of domestic and trade policy instruments regulating both prices (*e.g.*, support prices) and quantities (*e.g.*, tariff-rate quotas and quantitative export subsidy limits). In particular, it is highly useful (if not essential) for product-specific models to address discriminatory *ad valorem* tariffs (tariffs that vary by country of origin), the principal mechanism for trade liberalization through "tariffication" under the last round of WTO negotiations.

A few recently constructed models (*e.g.*, Cox and Zhu, 1997) have incorporated a higher degree of component and product disaggregation than the dairy trade models commonly in use a decade ago (*e.g.*, OECD, 1991; Baker, 1991). However, even recently developed dairy trade models do not include explicit representation of flows of intermediate dairy products (*i.e.*, those used in subsequent dairy processing) among countries. Modeling of *ad valorem* tariffs by Cox and Zhu (1997) relies upon iterative solution of the model with unit tariffs. The use of a mixed complementarity framework has great potential to incorporate characteristics of dairy trade not yet adequately addressed by existing empirical models. These characteristics include direct modeling of *ad valorem* tariffs, imperfectly competitive international markets (including state trading enterprises such as the New Zealand Dairy Board), nonlinearities in component balance equations due to variations in raw milk component content by region, and development of new intermediate products that circumvent existing trade barriers.

The objectives of this paper are to describe a model of world dairy trade using the mixed complementarity approach, and to discuss its advantages over existing model formulations. We focus on the development of the model structure that accounts for relevant factors influencing dairy trade, and contrast our structure with that of previous modeling efforts. Empirical implementation of our model structure is ongoing, so no numerical results are presented herein. Our model is a joint-input (*i.e.*, multiple-component), multiple-product spatial trade model. Conceptually, the model derives an equilibrium across spatially dispersed markets for raw milk and the range of products derived from milk. In equilibrium, prices are related across regions and market levels subject to transfer costs, policy, and institutional impediments. The model includes an explicit representation of the dairy processing sector in each supply region. As a result, the equilibrium conditions ensure that milk component quantities are balanced, and that currently feasible technical relationships in dairy processing are maintained.

The model explicitly incorporates key trade policies such as product-specific tariffs, quotas, and export subsidy limitations. Domestic economic policies that can be specified as restrictions on prices or quantities, such as price supports or production quotas, are included where these have a material impact and are quantifiable. The structure incorporates multilateral and bilateral agreements. The complementarity framework readily allows the computational innovations related to imperfect competition of Hashimoto (1984) and Kolstad and Burris (1986) to be implemented in a full scale applied model (Ferris and Pang, 1995). The outcomes of various strategies that might be employed by state trading enterprises focusing on exports (*e.g.,* in New Zealand) and imports (*e.g.*, in Mexico and China) can be analyzed within this framework. Although our structure is specific to dairy products, the MCP approach has great potential for a broad range of product-specific trade analyses (Harrison *et al.*, 1997).

The Mixed Complementarity Problem

The complementarity problem is essentially a way to find a solution to a square system of nonlinear equations. As Ferris and Munson (2000) note, the complementarity problem adds a "combinatorial twist" to the classic square system of nonlinear equations. Of $2n$ equalities in a system, a subset n will be chosen that will hold as equalities. More formally, the nonlinear complementarity problem (NCP) can be specified as:

$$\text{Given a nonlinear function } F : \Re^n \rightarrow \Re^n \text{, find } z \in \Re^n \text{ such that}$$

$$0 \leq z \; or \; F(z) \geq 0.$$

Thus, only one of the inequalities is satisfied as an equality, or equivalently for individual components, $z_i F_i(z)=0$. This property is typically referred to as z_i being "complementary" to $F_i(z)$. As an extension to this NCP, we may sometimes wish to specify certain "intermediate" variables, for example, y_i, where

$$y_i = f(z) \text{ for } i = 1,\ldots,I$$

Then the NCP then becomes

$$\text{Given a nonlinear function } F : \Re^n \to \Re^n \text{, find } z \in \Re^n \text{ such that}$$

$$0 \le z \text{ or } F(z) \ge 0$$

$$\text{and}$$

$$y_i = f(z).$$

The problem now involves a mixture of equations (for the y_i) and complementarity contstraints. The "mixed" nature of this problem results in the name mixed complementarity problem. More formally, following Ferris and Munson (2000) the mixed complementarity problem can be defined as:

Given lower bounds $l \in \{\Re \cap \{-\infty\}\}^n$, upper bounds $u \in \{\Re \cap \{\infty\}\}^n$, and a function F:$\Re^n \to \Re^n$, find $z \in :\Re^n$ such that precisely one of the following holds for each $i \in \{1,\ldots n\}$:

$$F_i(z) = 0 \text{ and } l_i \le z_i \le u_i$$

$$F_i(z) > 0 \text{ and } z_i = l_i$$

$$F_i(z) < 0 \text{ and } z_i = u_i\ .$$

Often in trade modeling, non-negativity constraints will be appropriate, implying that $li = 0$. Note also that if $li = zi = u_i$, then the function $F_i(z)$ is unrestricted and can be omitted from the model.

In the typical simple spatial price equilibrium (Samuelson, 1952; Takayama and Judge, 1964) model with unit transportation costs and no other trade barriers, a nonlinear objective function is maximized subject to a set of constraints to calculate a market equilibrium. When the objective function is formulated in terms of inverse demand and supply functions, the model variables are the quantity produced in each region, the quantity demanded in each region, and the quantity shipped from each supply region to each demand region. The "dual" values in this formulation are the supply and demand prices in each region. In contrast, the MCP framework permits the construction of models with explicit representation of both prices and quantities as variables. For example, the basic spatial price equilibrium (SPE) model would be expressed as:

$$
\begin{aligned}
&\sum_j x_{ij} \le Q_i^s \text{ or } P_i^s \ge 0 \\
&Q_j^d \le \sum_i x_{ij} \text{ or } P_j^d \ge 0 \\
&g_j^d(Q_j^d) \le P_j^d \text{ or } Q_j^d \ge 0 \\
&g_i^s(Q_i^s) \le P_i^s \text{ or } Q_i^s \ge 0 \\
&P_j^d \le P_i^s + c_{ij} \text{ or } x_{ij} \ge 0
\end{aligned}
$$

where

Q_j^d = quantity demanded in region j

$\mathrm{Q}_\mathrm{i}^\mathrm{s}$ = quantity supplied in region i

x_{ij} = quantity shipped from supply region i to demand region j

P_j^d = demand price in region j

P_i^s = supply price in region i

c_{ij} = constant unit transport costs from supply region i to demand region j

$g_i^s(Q_i^s)$ = inverse supply function in supply region i

$g_j^d(Q_j^d)$ = inverse demand function in demand region j

The MCP framework exploits Kuhn-Tucker complementary slackness conditions to provide an explicit representation of both ‘primal’ and ‘dual’ variables in the model structure. Although primal-dual methods also exploit this complementarity, the MCP approach can be extended to create new problems for which no equivalent optimization problem exists. For example, Nicholson *et al.* (1994) have shown that the SPE model with discriminatory *ad valorem* tariffs (*i.e.*, tariffs on imports that differ by exporting region) cannot be directly solved using an optimization model, because the value of the tariff depends on the endogenously-determined supply price[1]. In the MCP framework, this is easily handled by modifying the condition relating supply and demand prices as follows:

$$P_j^d \leq (P_i^s + c_{ij})(1+\tau_{ij}) \text{ or } x_{ij} \geq 0$$

where the τ represent *ad valorem* tariffs imposed by demand region *j* on imports from supply region *i*. The essential points are that both price and quantity values can be simultaneously and directly constrained, and that relationships among these variables need not conform to the first-order conditions of an optimization problem.

Because both prices and quantities can be simultaneously constrained, policy instruments that target prices or quantities (*e.g.*, price supports, *ad valorem* tariffs, tariff rate quotas) can be modeled simultaneously and directly. Complementarity also makes mute the issue of integrability (*e.g.*, the need for symmetry of cross-price terms in demand equations) which is a major restriction required by many of the algorithms for solving conventional optimization problems. For the world dairy industry, the relevant set of spatial price equilibrium conditions can be formulated and solved as a mixed complementarity problem

---

[1] As noted earlier, however, it is possible to iteratively solve the SPE as an optimization problem to obtain unit tariff values equivalent to the applicable *ad valorem* tariffs.

(MCP) to yield supply and demand prices and quantities, milk component values, and interregional dairy product trade flows.

The World Dairy Trade Model Formulation

In this section, we provide a detailed mathematical description of the MCP model equations and variables. To help place the mathematics into perspective, a conceptual representation is provided for a simplified two-region, three product version of the model (Figure 1). Milk produced (circle) in region 1 can flow to processing plants in regions 1 or 2. In countries that have a raw milk supply quota, milk production can not exceed the quota quantity. The arrows connecting the raw milk supply and processing plants (triangles) represent raw milk assembly flows. In the processing sector, milk components are balanced. As a result, milk components, in the form of intermediate products, move between the plants. All intermediate and final products can potentially be traded between regions. Government policies and support programs (e.g., tariffs, quotas, tariff-rate quotas, export subsidies and price supports) are primarily administered through the processing sector. The label "final product trade" applies to the arrows connecting processing to demand both within a region and across regions. Products are demanded at wholesale level by "consumers" and the non-dairy industry (squares).

For the mathematical representation of the model, the sets, or indices, upon which the model is specified are as follows:

Regions: $R = \{i,j \mid i,j = (1, 2, ..., J)\}$.

Products: $P = (k,k',k'' \mid k,k',k'' = (1, 2, ..., K)\}$. $IP \in P$ denotes the set of intermediate products and $FP \in P$ denotes the set of final products. This specification enables, but does not require, a product to be both an intermediate and a final product. k = $FP \in P$ also denotes processing plant types, *i.e.* plant types correspond to final product types. Conversely, intermediate products must be produced at a final product plant type. The intermediate product shipments allowable in the model are summarized in Table 2.

Components: $C = \{m \mid m = (1, 2, ..., M)\}$.

Quota levels: $L = \{l \mid l = (1, 2, ..., L)\}$.

The parameters in the model are defined as:

α_i = slope coefficient in raw milk supply function in region i;

ε_i = own price elasticity of raw milk supply in region i;

β_{ik} = slope coefficient in demand function in region i for product $k \in FP$;

η_{ik} = own price elasticity in demand function in region i for product $k \in FP$;

ψ_{im} = proportion of milk component m contained in raw milk in region i;

δ_{km} = proportion of milk component m contained in intermediate product $k \in IP$;

γ_{ikm} = proportion of milk component m contained in final product $k \in FP$ in region i;

tc_{ijk} = per unit transportation cost to ship product k from region i to region j;

tcr_{ij} = per unit transportation cost to ship raw milk from region i to region j;

pc_{ik} = constant per unit processing cost for product k in region i;

t_{ijkl} = per unit import tariff imposed on the l^{th} level of the quota schedule by region j on imports of product k from region i;

τ_{ijkl} = *ad valorem* import tariff imposed on the l^{th} level of the quota schedule by region *j* on imports of product *k* from region *i*;

s_{ijk} = per unit export subsidy imposed by region *i* on exports of product *k* to region *j*;

rq_i = raw milk supply quota in region *i*;

bq_{ijkl} = bilateral import quota imposed by region *j* on the l^{th} level of the quota schedule on imports of product *k* from region *i*;

mq_{jkl} = multilateral import quota imposed by region *j* on the l^{th} level of the quota schedule on imports of product *k* from all regions; and

sv_{ik} = maximum export volume of product *k* that region *i* may subsidize.

The variables in the model are defined as:

QRM_i = quantity of raw milk produced in region *i*;

QCR_{ikm}= quantity of milk component *m* received at plant type $k \in FP$ in region *i*, and which arrives at the plant in the form of raw milk, *i.e.* it is also possible for components to arrive at plants in the form of intermediate products;

QCP_{ikm}= quantity of milk component *m* processed at plant type $k \in FP$ in region *i*. If components are *processed*, it implies they are used in the production of final products;

QPP_{ik} = quantity of product type *k* produced in region *i*. Unlike components that are *processed* at plants, the quantity of product *produced* at a plant is defined on all *k*, *i.e.* $k \in IP$ and $k \in FP$. This distinction between processing and producing is somewhat artificial and can be confusing. It is really only necessary to allow processing costs to be applied per unit of product;

QFP_{ik} = quantity of final product $k \in FP$ demanded in region *i*;

XRM_{ijk} = quantity of raw milk shipped from region i to plant type $k \in FP$ in region j;

$XIP_{ik'jk''kl}$ = quantity of intermediate product $k \in IP$ shipped from plant type $k' \in FP$ in region i to plant type $k'' \in FP$ in region j, on the l^{th} level of the quota schedule. There is only a single non-binding level to the quota schedule for all intra-regional shipments, *i.e.* when $i = j$; (See Table 2)

XFP_{ijkl} = quantity of final product $k \in FP$ (shipped from plant type $k \in FP$) in region i to region j, on the l^{th} level of the quota schedule. There is only a single non-binding level to the quota schedule for all intra-regional shipments, *i.e.* when $i = j$;

PRM_i = market price of raw milk in region i;

PCR_{ikm} = market price of milk component m received at plant type $k \in FP$ in region i;

PCI_{ikm} = market price of milk component m in interplant transfers of intermediate products at plant type $k \in FP$ in region i;

PCP_{ikm} = market price of milk component m processed at plant type $k \in FP$ in region i;

PRQ_i = market price of raw milk production quota in region i;

PQP_{ik} = market price of processing product type k in region i;

PXS_{ik} = market price of quantitative restriction on subsidized exports of product type k from region i;

PMQ_{ikl} = market price of the multilateral import quota imposed by region i on imports of product k on the l^{th} level of the quota schedule;

PBQ_{ijkl} = market price of the bilateral import quota imposed by region j on imports of product k from region i on the l^{th} level of the quota schedule; and

PFP_{ik} = market price of final product k demanded in region i.

Employing the notation set out above, the model is defined as follows:

$$QRM_i \geq \sum_j \sum_{k \in FP} XRM_{ijk} \qquad \forall i \in R \tag{1}$$

$$\sum_j \left(\psi_{jm} * XRM_{jik}\right) \geq QCR_{ikm} \qquad \forall i \in R, k \in FP, m \in C \tag{2}$$

$$\begin{aligned} QCR_{ik'm} + \sum_j \sum_{k'' \in FP} \sum_{k \in IP} \sum_l \left(\delta_{km} * XIP_{jk''ik'kl}\right) \geq \\ \sum_j \sum_{k'' \in FP} \sum_{k \in IP} \sum_l \left(\delta_{km} * XIP_{jk'ik''kl}\right) + QCP_{ik'm} \qquad \forall i \in R, k \in FP, m \in C \end{aligned} \tag{3}$$

$$QCP_{ikm} \geq \sum_j \sum_l \left(\gamma_{jkm} * XFP_{ijkl}\right) \qquad \forall i \in R, k \in FP, m \in C \tag{4}$$

$$QPP_{ik} \geq \sum_j \sum_{k' \in FP} \sum_{k'' \in FP} \sum_l XIP_{ik'jk''kl} + \sum_j \sum_l XFP_{ijkl} \qquad \forall i \in R, k \in P \tag{5}$$

$$\sum_i \sum_l XFP_{ijkl} \geq QFP_{jk} \qquad \forall j \in R, k \in FP \tag{6}$$

$$rq_i \geq QRM_i \qquad \forall i \in R \tag{7}$$

$$s_{ijk} \geq \sum_{i \neq j} \sum_{k' \in FP} \sum_{k'' \in FP} \sum_l XIP_{ik'jk''kl} + \sum_{i \neq j} \sum_l XFP_{ijkl} \qquad \forall i \in R, \forall k \in P \tag{8}$$

$$mq_{jkl} \geq \sum_i \sum_{k' \in FP} \sum_{k'' \in FP} XIP_{ik'jk''kl} + \sum_i XFP_{ijkl} \qquad \forall i \neq j \in R, \forall k \in P, l \in L \tag{9}$$

$$bq_{ijkl} \geq \sum_{k' \in FP} \sum_{k'' \in FP} XIP_{ik'jk''kl} + XFP_{ijkl} \qquad \forall i \neq j \in R, \forall k \in P, l \in L \tag{10}$$

$$\left(\tfrac{1}{\alpha_i}\right)^{\varepsilon_i} * QRM_i^{1/\varepsilon_i} + PRQ_i \geq PRM_i \qquad \forall i \in R \tag{11}$$

$$PRM_i + tcr_{ij} \geq \sum_m \left(\psi_{im} * PCR_{jkm}\right) \qquad \forall i, j \in R, k \in FP \tag{12}$$

$$PCR_{ikm} \geq PCI_{ikm} \qquad \forall i \in R, k \in FP, m \in C \tag{13}$$

$$\left(\sum_m \left(\delta_{km} * PCI_{ik'm}\right) + PQP_{ik} + tc_{ijk}\right) * \left(1 + \tau_{ijkl}\right) - s_{ijk} + t_{ijkl} + PXS_{i \neq jk} + PMQ_{i \neq jkl} + PBQ_{ijkl} \geq \sum_m \left(\delta_{km} * PCI_{jk''m}\right) \qquad \forall i, j \in R, k \in FP, l \in L \tag{14}$$

$$PCI_{ikm} \geq PCP_{ikm} \qquad \forall i \in R, k \in FP, m \in C \tag{15}$$

$$pc_{ik} \geq PQP_{ik} \qquad \forall i \in R, k \in P \tag{16}$$

$$\left(\sum_m \left(\gamma_{jkm} * PCP_{ikm}\right) + PQP_{ik} + tc_{ijk}\right) * \left(1 + \tau_{ijkl}\right) - s_{ijk} + t_{ijkl} + PXS_{i \neq jk} + PMQ_{i \neq jkl} + PBQ_{ijkl} \geq PFP_{jk} \qquad \forall i, j \in R, k \in FP, l \in L \tag{17}$$

$$PFP_{ik} \geq \left(\tfrac{1}{\beta_{ik}}\right)^{\eta_{ik}} * QFP_{ik}^{1/\eta_{ik}} \qquad \forall i \in R, k \in FP \tag{18}$$

In order to exploit complementary slackness when solving the model, it is necessary to associate each equation with its complementary variable. The complementarity pairings are defined as follows:

Equation	Variable		Equation	Variable
(1)	*PRM*		(11)	*QRM*
(2)	*PCR*		(12)	*XRM*
(3)	*PCI*		(13)	*QCR*
(4)	*PCP*		(14)	*XIP*
(5)	*PQP*		(15)	*QCP*
(6)	*PFP*		(16)	*QPP*
(7)	*PRQ*		(17)	*XFP*
(8)	*PXS*		(18)	*QFP*
(9)	*PMQ*			
(10)	*PBQ*			

Some notes to explain the model are warranted. Equations (1) through (6) are essentially the underlying "primal" constraints. Together with an appropriate objective function, they would constitute an NLP formulation of the classic Samuelson-Takayama-Judge SPE type of model. Equations (7) through (10) are policy conditions, again, operating on the quantity or primal variables. Equations (10) through (18) are the "dual" conditions in the MCP formulation of the model. Alternatively, they can be thought of as zero profit, or arbitrage, conditions.

The raw milk supply and final product demand functions, equations (11) and (18) respectively, are inverted for convenience. In other words, the first term in (11) yields the supply price, while the right-hand side of (18) yields demand prices.

We now briefly describe each equation in turn. Equation (1) simply says that the quantity of raw milk produced in region *i* must exceed the quantity shipped to plants. While raw milk is theoretically able to cross regional boundaries, it is expensive to transport large distances, and may encounter hygiene-related barriers to trade. Equation (2) translates the raw milk delivered to plants into a quantity of *m* milk components. Even when raw milk is shipped interregionally, it is the composition of milk at the point of supply, and the quantity of milk shipped, which determines the quantity of each milk component received at plants. Equation (3) appears quite complex; it is the component balancing constraint associated with interplant shipments of intermediate products. It says that for each of the *m* component types in milk, the quantity received at a plant in the form of raw milk, plus the quantity received in the form of interplant shipments, must be greater than or equal to the quantity shipped out as interplant shipments, plus the quantity processed into final products. Equation (4) is another balancing constraint. It ensures that the quantity of each milk component shipped out of a plant in the form of final products does not exceed the quantity actually processed at that plant. Equation (5) is included in the model for convenience; it allows us to compute the variable *QPP* so that we can assign processing costs on a per unit of product (final or intermediate) basis, rather than trying to estimate such cost on a component basis. Equation (6) simply says that the quantity of final product demanded in a region can be no more than the quantity shipped to that region (including from itself).

Equation (7) imposes raw milk supply quotas where they exist; equation (8) imposes quantitative restrictions on subsidized exports; and equations (9) and (10) impose, respectively, multilateral and bilateral import quotas. As already alluded to, the import quota schedule can have many levels or steps to it. The sum of all bilateral import quotas that any region may impose is, by definition, less than or equal to (usually less than) that region's multilateral import quota. Incidentally, the tariffs associated with each step of the tariff-rate quota schedule must be monotonically increasing.

Equation (11) states that for each region, the raw milk supply price plus the raw milk supply quota value must be greater than or equal to the market price of raw milk. Equation (12) says that the market price of raw milk plus the cost of shipping milk to a plant must be at least as great as the price of milk at the plant. The plant price of milk is computed from the sum of the component values each multiplied by their respective composition parameters. Equation (13) requires that the price of a milk component, m, at the point of receipt at a plant is equal to or greater than its price when transferred elsewhere as an intermediate product shipment. Equations (14) and (17) are similar; (14) is the zero profit condition for intermediate products while (17) is the same condition for final products. Essentially, they specify the wedges that exist between prices at plants and/or plants and demand markets in terms of import tariffs (*ad valorem* and specific), export subsidies, transportation costs, and quota rental values. It is these two constraints that enable discriminatory *ad valorem* tariffs to be modeled (τ_{ijkl} is defined bilaterally). As noted earlier, this is not possible in an NLP formulation.

Like equation (13), (15) is just an accounting identity that emerges from the underlying profit maximizing behavior assumed on the part of processing firms. Equation (16) says that the per unit cost of processing each product type must be greater than or equal to the market price of that processing activity. Finally, equation (18) requires that, for each region, the market price of a final product is consistent with the quantity of that product demanded and the specified inverse demand function. Government purchase prices for specific products can be established by fixing lower bounds on the price variable PFP.

The MCP can be solved in GAMS using the PATH solver (Dirske and Ferris, 1995; Ferris and Munson, 2000), and can be more computationally efficient than optimization formulations for some problems.

Data Considerations

Although the focus of this paper is on the mathematical structure of the model, a brief discussion of data is relevant. The data requirements of our model present a significant challenge due to the high degree of disaggregation in product, spatial, and policy dimensions. These data and potential sources are summarized in Table 3. The data required can be categorized as economic parameters, technical parameters, and policy or institutional parameters. Secondary sources can be used to obtain many of these parameters, although in some cases available data must be used to develop estimates of the necessary coefficients. Key sources for the required information are appropriate country-level agencies and the relevant literature. International agencies that collate data are also sources, *e.g*., WTO and APEC. As a last resort, the popular sources such as FAO and OECD can be used. Despite the challenges, Nicholson (1996) has shown the feasibility of collecting and assessing the detailed information on dairy production, processing, and consumption required for a model of the type proposed.

Conclusion

The world dairy industry currently faces major domestic and trade policy reform and technological changes that have the potential to markedly alter existing dairy trade patterns. To adequately analyze this potential, current dairy trade models must be modified to incorporate additional essential characteristics of the industry. This paper has described the mathematical structure of a mixed complementarity formulation that includes many of these essential characteristics, demonstrating the potential of a MCP to extend product-specific dairy trade modeling in relevant directions.

References

Baker, A.D. *An Investigation of the Dairy Subsector Impacts of a Free Trade Agreement Between Canada and the United States*. Unpublished Ph.D. Dissertation, The Pennsylvania State University, University Park, Pennsylvania, 1991.

Bishop, P.M., J.E. Pratt, and A.M. Novakovic. *Using a Joint-Input, Multi-Product Formulation to Improve Spatial Price Equilibrium Models.* SP 94-06, Department of Agricultural, Resource, and Managerial Economics, Cornell University, 1994. Presented at the "New Dimensions in North American-European Agricultural Trade Relations" conference, Calabria, Italy, June 20-23, 1993.

Cox, T.L. and Y. Zhu. *Assessing the Impacts of Liberalization in World Dairy Trade*. No. 406 Agricultural and Applied Economics Staff Paper Series, University of Wisconsin, Madison, January 1997.

Dirske, S.P. and M.C. Ferris. "The PATH Solver: A Non-Monotone Stabilization Scheme for Mixed Complementarity Problems." *Optimization Methods and Software* 5:123-159, 1995.

Ferris M. C. and J. S. Pang. Engineering and Economic Applications of Complementarity Problems. MP-TR-95-07, Computer Sciences Department, University of Wisconsin, Madison, May 1995.

Ferris, M. C., and T. S. Munson. GAMS/PATH User Guide, Version 4.3 GAMS Development Corporation, 2000.

Harrison, G. W., T. F. Rutherford, and D. Tarr. Quantifying the Uruguay Round. The *Economic Journal*, 107:1405-1430, 1997.

Hashimoto, H. "A Spatial Nash Equilibrium Model," in *Spatial Price Equilibrium: Advances in Theory, Computation, and Application*, P.T. Harker (editor), Springer–Verlag, 1984.

Kolstad, C.D. and A.E. Burris. "Imperfectly Competitive Equilibria in International Commodity Markets," *American Journal of Agricultural Economics,* 68:27-36, 1986.

Nicholson, C. F. Economic Liberalization, Free Trade, and Structural Change: A Spatial Economic Model of Mexico's Dairy Sector. Ph. D. dissertation, Cornell University, 1996.

Nicholson, C. F., P. M. Bishop, and A. Nagurney. Using Variational Inequalities to Solve Spatial Price Equilibrium Models with Ad Valorem Tariffs and Activity Analysis. Department of Agricultural, Resource, and Managerial Economics, Cornell University, 1994. [Working Paper 94-14]

Organization for Economic Cooperation and Development (OECD). *Changes in Cereals and Dairy Policies in OECD Countries: A Model-Based Analysis*. OECD, Paris, 1991.

Rizvi, S. S. H. Supercritical Fluid Extraction of Dairy Foods. *New York Food and Life Sciences Quarterly,* 17:23–26, 1987.

Rizvi, S. S. H., and A. R. Bhaskar. Supercritical Fluid Processing of Milk Fat: Fractionation, Scale-up, and Economics. *Food Technology,* 49:90-97,100, 1995.

Samuelson, P.A. "Spatial Price Equilibrium and Linear Programming." *American Economic Review,* 42:283–303, 1952.

Takayama, T. and G.G. Judge. "An Interregional Activity Analysis Model of the Agricultural Sector." *Journal of Farm Economics,* 46:349–365, 1964.

Figure 1. Simplified Conceptual Representation of the World Dairy Trade Model

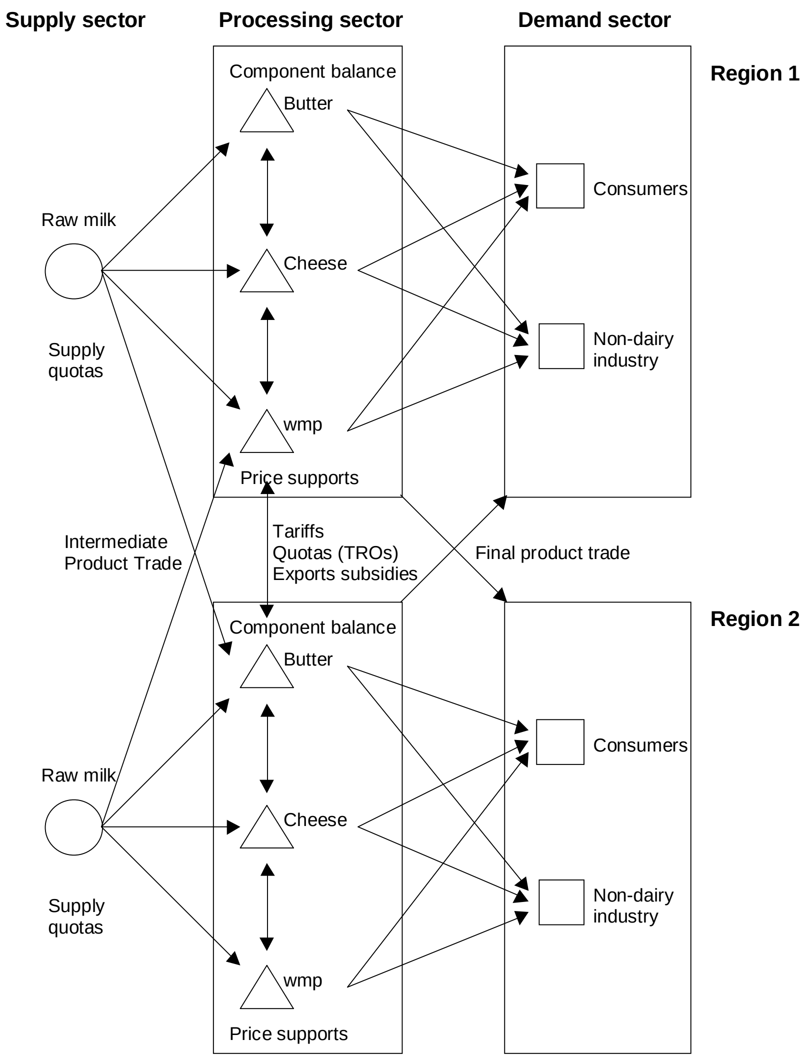

Table 1. Minimum Characteristics Required for Modeling Dairy Trade

Characteristic	Example of representation in a dairy trade model
Jointness in production, component disaggregation	Dairy products characterized as two or more components (*e.g.*, fat, protein and other solids), not as milk equivalents.
Intermediate products	Allow dairy products traded among processing firms (*e.g.*, milk powders, butteroil, whey powders) to be manufactured and traded in addition to products for final demand.
Explicit processing sector	For each region, specify a processing plant or plants. Plants serve to mediate supplies of raw milk and intermediate products to meet final demands. Model constraints ensure that milk component inflows and outflows are balanced, and that currently feasible technical relationships in dairy processing are maintained.
Trade policy specificity	Tariffs (*ad valorem* and unit, discriminatory and non-discriminatory), quotas, TRQs and export subsidies modeled explicitly for specific dairy products, rather than aggregated measures. Trade policies modeled with constraints on the price and quantity relationships in model formulation.
Domestic policy specificity	Key price supports, production quotas, and price controls in the dairy sector modeled with constraints on prices and quantities in the model formulation.
Bilateral trade flows	Use of price-responsive domestic supply and demand functions, rather than excess supply and demand functions, allows regions to switch from net importer (exporter) to net exporter (importer)
Alternative market structure assumptions	Market imperfections of the types described in Hashimoto (1984) and Kolstad and Burris (1986) with relevance to examination of STEs modeled as constraints in the model formulation.
Product disaggregation	Examine intermediate and final dairy product types, rather than the small number in many previous models.
Regional disaggregation	Specify at least 10 production, processing, and consumption regions, based on the importance of countries in current world production, consumption, or trade.

Table 2: Interplant (Intermediate Product) Shipments Allowed in the World Dairy Trade Model

Intermediate product type	From plant type	To plant type
Skim Milk Powder (SMP)[1]	SMP	Cheese
SMP	SMP	Fluid
SMP	SMP	Soft products
Whole Milk Powder (WMP)	WMP	Cheese
WMP	WMP	Fluid
WMP	WMP	Soft products
Anhydrous Milk Fat (AMF)	Butter	Fluid
AMF	Butter	Soft products
Cream	Fluid	Butter
Cream	Fluid	WMP
Cream	SMP	Butter
Cream	SMP	WMP
Cream	WMP	Butter
Skim milk	Casein	Cheese
Skim milk	Casein	Fluid
Skim milk	Casein	Soft products
Skim milk	Casein	SMP
Skim milk	Butter	Casein
Skim milk	Butter	Cheese
Skim milk	Butter	Fluid
Skim milk	Butter	Soft products
Skim milk	Butter	SMP
Skim milk	WMP	Casein
Skim milk	WMP	Cheese
Skim milk	WMP	Fluid
Skim milk	WMP	Soft products
Skim milk	WMP	SMP
Butter milk	Butter	Soft products
Butter milk	Butter	WMP

[1] The intermediate product skim milk powder, for example, can flow from a skim milk powder plant to a cheese plant, a fluid plant and a soft product plant. Intermediate product anhydrous milk fat is processed in a butter plant. It can flow from a butter plant to a plant for fluids and to a soft product plant.

Table 3. Data Requirements for the World Dairy Trade Model

Type of data	Examples	Sources
Economic parameters		
Supply elasticities	Raw milk, inputs (grain) and complementary outputs (sheep, beef)	Existing estimates (*e.g.*, SWOPSIM, FAO), academic and government literature in each region
Demand elasticities	Final products, income	Existing estimates (*e.g.*, SWOPSIM, FAO), academic and government literature in each region
Price-quantity pairs in base period	Raw milk and final products	FAO, national statistics, EU Commission, ABARE
Transformation costs	Processing costs for intermediate and final products	Academic, government, and industry sources in each region, contacts with key dairy industry leaders
Transportation costs	Milk hauling, ocean freight (refrigerated vs. non-refrigerated; container vs. pallet), land-based transportation costs for manufactured products	Key industry contacts
Technical parameters		
Product composition	Component content of raw milk, intermediate products, and final products	USDA, FAO and national statistics data to construct component balances for key regions; contacts with dairy industry leaders
Transformation coefficients	Yield relationships in dairy processing, possibilities for interplant shipments and joint production	Same as above.
Institutional and policy parameters		
Tariffs	Traded dairy products by region (including discriminatory tariffs)	U.S. Department of Commerce, APEC, national agencies, fee-based sources
Quotas	Traded dairy products	Same as above
Export subsidies	Subsidized exports	National and regional policy documents
Levels of domestic price and quantity-related policy instruments	Support prices and associated government purchases, production quotas, retail price controls	National and regional policy documents