Factor Demand in the Sawmill Industry of the Lake States

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Abstract

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This study examined the production structure of the sawmilling industry of the Lake States (Michigan, Minnesota and Wisconsin) in order to determine elasticities of substitution and elasticities of demand. A homogeneous translog cost function was estimated using pooled time-series data for the period 1963-1996 with inputs labor, materials and capital.

Based on the model selection process, the estimated model imposes constant returns to scale and allows for nonunitary elasticities of substitution amongst the inputs. The hypothesis of constant returns to scale could not be rejected at the 1% level. This was common for studies of the sawmill industry but seems particularly common to regions where the industry was made up primarily of small mills. Constant returns to scale in a mature sawmill industry would lead to the outcome of mills of similar size as all economies of scale have been exhausted and the industry has settled into an equilibrium firm size near the minimum of the long run average cost curve. Nevertheless, this does not explain why the average mill size in the Lake States is small compared to the Pacific Northwest and Southeastern U.S.

Results for the Allen Partial Elasticity of Substitution (AES) indicate that labor and materials were inelastic substitutes with an elasticity of 0.76 while labor and capital were elastic complements with an elasticity of -1.38. Materials and capital were also inelastic substitutes and had an elasticity of substitution of 0.80. The labor/materials
elasticity of substitution is high compared to almost all studies of softwood lumber producing regions.

The Morishima Elasticity of Substitution (MES) results indicate that labor/materials, materials/labor, materials/capital, capital/materials and labor/capital were inelastic substitutes with the greatest substitutability between labor/materials. The MES for capital/labor was -0.017 indicating a complementary relationship.

The own-price elasticities of demand were all inelastic and negative indicating downward sloping demand curves. All other elasticities were inelastic and indicate that materials was a substitute for labor and capital but labor and capital were complements. Changes in the price of materials had a relatively large, but inelastic, effect on the demand for capital with a cross-price elasticity of 0.51. Changes in the price of labor also had a relatively large effect on the demand for capital, but in a complementary fashion, with an elasticity of -0.44. Changes in the price of materials had a greater effect on the demand for labor than the other way around with cross-price elasticities of 0.48 and 0.24, respectively.
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Introduction

This is a study of the production structure of the sawmilling industry of the Lake States (Michigan, Minnesota and Wisconsin). A translog cost function was estimated to determine own and cross-price elasticities of demand, Allen-Uzawa partial elasticities of substitution and Morishima elasticities of substitution for the inputs to the sawmill industry. State level data for Standard Industrial Classification (SIC) 242 (sawmills and planning mills) were collected for each state for the years 1963-1996.

The sawmill industry in the Lake States is characterized by many small mills producing hardwood lumber. Seventy five percent of the lumber produced by volume in the region is hardwood lumber compared to an average of twenty four percent nationwide (Census Bureau, 1997). Softwood lumber is produced primarily in the northern part of the region. The industry in the Lake States also differs from that in other regions of the US, particularly the Northwest and Southeast, in that more than fifty percent of the timber harvested comes from nonindustrial private forestland (NIPF).

There have been no production studies of the sawmill industry of the Lake States but there is a wealth of research on the industry in other lumber producing regions including Baardsen 2000 (Norway), Bigsby 1994 (Australia), Puttock and Prescott 1992 (Southern Ontario), Meil and Nautiyal 1988 (various regions of Canada), and Abt 1987 (three regions of the U.S.). Only the Puttock and Prescott, and Abt studies include regions where hardwoods predominate.

One concern in the Lake States is that timber is becoming scarcer and this could lead to increasing prices. Changes in attitudes of private landowners in the future may
restrict timber supply from NIPF. Fifty-five percent of the forestland in the study region is NIPF and there is a trend toward preservation of forest cover on these lands by new landowners which could affect timber supply. Overall, forest managers in the Lake States foresee an 8 percent decrease in area of land available for harvest by 2020 (Vasievich, et al. 1997).

Measurement of price elasticities and elasticities of substitution can be useful to policy makers by providing information regarding the ability of the industry to react to input price changes. The more substitutable inputs are for each other the smaller the effect of price changes.

**Theoretical and Conceptual Model**

The primary behavioral assumption in this study is that sawmill managers will minimize costs with respect to a given output level and factor prices. This is a reasonable assumption that is more behaviorally restrictive than the assumption that they are profit maximizers. If we assume that sawmill managers are profit maximizers then they are able to adjust the quantity of inputs and output to maximize profit. If we assume that they are cost minimizers, then they are only able to adjust the quantity of inputs to minimize costs given a fixed output level.

By minimizing costs, the producers are being both technically and allocatively efficient given an output level, and input prices. When relative input prices change, the rational producer will use different amounts of each input in order to produce the given output level at the minimum cost. In this case, we are allowing producers to vary the quantities of labour, materials (sawlogs) and capital used in the production process. The
interest here is in the degree to which one input will substitute for another when relative prices change, or in other words, the elasticities of substitution.

Another assumption of the model is that producers are price takers for inputs and outputs. In other words, the input and output markets are perfectly competitive. There can still be variability in input and output prices though as a result of an institutional factor such as unionized labour forces in some areas or heterogeneity of the wood input based on species composition, age and size and also transportation costs. The point is that those factors are largely beyond the control of the mills and therefore they remain price takers (Banskota et al. 1985).

Estimating factor demand using a cost function makes use of the dual relationship between the production and cost functions. Rather than estimate a production function, this model estimates the dual cost function to obtain results regarding price elasticities and elasticities of substitution.

The implicit production function is:

\[ Q = F(L,M,K,t) \]  \hspace{1cm} (1)

Where:
- \( Q \) = lumber output
- \( L \) = labor
- \( M \) = materials (sawlogs)
- \( K \) = capital
- \( t \) = time trend

Labor, materials and capital are variable inputs and the rational producer adjusts the use of those inputs based on their relative prices to minimize costs given an output level (Q).

The dual cost function for the above production function is:

\[ VC = F(P_L,P_M, P_K ,Q,t) \]  \hspace{1cm} (2)
Where:

\[ VC = \text{variable cost} = \sum P_i X_i \]

\( X_i = \) cost-minimizing variable factor quantity
\( P_i = \) variable factor price
\( Q = \) lumber output
\( t = \) time trend

**Empirical Model**

The above cost function was estimated as an unrestricted translog variable cost function. The translog functional form was chosen for its flexibility with regards to returns to scale, elasticities of substitution and the ability to incorporate biased technical change. Each of these properties can be tested for validity by using the likelihood ratio test between the unrestricted model and a model restricted to constant returns to scale, unitary elasticities of substitution, and unbiased technological change in turn.

The unrestricted model is shown below.

\[
\ln VC = \beta_c + \beta_L \ln LP + \beta_M \ln MP + \beta_K \ln KP + \beta_t t + \beta_Q \ln Q \\
+ \frac{1}{2} [\beta_{LL} \ln LP^2 + \beta_{MM} \ln MP^2 + \beta_{KK} \ln KP^2 + \beta_{tt} t^2 + \beta_{QQ} \ln Q^2 ] + \beta_{LT} \ln LP^* t + \beta_{LQ} \ln LP \ln Q + \beta_{MT} \ln MP^* t + \beta_{MQ} \ln MP \ln Q + \beta_{KT} \ln KP^* t + \beta_{KQ} \ln K \ln Q + \beta_{QV} \ln Q + \beta_{L} \ln LP \ln MP + \beta_{LK} \ln LP \ln KP + \beta_{MK} \ln MP \ln KP + \mu \\
\]

\( S_L = \beta_L + \beta_{LL} \ln LP + \beta_{LM} \ln MP + \beta_{LK} \ln KP + \beta_{Lt} t + \beta_{LQ} \ln Q \)  
\( S_K = \beta_K + \beta_{KK} \ln LP + \beta_{LK} \ln LP + \beta_{MK} \ln MP + \beta_{Kt} t + \beta_{KQ} \ln Q \)

Where:
\( VC = \) variable cost defined as the sum of labor, materials and capital costs.
\( LP = \) labor price ($/hour)
\( MP = \) materials price (sawlog price) ($/MBF)
\( KP = \) capital price defined as the ratio between gross quasirent and capital stock
\( t = \) time trend (year)
\( Q = \) lumber output (millions of board feet)
\( S_L = \) labor cost share (labor cost divided by \( VC \))
\( S_K = \) capital cost share (capital cost divided by \( VC \))
The cost function can be estimated on its own but estimating it as a system along with two of the three share equations improves the efficiency of the estimates. The cost shares sum to one so only two are linearly independent so one must be dropped. Theoretically it doesn’t matter which one is dropped but computationally it may matter. Capital has a small share in costs so the sum of labor cost and materials cost is near one. For this reason, the materials cost share equation was dropped and only the labor and capital cost shares were used.

In order to derive the input demand equation for input $i$, Shephard’s Lemma was used (Shephard 1953):

$$x_i^* = \frac{\partial VC_i(Q, p)}{\partial x_i}, \quad (5)$$

Where $x_i^*$ is the optimal quantity of input $i$.

For the translog function used here, the equation is:

$$\frac{\partial \ln VC}{\partial \ln p_i} = \frac{\partial VC}{\partial p_i} \frac{p_i}{VC} = \frac{p_i x_i}{VC} = S_i \quad (6)$$

Where $p_i$ is the price of input $i$.

This leads to the share equations used to estimate the model above. The derivation of the materials share equation is analogous to the others:

$$S_M = \beta_M + \beta_{MM} \ln MP + \beta_{LM} \ln LP + \beta_{MK} \ln KP + \beta_{Mt} + \beta_{Mt} \ln Q \quad (7)$$

**Elasticities**

Several types of elasticities were calculated in order to describe the sawmill industry of the Lake States. Elasticities of substitution between input pairs were
calculated to determine the extent to which inputs are technically substitutable for each other. There are two common forms of these elasticities: Allen-Uzawa Partial Elasticity of Substitution (AES) and Morishima Elasticity of Substitution (MES). Traditionally, the AES (Allen and Hicks 1934; Uzawa 1962) has been used, but following Blackorby and Russell (1989), there has been increasing use of the MES. In addition to the elasticities of substitution, own and cross-price elasticities were calculated.

Explanation of the AES and MES elasticities of substitution and their calculation is given below along with methods for calculating the own and cross-price elasticities.

**Allen-Uzawa versus Morishima Elasticity of Substitution**

The AES is calculated from the cost function as:

$$A_{ij}(Q, p) = \frac{VC(Q, p)VC_{ij}(Q, p)}{VC_i(Q, p)VC_j(Q, p)}, \quad (8)$$

where subscripts represent partial derivatives with respect to inputs $i$ and $j$, $Q$ is output quantity and $p$ is the vector of input prices.

Then from this we can write:

$$A_{ij}(Q, p) = \frac{\xi_{ij}(Q, p)}{S_j(Q, p)} \quad (9)$$

Where $\xi_{ij}(Q, p)$ is the constant-output cross-price elasticity of demand and

$$S_j(Q, p) = p_jVC_j(Q, p) / VC(Q, p)$$

is the cost share of input $j$ in total cost (Blackorby and Russell 1989).

The criticisms of the AES are threefold:

1) It does not measure the curvature of the isoquant

2) Provides no information about relative factor shares
3) Cannot be interpreted as the derivative of a quantity ratio with respect to a price ratio

Blackorby and Russell argue that the AES provides no information that is not provided by the constant-output cross-price elasticity. The AES has been used to classify net substitutes and complements but this application can be accomplished by the constant-output cross-price elasticity. The constant-output cross-price elasticity is both unit free and has a clear economic meaning. The AES is merely the constant-output cross-price divided by the cost share of input $j$ and they argue that this is meaningless.

Blackorby and Russell propose an alternative elasticity of substitution originally derived by Morishima (1967). The MES is given by:

$$M_{ij}(Q, p) = \frac{p_i VC_{ij}(Q, p)}{VC_j(Q, p)} - \frac{p_j VC_{ji}(Q, p)}{VC_i(Q, p)} = \xi_{ji}(Q, p) - \xi_{ij}(Q, p) \quad (10)$$

One desirable property of the MES is that it allows for asymmetrical elasticities of substitution (i.e. $M_{ij} \neq M_{ji}$) in cases with more than two inputs. The AES imposes symmetry on the elasticity of substitution. This is counterintuitive. Variation of $p_i$ in the ratio $p_i/p_j$ will have two corresponding effects on the ratio $x_i^*/x_j^*$: the change in $x_j^*$ given by $\xi_{ji}(Q, p)$ and the change in $x_i^*$ given by $\xi_{ij}(Q, p)$. However, the effect of a change in the price ratio $p_i/p_j$ by holding $p_i$ constant and varying $p_j$ is given by

$$M_{ji}(Q, p) = \xi_{ji}(Q, p) - \xi_{ij}(Q, p),$$

thus, there is no requirement for $M_{ij} = M_{ji}$ (Blackorby and Russell 1989).

In recent production studies of the forest industry (Smith and Munn (1998), Baardsen (2000)) these criticisms of the AES were broached. In the case of Smith and Munn (1998) only the MES were presented and in Baardsen (2000) both the AES and
MES were presented. In this study, both types are presented in order to allow comparison with other studies.

**Elasticity Calculations**

Following Nautiyal and Singh (1985), the own-price and cross-price elasticities are calculated based on the following formulae:

\[
\xi_{ii} = A_{ii} \times S_i \\
\xi_{ij} = A_{ij} \times S_j
\]  

(11)

Where \( S_i \) is the cost share for input \( i \) and \( A_{ij} \) and \( A_{ji} \) (the AES) are given by:

\[
A_{ii} = \frac{\beta_{ii} + S_i^2 - S_j}{S_i^2} \\
A_{ij} = \frac{\beta_{ij} + S_i S_j}{S_i S_j}
\]  

(12)

Notice that the AES will vary with the relative factor shares \( S_i \) and \( S_j \), so its value will depend on whether it is calculated at mean factor share levels or with individual yearly observations (Nautiyal and Singh 1985).

From Blackorby and Russell (1989) the MES is calculated as such:

\[
M_{ij} = \xi_{ji} - \xi_{ii} \\
M_{ji} = \xi_{ij} - \xi_{jj}
\]  

(13)

Once the estimation is complete the estimated parameters are used to calculate own-price and cross-price elasticities using the above formulae.

**Data**

The data required for the model include: quantity of labor and logs used in the milling process, the prices of those inputs, new capital expenditures, the value of the
capital stock of the milling industry, the price of capital, user cost of capital and the volume of output (lumber). Data were collected for three states (Michigan, Minnesota and Wisconsin) for the period 1963-1996. All financial data was discounted to 1996 using the PPI for all commodities. There are a total of 66 observations. For Michigan, there are 27 observations, 17 for Minnesota and 22 for Wisconsin. Each state had a different number of observations because for some years, data for SIC 242 were not published for some states. It was felt that rather than try to generate the missing data from reported data for SIC 24 for each state or national SIC 242 data it was better to just omit those years from the dataset.

The time period of the study was chosen partly due to data constraints. Prior to 1963, data from the Annual Survey of Manufacturers were not as readily available for SIC 242 at the state level. The time series ends at 1996 because at that time the Census Bureau changed from the SIC system to the North American Industrial Classification System (NAICS) and there is poor correspondence between SIC 242 and the new classifications.

There is also the issue of the level of aggregation of the data and the assumption of exogenous input prices. At the state level of aggregation it could be possible that the industry itself determines prices of at least some inputs (particularly wood). Baardsen (2000) claims that aggregating data beyond the mill level was inappropriate given the assumption of exogenous input prices. He quotes Varian (1984) as saying this is “unrealistic” (Varian 1984, p. 179). What Varian actually says is it “seems unrealistic” (ibid). This is a slightly more mild criticism and may not be completely applicable to the Lake States. In the case of the Lake States this may not be the case because there are no
restrictions for the transport of wood to neighboring states and so timber sellers can simply sell to buyers across the state line if they feel that they can get a better price. This does not apply to all areas of all the states because transportation costs and information costs may become limiting factors for this movement of wood but there is export of timber to Indiana and Illinois which are not part of this study. In addition, Stone (1997), states that “changes in the supply of timber in other regions of the country (and world) influences Lake States timber markets”. This is another indication that the timber market in the Lake States is not a closed system and that the industry in the region has limited influence on the prices it pays for inputs. Likewise, for labor, in most parts of the Lake States there are more employment opportunities available outside the sawmill sector or the wood products industry in general than there might be in the Interior of British Columbia, so the labor price can be taken to be exogenous to the sawmill sector. With 11,000 employees total in the Lake States in 1996 it seems unlikely that the industry as a whole can influence wage rates to any great degree. Certainly capital prices are not determined within a market endogenous to the Lake States sawmill industry. Studies aggregated at the national level or studies of regions such as the Pacific Northwest or British Columbia may be more subject to this criticism.

**Labor Quantity**

Labor quantity is defined as man-hours of production labor in SIC 242 (sawmills and planing mills) for each year. The data source was the Annual Survey of Manufactures (ASM) and the Census of Manufactures (CoM) from the Census Bureau.
**Labor Price**

Labor price, or wages, is calculated easily from the Census Bureau data for SIC 242. Labor price was calculated as dollars per hour for production workers. It was simply the payroll expense divided by the number of hours worked.

**Sawlog Price**

Sawlog price was calculated as the quotient of the cost of materials and volume of sawlogs entering mills in each state for years when the Forest Service Timber Product Output (TPO) data were available (USDA Forest Service North Central Forest Experiment Station Resource Bulletins, various years). The Census Bureau collects data on the gross cost of material inputs to the industry. This cost was almost entirely made up of sawlog costs. In order to calculate a price per thousand board feet (MBF), the total material cost was divided by the volume of sawlogs consumed in each year.

Sawlog prices for years when Forest Service TPO data for sawlog receipts were not available (see Table A-5) were calculated based on price data from Timber Mart North, state Departments of Natural Resources stumpage data, Forest Service stumpage data, Minnesota Forest Products Price Report, Wisconsin County stumpage data, the Wisconsin Forest Products Price Review and U.S. Timber Production, Trade, Consumption, and Price Statistics, 1950-85 (Ulrich 1987). A weighted average price was calculated based on proportions of each species and grade harvested in each state.

**Capital Stock**

The capital stock series was created using the perpetual inventory method developed by Christensen and Jorgenson (1969).
With the perpetual inventory method, capital stock in the current period \( (K_t) \) is a function of the investment in the current period \( (I_t) \), capital stock in the previous period \( (K_{t-1}) \), and the depreciation rate \( (\mu) \):

\[
K_t = I_t + (1-\mu)K_{t-1}
\]  \hspace{1cm} (14)

Investment was taken to be new capital expenditures (NCE) for structures and equipment. Total NCE were available from both the CoM and the ASM for SIC 242 at the state level. Nevertheless, not all years were available for all states. In years for which NCE data were not available, data were generated by calculating the proportion of each state’s NCE for SIC 242 to national NCE for SIC 242. The proportion for the years immediately preceding and following the missing data were then averaged and multiplied by the national NCE to arrive at NCE at the state level. NCE were further broken down into NCE for buildings and structures, and NCE for machinery and equipment due to different service lives for each type of capital. The breakdown into the two categories of capital was accomplished by using the national SIC 242 data on NCE for each category. Over the timeframe of the study, 17.5% of NCE were for buildings and structures and 82.5% for machinery and equipment. These proportions were fairly constant during this period.

There are three types of depreciation methods that could be used to depreciate the capital stock series: straight-line, geometric (of which double-declining balance depreciation is one type) and hyperbolic. Computationally, all require an estimate of the service life of the capital and in the case of geometric and hyperbolic depreciation, a parameter that controls the rate of depreciation.
As the name suggests, straight-line depreciation is linear so the depreciation rate is constant and equal to $1/L$ where $L$ is the service life of the capital. This assumes the productive capacity of the capital decreases equally each year of its life.

Geometric depreciation is a form of accelerated depreciation which assumes that the productive capacity of the capital decreases more rapidly in the early stages than it does near the end of its service life. With geometric depreciation, the declining-balance rate is the same every year: $\delta_i = R/L$

where $R$ is rate relative to the straight-line rate of $1/L$. Therefore, the double-declining-balance rate is $2/L$. As $R$ increases, the depreciation rate increases and depreciation in the early stages of the life of the capital is increased. Geometric depreciation is measured as: $d_{x,G} = \delta G (1 - \delta G)^{x-1}$

Where $x=1,2,3,\ldots,L$

Hyperbolic depreciation is the opposite of accelerated depreciation. With hyperbolic depreciation, the capital retains more of its productive capacity in the early stages and as the capital ages, the depreciation accelerates. That is, the depreciation is delayed (as opposed to accelerated) in the early stage of its life. The hyperbolic depreciation rate is calculated as:

$$d_{x,D} = \frac{L - (x - 1)}{L - \beta(x - 1)} - \frac{L - x}{L - \beta x}$$

(15)

Where $x=1,2,3,\ldots,L$ and $\beta$ is the curvature parameter.

With a $\beta$ of zero, hyperbolic depreciation reduces to straight-line depreciation. If $\beta$ is 1, there will be no depreciation throughout the course of the service life of the capital with the exception of the final year when all depreciation will occur.
Intuitively, hyperbolic depreciation most closely matches the actual degradation of productive services from buildings and machinery in the sawmill industry. As capital ages, repairs become more frequent and more extensive. Abt (1987) employed hyperbolic depreciation to create the capital stock series for this reason.

The Bureau of Labor Statistics has established procedures for estimating net capital stock and depreciation profiles using hyperbolic depreciation (BLS 1997). The curvature parameter for depreciation of structures is assumed to be 0.75 and for equipment it is 0.5.

The service life of buildings and machinery for SIC 242 was also not easy to estimate but BLS procedures assume buildings and structures in SIC 242 have a service life of 28 years and machinery and equipment have a service life of 13 years.

In order to employ the perpetual inventory method of capital stock estimation, a benchmark, or starting value of the capital stock is required. Once a starting value of $K$ is established, NCE and the depreciation rate are used to calculate the capital stock in each subsequent period. The path of the capital stock series is somewhat sensitive to the benchmark capital stock level at the beginning of the series, but any errors in the estimation of the benchmark are quickly dissipated by the NCE data.

In this case, gross book value (GBV) data were not available for SIC 242 for the Lake States. Capital stock data for SIC 242 in Kentucky and West Virginia are available and the starting values for each of the Lake States are based on the capital stock levels in those states. The majority of lumber produced in the Lakes States is hardwood and the same is true of Kentucky and West Virginia. Also, the production levels are relatively
close as well. The input factor mix is assumed to be the same for the Lake States and Kentucky and West Virginia for the purposes of calculating the initial capital stock.

The starting value for the capital stock series for each state was calculated by determining the proportion of each of the Lake States’ lumber production to the lumber production in Kentucky and also in West Virginia in 1963. After the proportions were calculated, they were multiplied by the 1963 capital stock data for Kentucky and West Virginia. The resultant figures were then averaged to arrive at an estimate of the capital stock in 1963 for each of the Lake States. These figures were then used to begin the estimation of capital stock in subsequent years using NCE data in the perpetual inventory method. Once capital stock for each category of capital was calculated, the figures were summed for each year and state to arrive at total capital stock.

After capital stock was calculated for each year and state, the price of capital was calculated. Following Stier (1985) capital price was calculated as the ratio between gross quasi-rent and capital stock. Gross quasi-rent was defined as value of shipments minus total employee compensation minus materials costs. Capital price is a ratio of two dollar values and as such has no units. It is the ratio of revenues (value of shipments), net of labor and materials cost, to capital stock. Value of shipments and employee compensation data came from the Census of Manufactures and the Annual Survey of Manufactures. Materials cost data were constructed as described above. Capital price does not need to be calculated for every year for every state because the final data set had some years missing for each state due to missing Census Bureau data. For Michigan, there were no Census Bureau data for SIC 242 for the years 1968, 1979-1981 and 1984-1986. Minnesota was missing Census Bureau data for SIC 242 for the years 1966, 1968-1971,

Gross-quasi rent (returns to capital) may be negative in some years and so it was necessary to smooth the capital price series by fitting a line using OLS estimation. The data for capital price presented in Appendix A is the smoothed capital price trend. It was necessary to calculate NCE for every year in order to employ the perpetual inventory method of estimating capital stock.

The user cost of capital was also calculated in order to create the cost variable for the left hand side of the cost function and to determine cost shares for each of the three inputs. Bigsby (1994) calculates the user cost of capital services in the Australian sawmilling industry as:

\[ UC = (r + g_n - \frac{dP_{nt}}{P_{nt}})P_{nt}K_{nt} \]  

(16)

Where,

- \( UC \) = user cost of capital,
- \( r \) = the interest rate on 10-year government bonds
- \( g_n \) = the declining balance depreciation rate for capital \( n \)
- \( dP_{nt}/P_{nt} \) = the annual rate of change of capital price for capital \( n \)
- \( P_{nt} \) = the price of capital \( n \) at time \( t \) and,
- \( K_{nt} \) = the value of the capital stock of capital \( n \) at time \( t \).

For this study, Bigsby’s formula needed to be modified to incorporate the hyperbolic depreciation scheme for depreciating the two kinds of capital used to construct the capital stock variable.

The final values were then converted to 1996 dollars using the PPI for all commodities.
Lumber Output

Output volumes came from the Census Bureau publication MA24T: Lumber production and mill stocks. Volumes are in MMBF International 1/4” log rule. A point to note regarding the lumber output statistics is that in 1994 the Census Bureau undertook a reconciliation among the MA24T, the 1992 Census of Manufactures, and state sawmill directories (Census Bureau 1994). The results of this reconciliation were large upward revisions of the output statistics from 1993. This may have the effect of underestimating productivity gains earlier in the study period relative to later in the study period.

Variable Cost and Input Cost Shares

The left-hand side variables of the translog cost function and share equations were derived from the cost data for the inputs labor, materials and capital. Variable cost was the sum of labor, material and capital costs which were in turn the product of the price and quantity of each input. The input cost shares were the proportion of variable cost made up by each of the three inputs.

Model Estimation and Results

There are several restrictions imposed on the model in order for it to conform to the theoretical requirements of a cost function. A cost function must be homogeneous of degree one in input prices. This means that if all input prices double, cost will double, holding output constant. In order for that condition to hold, the following parameter restrictions are imposed on the translog cost function (Nautiyal and Singh 1985):
\[ \sum_{i} \beta_i = 1 \]
\[ \sum_{i} \beta_{ij} = 0 \]
and
\[ \sum_{i} \beta_{ij} = \sum_{j} \beta_{ji} = \sum_{i} \beta_{ii} = 0 \]

Where \( i \) and \( j \) represent the inputs labor, materials and capital, \( Q \) is production and \( t \) is the time trend.

These restrictions are necessary for theoretical reasons but they provide the benefit of reducing the number of parameters to estimate. The unrestricted model has twenty-one parameters and the imposition of the above restrictions reduces the number of parameters to estimate to fifteen.

The original model that was estimated is shown below with the parameter restrictions imposed.

\[
\ln VC = \beta_C + (1 - \beta_M - \beta_K) \ln LP + \beta_M \ln MP + \beta_K \ln KP + \beta_t t + \beta_Q \ln Q \\
+ \frac{1}{2} \left[ (-\beta_{LM} - \beta_{LK}) \ln L P^2 + (-\beta_{LM} - \beta_{MK}) \ln M P^2 + (\beta_{LK} - \beta_{MK}) \ln K P^2 + \beta_M t^2 + \beta_Q Q^2 \right] + \\
(\beta_{LQ} - \beta_{KQ}) \ln L P t + (\beta_{MQ} - \beta_{KQ}) \ln M P Q + \beta_{LM} \ln L P + \beta_{MK} \ln M P + \beta_{LQ} \ln L P Q + \beta_{M} \ln M P Q + \beta_{K} \ln L P Q + \\
\beta_{LQ} \ln K P Q + \beta_{K} \ln L P Q + \beta_{LM} \ln L P + \beta_{MK} \ln M P + \beta_{LQ} \ln M P Q + \beta_{M} \ln M P Q + \beta_{K} \ln M P Q
\]

\[
S_L = (1 - \beta_M - \beta_K) \ln LP + \beta_M \ln MP + \beta_L K \ln KP + \\
(\beta_{LM} - \beta_{LQ}) t + (\beta_{MQ} - \beta_{KQ}) \ln Q
\]

\[
S_K = \beta_K + (\beta_{LK} - \beta_{MK}) \ln KP + \beta_K t + \beta_{LQ} \ln Q + \beta_{LK} \ln LP + \beta_{MK} \ln MP
\]

(18)

**Model Selection Procedure**

The model selection procedure is a stepwise testing of model assumptions in order to select the most appropriate model for the data set. The translog cost function used to model the production structure of the sawmill industry of the Lake States is flexible in that it allows for nonconstant returns to scale and nonunitary elasticities of substitution.
among the inputs. The modeling procedure starts with the unrestricted model and then adds successively more restrictive assumptions. Model (1) is the model that includes restrictions on the parameters that impose homogeneity of degree one in input prices. Considering that a well-behaved cost function must behave in that way, we take this model to be the unrestricted model in relation to the more restrictive assumptions of the other models. Model (2) imposes homogeneity of degree one in output. In other words, it imposes constant returns to scale. Model (3) imposes unitary elasticity of substitution among the inputs but does not impose constant returns to scale.

To restrict the model to constant returns to scale, (Model 2, below) the following parameter restrictions are imposed:

\[ \beta_{iQ} = 0 \text{ and } \beta_{QQ} = 0 \]  \hspace{1cm} (19)

Where \( i \) equals labor, materials and capital.

In order to impose unitary elasticity of substitution restrictions, (Model 3, below) the following parameter restrictions are imposed (Nautiyal and Singh 1985):

\[ \beta_{ij} = 0 \]  \hspace{1cm} (20)

Where \( i \) and \( j \) equal labor, materials and capital.

The parameters of the translog cost function and the labor and capital share equations were estimated as a system of equations using Full Information Maximum Likelihood (FIML) estimation with EViews 4.1© software. Table-1 shows the values of the estimated parameters for each model as well as the parameters that were eliminated based on the restrictions of equation (17).
# Table-1. Estimates of the Cost Function Parameters (1963-1996)

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Model 1</th>
<th>Model 2</th>
<th>Model 3</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\beta_C$</td>
<td>-12.83**</td>
<td>-8.37***</td>
<td>-13.81***</td>
</tr>
<tr>
<td></td>
<td>(5.83)</td>
<td>(1.91)</td>
<td>(4.85)</td>
</tr>
<tr>
<td>$\beta_L$</td>
<td>0.29</td>
<td>0.38</td>
<td>0.42</td>
</tr>
<tr>
<td>$\beta_M$</td>
<td>0.48***</td>
<td>0.46***</td>
<td>0.53***</td>
</tr>
<tr>
<td></td>
<td>(0.16)</td>
<td>(0.11)</td>
<td>(0.097)</td>
</tr>
<tr>
<td>$\beta_K$</td>
<td>0.23*</td>
<td>0.16</td>
<td>0.046</td>
</tr>
<tr>
<td></td>
<td>(0.12)</td>
<td>(0.099)</td>
<td>(0.063)</td>
</tr>
<tr>
<td>$\beta_t$</td>
<td>0.21***</td>
<td>0.22***</td>
<td>0.21***</td>
</tr>
<tr>
<td></td>
<td>(0.062)</td>
<td>(0.066)</td>
<td>(0.052)</td>
</tr>
<tr>
<td>$\beta_Q$</td>
<td>3.50*</td>
<td>1.80***</td>
<td>3.73**</td>
</tr>
<tr>
<td></td>
<td>(1.98)</td>
<td>(0.36)</td>
<td>(1.68)</td>
</tr>
<tr>
<td>$\beta_{Lt}$</td>
<td>-0.00066</td>
<td>-0.0007</td>
<td>-0.00278</td>
</tr>
<tr>
<td>$\beta_{Mt}$</td>
<td>0.0006</td>
<td>0.0005</td>
<td>0.0030***</td>
</tr>
<tr>
<td></td>
<td>(0.0017)</td>
<td>(0.0017)</td>
<td>(0.0009)</td>
</tr>
<tr>
<td>$\beta_{Kt}$</td>
<td>0.00006</td>
<td>0.0002</td>
<td>0.0005</td>
</tr>
<tr>
<td></td>
<td>(0.0008)</td>
<td>(0.0009)</td>
<td>(0.0006)</td>
</tr>
<tr>
<td>$\beta_{tQ}$</td>
<td>-0.041***</td>
<td>-0.044***</td>
<td>-0.043***</td>
</tr>
<tr>
<td></td>
<td>(0.013)</td>
<td>(0.014)</td>
<td>(0.011)</td>
</tr>
<tr>
<td>$\beta_{tt}$</td>
<td>0.0009</td>
<td>0.0009</td>
<td>0.0014</td>
</tr>
<tr>
<td></td>
<td>(0.0012)</td>
<td>(0.0012)</td>
<td>(0.0014)</td>
</tr>
<tr>
<td>$\beta_{LL}$</td>
<td>0.093</td>
<td>0.082</td>
<td>NA</td>
</tr>
<tr>
<td>$\beta_{MM}$</td>
<td>0.052</td>
<td>0.055</td>
<td>NA</td>
</tr>
<tr>
<td>$\beta_{KK}$</td>
<td>0.045</td>
<td>0.039</td>
<td>NA</td>
</tr>
<tr>
<td>$\beta_{LM}$</td>
<td>-0.050</td>
<td>-0.049</td>
<td>NA</td>
</tr>
<tr>
<td></td>
<td>(0.030)</td>
<td>(0.030)</td>
<td>(NA)</td>
</tr>
<tr>
<td>$\beta_{LK}$</td>
<td>-0.043*</td>
<td>-0.033</td>
<td>NA</td>
</tr>
<tr>
<td></td>
<td>(0.022)</td>
<td>(0.022)</td>
<td>(NA)</td>
</tr>
<tr>
<td>$\beta_{MK}$</td>
<td>-0.0023</td>
<td>-0.0056</td>
<td>NA</td>
</tr>
<tr>
<td></td>
<td>(0.019)</td>
<td>(0.020)</td>
<td>(NA)</td>
</tr>
<tr>
<td>$\beta_{QQ}$</td>
<td>-0.32</td>
<td>NA</td>
<td>-0.35</td>
</tr>
<tr>
<td></td>
<td>(0.34)</td>
<td></td>
<td>(0.29)</td>
</tr>
<tr>
<td>$\beta_{LQ}$</td>
<td>0.011</td>
<td>NA</td>
<td>-0.0042</td>
</tr>
<tr>
<td>$\beta_{MQ}$</td>
<td>-0.0015</td>
<td>NA</td>
<td>0.0065</td>
</tr>
<tr>
<td></td>
<td>(0.019)</td>
<td></td>
<td>(0.018)</td>
</tr>
<tr>
<td>$\beta_{KQ}$</td>
<td>-0.0097</td>
<td>NA</td>
<td>-0.0023</td>
</tr>
<tr>
<td></td>
<td>(0.011)</td>
<td></td>
<td>(0.012)</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Number of restrictions</th>
<th>none</th>
<th>3</th>
<th>3</th>
</tr>
</thead>
<tbody>
<tr>
<td>Log of likelihood function</td>
<td>296.34</td>
<td>292.21</td>
<td>284.07</td>
</tr>
<tr>
<td>Likelihood ratio statistic</td>
<td>NA</td>
<td>8.26</td>
<td>24.54</td>
</tr>
<tr>
<td>Critical $\chi^2$ at (1%)</td>
<td>NA</td>
<td>11.341</td>
<td>11.341</td>
</tr>
</tbody>
</table>
From the results of the model estimation and likelihood ratio tests, we cannot reject the hypothesis of constant returns to scale at the 1% level of significance. We can reject the hypothesis of unitary elasticity of substitution at the 1% level of significance. Therefore, the calculations for the Allen and Morishima elasticities of substitution and the price elasticities were done using the parameter estimates for the constant returns to scale model (Model 2).

<table>
<thead>
<tr>
<th>Allen Partial Elasticity of Substitution</th>
<th>Labor</th>
<th>Materials</th>
<th>Capital</th>
</tr>
</thead>
<tbody>
<tr>
<td>Labor</td>
<td>0.76</td>
<td>-1.38</td>
<td></td>
</tr>
<tr>
<td>Materials</td>
<td>0.76</td>
<td>0.80</td>
<td></td>
</tr>
<tr>
<td>Capital</td>
<td>-1.38</td>
<td>0.80</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Morishima Elasticity of Substitution</th>
<th>Labor</th>
<th>Materials</th>
<th>Capital</th>
</tr>
</thead>
<tbody>
<tr>
<td>Labor</td>
<td>0.67</td>
<td>-0.017</td>
<td></td>
</tr>
<tr>
<td>Materials</td>
<td>0.91</td>
<td>0.57</td>
<td></td>
</tr>
<tr>
<td>Capital</td>
<td>0.0059</td>
<td>0.10</td>
<td></td>
</tr>
</tbody>
</table>

The AES results show that labor and materials are complements but the MES indicate that an increase in the use of capital will result in a decrease in the use of labor but an increase in the use of labor will result in an increase in the use of capital. These results should be interpreted with caution because none of the parameter estimates involving capital were significant at the 5% level. The AES results also show that materials and capital are complements in the production process. The MES results indicate a complementary relationship between labor and materials in both directions but the change in the quantity of materials used is greater given an increase in the quantity of
labor used than the other way around. For materials and capital, the MES results show them to be complements and as with labor and materials, the complementarity is not symmetric. An increase in the use of capital results in a larger increase in the use of materials than an increase in the use of materials results in an increase in the use of capital.

Table-3 shows the results of this study compared to those of other studies of the sawmill industry. The labor/materials elasticity is the largest amongst the presented results. The labor/capital result is the only one to show that the two inputs are substitutes whereas the capital/materials results vary among the presented studies.

<table>
<thead>
<tr>
<th>Study</th>
<th>Labor/Materials</th>
<th>Labor/Capital</th>
<th>Capital/Materials</th>
</tr>
</thead>
<tbody>
<tr>
<td>Stier, 1980</td>
<td>NA</td>
<td>0.105</td>
<td>NA</td>
</tr>
<tr>
<td>Nautiyal and Singh, 1985</td>
<td>0.60</td>
<td>0.93</td>
<td>1.24</td>
</tr>
<tr>
<td>Singh and Nautiyal, 1985</td>
<td>0.24</td>
<td>2.58</td>
<td>-0.62</td>
</tr>
<tr>
<td>Banskota et al., 1985</td>
<td>0.0614</td>
<td>1.7274</td>
<td>-0.0544</td>
</tr>
<tr>
<td>Martinello, 1985</td>
<td>0.00</td>
<td>0.226</td>
<td>0.575</td>
</tr>
<tr>
<td>Martinello, 1987&lt;sup&gt;1&lt;/sup&gt;</td>
<td>0.203</td>
<td>1.669</td>
<td>0.246</td>
</tr>
<tr>
<td></td>
<td>0.053</td>
<td>1.254</td>
<td>0.572</td>
</tr>
<tr>
<td>Meil and Nautiyal, 1988&lt;sup&gt;2&lt;/sup&gt;</td>
<td>0.337</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Puttock and Prescott, 1992</td>
<td>0.595</td>
<td>NA</td>
<td>NA</td>
</tr>
<tr>
<td>Baardsen, 2000</td>
<td>-0.10</td>
<td>0.73</td>
<td>0.71</td>
</tr>
<tr>
<td>McQueen, Potter-Witter 2004</td>
<td>0.76</td>
<td>-1.38</td>
<td>0.80</td>
</tr>
</tbody>
</table>
Table-3 continued,
1Top numbers are for BC Coast and bottom numbers are for BC Interior.
2Number is for smallest mill-size class in Ontario.

The price elasticity results presented in Table-4 show that all elasticities are inelastic and that all input pairs are substitutes with the exception of labor and capital which exhibit a complementary relationship. Among the own-price elasticities, capital is the least elastic while demand for labor is the most elastic.

<table>
<thead>
<tr>
<th></th>
<th>Labor Price</th>
<th>Materials Price</th>
<th>Capital Price</th>
</tr>
</thead>
<tbody>
<tr>
<td>Labor</td>
<td>-0.42</td>
<td>0.48</td>
<td>-0.060</td>
</tr>
<tr>
<td>Materials</td>
<td>0.24</td>
<td>-0.28</td>
<td>0.034</td>
</tr>
<tr>
<td>Capital</td>
<td>-0.44</td>
<td>0.51</td>
<td>-0.065</td>
</tr>
</tbody>
</table>

All these results are indicative of the rather small role capital plays in the production process in relation to sawmill industries in other jurisdictions. It is accepted that mills in the Lake States are less capital intensive than those in softwood regions of the U.S. and Canada but the reasons for this are unclear. The short answer is that the relative prices and marginal products of inputs in the Lake States lead to this situation but the determinants of those factors are also unclear. It may be a result of the different physical characteristics between softwood and hardwood sawlogs. Hardwood logs are typically less symmetrical than softwood logs which would necessitate more complicated sawmill technology to deal with in order to reduce the labor requirement. This aspect of the production process requires further study.

**Conclusions**

The sawmilling industry of the Lake States can be characterized with a homogenous in output (constant returns to scale) technology with nonunitary elasticity of substitution amongst the inputs labor, materials and capital. All input pairs in the
production process are substitutes with the exception of labor and capital which are complements. According to the price elasticity results, increasing scarcity and consequently increasing price of materials could lead to greater capital intensity in the industry as well as greater employment.

The findings of the study show that there is greater substitutability between labor and materials in the Lake States than in the softwood lumber regions. Therefore, policies affecting the sawmill industry should be tailored to the region they are located. The same policy will affect sawmills differently in Washington than it will in Michigan. For example, in the Lake States and other hardwood regions, policies that promote research and development on sawmill technology may help increase the productivity of the industry more than similar investments in softwood regions which typically are already more capital intensive than the hardwood industry.

To the extent that decreased harvest levels on public land increases timber prices, such a policy may actually increase employment given the substitution effect reported in this study. It is difficult to say to what extent this effect may apply because the results for the substitution between inputs are contingent on output levels remaining constant. Large decreases in timber harvests could lead to the shutdown of mills that have less of an ability to substitute away from wood and consequently have higher costs.

Given that such a large amount of forestland in the Lake States is privately owned, policies affecting harvest levels on public land are only part of the picture. As was discussed, there is concern that harvests from private lands will decrease as landowners’ preferences shift towards the aesthetic value of standing timber as opposed to the financial value of harvested timber. In this case, government policymakers have no
direct control over harvest patterns and so instruments such as tax incentives for forest management would be necessary. These types of incentives already exist to help achieve forest management goals on private land. Using the results of this study it may be possible to gauge the effects of such policies more accurately.

The finding that labor and capital are complements in the sawmilling process of the Lake States also differentiates the region from other lumber producing regions of the United States. Changes in the price of all inputs affects the demand for all other inputs and so policymakers must be cognizant of this. In the case of labor and capital, the effects are synergistic. Apart from the direction of demand changes when the price of labor or capital changes, the magnitude is important. Changes in the price of capital have less of an effect on the demand for labor than the other way around. Therefore policies affecting one or other of these two inputs will have different effects depending on which input they apply to. For example, changes in the Federal Reserve prime rate will not directly affect employment in the sawmill industry as much as changes in payroll taxes will affect demand for capital.

The industry exhibits constant returns to scale although the exact reason for this is unclear. Constant returns to scale in a mature sawmill industry would lead to the outcome of mills of similar size as all economies of scale have been exhausted and the industry has settled into an equilibrium firm size near the minimum of the long run average cost curve. Nevertheless, this does not explain why the average mill size in the Lake States is small compared to the Pacific Northwest and Southeastern U.S. It perhaps has to do with the ability of mill owners to acquire timber from nonindustrial private forest land, which is the predominant forest land ownership type in the region.
**Literature Cited**


Minnesota Department of Natural Resources. Various Years. Minnesota forest products price report. Division of Forestry, Minnesota Department of Natural Resources.


Puttock, G.D., and D.M. Prescott. 1992. Factor substitution and economies of scale in the


Wisconsin Department of Natural Resources. 1999. NR 46.30 Stumpage rates-2001. Wisconsin Department of Natural Resources.