A statistical model for consumer preferences: the case of Italian extravirgin olive oil

Marcella Corduas

Department of Political Sciences
University of Naples Federico II, Naples (Italy)
corduas@unina.it
Abstract

The CUB model is a mixture distribution recently proposed in literature for modelling ordinal data. In this article we investigate how this univariate distribution can be exploited in order to model the margins of multivariate ordinal data. In particular, we take the theory of discrete copula into consideration and we show how use the Plackett distribution in order to construct a one parameter bivariate distribution from CUB margins.

Keywords: Ordinal data, CUB model, Multivariate distribution, Plackett’s distribution, Food preferences.

1 Introduction

Empirical studies on consumer behaviour often analyze data obtained through a questionnaire where interviewees are requested to rate the importance of certain factors in their choices by means of a Likert scale. This type of data deserve special attention because the judgments may depend on covariates characterizing the raters which may be clustered into sub-groups exhibiting more homogeneous choices. In addition, the judgements about connected items may be correlated.

In this work, we present an innovative technique for modelling multidimensional ordinal data. In particular, following the approach proposed by Corduas (2014), we consider the method introduced by Plackett for constructing a one parameter bivariate distribution from given margins and we apply it in order to represent correlated ordinal variables which individually follows a CUB model. This is a univariate mixture distribution defined by the convex combination of a Uniform and a shifted Binomial distribution whose parameters may be related to rater’s covariates (Piccolo, 2006; Corduas et al. 2009). Various contributions have investigated the use of such model for consumer behaviour analysis providing applications to numerous products (Cicia et al., 2010; Corduas, 2011; Corduas et al. 2013; Iannario et al. 2012; Manisera et al. 2011; Piccolo and D’Elia, 2008).

In this article, we shows how the bivariate distribution can be defined and how its characterizing parameter, which describes the association between the component random variables, can be related to the subject’s covariates. The proposed technique will be applied to the study of key drivers of extra virgin
olive oil consumption in Italy. The technique allows a representation of the data whose meaning can be easily interpreted providing useful information for management support.

2 Modelling correlated bivariate ordinal data

Firstly, we recall that the bivariate Plackett random variable \((X, Y)\) is characterized by a joint cumulative distribution function \(H(x, y; \psi)\), \(\psi \in (0, \infty)\), such that:

\[
H(x, y; \psi) = C(F(x), G(y)) = \frac{M(x, y) - [M^2(x, y) - 4\psi(\psi - 1)F(x)G(y)]^{1/2}}{2(\psi - 1)},
\]

where \(F(x)\) and \(G(y)\) are the pre-defined marginal distribution functions defined on the support \(S_x\) and \(S_y\), respectively (Plackett, 1965). Moreover, \(M(x, y) = 1 + (F(x) + G(y))(\psi - 1)\) (Mardia, 1970).

The parameter \(\psi\) is a measure of association between \(X\) and \(Y\); in particular, \(\psi = 1\) implies that \(X\) and \(Y\) are independent (so that \(H(x, y; \psi) = F(x)G(y)\)), whereas \(\psi < 1\) and \(\psi > 1\) refer to negative and positive association, respectively.

The Plackett distribution family has found numerous applications being the base for new types of models for continuous/discretized and discrete data. Moreover, overcoming the restriction on dimensions, which were initially limited to the bivariate or trivariate case, Molenberghs (1992) successfully extended the results to the multivariate case. Furthermore, Molenberghs and Lesaffre (1994) exploited that result for proposing a modelling approach to account the dependence of the association parameter from explanatory variables.

Corduas (2011, 2014) investigated the usefulness of the method introduced by Plackett for representing correlated ordinal variables which individually follows a CUB model. As mentioned above, one of the positive feature of this model is given by the possibility of including covariates describing special characteristics of the raters. This originates two possible models for bivariate data that we briefly illustrate. We will assume that \((X, Y)\) is a discrete random variable with support \(S_{xy} = \{(x, y) : x = 1, 2, ..., m; y = 1, ..., m\}\)

- **Bivariate model with no covariates**
In this case, the marginal distribution functions $F$ and $G$ entering the Plackett copula are simply derived from CUB models without covariates.

For brevity, we report the definitions for $X$ (and similar results applies to $Y$) and we describe the CUB model introducing the the probability mass distribution (rather than the cumulative distribution function) since that highlights the role of the two characterizing parameters. CUB model is, in fact, the mixture distribution:

$$p(x; \theta_x) = \pi_x \left( \frac{m - 1}{x - 1} \right) (1 - \xi_x)^{x-1} \xi_x^{m-x} + (1 - \pi_x) \frac{1}{m}, \quad x = 1, 2, ..., m. \quad (2)$$

where $\xi_x \in [0, 1]$, $\pi_x \in (0, 1]$ and identifiability is ensured when $m > 3$ (Iannario, 2010). The weight $\pi_x$ determines the contribution of the Uniform distribution in the mixture, therefore, $(1 - \pi_x)$ is interpreted as a measure of uncertainty which is intrinsic to any judgment. Besides, the parameter $\xi_x$, characterizes the shifted Binomial distribution and $(1 - \xi_x)$ denotes the degree of liking expressed by raters with respect to the item. In the former case $(1 - \xi_x) > 0.5$; the skewness is negative so that the portion of raters which give a favourable judgement about the item under evaluation is large. The opposite is verified when $(1 - \xi_x) < 0.5$. Various statistical properties and extensions together with an efficient estimation algorithm (Piccolo, 2006; Iannario and Piccolo, 2013) make this univariate model of particular interest for real applications (see, Iannario and Piccolo, 2012, and references therein).

Then, we denote with $F(x; \theta_x)$ being $\theta_x = (\pi_x, \xi_x)'$ the distribution function of $X$ (similarly, $G(y; \theta_y)$ for $Y$).

Thus, the bivariate copula (1) can be applied so that the joint distribution of $(X, Y)$ is given by:

$$H(x, y; \psi, \theta_x, \theta_y) = C(F(x; \theta_x), G(y; \theta_y)) \quad (3)$$

Given an observed sample of ordinal data, $(y_i, x_i)$, for $i = 1, 2, ..., n$, the estimation is performed by means of the two step procedure proposed by Joe and Xu (1996), the so called inference for the margins (IFM) method. Specifically, in the first stage only the parameters in the univariate margins, that is the CUB models are estimated by maximum likelihood. This step leads to: $\hat{\theta}_x$ and $\hat{\theta}_y$. The second stage involves maximum likelihood of the dependence parameter, $\psi$, with the univariate parameters held fixed from
the first stage. The estimation, therefore, is performed by maximizing each of the following log-likelihood functions separately:

\[
    l_1(\theta_x; x) = \sum_{x=1}^{m} n_x \ln(p(x; \theta_x)),
\]

\[
    l_2(\theta_y; y) = \sum_{y=1}^{m} n_y \ln(p(y; \theta_y)),
\]

\[
    l_3(\psi; x, y) = \sum_{x=1}^{m} \sum_{y=1}^{m} n_{xy} \ln(h(x, y; \psi)),
\]

where, according to standard notation, \( n_{xy} \) is the frequency of the occurrence of \((x, y)\) in the observed sample, \( n_x \) and \( n_y \) are the related marginal frequencies, and \( h(x, y; \psi) \) is the probability mass distribution implied by (1).

In this respect, Joe (1997) showed that the IFM estimator is consistent, asymptotically Normal under regular conditions. In addition, Joe (2005) studied the asymptotic relative efficiency of IFM procedure compared with maximum likelihood estimation and considered some specific models indicating the typical level of efficiency.

- **Bivariate model with covariates**

The uncertainty and feeling, that interviewees have towards the object under judgement, can be interpreted in terms of the subject’s characteristics. The CUB model can be extended so that covariates are included:

\[
    p(x|z_i, w_i) = \pi_i \left( \frac{m-1}{x-1} \right) (1-\xi_i)^{x-1}\xi_i^{m-x} + (1-\pi_i) \frac{1}{m},
\]

\[
    x = 1, 2, ..., m.
\]

with:

\[
    \pi_i = \frac{1}{1+e^{-z_i^\prime \beta_x}}; \quad \xi_i = \frac{1}{1+e^{-w_i^\prime \gamma_x}}; \quad i = 1, 2, ..., n,
\]

where \( z_i = (1, z_{i1}, ..., z_{ip})' \) and \( w_i = (1, w_{i1}, ..., w_{iq})' \) are the row vectors containing the covariates associated to the \( i \)-th rater. Moreover, \( \beta_x = (\beta_0, \beta_1, ..., \beta_p)' \) and \( \gamma_x = (\gamma_0, \gamma_1, ..., \gamma_q)' \) are the vectors of parameters. For convenience, in the following, we set: \( \theta_x = (\beta_x', \gamma_x')' \) and we will denote the set of all distinct subject’s covariates which have an effect on the CUB parameters with \( d_{xi} \). Moreover, we assume that \( z_i \) and \( w_i \), for \( i = 1, ..., n \), are subsets of such a collection which do not necessarily coincide.
Although no restriction is needed for the marginal models, in the following we will assume that the subject’s features are only measured by discrete (or discretized) variables. Moreover, with reference to the \(i\)-th rater, as done above, we will denote the observations of those covariates (including constant) as a vector row: \(\mathbf{t}_i\). Thus, the definition of the Plackett bivariate distribution in (1), has to be modified in order to take into account the effect of covariates on the margins and the relationship that connects the association parameter to those explanatory variables. Specifically, we consider:

\[
H(x, y; \psi_i) = C(F(x|\theta_x, d_{xi}), G(y|\theta_y, d_{yi}))
\]

where, in general:

\[
\ln(\psi_i) = \mathbf{t}_i \eta.
\]

The model (9) allows the description of the joint distribution of judgements about connected items in presence of respondents that are grouped in clusters. As a matter of fact, given a sample of observed ordinal data and subjects’ covariates \((x_i, y_i, \mathbf{t}_i), i = 1, ..., n\), we regard the observations that share the same covariate values as a cluster. Again, IFM procedure can be applied.

In the first step the CUB model with covariates are fitted to each marginal variable. This will provides the ML estimate \(\hat{\theta}_x\) and \(\hat{\theta}_y\). Then, considering each distinct subject profile (as determinde by the values of the covariates), the CUB estimated parameters are computed. For the generic \(c\)-th cluster, these will be denoted as \(\hat{\pi}_{xc}\) and \(\hat{\xi}_{yc}\). Finally, the estimated CUB distribution for each marginal random variable is obtained.

In the second step, we search for the ML estimate of \(\eta\). Specifically, the log-likelihood function: \(L(\eta; \hat{\theta}_x, \hat{\theta}_y)\) is written as the sum of \(c\) components:

\[
L(\eta; \hat{\theta}_x, \hat{\theta}_y) = \sum_{c=1}^{k} L_c(\psi_c)
\]

where: \(\log \psi_c = \mathbf{t}_c \eta\) defines the relationship ruling the association parameter of the Plackett distribution of the \(c\)-th cluster (corresponding to the subjects’ profile \(\mathbf{t}_c\)) and :

\[
L_c(\psi_c) = \sum_{j=1}^{\nu_c} \log p(x_j, y_j; \psi_c, \hat{\pi}_{xc}, \hat{\xi}_{xc}, \hat{\pi}_{yc}, \hat{\xi}_{yc})
\]

is the component of the log-likelihood function corresponding to the \(c\)-th cluster consisting of \(\nu_c\) subjects having the same covariates profile, \(\mathbf{t}_c\). Finally, the asymptotic covariance matrix and standard errors of the estimated
parameters are evaluated using the jackknife method, as suggested by Xu (1996).

3 Application: the extra virgin olive oil

As an illustration, we present a study about the perception of Italian consumers on extra virgin olive (EVO) oil quality.

Italy is one of the major producing and consuming countries of olive oil. However, the domestic production is not sufficient to cover the demand and, for this reason, a large amount of olive oil that purchasers find on the shelves are originated from raw materials grown in other countries. The factors that affect olive oil purchasing behaviour are not clear because consumers are not accustomed to associate the organoleptic properties to quality signals and, in addition, they are misdirected by the high frequency of use and by the huge number of different cooking preparations and combining ingredients. Numerous contributions have investigated the liking/disliking of consumers about EVO oil focusing the attention on various consumption driving factors, such as the perceived health benefits, the importance of the region of origin, the role of sensory cues (Dekhili and d’Hauteville, 2009, Dekhili et al., 2011; Caporale et al., 2006; Fotopoulos and Krystallis, 2001).

In the following we consider some sensory characteristics (colour, flavour, pungency/bitterness) and the two extrinsic attributes that typify the production: the geographical certification (PDO-protected designation of origin and PGI-protected geographical indication) and the use of organic farming practices.

The survey was carried out between February and March 2012 and involved a sample of 1000 subjects belonging to the Italian consumers’ panel of AC Nielsen. All the recruited interviewees consumed EVO oil and, in addition, were in charge of the purchases of the product for themselves and their family. Each interviewee was asked to rate the importance of the EVO oil attributes in determining his/her purchase decision on a 7 point Likert scale (where 1 denoted “not important at all” and 7 "extremely important"). In addition, the respondents gave a self-assessment of the level of awareness of the information about the product’s features over a 7 point scale. These ratings were then organized in a binary variable denoting low-medium (L)
Table 1: CUB models for EVO oil attributes

<table>
<thead>
<tr>
<th>Variable</th>
<th>$\hat{\pi}$</th>
<th>$\hat{\gamma}_0$</th>
<th>$\hat{\gamma}_1$</th>
<th>$\xi_L$</th>
<th>$\xi_H$</th>
<th>$E(V_L)$</th>
<th>$E(V_H)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Color</td>
<td>0.902</td>
<td>-0.524</td>
<td>-0.532</td>
<td>0.372</td>
<td>0.258</td>
<td>4.693</td>
<td>5.310</td>
</tr>
<tr>
<td></td>
<td>(0.023)</td>
<td>(0.048)</td>
<td>(0.065)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Flavour</td>
<td>0.915</td>
<td>-0.801</td>
<td>-0.706</td>
<td>0.310</td>
<td>0.181</td>
<td>5.04</td>
<td>5.75</td>
</tr>
<tr>
<td></td>
<td>(0.020)</td>
<td>(0.050)</td>
<td>(0.071)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Taste (Pungency and bitterness)</td>
<td>0.519</td>
<td>-0.198</td>
<td>-0.777</td>
<td>0.450</td>
<td>0.274</td>
<td>4.154</td>
<td>4.704</td>
</tr>
<tr>
<td></td>
<td>(0.038)</td>
<td>(0.084)</td>
<td>(0.122)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Geographical certification</td>
<td>0.838</td>
<td>-0.878</td>
<td>-1.166</td>
<td>0.293</td>
<td>0.115</td>
<td>5.038</td>
<td>5.937</td>
</tr>
<tr>
<td></td>
<td>(0.023)</td>
<td>(0.056)</td>
<td>(0.093)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Organic farming</td>
<td>0.606</td>
<td>-0.199</td>
<td>-1.120</td>
<td>0.450</td>
<td>0.210</td>
<td>4.181</td>
<td>5.051</td>
</tr>
<tr>
<td></td>
<td>(0.036)</td>
<td>(0.080)</td>
<td>(0.114)</td>
<td></td>
<td></td>
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<td></td>
</tr>
</tbody>
</table>

and high (H) level of knowledge:

$$w_{1i} = \begin{cases} 
1, & \text{if the self-assessed score is } \leq 5; \\
0, & \text{otherwise}; 
\end{cases}$$

In particular, there are 556 subjects out of 1000 respondents that qualify themselves as more attentive to product information.

In Table 1 the estimated parameters of the univariate CUB models for each attribute are reported (asymptotic standard errors are shown in parentheses). The models include the self-assessed measure of product awareness as a covariate for determining the $\xi$ parameter. Figure 1 shows the estimated CUB distributions for each attribute for the two clusters.

In general, respondents show very low uncertainty when judging colour, flavour and geographical certification. These are cues that can be easily recognized even by inexpert consumers. Actually, colour is not a reliable sign of quality since it may depend on micro-components which are irrelevant for the flavour and taste. But as other studies prove (see for instance Wang et al., 2013), consumers usually pay a remarkable attention to this characteristic. This behaviour also is widespread among Italian consumers.

Moreover, the feeling that respondents have towards the EVO oil is affected by the degree of awareness of the related attributes. As a matter of facts, the covariate is very significant in all the models. Figure 1 clearly show that, depending on their level of product knowledge, consumers express different ratings. Note that despite the CUB random variable is discrete, the
estimated probability distribution is represented by means of a solid line for facilitating reading. The rating distribution locates towards lower scores when the consumer is inattentive to the product features whereas the distribution moves gradually towards medium or high evaluations as far as the level of knowledge increases. Accordingly, the average score increases between 0.7 and 0.9.

These findings are consistent with well-established consumer behaviour theories (see, for instance, Marks and Olson, 1981; Caporale et al., 2006; Espejel et al., 2008) suggesting that consumers with various levels of product knowledge differ in the perception of attributes and, in addition, they tend to have better-formulated decision criteria.

The feeling is generally higher for attentive consumers with respect to the others though the difference between the two groups is larger when the interviewees are requested to judge the oil texture. Moreover, consumers seem to be more uncertain when judging the importance of the organic agricultural practices and taste for their purchase decisions with respect to all the other items. This fact could be justified by considering that bitterness and pungency are not universally recognized as a sign of quality since inexpert consumers tend to dislike them. In addition, despite the Italian organic food market is in a growing phase, the size of the certified organic EVO oil market
Table 2: Bivariate Plackett distribution

<table>
<thead>
<tr>
<th>Variables</th>
<th>$\hat{\psi}_L$</th>
<th>$\hat{\psi}_H$</th>
<th>$Diss_L$</th>
<th>$Diss_H$</th>
<th>$P_L$</th>
<th>$P_H$</th>
</tr>
</thead>
<tbody>
<tr>
<td>(Color, Flavour)</td>
<td>4.663</td>
<td>6.128</td>
<td>0.119</td>
<td>0.186</td>
<td>0.183</td>
<td>0.420</td>
</tr>
<tr>
<td></td>
<td>(0.690)</td>
<td>(0.838)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(Color, Taste)</td>
<td>3.440</td>
<td>3.594</td>
<td>0.140</td>
<td>0.217</td>
<td>0.109</td>
<td>0.263</td>
</tr>
<tr>
<td></td>
<td>(0.534)</td>
<td>(0.521)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(Color, Geog.certif.)</td>
<td>7.024</td>
<td>4.657</td>
<td>0.155</td>
<td>0.205</td>
<td>0.206</td>
<td>0.439</td>
</tr>
<tr>
<td></td>
<td>(1.131)</td>
<td>(0.728)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(Color, Organic farming)</td>
<td>5.075</td>
<td>3.693</td>
<td>0.150</td>
<td>0.215</td>
<td>0.119</td>
<td>0.322</td>
</tr>
<tr>
<td></td>
<td>(0.841)</td>
<td>(0.526)</td>
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<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(Flavour, Taste)</td>
<td>6.311</td>
<td>4.791</td>
<td>0.190</td>
<td>0.254</td>
<td>0.161</td>
<td>0.329</td>
</tr>
<tr>
<td></td>
<td>(0.819)</td>
<td>(0.659)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(Flavour, Geog.certif.)</td>
<td>2.180</td>
<td>2.613</td>
<td>0.139</td>
<td>0.175</td>
<td>0.205</td>
<td>0.549</td>
</tr>
<tr>
<td></td>
<td>(0.354)</td>
<td>(0.351)</td>
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<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(Flavour, Organic farming)</td>
<td>2.634</td>
<td>2.351</td>
<td>0.149</td>
<td>0.190</td>
<td>0.121</td>
<td>0.375</td>
</tr>
<tr>
<td></td>
<td>(0.379)</td>
<td>(0.334)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(Taste, Geog.certif.)</td>
<td>1.807</td>
<td>1.744</td>
<td>0.166</td>
<td>0.200</td>
<td>0.116</td>
<td>0.317</td>
</tr>
<tr>
<td></td>
<td>(0.295)</td>
<td>(0.239)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(Taste, Organic Farming)</td>
<td>2.217</td>
<td>2.608</td>
<td>0.157</td>
<td>0.196</td>
<td>0.078</td>
<td>0.246</td>
</tr>
<tr>
<td></td>
<td>(0.334)</td>
<td>(0.393)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(Geog. certif., Organic farming)</td>
<td>6.678</td>
<td>4.431</td>
<td>0.158</td>
<td>0.201</td>
<td>0.159</td>
<td>0.438</td>
</tr>
<tr>
<td></td>
<td>(1.026)</td>
<td>(0.656)</td>
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</tbody>
</table>

is still very limited.

Finally, the geographical certification (PDO/PGI) is surely recognized as an indication of sensory quality and the importance is more appreciated by consumers with high level of product awareness.

The judgements about the considered items are positively correlated. Table 2 shows the estimated value of the association parameter ($\psi$) of the related joint probability distributions having introduced the level of knowledge as an explanatory variable. The standard errors (in parentheses) are estimated by jackknife method (Xu, 1996). Furthermore, we report the probability that the rater assigns a score larger than 5 to both items ($P_L$ and $P_H$) and the normalized dissimilarity index:
\[ Diss = 0.5 \sum_{x=1}^{k} \sum_{y=1}^{k} \left| p(x, y|\hat{\psi}) - \frac{n_{xy}}{n} \right| \]  

in order to measure the distance between the fitted probability and the observed distribution \( (n_{xy}/n) \).

We comment first on \textit{colour} and \textit{flavour} and then on the remaining items. It is evident that purchasers with a low level of knowledge about the product characteristics tend to give to \textit{colour} and \textit{flavour} a lower importance than that expressed by the other group and, in addition, their judgements on the two attributes produce a lower association coefficient. Figure 2 shows the joint bivariate distributions estimated for the two clusters of respondents.

\( P(x,y|w=0) \quad P(x,y|w=1) \)

Figure 2: Joint probability distribution of EVO oil colour and flavour (Low-medium product knowledge=left panel, High product knowledge=right panel)

In this respect, it worth noting that, for instance, the probability that inattentive consumers assign a rate higher than 5 to both features is only 0.183 whereas for more conscious consumers that probability increased to 0.420. This type of consideration could be useful in order to implement marketing actions (improving label information, food advertising, etc.) aimed at building a brand image for EVO oil. The goodness of fit is generally good as shown by the low value of the dissimilarity index.
It is useful to recall that the strength of the association parameter in the joint distribution measures the fact that respondents tend to give similar judgements on the two items but, of course, it does not imply that the larger probability mass is concentrated on the higher rates.

As a matter of facts, most bivariate distributions show a decrease of the parameter $\psi$ when the level of product knowledge increases. This could be just the effect of more thoughtful judgements since more conscious raters tend not to replicate the same rating for both items. The probability that attentive consumer give a rate higher than 5 to both items is generally larger than that concerning respondents with a low level of product awareness. When the former group of consumers jointly assesses territorial certification with colour or flavour or organic farming, the above mentioned probability is 0.4 or more. The latter group which gathers inattentive consumers, instead, very rarely assign high and positive evaluation to each couple of attributes, the probability of that event is generally about or less than 0.2. This of course suggest the implementation of better policies for improving the level of knowledge of consumers about the features of the extravirgin olive oil.

**Final remarks**

The results described in the previous section encourage further studies on the proposed model for correlated ordinal data. On the one hand, CUB models represent an effective statistical tool which helps to identify the role of two latent components: the uncertainty of respondents in rating product attributes and the strength of attraction each attribute arouses. On the other hand, the joint modelling of ratings allows the study of the bonds that connect consumer preferences about alternative products providing further insights into consumer behaviour.

Further studies are needed in order to implement the approach to the $k$-variate case. The Plackett distribution has in fact been generalised by Molenberghs (1992). However, the definition of such a distribution becomes computationally cumbersome for high-dimensional applications and because of the discrete nature of the involved random variables. The investigation of d-vine approach for estimating multivariate copula seems to be an interesting future line of research.
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References


