Delay Propagation and Multiplier

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Abstract
In the National Airspace System (NAS), many flights are delayed daily as reported in the Air Travel Consumer Report (monthly) by the Bureau of Transportation Statistics, the U.S. Department of Transportation. An initial delay often causes another delay in a subsequent flight, or a propagated delay. This paper examines the delays observed in a sequence of the flights operated with the same aircraft, or the same tail number among the busiest airports and produces a delay multiplier to assess the repercussion of the initial delay on the entire sequence of the flights during the day.

The propagation multiplier is frequently used in the context of the cost-benefit analysis to demonstrate how the reduction of the initial delay leads to a greater reduction in the propagated delays and, thus, greater benefits.

Furthermore, based on the delay propagation sequences constructed by a tail number tracking methodology for the flight data for the calendar year 2007, we tested the hypothesis that the propagated delays are exponentially distributed by fitting the Weibull or Gamma probability density function and, then, examining to see how close the estimates of the shape parameter are 1. We found that these two distributions over the propagated delays closely follow the exponential distribution.
1. Introduction
In the National Airspace System, multiple flights experience numerous delays daily. In a sequence of flights operated with the same aircraft (the tail number), a propagated delay results from an earlier delay in the form of a ripple effect. A delay multiplier is a ratio of the propagated delay to the earlier delay. Several researchers studied this multiplier effect in the context of the delay propagation, as described briefly in the next section.

Since the ASQP data are recorded in the local times at the departure and arrival airports, all the local times had to be converted to the Universal Time Coordinate (UTC). This conversion is critical since we have to construct each flight sequence with the individual flight legs not only chronologically but also logistically in a correct fashion. In other words, all the departure and arrival airports have to be placed logically under the same tail number, namely, a series of arrival and departure airports must be lined up consistently as in LGA-BOS, BOS-DCA, DCA-LGA, not LGA-BOS, DCA-BOS.

The delay propagation multiplier is important because it is used to capture additional benefits resulting from the reduction of the initial delays which started a propagation process. In this research, we tackle this problem by constructing a database of propagation sequences. We, then, compute the propagated delays and multipliers according to the definitions discussed below.

2. Previous Work
Beatty et al. (1999) examined, using a concept of delay trees, the effect of the initial delay on multiple departures. The American Airlines provided groups of flights forming the delay trees. This study was able to use “more than 500 delay trees, involving thousands of delayed flights.” It tracked the flights by either the airframes (tail numbers) or the flight crews. It produced a two-way table of the time of the day and the magnitude of the initial delay.

Abdel-aty et al. (2007) used a two-stage approach: (1) a frequency analysis method; (2) statistical methods and concluded that both methods detected “seasonal, weekly and daily patterns of arrival delay using daily average delay data.” But, the authors also discovered that all the patterns were not significant in both methods.

Allan et al. (2001) studied to determine causes of delay at EWR in 1998-2001. This study also examined “the correlation between the types of weather events and the resulting type of delay.” It further argued that a simple “IMC vs. VMC airport capacity model since it fails to capture the causal relationship between the environment and delay and therefore does not explain the actual delays at an airport.

Allan et al. (2004) studied the relationship between the causes of Ground Delay Program (GDP) and the hours of delay computed based on the Estimated Departure Clearance Time (EDCT) in the New Area Airports in 2004. The authors summarized four products: Integrated Terminal Weather System (ITWS), Corridor Integrated Weather System (CIWS), Route Availability Planning Tool (RAPT), Terminal Convective Weather Forecast (TCWF). This study concluded that the application of these products, inter alia, would result in significant benefits by adding “incremental improvements in the ceiling and visibility forecast.”

Baden et al. (2005) studied the flight delay propagation by proposing a backtracking algorithm after constructing a flight sequence of flights by the same aircraft (Tail Number Tracking).

Xu et al. described “a stochastic Bayesian Network model to analyze the relationship between: (1) delay variables, and (2) the factors that cause delays.” This paper further provides
several useful references for the delay propagation which is relevant to the current paper and useful ideas for future work extending our current paper.

Evans et al. (2004) cited 1.8 as an example of the multiplier from a study conducted by Beatty et al. (1999). Evans et al. described 1.8 as applicable to a combination of a mid afternoon arrival and one hour of an initial delay.

Boswell et al. examined a carryover delay which is incurred when a plane is delayed on one of the day’s flights. They developed the statistical models to predict: 1. The downstream delays that occur when a flight experiences an initial delay, and 2. The likelihood of flight cancellation as a function of the initial delay.

3. Determination of Propagated Delays, Propagation Multiplier, and Leap Counts

Example 1
Based on Beatty et al. conclusions, the leading question is in this study is whether we compute the propagated delay multipliers differentiated for each airport and the time of the day, based upon the delay data available to us. To introduce the propagated delay, let us use an example of eight consecutive flight legs traveled by the same aircraft identified by a tail number mentioned in Table 1. Observe that the first flight leg, LGA-DCA, arrived at DCA 20 minutes late, departed for BOS 21 minutes late, and arrived at the second destination, (BOS), 45 minutes. In this example, the 20-minute arrival delay at DCA is the initial delay and the 45-minute arrival delay at BOS is a propagated delay.

| TABLE 1 A Flight Sequence by a Single Tail Number |

<table>
<thead>
<tr>
<th>Flight Propagation Detail Report</th>
</tr>
</thead>
<tbody>
<tr>
<td>Tail Number: N765US</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Fit Num</th>
<th>Dep</th>
<th>Arr</th>
<th>Sch Dep Time</th>
<th>Act Dep Time</th>
<th>Sch Arr Time</th>
<th>Act Arr Time</th>
<th>Sch Block Time</th>
<th>Act Block Time</th>
<th>Sch Turn Min</th>
<th>Act Turn Min</th>
<th>Gate Dep Dly</th>
<th>Gate Arr Dly</th>
<th>Prop Dly</th>
<th>Share For Each Stop</th>
<th>Now Dly</th>
</tr>
</thead>
<tbody>
<tr>
<td>2145 (Map-It)</td>
<td>LGA</td>
<td>DCA</td>
<td>08:00</td>
<td>09:18</td>
<td>09:10</td>
<td>09:30</td>
<td>70</td>
<td>72</td>
<td>0</td>
<td>0</td>
<td>18</td>
<td>20</td>
<td>0</td>
<td>0.0%</td>
<td>0</td>
</tr>
<tr>
<td>2020 (Map-It)</td>
<td>DCA</td>
<td>BOS</td>
<td>09:45</td>
<td>10:06</td>
<td>11:06</td>
<td>11:57</td>
<td>83</td>
<td>111</td>
<td>35</td>
<td>36</td>
<td>21</td>
<td>49</td>
<td>20</td>
<td>5.3%</td>
<td>29</td>
</tr>
<tr>
<td>2127 (Map-It)</td>
<td>BOS</td>
<td>LGA</td>
<td>12:00</td>
<td>12:34</td>
<td>13:06</td>
<td>13:51</td>
<td>66</td>
<td>77</td>
<td>52</td>
<td>37</td>
<td>34</td>
<td>45</td>
<td>34</td>
<td>9.0%</td>
<td>11</td>
</tr>
<tr>
<td>2177 (Map-It)</td>
<td>LGA</td>
<td>DCA</td>
<td>14:00</td>
<td>14:27</td>
<td>15:09</td>
<td>15:56</td>
<td>69</td>
<td>89</td>
<td>54</td>
<td>36</td>
<td>27</td>
<td>47</td>
<td>27</td>
<td>7.2%</td>
<td>20</td>
</tr>
<tr>
<td>2040 (Map-It)</td>
<td>DCA</td>
<td>BOS</td>
<td>15:45</td>
<td>16:54</td>
<td>17:10</td>
<td>18:36</td>
<td>85</td>
<td>102</td>
<td>36</td>
<td>56</td>
<td>69</td>
<td>86</td>
<td>47</td>
<td>12.5%</td>
<td>39</td>
</tr>
<tr>
<td>2139 (Map-It)</td>
<td>BOS</td>
<td>LGA</td>
<td>18:00</td>
<td>19:50</td>
<td>19:13</td>
<td>21:20</td>
<td>73</td>
<td>90</td>
<td>50</td>
<td>74</td>
<td>110</td>
<td>127</td>
<td>86</td>
<td>22.9%</td>
<td>41</td>
</tr>
<tr>
<td>2109 (Map-It)</td>
<td>LGA</td>
<td>DCA</td>
<td>20:00</td>
<td>21:47</td>
<td>21:15</td>
<td>22:36</td>
<td>75</td>
<td>49</td>
<td>47</td>
<td>27</td>
<td>107</td>
<td>86</td>
<td>81</td>
<td>21.5%</td>
<td>0</td>
</tr>
<tr>
<td>2052 (Map-It)</td>
<td>DCA</td>
<td>BOS</td>
<td>21:45</td>
<td>23:17</td>
<td>23:08</td>
<td>00:35</td>
<td>83</td>
<td>78</td>
<td>30</td>
<td>41</td>
<td>92</td>
<td>87</td>
<td>81</td>
<td>21.5%</td>
<td>6</td>
</tr>
</tbody>
</table>

Overall: 604 685 304 308 376 53.71

8 flights found.

In this paper, we focus on the arrival side where the propagated delay is taking place. This scope is based on the notion that the passengers are mainly affected by arrival delays, and then, departure delays. The assumption is that departure delays can be more “tolerable” than the arrival delay.
**Propagated Delay**

For a given sequence of flights, the following three conditions must hold simultaneously to record a propagated delay:

- A flight arrives late;
- It departs late in the subsequent flight leg;
- It arrives late at the next destination.

As long as the three conditions are met, propagation exists. These conditions can be simply depicted by a Venn diagram as Figure 1.

It has been suggested by a reviewer that “this definition does not distinguish between true ‘propagated’ delay and down-stream operational delay.” This is a relevant observation since we view the delays from the consumers’ point of view and focus on how much delay the consumers must bear. In the sense of the true propagated delay, those delays generated as a result of a ground delay program could terminate a propagation and initiate a new propagation process as an operational delay as a starter. This aspect would be surely addressed in a future implementation of the operational delay propagation system in the Aviation System Performance Metrics (ASPM), including needs for extensive statistical analyses. Nonetheless, the focus of the current implementation remains with the intersection of the three conditions in the definition.

![Venn diagram depicting the three conditions for a propagated delay](image-url)
Tail Number Tracking Methodology

The source of the data is the Airlines Service Quality Performance (ASQP) files published by the Bureau of Transportation Statistics of the U.S. Department of Transportation (http://www.bts.gov). The initial source data consisting of the city-pair flights cover the period from January, 2007 to September, 2007 with on-going augmentation as a new ASQP monthly file becomes available (approximately 3 weeks after an end of each month).

Having sequenced all the flight legs related to each tail number and, then, determined all the propagated delays according to the three conditions, let us now turn to illustrate and, subsequently, define the three indicators: a leap count, a delay propagation multiplier, and a delay propagation accelerator.

The leap count, \( l(i) \), is defined at the \( i^{th} \) arrival airport with \( i \) taking on a value from \( 1, 2, \ldots, \text{idx}(tn) \) where \( \text{idx}(tn) \) is the number of the flight legs in the sequence and indicates how far a propagation sequence continues. The leap count itself could take on a natural number, 0, 1, 2, up to \( \text{idx}(tn) - 1 \). A value of 0 implies no propagated delay at an arrival airport, \( (A_1) \), whereas a value of 1 at the same arrival airport, \( (A_1) \), indicates that there was a propagated delay at the second arrival airport, \( (A_2) \) but the propagation terminated itself. In this case, the leap count, \( l(2) \), at the airport, \( A_2 \), takes on a value of 0. Hence, the leap count of 2 means that there is a three leg sequence with two propagated delays, and so on.

It does not necessarily mean that there can be only one propagation sequence. This process can display a stop-and-go behavior. In a long series of legs, there can be more than one propagation sequence.

Next, let us define the delay propagation multiplier as follows. The delay propagation multiplier is \( m(i, j) = a(j) / a(i) \) where \( a(i) \) and \( a(j) \) are the arrival delays at the \( i^{th} \) and \( j^{th} \) arrival airports, respectively, in the flight leg sequence with \( i \) going from 1 to \( \text{idx}(tn) \) and \( j \) going from \( i+1 \) to \( l(j) \).

Lastly, let us define the delay propagation accelerator. The delay propagation accelerator is \( v(i, j) = p(j) / p(i) \) where \( p(i) \) and \( p(j) \) are the arrival delays at the \( i^{th} \) and \( j^{th} \) arrival airports, respectively, in the flight leg sequence with \( i \) going from 2 to \( \text{idx}(tn) \) and \( j \) going from \( i+1 \) to \( l(j) \). Since we have only conducted rather limited literature search where there have been a huge number of publications in this subject, we humbly think that the implementations of the leap counts and the delay propagation accelerator are original whereas the multiplier as a ratio of the two consecutive arrival delays is interesting and quite useful but not unique.

Example 2

Table 2 below is the same as Table 1 except for the last two columns exhibiting the multiplier and the leap count.
TABLE 2 A Flight Sequence by a Tail Number

Delay Propagation Detail Report

<table>
<thead>
<tr>
<th>Flt Num</th>
<th>Dep</th>
<th>Arr</th>
<th>Sch Dep Time</th>
<th>Act Dep Time</th>
<th>Sch Arr Time</th>
<th>Act Arr Time</th>
<th>Sch Block Time</th>
<th>Act Block Time</th>
<th>Sch Turn Min</th>
<th>Act Turn Min</th>
<th>Gate Dep Dly</th>
<th>Gate Arr Dly</th>
<th>Prop Dly</th>
<th>Share For Each Stage</th>
<th>New Dly</th>
<th>Multiplier</th>
<th>Prop. Accelerator</th>
<th>ASGP Delay Cause</th>
<th>Stage Cnt</th>
</tr>
</thead>
<tbody>
<tr>
<td>2165</td>
<td>LGA</td>
<td>DCA</td>
<td>08:00</td>
<td>08:16</td>
<td>09:10</td>
<td>09:30</td>
<td>70</td>
<td>72</td>
<td>0</td>
<td>0</td>
<td>18</td>
<td>20</td>
<td>0</td>
<td>0.0%</td>
<td>0</td>
<td>0.00</td>
<td>0.00</td>
<td>N</td>
<td>7</td>
</tr>
<tr>
<td>2033</td>
<td>DCA</td>
<td>BOS</td>
<td>09:45</td>
<td>10:06</td>
<td>11:00</td>
<td>11:57</td>
<td>83</td>
<td>111</td>
<td>35</td>
<td>38</td>
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<td>49</td>
<td>20</td>
<td>5.3%</td>
<td>29</td>
<td>2.45</td>
<td>0.00</td>
<td>NIL</td>
<td>0</td>
</tr>
<tr>
<td>2127</td>
<td>BOS</td>
<td>LGA</td>
<td>12:00</td>
<td>12:34</td>
<td>13:06</td>
<td>13:51</td>
<td>66</td>
<td>77</td>
<td>52</td>
<td>37</td>
<td>34</td>
<td>45</td>
<td>34</td>
<td>9.0%</td>
<td>11</td>
<td>0.92</td>
<td>1.70</td>
<td>LN</td>
<td>5</td>
</tr>
<tr>
<td>2177</td>
<td>LGA</td>
<td>DCA</td>
<td>14:00</td>
<td>14:27</td>
<td>15:09</td>
<td>15:56</td>
<td>69</td>
<td>59</td>
<td>54</td>
<td>36</td>
<td>27</td>
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<td>20</td>
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<td>0.79</td>
<td>LN</td>
<td>4</td>
</tr>
<tr>
<td>2040</td>
<td>DCA</td>
<td>BOS</td>
<td>15:45</td>
<td>16:54</td>
<td>17:10</td>
<td>18:36</td>
<td>85</td>
<td>102</td>
<td>36</td>
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<td>1.74</td>
<td>LN</td>
<td>3</td>
</tr>
<tr>
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<td>BOS</td>
<td>LGA</td>
<td>18:00</td>
<td>19:30</td>
<td>20:13</td>
<td>21:20</td>
<td>73</td>
<td>90</td>
<td>50</td>
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<td>127</td>
<td>86</td>
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<td>21:15</td>
<td>22:36</td>
<td>75</td>
<td>49</td>
<td>47</td>
<td>27</td>
<td>107</td>
<td>81</td>
<td>81</td>
<td>21.5%</td>
<td>0</td>
<td>0.64</td>
<td>0.94</td>
<td>L</td>
<td>1</td>
</tr>
<tr>
<td>2052</td>
<td>DCA</td>
<td>BOS</td>
<td>21:45</td>
<td>22:17</td>
<td>23:08</td>
<td>00:35</td>
<td>83</td>
<td>76</td>
<td>30</td>
<td>41</td>
<td>92</td>
<td>87</td>
<td>81</td>
<td>21.5%</td>
<td>6</td>
<td>1.07</td>
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<tr>
<td>Overall</td>
<td></td>
<td></td>
<td>604</td>
<td>668</td>
<td>304</td>
<td>309</td>
<td>376</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>1.23</td>
<td>1.26</td>
<td></td>
<td></td>
<td>5.71</td>
<td></td>
</tr>
</tbody>
</table>

8 flights found.

* - Overall number is computed as geometric mean.
Note: ASGP causes of delay are: C = Caterer, N = TAA, L = Late Arrivals, S = Security, W = Weather.

Let us select a flight sequence for the purpose of illustrating how the leap count, \( l(i) \) and the delay propagation multiplier \( m(i, j) \) function. Table 2 illustrates an example of a flight sequence serviced by the tail number, N768US, on May 20, 2007, going from LGA via DCA, BOS, LGA, DCA, BOS, LGA to BOS, (its final destination for that day). The aircraft, N768US, actually landed at 00:27 a.m. and checked in at 00:35 a.m. in BOS in the next day. This example was deliberately chosen because the propagated delays were experienced by every eligible arrival airport.

In this particular example, \( idx(N768US) = 8 \), \( l(1) = 7 \), \( l(2) = 6 \), \( i(3) = 5 \), ..., \( i(8) = 0 \). A multiplier for a trip from LGA to BOS (the last arrival in the sequence) is \( m(1, 8) = a(8)/a(1) = 4.35 \). By the definition, \( m(1, 8) \) is also \( m(1, 2) \) * \( m(2, 3) \) * \( m(3, 4) \) * ... * \( m(7, 8) = 4.35 \). In other words, \( m(1, 8) = (a(2)/a(1))(a(3)/a(2))(a(4)/a(3))(a(5)/a(4))...(a(8)/a(7)) \). The two sets, one consisting of eight arrival delays and another consisting of the initial arrival delay and seven multipliers are interchangeable in their informational contents. The averages for the multiplier and accelerator are computed as the geometrical means of all the arrival and propagated delays in this propagation sequence. In this particular example, 1.23 and 1.26 in Table 2 are the seventh and sixth roots of the products of all the available arrival and propagated delays, respectively.

Furthermore, the multipliers enable us to select a part of a propagation sequence from any point to any point in order to study a specific routing such as ATL ORD LAX regardless of where the sequence began and ended.

4. Distribution of the Propagated Delays

The distribution of the propagated delays came into focus in order to respond to the author’s interest to getting the delays lead to a candidate(s) for a probability density instead of assuming a distributional form beforehand, for example, a normal distribution. A conjecture was proposed
to let the data in the form of a histogram to get us a most fitting distribution and, then, parameter estimates for a corresponding probability density function. For example, it was hoped that the estimates of the shape parameter of the Weibull probability density provide us with three clearly distinct distributions corresponding to differences due to the time of the day at an airport or among a group of airports. We regret to state that, in this paper, the conjecture was not borne out. Therefore, in this section, we shall explore the distribution of the propagated delays throughout the course of a day at two of the Nation’s busiest airports, ORD and ATL, in order to demonstrate how well the Weibull and Gamma distributions emulate the Exponential distribution at ORD while this is not the case at ATL. In order to carry out this task, we let the propagated delays themselves lead us to an appropriate probability density function. It is important to know if and how the distributions of the propagated delays change to affect the propagation multipliers during the day. We must investigate if the lengthy delays tend to generate large values of the multipliers. For this purpose, the Weibull distribution was chosen along with the Gamma distribution with the well-known properties that it can be applicable to several distribution forms. The estimates of the shape parameters distinguish forms of the distributions at different hour during the day.

Next, let us examine a probability density function (pdf) for the propagation delays by the local departure hour.

![Figure 2](image)

**FIGURE 2. Histogram and four fitted probability density functions for propagated delays at the second arrival airport with the flight sequences beginning from ORD**

Figure 2 shows the histogram of the propagated delays observed at the second arrival airports for the flight leg sequences originating from Chicago O’Hare Airport between 1800 and 1900 with four probability functions: Lognormal, Exponential, Weibull, and Gamma distributions for the calendar year 2007. It turns out that these probability density functions have the shape parameters which define the shape of the curves and other properties and, further, are identical to the exponential probability density function when the shape parameter is 1. The values of the shape parameter estimates for the fitted Weibull and Gamma probability density
functions are 1.06 and 1.11, respectively. This phenomenon is observed consistently throughout the busy time periods at the large airports. On the other hand, when the values of the shape parameter differ from 1 even by a fairly small amount, there is a clear distinction; Figure 3 below provides an example for such a case.

![Summary Statistics](image)

<table>
<thead>
<tr>
<th>Summary Statistics</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>N</td>
<td>62</td>
</tr>
<tr>
<td>Mean</td>
<td>35.05</td>
</tr>
<tr>
<td>Std Dev</td>
<td>27.26</td>
</tr>
<tr>
<td>Skewness</td>
<td>1.049</td>
</tr>
</tbody>
</table>

**FIGURE 3.** Histogram and four fitted probability density functions for propagated delays at the second arrival airport with the flight sequences beginning from ATL

Notice that the deviation away from the exponential distribution is quite significant with both the Weibull and Gamma distributions. The values of the shape parameter estimates for the fitted Weibull and Gamma probability density functions are 1.30 and 1.66, respectively.

Having observed that the estimates of the shape parameters are a key to statistical variations of the propagated delays by the local departure hours, the hourly estimates of the shape parameter are shown below.

Figure 4 Shows that such a distribution is equally applicable at other airports. While the observed nature of the propagated delays point to the exponential distribution, the versatility of the Weibull distribution enables us to let the data determine the “shape” of the distribution where we need not assume (or presuppose) the shape parameter to be exactly 1, *a priori.*
Figure 4 exhibits the estimates of the Weibull shape parameters decreasing from 10 a.m. (local) to the hours in the afternoon and stabilizing in the evening. It will be shown in the next section that this behavior of the shape parameter estimates is consistent with those of the secondary and tertiary propagation multipliers.

5. Delay Propagation Multiplier
Let us discuss the multi-stage multipliers as defined in the section 3. Figure 5 exhibits the delay propagation multipliers for the first three stages, \( m(1,2) \), \( m(2,3) \), and \( m(3,4) \). Since the delay propagation sequences are organized by the local departure hour, it is possible to compute a set of the delay propagation multiplier for each length of the leap count as long as the propagation sequences continue. The multiplier at the second arrival airport, \( m(1,2) \), are consistently in a narrow band whereas the secondary and tertiary multipliers, \( m(2,3) \) and \( m(3,4) \), respectively, exhibit quite large fluctuations. By their definition, the delay propagation multipliers, \( m(1,3) = m(1,2) \times m(2,3) \) or \( m(1,4) = m(1,2) \times m(2,3) \times m(3,4) \) and so on.

In the early parts of the day, if the duration of the propagation sequences threaten to become unmanageable and, in fact, the propagation delays become even lengthier and lengthier, there are some measures that the airlines can implement: an aircraft substitution (a remedy) which forces the termination of the propagation for the same tail number (the same aircraft) or re-sequencing departing aircraft by passing other aircraft in the departure queue with assistance and accommodation by the air traffic control (an improvement). Examining such operational modifications, however, goes beyond the scope of this paper and is left to future studies.
FIGURE 5. Delay propagation multipliers (at the second, third, and fourth airports) plotted against the time of the day at Chicago O’Hare Airport

Relationship between multipliers and leap counts
The leap counts are recorded in a descending order at each of the consecutive arrival airports. Namely, the leap count at the \((i + 1)th\) arrival airport is, \(l(i+1) = l(i) - 1\) by the definition. A multiplier value of 7 means that, if \(l(i) = 7\), the delay propagation sequence has eight legs included in the process. For the rest of the propagation, \(l(i+1) = 6, \ldots, l(i+7) = 0\).

An example of such relationship is exhibited in Figure 6 below for the Chicago O’Hare Airport. In the x axis, a value of 1 for \(l(i)\) with any eligible \(i\) obviously indicates that a delay propagation sequence has only one arrival airport where a propagated delay is observed and the propagation process is terminated. At the right end, a value of 7 means that the delay propagation process reached the eight arrival airport where the multiplier, \(l(8) = 0\).

FIGURE 6. Delay propagation multipliers plotted against the length of the delay propagation process at Chicago O’Hare Airport

Based upon the multipliers for which the summary is presented in Figure 6, the mean multiplier is 2.31. For the OEP 34 with excluding HNL, the mean multiplier is 2.26.
6. CONCLUSIONS AND RESULTS
Using the ASQP data, we restructured the information contained according to our notion of the delay propagation, defined the delay propagation multiplier and the leap count in the flight leg sequence.

We showed that, during the busy hourly periods, the probability distribution of the first stage propagated delays follows the Weibull probability density function throughout the most of the periods and, furthermore, quite frequently, it behaves closely like the exponential distribution during peak times. This result enables us to customize the propagation multiplier to particular circumstances which arise in a cost-benefit study associated with a capital investment in a chose airport or a regional economic area.

We have created a new online data base system as a subsystem in the ASPM. In this new system, the basic entity is a flight leg sequence formulated according to the definitions for the afore-mentioned propagated delays, propagation multipliers, and leap counts. Some of the initial user interface screens are illustrated in the appendix in order to introduce a user to this system.

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REFERENCES


APPENDIX
Those concepts discussed in this article: the delay propagation, multiplier, and leap count, are being implemented in the Aviation System Performance Metric (ASPM) system as the Delay Propagation subsystem. Three images from the user screens are exhibited in this appendix as signs of what is to come in the near future without delving into excessive detail of the user interface.

**FIGURE A-1. Delay propagation city-pair main page**

Figure 1 shows the initial screen where all choices for the departure and arrival airports, air carriers, dates, and filters in order to search the delay propagation sequences satisfying criteria. The filters let a user put restrictions to the search such as a tail number or an airport routing as in ‘ATL ORD SFO’ to extract all the delay propagation sequences containing the routing anywhere in the propagation.
Delay Propagation Base Report

From 12/2007 To 12/2007 | Airport Chain= ‘ATL ORD SFO’

FIGURE A-2. Delay Propagation Flight Sequences by the tail number

Figure A-2 displays the result of the search with the conditions: all the delay propagation sequences with the routings of ‘ATL ORD SFO’ in December 2007. This table shows three delay propagation sequences which were all operated with the aircraft belonging to the American Airlines.

Delay Propagation Detail Report

Tail Number : N566AA | Date : 12/09/2007 | Carrier : AAL | Aircraft : MD80

FIGURE A-3. Detail flight segments with the multiplier and the leap count

Figure A-3 displays all the relevant detail needed to follow this particular delay propagation sequence. The arrival and propagated delays, the delay propagation multiplier, and the leap count are provided in the last columns.