Environmentally Adjusted Productivity and Efficiency Measurement: A New Direction for the Luenberger Productivity Indicator

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Abstract

The study proposes a new way of measuring productivity and efficiency, with and without considering environmental effects from a production activity, by modifying the conventional Luenberger productivity indicator. The Luenberger approach has so far been applied in productivity and efficiency measurement in time-varying contexts. It has been mainly used in comparisons of international productivity growth and efficiency over a period of time. This study proposes the use of the Luenberger approach in an alternative way by constructing two new indicators: the Luenberger environmental indicator and the Luenberger spatial indicator. These two indicators take a spatial orientation, as opposed to the temporal orientation of the traditional Luenberger indicator. The Luenberger environmental indicator is employed to measure relative performance of productive units across space by incorporating environmental impacts in the production model. The Luenberger spatial indicator does not include environmental impacts. To compare the performance of a unit of observation to a meaningful reference, a new concept of a reference frontier, an infrafrontier, is proposed. An empirical application of these indicators is to the Australian irrigation agriculture sector taking place in eleven natural resource management regions within the Murray-Darling Basin. These newly developed indicators can be widely used in any sector of the economy to measure relative productivity and environmental efficiency.

Key words: environmentally adjusted productivity, environmentally adjusted efficiency, infrafrontier, Luenberger indicators

JEL Codes: D24, Q50, Q55, Q57

1. Introduction

Over the last twenty or so years environmental effects from economic activities have been incorporated in the productivity and efficiency modelling framework (Tyteca, 1996). A number of environmentally adjusted productivity and efficiency models have been developed and applied in various contexts (Azad, 2012). Environmentally adjusted efficiency measurements allow researchers to identify productive activities that create high economic value and have relatively small environmental impacts, as well as productive activities that create large environmental impact, but only create modest economic value. Recent work includes applications of a quantity index approach to environmental performance (Färe and Grosskopf, 2004), the environmental efficiency and environmental productivity (Kumar and Khanna, 2009), the environmental performance index (Azad and Ancev, 2010) and the environmental total factor productivity (Hoang and Coelli, 2011). Most of these studies are using the standard, ratio-based indexes, e.g. Malmquist type indexes, in incorporating environmental effects in measuring efficiency or analysing productivity.[1] However, the very nature of the ratio-based indexes creates a problem with evaluation of the actual environmental impacts. Ratio-based indexes can only indicate a relative difference in environmental performance. For instance, based on using a ratio-based index two production units might be found to have same environmentally adjusted efficiency score even though one of them causes many times greater environmental damage than the other. When it comes to evaluating environmental effects, the extent of the environmental impact is often more important than the relative trade-off between economic and environmental efficiency. We are often interested in identifying those production units for which the difference between environmentally adjusted and unadjusted performance is the

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[1] There are two general types of primal productivity indexes that are used in productivity analysis and efficiency measurement: ratio-based indexes and difference-based indexes. While the ratio-based indexes have been employed in a large number of empirical applications (Färe et al., 1998; Fethi and Pasiouras, 2010), few studies have applied the difference-based indexes (Chambers, 2002). A difference-based index measures productivity and efficiency of an economic activity in terms of differences of distance, or directional distance functions, rather than their ratios, as is the case with ratio-based indexes.

smallest. Effectively, we want to measure the economic contribution made by an individual production unit net of the environmental degradation caused by making that contribution. Difference-based indexes lend themselves well for this purpose.

In addition, there are several known limitations of using ratio-based indexes. For instance, one source of nuisance with a ratio-based index obviously occurs when the denominator of the index has a zero value. Some empirical studies (i.e., Boussemart et al., 2003; Briec and Kerstens, 2004; Managi, 2003) showed that ratio-based productivity indexes overestimate productivity change compared to other productivity indicators.

A candidate difference-based index to overcome these problems is the Luenberger productivity indicator introduced by Chambers et al. (1996). There are strong justifications for applying the Luenberger productivity indicator in the productivity and efficiency analysis, in general, and in environmentally adjusted analysis in particular. Firstly, this indicator is more general than the Malmquist index developed by Caves et al. (1982). While the Malmquist index focuses on either cost minimization or revenue maximization, the Luenberger productivity indicator is the dual to the profit function, and implies profit maximisation (Boussemart et al., 2003; Chambers et al., 1996). Secondly, using the Malmquist approach requires a choice to be made between an input or an output perspective (Färe et al., 1985; Chambers et al., 1996), whereas the Luenberger indicator can address simultaneously input contraction and output expansion (Boussemart et al., 2003; Managi, 2003). Therefore, the Luenberger productivity indicator requires less restrictive assumptions than the other standard non-parametric productivity indexes (Williams et al., 2011).

The Luenberger productivity indicator has thus far been mainly used as a time-series based productivity measurement approach that can be employed to measure productivity growth. It is useful for estimating productivity and efficiency changes of units of observation (i.e., firms, industries or countries) over a period of time. A number of studies have recently used the Luenberger productivity indicator to estimate the change in productivity and efficiency for various economic units, but without the adjustments for environmental performance (e.g Epure et al., 2011; Williams et al., 2011; Brandouy, et al., 2010; Nakano and Managi, 2008). The conventional Luenberger indicator is typically applied when time-series data are available. However, in many situations that are pertinent to environmentally adjusted productivity analysis and efficiency measurement, time-series data are not readily available. For example, environmental measurements might be taken irregularly across time, or might be estimated based on data from a single time period. Also, when it comes to environmentally adjusted productivity analysis and efficiency measurement, there are many instances where researchers might be more interested in cross-sectional variation, rather than variation across time. Examples of such instances are situations where the environmental impacts from productive activities are dependent on variables that vary across units of observations, and not necessarily across time. The quality and abundance of environmental assets or the significance of ambient environmental quality in an area where a unit of observation operates can be very different compared to another area where another unit of observation operates. Likewise, the availability of environmentally sensitive resources, often used as inputs in productive activities (e.g. water, soil, fish, and forests) can vary significantly across areas where different units of observation operate. [2]

Confronted with the need to measure this type of variation across space, the standard, time-series oriented Luenberger approach fails to be an appropriate index, not the least because it relies on estimation of a production frontier for each time period in the

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[2] There certainly are environmental variables that can vary significantly across time, e.g. temperature. In those instances, the use of time series data and the adequately oriented Luenberger index would be appropriate. However, this paper is predominantly interested in cross-sectional variation, which is the reason why it develops spatially oriented Luenberger indexes. In a more general sense, one would ideally want to account for both temporal and spatial variation, not unlike the use of panel data with parametric estimation methods. At this stage, this is beyond the scope of the current paper, but it is of an imminent research interest.

sample. This is not particularly useful, as we rather need to estimate a frontier for each area that has particular environmental characteristics that in turn vary among the areas. Further, the time-series oriented Lunberger approach naturally uses the estimated frontier for the first (or the earliest) time period in the sample as a reference frontier. However, when one is interested in variation across space, the reference frontier becomes much more elusive, as there is no natural reference that could be designated in a clear-cut way. We will come back to this point in some detail below, with a discussion of a new type of reference frontier – the infrafrontier.

In the present study, we propose a new methodological approach – a spatially oriented Luenberger indicator – which is suited for measuring the efficiency of production units across spatially diverse environments. We also introduce a new type of a reference production frontier – an infrafrontier. These new concepts enable the researchers to estimate comparative efficiency of units of observation across regions, states or countries. They can also be employed to measure environmentally adjusted performance of productive units across space by incorporating environmental impacts in the production model. These new concepts are empirically applied to the measurement of both environmentally adjusted and un-adjusted efficiency of irrigation enterprises in the Murray-Darling Basin, Australia.

The paper has six sections, and is organised as follows. Section 2 sets out the conceptual framework for the environmental Luenberger productivity indicator and the Luenberger spatial productivity indicator. The estimation of Luenberger environmental indicator is outlined in section 3. Sources of data and variables used in the study are described in section 4. The results obtained from using both productivity indicators are discussed in section 5. The ultimate section offers some conclusions and policy implications.

2. Conceptual Framework

2.1 The Environmental Luenberger productivity indicator: A new direction

The Luenberger productivity indicator (LPI) is based upon the shortage function established by Luenberger (1992). The LPI can be modelled with a set of directional distance functions. The advantage of using directional distance functions is that they can be used to accommodate simultaneously desirable and undesirable outputs of a production technology. In addition, it allows researchers to evaluate the performance of production units in terms of expanding desirable outputs, and in terms of contraction of undesirable outputs in a multi-output production process. To model a multi-output production technology, suppose that we have a sample of K production units, each of which uses a vector of inputs $x = (x_{1,} \dots, x_N) \in \Re_+^N$ to produce a vector of desirable outputs $d = (d_1, \dots, d_M) \in \Re_+^M$. Some amount of environmental degradation, termed here as undesirable outputs $u = (u_1, \dots, u_J) \in \Re_+^J$ are also produced as a consequence of the production process. The production technology can be defined by its output set as follows:

$$P(x) = \{(d, u): x \text{ can produce } (d, u)\}. \quad (1)$$

We assume that this technology is characterised with weak disposability of outputs, which implies that a reduction in any output (desirable or undesirable) is feasible by reducing the production of all other outputs proportionately. In addition to weak disposability, we assume null-jointness, which states that desirable outputs cannot be produced without undesirable outputs as by-products.

By defining a direction vector $\mathrm{g} = (\mathrm{g}_{d,}\, \mathrm{g}_u)$, the directional output distance function can be written as:

$$\vec{D}_o(x, d, u;\ \mathrm{g}_{d,} - \mathrm{g}_{u,}) = \sup\{\beta: (d + \beta \mathrm{g}_{d}, u - \beta \mathrm{g}_u) \in P(x)\}. \quad (2)$$

This directional output distance function seeks to find the maximum feasible expansion of desirable outputs in the g_d direction, but at the same time the largest possible contraction of undesirable outputs in the g_u direction. The directional output distance function is graphically illustrated in Figure 1. Unlike the output distance function that expands output vector (d, u) towards the frontier technology at point t, the directional output distance function scales the output vector (d, u) in correspondence to the directional vector $(g_{d,} g_u)$, and places it at point s on the production technology boundary. The increase in desirable outputs and decrease in undesirable outputs of the output vector towards the directional vector are given by $(d + \beta g_{\mathrm{d}},\ u - \beta g_{\mathrm{u}})$. If the output vector lies on the boundary of $P(x)$, the value of directional output distance function would be zero, and the observation will be deemed as being efficient. In contrast, the value of the function will always be positive for an inefficient observation. The more inefficient the observation, the higher the value of the directional output distance function.

As the present study aims to construct spatially oriented Luenberger productivity indicators in order to analyse productivity and measure efficiency for units of observation across space, the directional output distance functions (the components of the Luenberger productivity indicator) are required to be structured in a spatially referenced form. Suppose that we aim to compare efficiency of a production activity between two regions (or other spaces), say region a and b. The spatially referenced directional output distance function can then be defined in the following manner. For a given region a it can be written as:

$$\vec{D}_o^a\left(x^a, d^a, u^a;\ g_{d,} {-g_u}\right) = \sup\{\beta\colon (d^a + \ \beta g_d\,,\ u^a{-}\beta g_u) \in P^a(x^a)\}. \qquad (3)$$

Suppose that we are interested in comparing the performance of an enterprise located in region a with that of the same type of enterprise located in a region b.[3] We will call

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[3] The term ‘enterprise’ is used throughout the text to denote an agricultural production unit (e.g. cotton growing, or growing vegetables) in its entirety, comprising the technology, type of crops, and location in this case.

region b as a reference region.[4] The form of the directional distance function for region b $\vec{D}_o^b$ is similar to that in region a. Consequently, the environmental Luenberger productivity indicator can be formulated as:

$$LEI_a^b = \frac{1}{2}[\vec{D}_o^b(x^a, d^a, u^a; \mathrm{g}_{d,} {-\mathrm{g}_u}) - \vec{D}_o^b(x^b, d^b, u^b; \mathrm{g}_{d,} {-\mathrm{g}_u}) + \vec{D}_o^a(x^a, d^a, u^a; \mathrm{g}_{d,} {-\mathrm{g}_u}) - \vec{D}_o^a(x^b, d^b, u^b; \mathrm{g}_{d,} {-\mathrm{g}_u})] \qquad (4)$$

The above productivity indicator can be used to measure the comparative performance of production units that simultaneously use a fundamentally identical production technology (with possible regional specificities) but are located in regions a and b, respectively.[5] If the value of LEI_a^b is greater than zero, it implies that the environmentally adjusted efficiency of a production unit located in region b is greater than that located in region a. If the indicator takes value less than zero, it implies that the production unit in region b is comparatively less efficient than that in region a.

The Luenberger environmental indicator constructed in equation (4) possesses two distinguishing features: (i) it incorporates environmental degradation variables that can be treated as undesirable outputs in the production model, and (ii) it compares relative performance of units of observation across space. The later feature of the Luenberger environmental indicator modifies the time-varying orientation of the conventional Luenberger indicator into a spatially varying orientation.

To explore possible applications of the environmental Luenberger indicator in productivity analysis and efficiency measurement, let us consider a set of enterprises that operate in areas with different environmental characteristics. Within a given area we might have several different enterprises. We may also have an enterprise of the

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[4] We will look in detail at the issue of how to go about formulating a reference region in the following section.

[5] An identical production technology is defined as the same type of a production system (i.e., growing cotton with sprinkler irrigation system) that units of observations of that enterprise type use in their production process.

same type that operates in different areas (e.g. particular type of agricultural activity or manufacturing). Suppose that we would like to compare the productivity and efficiency of units of observations that belong to a same type of enterprise, but are located in different places. We may expect that the productivity or efficiency of a particular enterprise may vary dependent on the area where it is located. The same type of enterprise may be more (or less) productive in one region compared to what it would have been the case, had it existed in another region.[6] How productive or efficient an enterprise is, will depend on the entire production environment in a given area. This can comprise of many factors, including: accessibility to essential resources (physical, technical or financial resources), and the characteristics of the surrounding environment (natural, institutional or cultural characteristics). Substantial variation in these factors across space can have significant effect on the productivity and efficiency of the enterprises under consideration.

In order to contrast relative performance of a unit of observation across space we must have a meaningful reference point to which efficiency of production unit from each and every location can be compared. In other words, we have to construct a special type of reference production technology to which units of observation – possibly found in many different areas – can be compared. The method of construction of such reference production technology is described in the following section.

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[6] While we are not able to observe the exact same unit of observation (an enterprise in the current context) operating in two distinct places simultaneously, we might be able to observe two units of observation that are very similar to each other (in terms of technology employed, inputs used and types of outputs produced) that exist simultaneously in two different areas. The case in point is agriculture, where we can observe very similar enterprises operating in two areas that are very different in terms of their environmental sensitivity or the significance of environmental quality.

2.2. *The reference frontier: An Infrafrontier*

In general, efficiency scores measured relative to one frontier cannot be directly compared with efficiency scores measured relative to another frontier (Battese et al. (2004; O'Donnell et al., 2008). Productivity and efficiency comparisons are only meaningful when frontiers for different groups of units of observation are identical. To address this issue, Battese et al. (2004) proposed a metafrontier to compare technical efficiencies of firms operating under different technologies. A metafrontier can be used as a reference production technology to compare performance of units of observation from various regions. However, the application of the metafrontier has so far been limited to analyses using ratio-based indexes. In addition, a metafrontier cannot readily accommodate production models that incorporate undesirable outputs. Furthermore, estimation using Data Envelopment Analysis (DEA) may be infeasible when group frontier outputs exceed the metafrontier (Battese et al. 2004).

To address these shortcomings, we propose to construct a new reference production technology, a concept similar to the metafrontier, using the so called minimum-maximum (min-max) approach. We term this reference frontier as an *infrafrontier*. The infrafrontier is derived by taking the minimum amount of desirable output, and the maximum amount of input and undesirable output for each of the individual enterprise types within the data set. These constructed worst performing enterprises are subsequently grouped together to form the infrafrontier.

To construct an infrafrontier for a multi-output production model, suppose we have a vector of inputs, $x = \left(x_{1,} \dots, x_N\right) \in \Re_+^N$ that can produce a vector of desirable outputs $d = (d_1, \dots, d_M) \in \Re_+^M$ and also some undesirable outputs, $u = (u_1, \dots, u_J) \in \Re_+^J$. The infrafrontier entails a set of elements (observations), which can be defined as follows:

$$IF(x, d, u) = [\{\max(x, u), \min(d)\} \in T]$$

The production efficiency and environmental performance of an individual observation (an enterprise) is estimated by measuring the directional distance from each of the individual observation to its regional frontier, and also the distance to the infrafrontier.[7] The construction of the infrafrontier in this way makes it compatible with the Luenberger environmental productivity indicator and overcomes the infeasibility problem encountered with the use of metafrontier (Figure 2).

2.3 Decomposition of the Luenberger environmental indicator

The Luenberger environmental indicator (*LEI*) can be decomposed into an additive indicator of efficiency variation and an indicator of technological variation – just like the time-oriented Luenberger index (Chambers et al., 1996; Färe et al., 1994), as follows:

$$LEI_a^b = [\vec{D}_o^a(x^a, d^a, u^a; \mathrm{g}_{d,} {-\mathrm{g}_u}) - \vec{D}_o^b(x^b, d^b, u^b; \mathrm{g}_{d,} {-\mathrm{g}_u})] + \frac{1}{2}[\vec{D}_o^b(x^b, d^b, u^b; \mathrm{g}_{d,} {-\mathrm{g}_u}) - \vec{D}_o^a(x^b, d^b, u^b; \mathrm{g}_{d,} {-\mathrm{g}_u}) + \vec{D}_o^b(x^a, d^a, u^a; \mathrm{g}_{d,} {-\mathrm{g}_u}) - \vec{D}_o^a(x^a, d^a, u^a; \mathrm{g}_{d,} {-\mathrm{g}_u})] \qquad (5)$$

The expression in the first set of brackets of the above equation represents the efficiency variation (EV) between the regions, a and b, while the arithmetic mean of the difference between the two terms inside the second set of brackets expresses the technological variation (TV) component, which represents the variation of technology between the two regions. The EV is largely determined by the availability of resources and how they are used in the production process in a particular region. As resources are typically inputs in production, greater abundance of resources (implying lower shadow prices for natural resources, such as water) justifies their greater use. In contrast, scarcity of resources (higher shadow prices) in a particular region implies that these inputs should be used more sparingly. The implication for EV is that if two identical units of observation operating in two different regions are observed, then the one operating in the region that is more abundant in resources used as inputs is more likely to be more efficient.

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[7] Regional production frontiers (i.e., R1, R2, R3) are constructed from output (desirable and undesirable) and input data on all enterprises within a region.

The technological variation occurs due to differences in the characteristics of the surrounding environment where an enterprise operates. These can be the natural characteristics of the environment or the institutional characteristics (e.g. policies). For instance, an observed difference in the TV between two identical units of observation operating in two different regions might indicate that one of the regions is naturally better suited for the particular type of production, or that policy conditions are more favorable, thus rendering the same technology more productive in that region.

By evaluating the directional output distance functions for a given enterprise type that operates simultaneously in region a and b, the EV and TV of a production technology between the two regions can be measured, as illustrated in Figure 3. The EV component measures the difference in the position of a unit of observation relative to the best-practice frontier technology from one region to another region, which can be written as: $EV = MN - RS$.

In contrast, the TV of a production unit between regions a and b can be estimated by computing the distance between an observation in region a, (d^a, u^a, x^a) from the region b frontier, $P^b(x)$, and the distance between an observation in region b,(d^b, u^b, x^b) from the region a frontier, $P^a(x)$. Put differently, the EV component simply measures the "catching up" to the $P(x)$ frontier, and the TV component measures the difference in $P(x)$ between regions. From Figure 3 the technological variation can be written as: $TV = \frac{1}{2}[\mathrm{RS} - \mathrm{ST} + \mathrm{LN} - \mathrm{MN}] = \frac{1}{2}[\mathrm{LM} + \mathrm{RT}]$.

2.4 The Luenberger spatial productivity indicator

The discussion in the previous section focused on the Luenberger environmental indicator that includes undesirable outputs in the production model. Without considering undesirable outputs, a spatially-oriented Luenberger productivity indicator can also be constructed. The production technology without considering undesirable outputs can be written as:

$$P(x) = \{d\colon x \text{ can produce } d\}. \tag{6}$$

The directional output distance function can be defined as:

$$\vec{D}_o(x, d;\ \mathrm{g}_d) = \sup\{\beta : (d + \beta \mathrm{g}_d) \in P(x)\}. \tag{7}$$

When undesirable outputs are not considered in the model, the production technology is characterised with free disposability of undesirable outputs (i.e., desirable outputs can be produced without producing undesirable outputs), which is the basic difference from the environmental productivity model discussed in the earlier section.

Based on the spatial referencing, the Luenberger productivity indicator can now be constructed as:

$$LEI_a^b = \frac{1}{2}[\vec{D}_o^b(x^a, d^a; \mathrm{g}_d) - \vec{D}_o^b(x^b, d^b; \mathrm{g}_d) + \vec{D}_o^a(x^a, d^a; \mathrm{g}_d) - \vec{D}_o^a(x^b, d^b; \mathrm{g}_d)] \tag{8}$$

This productivity model can be defined as the 'Luenberger spatial productivity indicator' *(LSI)*. Like the Luenberger environmental indicator, and with similar implications, this model can be decomposed into its two constituents: efficiency variation and technological variation, as follows:

$$LEI_a^b = [\vec{D}_o^a(x^a, d^a; \mathrm{g}_d) - \vec{D}_o^b(x^b, d^b; \mathrm{g}_d)] + \frac{1}{2}[\vec{D}_o^b(x^b, d^b; \mathrm{g}_d) - \vec{D}_o^a(x^b, d^b; \mathrm{g}_d + \vec{D}_o^b\ (x^a, d^a; \mathrm{g}_d) - \vec{D}_o^a(x^a, d^a; \mathrm{g}_d)]. \tag{9}$$

The Luenberger spatial productivity indicator can be useful when environmental impacts of the production activities do not need to be considered in the productivity and efficiency measurement analysis. In policy studies, this model can also be applied along with the Luenberger environmental indicator to determine the contribution of the environmental effects on the productivity and efficiency levels across various regions or countries.

3. Estimation of the Luenberger environmental indicator

The Luenberger environmental indicator can be derived by estimating its component directional output distance functions that can be computed in several ways. The simplest, but a robust technique is the data envelopment analysis (DEA) or activity analysis model (Tyteca, 1996). In contrast to the parametric methods, such as the stochastic frontier approach, the DEA does not require a particular functional form to be imposed on the data.[8] DEA was formally introduced by Charnes et al. (1978) based on the work of Shephard (1953, 1970) and Farrell (1957). It facilitates the construction of a non-parametric piece-wise frontier over the existing data with the use of linear programming methods, and allows for efficiency measures to be calculated relative to this frontier (Coelli et al., 2005). DEA is an evaluation method by which the performance of a production unit can be compared with the best performing units of the sample (efficient frontier). The DEA procedure for the ensuing empirical analysis is presented below.

Suppose that there are $k = 1, \dots, K$ observations and two regions a and b. In order to estimate the first component of the Luenberger environmental indicator, $\vec{D}_o^b(x^a, d^a, u^a; \mathrm{g}_{d,} {-\mathrm{g}_u})$, the following linear programming model can be formulated:

$$\begin{aligned} &\vec{D}_o^b(x^{k',a}, d^{k',a}, u^{k',a}; \mathrm{g}_{d,} {-\mathrm{g}_u}) = \max \beta \\ &\quad s.t.\ \textstyle\sum_{k=1}^{K} z_k^b\, d_{km}^b \geq d_{k'm}^a + \beta \mathrm{g}_{dm}\,, && m = 1,\dots, M \\ &\quad \textstyle\sum_{k=1}^{K} z_k^b\, u_{kj}^b = u_{k'j}^a - \beta \mathrm{g}_{uj}\,, && j = 1,\dots, J \\ &\quad \textstyle\sum_{k=1}^{K} z_k^b\, x_{kn}^b \leq x_{k'n}^a\,, && n = 1,\dots, N \\ &\quad z_k^b \geq 0\,, && k = 1,\dots, K\,. \end{aligned} \tag{10}$$

The variables z_k are the weights assigned to each observation when constructing the production possibilities frontier. These intensity variables z_k $(k = 1,\dots, K)$ are non-

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[8] Aigner, Lovel and Schmidt (1977) and Meeusen and Broeck (1977) proposed the stochastic frontier production function model.

negative, which implies that the production technology satisfies constant returns to scale. To allow free disposability of inputs and desirable outputs, inequality constraints are imposed in the model. On the other hand, strict equality on the undesirable output constraints serves to impose weak disposability of undesirable outputs. Similarly, the three other components of the Luenberger environmental indicator can be estimated by the linear programming models stated below:

$$
\begin{aligned}
&\vec{D}_o^b(x^{k',b}, d^{k',b}, u^{k',b}; \mathrm{g}_{d,} {-\mathrm{g}_u}) = \max \beta \\
&\quad \textit{s.t.} \ \textstyle\sum_{k=1}^{K} z_k^b \, d_{km}^b \geq d_{k'm}^b + \beta \mathrm{g}_{dm} , && m = 1,\dots, M \\
&\quad \textstyle\sum_{k=1}^{K} z_k^b \, u_{kj}^b = u_{k'j}^b - \beta \mathrm{g}_{uj} , && j = 1,\dots, J \\
&\quad \textstyle\sum_{k=1}^{K} z_k^b \, x_{kn}^b \leq x_{k'n}^b , && n = 1,\dots, N \\
&\quad z_k^b \geq 0 , && k = 1,\dots, K .
\end{aligned}
\tag{11}
$$

$$
\begin{aligned}
&\vec{D}_o^a(x^{k',a}, d^{k',a}, u^{k',a}; \mathrm{g}_{d,} {-\mathrm{g}_u}) = \max \beta \\
&\quad \textit{s.t.} \ \textstyle\sum_{k=1}^{K} z_k^a \, d_{km}^a \geq d_{k'm}^a + \beta \mathrm{g}_{dm} , && m = 1,\dots, M \\
&\quad \textstyle\sum_{k=1}^{K} z_k^a \, u_{kj}^a = u_{k'j}^a - \beta \mathrm{g}_{uj} , && j = 1,\dots, J \\
&\quad \textstyle\sum_{k=1}^{K} z_k^a \, x_{kn}^a \leq x_{k'n}^a , && n = 1,\dots, N \\
&\quad z_k^a \geq 0 , && k = 1,\dots, K .
\end{aligned}
\tag{12}
$$

$$
\begin{aligned}
&\vec{D}_o^a(x^{k',b}, d^{k',b}, u^{k',b}; \mathrm{g}_{d,} {-\mathrm{g}_u}) = \max \beta \\
&\quad \textit{s.t.} \ \textstyle\sum_{k=1}^{K} z_k^a \, d_{km}^a \geq d_{k'm}^b + \beta \mathrm{g}_{dm} , && m = 1,\dots, M \\
&\quad \textstyle\sum_{k=1}^{K} z_k^a \, u_{kj}^a = u_{k'j}^b - \beta \mathrm{g}_{uj} , && j = 1,\dots, J \\
&\quad \textstyle\sum_{k=1}^{K} z_k^a \, x_{kn}^a \leq x_{k'n}^b , && n = 1,\dots, N \\
&\quad z_k^a \geq 0 , && k = 1,\dots, K .
\end{aligned}
\tag{13}
$$

When comparing the efficiency of production units across more than two regions, it is mandatory to have a common reference to which all regions can be compared. We estimate an infrafrontier as a reference technology to compare efficiencies of production units across regions. This ensures that a feasible solution for each activity analysis model can be found. Similar to the Luenberger environmental indicator, the components of the Luenberger spatial productivity indicator can be estimated by formulating linear programming models, which are shown in appendix A.

4. Data and Variables

With the aim of illustrating the concepts outlined above, the empirical part of the study considers six types of irrigated enterprises geographically grouped in 11 natural resource management regions within the Murray-Darling Basin (MDB), Australia. We construct efficiency measures for six types of enterprises across the NRM regions, and we use NRM regional level data.

The production technology for irrigated enterprises that are modelled within the framework of the Luenberger environmental indicator consisted of two inputs, one desirable output and one undesirable output. The volume of applied irrigation water was one input, while all production costs excluding the cost of irrigation water was treated as the other 'composite' input. Gross revenue obtained from an irrigated enterprise was defined as the desirable output. An aggregate indicator (ecologically weighted water withdrawal index) that Azad and Ancev (2010) developed for the measurement of environmentally adjusted efficiencies for irrigated agricultural enterprises was considered as an undesirable output in this study. Descriptive statistics of economic and environmental variables that are included in the efficiency model are presented in Table 1. Inputs and output data used for the production model were gathered for the fiscal year 2011-2012. Various reports, including those published by the Australian Bureau of Statistics (ABS, 2014), Bureau of Meteorology (BOM, 2014), Murray Darling Basin Authority (MDBA, 2014), and the Departments of Primary Industry of NSW, Queensland and Victoria were additional sources of data used for this study.

5. Results and discussion

Using the DEA techniques described above, efficiency scores for irrigated enterprises across NRM regions were computed following the Luenberger environmental indicator and Luenberger spatial productivity indicator, and the estimated results are presented in Table 2 and 3. Unlike the conventional Luenberger productivity indicator, the Luenberger environmental indicator (*LEI)* developed in this study does not produce negative values, since efficiencies of irrigated enterprises are compared to an infrafrontier.[9] A positive value of the Luenberger environmental indicator implies that environmentally adjusted efficiency of an irrigated enterprise for a given region is greater than the efficiency of that enterprise grown in the reference region.[10] Therefore, the higher value of *LEI* implies a better environmentally adjusted performance of an irrigated enterprise for a specific natural resource management region.

The findings reveal that there is a substantial variation in environmentally adjusted efficiency of irrigation enterprises across the NRM regions. This result is consistent with the previous findings drawn from the use of other non-parametric efficiency models (e.g. Azad and Ancev (2010)). Some irrigation enterprises were found to be relatively environmentally efficient in some NRM regions, but they are not efficient in others. For example, cereal crops for grain/seed had low *LEI* values for North Central (0.77), Central West (0.82) and Border River (QLD) (0.85), but comparatively higher *LEI* scores were observed in Border River-Gwydir (2.10), Condamine (1.30) and Namoi (1.24) NRM regions. On the other hand, vegetables produced in Border River (QLD), Border River-Gwydir and Murray regions had comparatively higher *LEI* score than other NRM regions, which imply that substantial economic benefits from vegetables production can be achieved without creating large scale of environmental damage in these regions.

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[9] The value of the Luenberger environmental indicator can only be negative if an observation lies beneath the infrafrontier. But that is not possible by the very construction of the infrafrontier.

[10] A reference region in this context can be defined as a hypothetical region where all worst performing enterprises are located. These hypothetica enterprises are grouped together to form the infrafrontier.

Findings reveal that environmental threats of irrigation water withdrawal in a specific region is largely dependent on both the existence and the significance of ecological assets that are affected by water withdrawals. For instance, Border River-Gwydir regions where cotton takes out substantial amounts of irrigation water from the river system (which would have otherwise fed the Ramsar wetlands within this region or within downstream regions) exert a greater environmental pressure in comparison to Lachlan, and Namoi regions, where Ramsar wetlands are not present within these regions, and/or within downstream regions.[11] Irrigated enterprises grown in a region that has large volume of surface water resources, but contains insignificant environmental assets, would be in a better position to attain high environmental performance than enterprises that are produced in a region where water resources are scarce, and environmental assets are significant. The findings also reveal that higher LEI for most of the irrigated enterprises were observed in Lachlan, Namoi, Border River (QLD), Condamine regions, where Ramsar wetlands did not exist.

The results from the decomposition of the Luenberger environmental model indicate that the greater TV is primarily due to the difference in environmental assets, which is consistent with the previous empirical evidence (Färe et al., 2012; Williams, 2011; Koutsomanoli-Filippaki, 2009; Casu et al., 2004; Weber and Weber, 2004). The influence of the TV on the efficiency model is substantially higher than that of EV on the model for most of the cases.

The decomposition analysis also indicates that TV is positive for most cases, whereas the efficiency variation showed mixed signs (positive or negative). A zero value of EV for some enterprises implies that there is no difference in resource abundance between the enterprise grown in a particular region and the enterprise that operates in the

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[11] Ramsar wetlands are internationally important sites that contain representative, rare or unique wetlands, or that are important for conserving biological diversity.

reference region. A higher value of *TV* indicates that there is a significant variation in prevailing natural or institutional conditions between regions, and therefore the productivity of the technology used for the particular irrigated enterprise varies across the NRM regions. For instance, the high value of *TV* for cereal crops for grain/seed in Goulburn Broken indicates that the Goulburn Broken region has an environment that is suited for this enterprise more than in other region.

The model for the Luenberger spatial productivity indicator that excludes undesirable outputs (environmental impacts) from the production function is estimated using DEA, and results are presented in Table 3. In most cases, the efficiency of irrigated enterprises calculated from the Luenberger spatial productivity model are found to be greater than those calculated by the Luenberger environmental indicator (Figure 4). For instance, the estimated efficiencies for cotton and pasture & cereal crops for grazing generated from the Luenberger spatial productivity model are higher than that those calculated from the Luenberger environmental indicator for most of the regions (Figure 4). This indicates that the estimated efficiencies of enterprises declined with the inclusion of environmental effects in the production model, as expected. The finding suggests that it is important to take into account the undesirable outputs (environmental effects of production activities) in the models of productivity analysis and efficiency measurement. In the case of total factor productivity measurement, the model without undesirable outputs may produce biased productivity growth results.

The Wilcoxon signed-rank test was conducted to examine whether the estimated differences in efficiency are significant for both the Luenberger environmental indicator and the Luenberger spatial indicator. This non-parametric test was used to verify the significance of the differences between paired efficiency scores for each enterprise obtained from the two productivity models. We tested a hypothesis that the median difference between a pair of observations is zero, and hence the null hypothesis will be rejected if the Wilcoxon test value (p-value) is small (i.e., less than 0.05). If the null hypothesis is rejected, then it can be concluded that the efficiency scores resulting from the two different productivity models are significantly different. The Wilcoxon signed-rank test results show that there is a significant difference in average productivity and efficiency scores that are estimated using the Luenberger environmental indicator and the Luenberger spatial productivity model in most cases (Table 4). This result implies that efficiency scores of irrigated enterprises will vary depending on which productivity model is chosen for efficiency analysis.

6. Conclusions and Policy Implications

This study reports on a new environmentally adjusted efficiency index, which is developed by modifying the traditional framework of the Luenberger productivity indicator. By modifying the time-oriented referencing into space-oriented referencing, this approach offers a new direction of measuring productivity and efficiency of units of observation across space. One of the key features of the Luenberger environmental indicator is that it can be employed to measure relative performance of units of observation across space incorporating environmental impacts in the production model. This allows direct comparison of environmentally adjusted performance among the same enterprise types that are located in different regions.

An additional concept developed in the paper is that of an infrafrontier. This is a new type of a reference frontier that is similar to the concept of a metafrontier in that it provides a reference for comparison across non-homogenous technologies. Unlike the metafrontier that envelopes the meta-production set from above, the infrafrontier envelops it from below.

An empirical application of the *LEI* and *LSI* is to the Australian irrigation sector considering 17 NRM regions within the Murray-Darling Basin. Findings show that environmentally adjusted efficiencies of irrigated enterprises vary considerably across the regions. The difference in environmentally adjusted efficiency is largely driven by technological rather than efficiency variation, which implies that the differences across regions is mostly due to the variation in the characteristics of the surrounding environment. This result is consistent with the previous empirical evidence.

Both the *LEI* and the *LSI* can be widely used in any sector of the economy to measure relative productivity and environmental efficiency across space. In recent years, there has been growing public concern about the environmental impacts of agricultural production activities. Since there is a large variation in availability of natural resources across space, and the interactions between agriculture, natural resources and the environment is very complex, policy approaches that take into account regional differences in environmental assets and their significance to society might be appropriate. The methods presented in this paper based on a spatially oriented Luenberger indicator can facilitate comparisons among agricultural production activities in specific areas and their associated environmental pressures. These indicators can provide policymakers with necessary information that can be useful as a guideline for formulating policies and strategies towards more sustainable agricultural production.

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Table 1. Descriptive statistics of the economic and environmental variables included in the efficiency model.

Irrigated enterprises	Volume of water applied (GL)	Gross cost (excluding water) (Million AUD)	Gross revenue (Million AUD)	Ecologically weighted water withdrawal index ('000')
Cotton				
Mean	227.08	125.85	200.50	179.09
Maximum	487.12	261.52	396.87	555.78
Minimum	11.50	6.97	8.29	0.026
Cereal crops for grain/seed				
Mean	48.05	10.54	28.23	27.70
Maximum	224.86	34.89	102.31	75.95
Minimum	13.58	3.07	9.07	0.004
Pasture and cereal crops cut for hay				
Mean	16.38	3.89	6.94	19.18
Maximum	48.82	10.85	19.97	124.02
Minimum	0.52	0.16	0.31	0.001
Pasture and cereal crops for grazing				
Mean	90.22	20.85	43.36	135.36
Maximum	396.66	75.08	181.35	1007.74
Minimum	1.21	0.18	0.44	0.001
Other broadacre crops				
Mean	5.14	1.55	3.63	2.97
Maximum	22.29	5.41	13.55	12.98
Minimum	0.06	0.02	0.04	0.001
Vegetables				
Mean	6.43	29.21	37.49	23.684
Maximum	15.11	65.58	73.45	38.39
Minimum	0.04	0.59	0.92	0.001

Table 2a. Estimated values of the Luenberger environmental indicator for irrigated enterprises across NRM regions

NRM Regions	Cotton			Cereal crops for grain/seed			Pasture & cereal crops cut for hay		
	LEI	*EV*	*TV*	*LEI*	*EV*	*TV*	*LEI*	*EV*	*TV*
Border River-Gwydir	1.247	0.512	0.736	2.101	0.984	1.117	1.431	0.875	0.556
Central West	1.283	0.576	0.714	0.822	-0.250	1.118	1.203	0.505	0.767
Lachlan	1.053	0.116	0.944	1.028	0.123	0.912	1.252	0.559	0.664
Murray	1.181	0.441	0.760	0.955	0.010	0.958	1.164	0.441	0.756
Murrumbidgee	0.995	0.023	0.949	1.046	0.120	0.935	1.320	0.687	0.651
Namoi	1.327	0.778	0.594	1.237	0.790	0.219	1.433	0.884	0.569
Goulburn Broken	-	-	-	0.995	0.123	0.889	1.296	0.652	0.646
North Central	-	-	-	0.770	0.123	0.960	1.371	0.714	0.737
North East (VIC)	-	-	-	-	-	-	1.084	-0.021	1.004
Border River (QLD)	1.194	0.392	0.802	0.849	-0.240	1.090	1.214	0.525	0.688
Condamine	1.394	0.778	0.611	1.297	0.564	0.667	1.360	0.680	0.630

Note: "-" indicates that there are no values for *LEI*, *EV* and *TV*, since inputs data are not available for the respective NRM regions.

LEI = Luenberger environmental indicator, *EV* = Efficiency variation, *TV* = Technological variation.

Table 2b. Estimated values of the Luenberger environmental indicators for irrigated enterprises across NRM regions

NRM Regions	Pasture & cereal crops for grazing			Other broadacre crops			Vegetables		
	LEI	*EV*	*TV*	*LEI*	*EV*	*TV*	*LEI*	*EV*	*TV*
Border River-Gwydir	1.485	0.993	0.492	1.386	0.789	0.597	1.273	0.560	0.714
Central West	1.146	0.340	0.865	1.212	0.463	0.953	0.995	0.000	0.991
Lachlan	1.254	0.580	0.660	1.363	0.825	0.555	1.167	0.345	0.827
Murray	1.120	0.341	0.811	1.407	0.881	0.544	1.271	0.552	0.713
Murrumbidgee	1.393	0.823	0.585	1.460	0.950	0.522	0.937	-0.114	1.057
Namoi	1.492	0.993	0.477	1.446	0.908	0.317	-	-	-
Goulburn Broken	0.974	-0.003	0.966	1.444	0.950	0.475	1.048	0.102	0.940
North Central	1.442	0.936	0.602	1.249	0.950	0.549	0.994	0.014	0.974
North East (VIC)	1.552	0.964	0.512	1.411	0.772	0.608	0.998	0.013	0.987
Border River (QLD)	1.473	0.993	0.480	1.456	0.950	0.506	1.276	0.560	0.720
Condamine	1.531	0.993	0.479	1.645	0.919	0.510	1.205	0.419	0.790

Note: "-" indicates that there are no values for *LEI*, *EV* and *TV*, since inputs data are not available for the respective NRM regions.

LEI = Luenberger environmental indicator, *ED* = Efficiency variation, *TV* = Technological variation.

Table 3a. Estimated values of the Luenberger spatial productivity indicator for irrigated enterprises across NRM regions

NRM Regions	Cotton			Cereal crops for grain/seed			Pasture & cereal crops cut for hay		
	LSI	*EV*	*TV*	*LSI*	*EV*	*TV*	*LSI*	*EV*	*TV*
Border River-Gwydir	3.578	0.110	3.468	4.243	0.113	4.130	4.380	0.529	3.850
Central West	3.023	0.118	2.905	6.816	-0.284	7.101	5.253	0.439	4.814
Lachlan	3.409	0.100	3.310	5.477	0.156	5.321	5.558	0.481	5.076
Murray	2.974	0.096	2.877	4.281	0.179	4.102	4.461	0.485	3.976
Murrumbidgee	3.892	0.099	2.781	2.290	0.125	0.452	4.510	0.489	4.211
Namoi	3.986	0.137	2.832	2.236	0.082	0.426	4.716	0.466	4.423
Goulburn Broken	-	-	-	5.684	0.124	5.560	5.764	0.487	5.276
North Central	-	-	-	2.270	0.088	0.450	4.214	0.499	3.958
North East (VIC)	-	-	-	-	-	-	3.184	-0.003	3.181
Border River (QLD)	4.087	0.077	4.010	10.252	-0.472	10.723	7.860	0.437	7.423
Condamine	3.770	0.148	3.622	5.400	-0.204	5.603	5.652	0.483	5.169

Note: "-" indicates that there are no values for *LSI*, *EV* and *TV*, since inputs data are not available for the respective NRM regions.
LSI = Luenberger spatial productivity indicator, *EV* = Efficiency variation, *TV* = Technological variation.

Table 3b. Estimated values of the Luenberger spatial productivity indicator for irrigated enterprises across NRM regions

NRM Regions	Pasture & cereal crops for grazing			Other broadacre crops			Vegetables		
	LSI	*EV*	*TV*	*LSI*	*EV*	*TV*	*LSI*	*EV*	*TV*
Border River-Gwydir	2.222	0.355	1.867	1.709	0.215	1.494	5.781	0.091	5.690
Central West	2.884	0.355	2.528	1.955	0.211	1.744	4.593	0.190	4.403
Lachlan	2.785	0.347	2.438	2.177	0.215	1.961	5.227	0.000	5.227
Murray	2.251	0.351	1.901	1.753	0.216	1.537	3.621	0.124	3.497
Murrumbidgee	2.340	0.351	2.139	1.689	0.217	1.580	4.597	0.022	4.586
Namoi	2.316	0.354	2.112	1.813	0.214	1.706	-	-	-
Goulburn Broken	2.886	0.351	2.535	2.256	0.217	2.039	3.232	0.012	3.219
North Central	2.261	0.354	2.057	1.577	0.217	1.469	3.942	0.097	3.894
North East (VIC)	1.651	0.039	1.604	1.223	0.027	1.209	4.428	0.001	4.428
Border River (QLD)	4.375	0.355	4.020	2.902	0.217	2.685	5.088	0.156	4.932
Condamine	2.835	0.355	2.480	2.213	0.215	1.998	4.494	0.056	4.438

Note: "-" indicates that there are no values for *LSI*, *EV* and *TV*, since inputs data are not available for the respective NRM regions.

LSI = Luenberger spatial productivity indicator, *EV* = Efficiency variation, *TV* = Technological variation.

Table 4. The Wilcoxon test to compare efficiency scores estimated under the Luenberger environmental indicator and Luenberger spatial productivity indicator

Enterprise	Number of Observations	Wilcoxon test value (Z value)	p-value
Cotton	8	2.521	0.012
Cereal crops for grain/seed	10	0.153	0.878
Pasture & cereal crops cut for hay	11	2.934	0.003
Pasture & cereal crops for grazing	11	2.934	0.003
Other broadacre crops	11	2.847	0.004
Vegetables	10	2.803	0.005

Figure 1. Directional output distance function

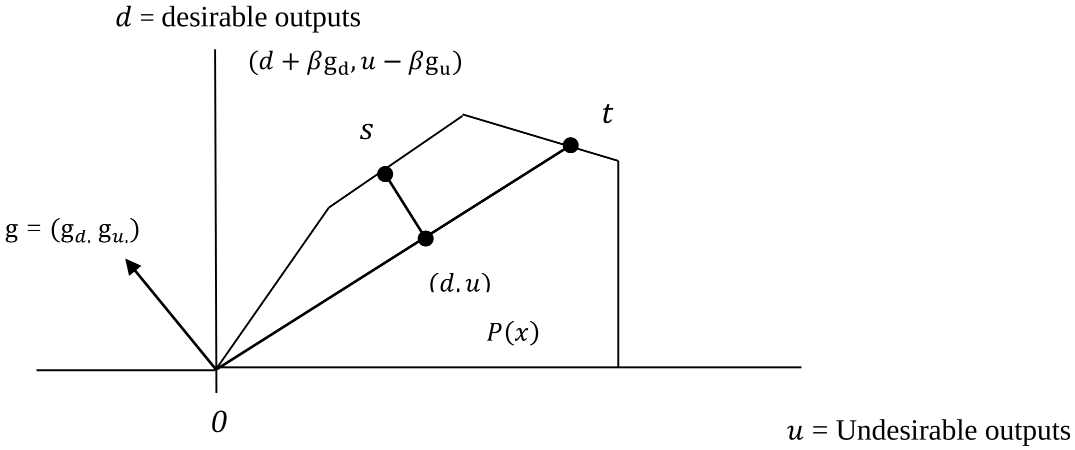


Figure 2. The infrafrontier

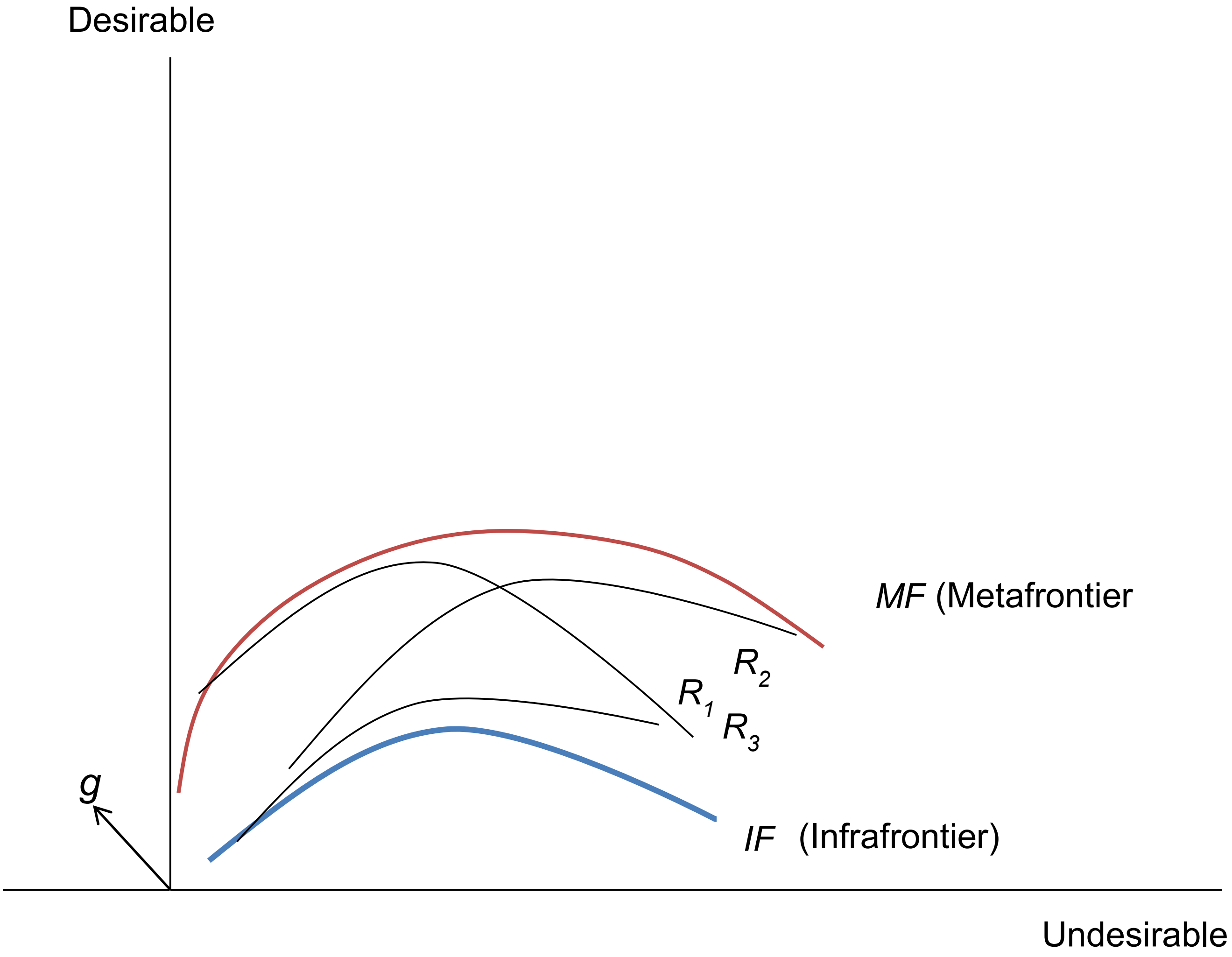


Figure 3. The Luenberger environmental indicator

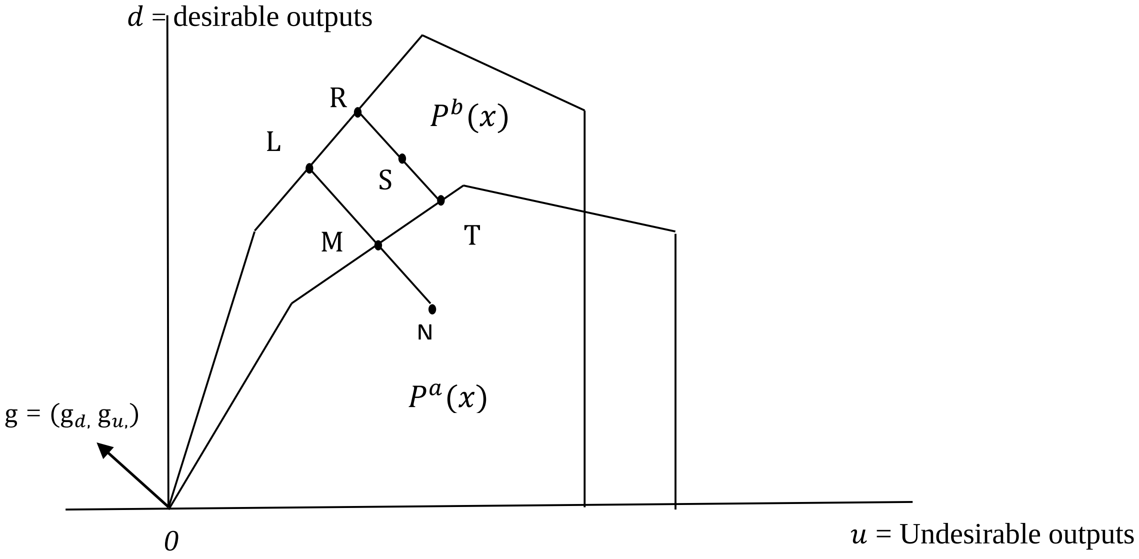
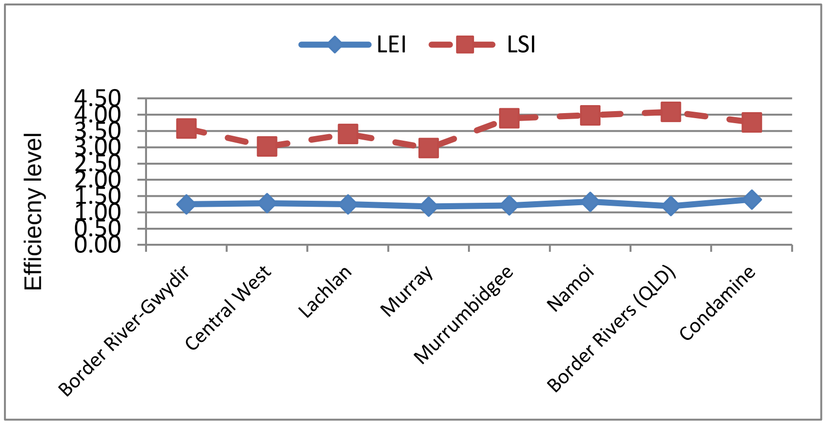
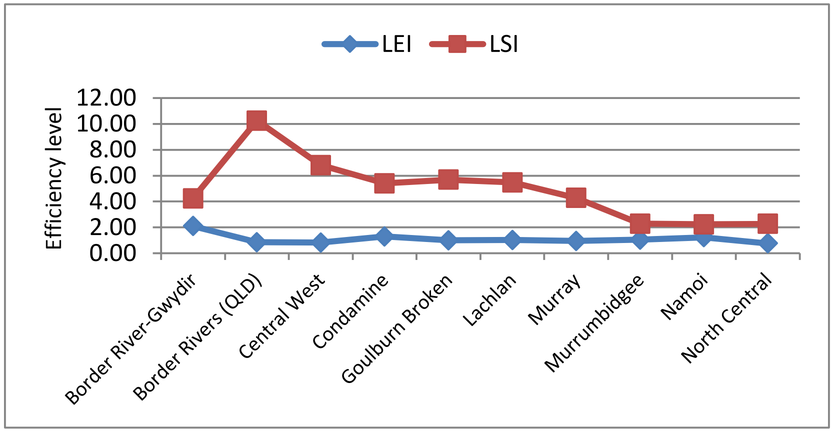


Figure 4a. Efficiency comparison: Luenberger environmental indicator (*LEI*) versus Luenberger spatial productivity indicator (*LSI*)

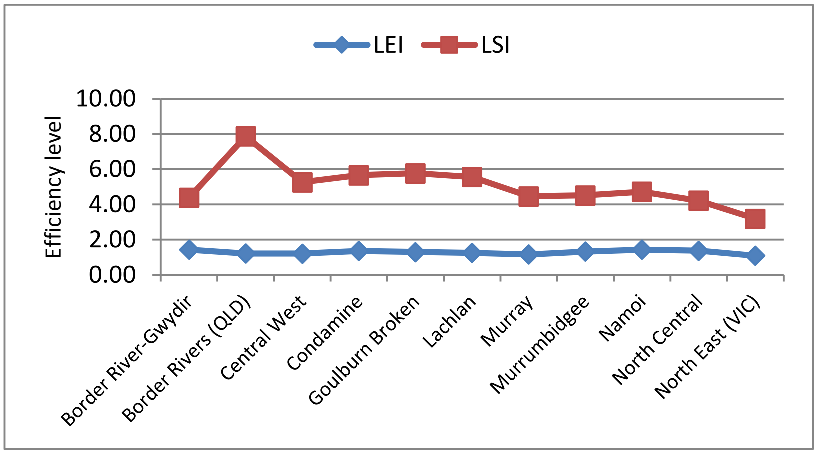
Cotton



Cereal crops for grain/seed



Pasture & cereal crops cut for hay



Pasture & cereal crops for grazing

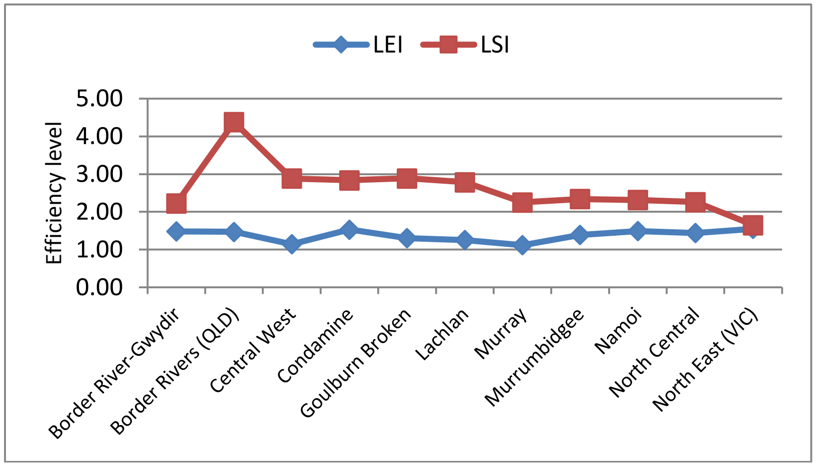
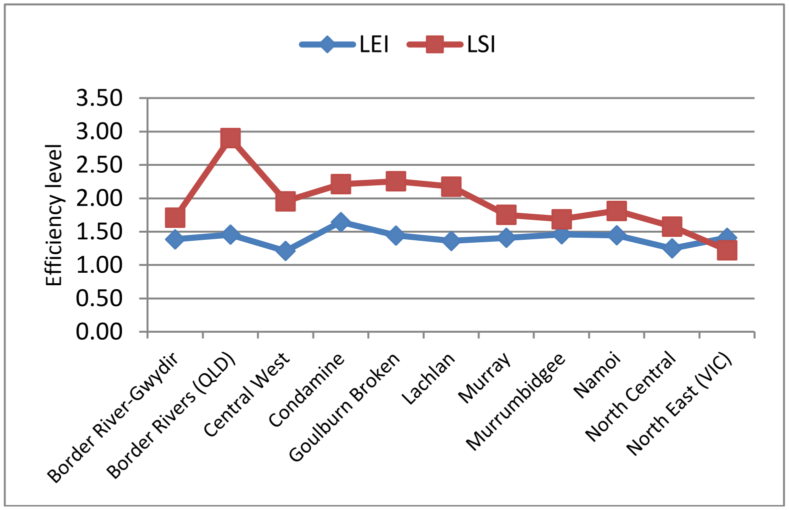
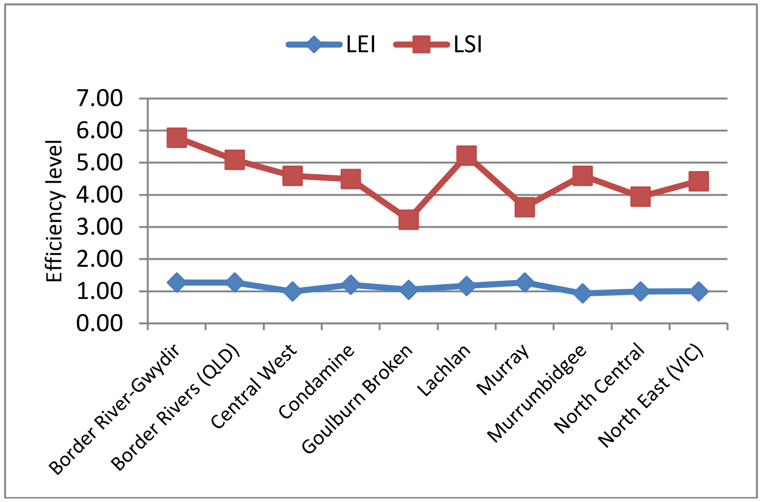


Figure 4b. Efficiency comparison: Luenberger environmental indicator (*LEI*) versus Luenberger spatial productivity indicator (*LSI*)

Other broadacre crops



Vegetables



Appendix A: Linear programming models to estimate the components of the Luenberger spatial productivity indicator.

$$\vec{D}_o^b(x^{k',a}, d^{k',a}; \mathrm{g}_d) = \max \beta$$

$$\begin{aligned} \text{s.t. } & \sum_{k=1}^{K} z_k^b\, d_{km}^b \geq d_{k'm}^a + \beta \mathrm{g}_{dm}, && m = 1,\dots, M \\ & \sum_{k=1}^{K} z_k^b\, x_{kn}^b \leq x_{k'n}^a, && n = 1,\dots, N \qquad (1) \\ & z_k^b \geq 0, && k = 1,\dots, K. \end{aligned}$$

$$\vec{D}_o^b(x^{k',b}, d^{k',b}; \mathrm{g}_d) = \max \beta$$

$$\begin{aligned} \text{s.t. } & \sum_{k=1}^{K} z_k^b\, d_{km}^b \geq d_{k'm}^b + \beta \mathrm{g}_{dm}, && m = 1,\dots, M \\ & \sum_{k=1}^{K} z_k^b\, x_{kn}^b \leq x_{k'n}^b, && n = 1,\dots, N \qquad (2) \\ & z_k^b \geq 0, && k = 1,\dots, K. \end{aligned}$$

$$\vec{D}_o^a(x^{k',a}, d^{k',a}; \mathrm{g}_d) = \max \beta$$

$$\begin{aligned} \text{s.t. } & \sum_{k=1}^{K} z_k^a\, d_{km}^a \geq d_{k'm}^a + \beta \mathrm{g}_{dm}, && m = 1,\dots, M \\ & \sum_{k=1}^{K} z_k^a\, x_{kn}^a \leq x_{k'n}^a, && n = 1,\dots, N \qquad (3) \\ & z_k^a \geq 0, && k = 1,\dots, K. \end{aligned}$$

$$\vec{D}_o^a(x^{k',b}, d^{k',b}; \mathrm{g}_d) = \max \beta$$

$$\begin{aligned} \text{s.t. } & \sum_{k=1}^{K} z_k^a\, d_{km}^a \geq d_{k'm}^b + \beta \mathrm{g}_{dm}, && m = 1,\dots, M \\ & \sum_{k=1}^{K} z_k^a\, x_{kn}^a \leq x_{k'n}^b, && n = 1,\dots, N \qquad (4) \\ & z_k^a \geq 0, && k = 1,\dots, K. \end{aligned}$$