VALUING COMPLEX POLICIES USING CONTINGENT VALUATION: FUNCTIONAL FORMS AND ESTIMATION PROCEDURES

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Abstract

A valid contingent valuation design accounts for the substitutions that households make across policy components. Flexible functional forms are specified that allow one to estimate valid Bradford bid curves and to test for substitution effects. Estimates conform to theory. Results indicate that regional environmental conditions are strict substitutes in valuation.
VALUING COMPLEX POLICIES USING CONTINGENT VALUATION:
FUNCTIONAL FORMS AND ESTIMATION PROCEDURES

Public policy is inherently multi-dimensional. A typical policy choice is not simply to provide more or less of a single service, but a decision as to what package or set of services to provide. The multiple dimensions of policy have important implications for valuing policy change. As policy shifts the package of public services, individual users make substitutions across a changing opportunity set. These substitutions affect the value that individuals place on both the policy and its components.

Conventional benefit cost procedures tend to ignore the substitutions made by households. With a conventional approach, each element of a multiple impact policy is evaluated independently, as if it were the only element to be changed by policy. A total valuation is obtained by summing across the independent valuations of the components.


This paper develops procedures for implementing the valid design with contingent valuation. The first section outlines the structure of a valid design. The key to a valid contingent valuation is a bid function that incorporates the relevant substitution relations between policy impacts. The second section identifies functional forms that conform to the requirements of a valid bid function. The third section examines estimation procedures and applies them to the valuation of regional environmental conditions.
I. Valid and Conventional Benefit Measures

Public policy provides services that range from physical services (e.g., parklands, environmental quality, roads and bridges), to social, legal, and educational services. The value of a change in public policy stems from the tradeoffs a household is willing to make between (1) the change in public services and (2) other desirable goods and services. These tradeoffs are summarized in terms of a household's expenditure function.

An expenditure function, \( e(q_1, q_2, u) \), states the minimum amount of income that a household requires in order to maintain a utility level \( u \) at public service level \( q = (q_1, q_2) \). Prices are left implicit in the definition of \( e(q_1, q_2, u) \) since changes in public services are the focus of this analysis. Under standard assumptions, the expenditure function is convex and strictly decreasing in \( q \) (Måler, 1972). At an initial \( q^0 \), initial income, \( m^0 \), is just enough to sustain initial utility, \( u^0 \); that is, \( m^0 = e(q^0_i, q^0_2, u^0) \).

The Hicksian compensating measure of benefit, \( HC \), is the amount of income, paid or received, that would leave a household at a pre-policy level of well-being at the post-policy level of public services. For a multiple impact policy that shifts \( q^0 \) to \( q^1 = (q^1_i, q^1_2) \), \( HC \) is

\[
(1) \quad HC(q^1; q^0) = m^0 - e(q^1_i, q^1_2, u^0)
\]

Equation (1) summarizes the restrictions of a valid benefit evaluation design. As the difference between initial income, \( m^0 \), and a well-defined function \( e(\cdot) \), \( HC \) is unique for any multi-dimensional change in policy.

\( HC \) may also be disaggregated into a set of component valuations by selecting a valuation path that begins at \( (q^0_i, q^0_2) \) and ends at \( (q^1_i, q^1_2) \). One
admissible sequential path values the change from \((q_1^0, q_2^0)\) to \((q_1^1, q_2^0)\), first, and the change from \((q_1^1, q_2^0)\) to \((q_1^1, q_2^1)\), second. Using this path, HC is

\[
(2.1) \quad HC(q_1^1; q_0^0) = m^0 - e(q_1^1, q_2^0, u^0)
\]

\[
(2.2) \quad + e(q_1^1, q_2^0, u^0) - e(q_1^1, q_2^1, u^0)
\]

The sequenced valuations vary with the selected path. With a valuation path that changes \(q_2\), first, and \(q_1\), second, the component valuations would be conditioned on a different level of the other environmental service. A change in conditioning variables changes the sequenced valuations.

Differences in the conditioning variables cause a valid design to result in component and aggregate valuations that are different from those of conventional benefit cost analysis (CBC) (Hoehn and Randall, 1989). For a small number of policy impacts, the relationship between HC and CBC depends upon the extent of substitution, independence, or complementarity.

Elements of \(q\) are substitutes, independent, or complements in valuation as the marginal valuation of \(q_i\) decreases, remains constant, or increases with a positive \(\partial e/\partial q_i\partial q_j\). CBC overstates the valid valuation if environmental services are substitutes. With complementarity, the opposite effect occurs and CBC understates the aggregate and component valuations. CBC results in the same valuation as HC only when \(q_1\) and \(q_2\) are independent.

II. Contingent Valuation and Admissible Functional Forms

Contingent valuation (CV) is adaptable to any level of a valid valuation design. The issue with contingent valuation is one of experimental design—of
how to develop adequate value information at the least cost. In some cases, the valuation problem may be simple. For instance, if the composition of a policy is predetermined, the overall policy may be valued with a one-step, holistic valuation consistent with equation (1). There is no need to disaggregate and value each of the components.

The valuation problem is typically more complex. An agency may have identified a candidate set of policy instruments but be uncertain as to the composition of a final policy. Benefit information is used to evaluate both the aggregate policy as well as the components. In this case, a costly but valid approach would be to identify all combinations of the candidate impacts and then evaluate each holistically according to equation (1).

An econometric approach reduces the cost of exploratory valuation by extracting a maximum amount of information from a sample of value data. The first step is to draw a representative sample of policy scenarios and to elicit contingent valuations for each of the scenarios as suggested by equation (1). The functional relationship underlying equation (1) may then be estimated in a manner analogous to that used with multiproduct cost functions (Baumol, Panzar, and Willig, 1982). The estimated relationship is a multidimensional Bradford bid curve (Brookshire, et al, 1980).

There are three considerations in selecting a functional form for equation (1). First, the predetermined variables should enter a bid function in a manner consistent with theory. The bid function be equal to zero when environmental quality remains at the initial level. Socioeconomic variables should enter the function so that the initial conditions hold.

Second, the convexity of the expenditure function in q implies a bid function that is concave in q. A concave bid function has a Jacobian that is
negative semi-definite. Since this restriction results from a convenient but not necessary utility assumption, a bid function should allow either concavity or convexity. Such flexibility permits a test for concavity.

Third, the function should admit either substitution, complementarity, or independence. Common functional forms fail this second criterion. The Cobb-Douglas form imposes strict substitution across all amenities and a CES imposes uniform substitution or uniform complementarity.

Admissible bid functions are obtained using the approach used to derive flexible production and cost functions. This approach approximates the true function with a second order Taylor series expansion (TSE) (Gallant, 1984).

A quadratic bid function is obtained by applying the TSE directly to equation (1). Before expanding, however, two alterations are appropriate. First, to remove an unobservable utility term, the indirect utility function, \( u^0 = v(q^0, m^0) \), is substituted into (1). Second, consistent with the household production literature, the utility and expenditure functions depend on the socioeconomic characteristics of a household, \( \bar{s} \). With these changes, the quadratic specification is

\[
H(q^1; q^0) = m^0 - c(q^1_1, q^1_2, q^0, s).
\]

where \( c(q^1_1, q^1_2, q^0, s) = e(q^1_1, q^1_2, \bar{s}, v(q^0, \bar{s}, m^0)) \) and \( s = (\bar{s}, m^0) \).

Define the nth policy scenario, \( n \in (1, \ldots, N) \), as a set of ordered pairs, \( ((q^w_1, q^w_2), (q^z_1, q^z_2)) \) where \( w \) and \( z \) are from a real interval that includes zero and one. For a sample of \( R \) respondents and \( N \) policy scenarios, a TSE taken about the initial values \( q^0 \) and the mean value of \( s, \bar{s} \), results in
\[
(4) \quad h_{crn} = \alpha_0 + ds_r'\alpha_s + dq_n\beta + ds_r'Adq_n + dq_n'Bdq_n + \epsilon_{rn}
\]

where \( h_{crn} \) is the \( r \)th respondent’s valuation of the \( n \)th policy scenario; \( \alpha_0 \) is a constant; \( ds_r = (s_r - \bar{s}) \) is a vector of differences between the \( r \)th respondent’s characteristics, \( s_r \), and \( \bar{s} \); \( \alpha_s \) is a conformable coefficient vector; \( dq_n = [(q_1^r - q_1^0), (q_2^r - q_2^0)] \) is a vector of differences between the \( r \)th post-policy level of environmental quality and the initial environmental quality level; \( \beta = (\beta_1, \beta_2) \) is a two-element coefficient vector; \( A \) is a coefficient matrix conformable to \( ds_r \) and \( dq_n \); \( B = [b_{ij}] \), \( i,j \in (1,2) \), are environmental interaction coefficients; and \( \epsilon_{rn} \) is a statistical error term.

The error \( \epsilon_{rn} \) may be correlated across observations if multiple responses are obtained from individual respondents. In this case, ordinary least squares are not minimum variance estimators. Efficient estimates are obtained using generalized least squares procedures developed for panel data.

The quadratic function (4) is adaptable to requirements of economic theory. First, the expected value of \( h_{crn} \) is equal to zero at the initial quality level if \( \alpha_0 \) and \( \alpha_s \) are equal zero. These restrictions on \( \alpha_0 \) and \( \alpha_s \) can be tested using routine statistical procedures.

Second, the quadratic permits either a concave or convex bid function. The quadratic is concave (convex) if \( B \) is negative (positive) semi-definite. Concavity (convexity) requires that the diagonal elements of \( B \) are non-positive (non-negative) and the principal minors of \( |B| \) alternate in sign (are non-negative). For two quality elements, this means that \( b_{11} \) is non-positive for concavity, \( b_{11} \) is non-negative for convexity, and \( |B| = b_{11}b_{22} - b_{12}^2 \) is non-negative in either case.

Third, the quadratic equation (4) admits substitution, independence, or
complementarity. Differentiation shows that environmental services are substitutes, independent, or complements as the symmetric, off-diagonal terms in $B$, $b_{ij}$, $i \neq j$, are negative, zero, or positive.

A semi-logarithmic quadratic (semi-log) form is obtained by taking a TSE about the logarithms of the right-hand side variables in equation (3). The semi-log quadratic specification is

$$\text{(5) } h_{cr} = \rho_0 + dls_r'\rho_s + dlq_n'\gamma + dls_r'Cdlq_n + dlq_n'Ddlq_n + \nu_{rn}$$

where $\rho_0$ is a constant; $dls_r$ is a vector of differences between the logarithms of the elements of $s_r$ and the logarithms of the elements of $\bar{s}$; $\rho_s$ is a conformable coefficient vector; $dlq_n = [\log(q^n_1) - \log(q^n_0), \log(q^n_2) - \log(q^n_0)]$ is the vector of differences between the logarithms of the nth post-policy level of environmental quality and of the initial environmental quality level; $\gamma = (\gamma_1, \gamma_2)$ is a two-element coefficient vector; $C$ is a coefficient matrix conformable to $dls_r$ and $dlq_n$; $D = [d_{ij}], i, j \in \{1, 2\}$, are environmental interaction coefficients; and $\nu_{rn}$ is an error term. Statistical properties of $\nu_{nr}$ are analogous to those of $\epsilon_{rn}$.

Equation (5) is adaptable to the restrictions of theory. $E(h_{cr})$ is equal to zero at the initial quality level if the estimated $\rho_0$ is equal to zero. The convexity restrictions on $D$ and the form of interaction between environmental quality levels are analogous, at the point of approximation $q^0$ and $\bar{s}$, to those of equation (4).

III. Empirical Tests for Interaction

The results of Hoehn and Randall (1989) suggest that interaction effects in valuation are routine. Given that CBC overstates the benefits of a valid
design, substitution effects are likely to dominate in applied valuation. In this section, the estimated quadratic and semi-log bid functions are reviewed and used to test the substitution hypothesis.

Data for the analysis came from a contingent valuation experiment conducted in Chicago, IL, during 1980 and 1981 (Tolley and Fabian, 1988). The experiment used contingent valuation to value visual air quality changes in both Chicago and the region surrounding the Grand Canyon. Changes in air quality were described using narrative and 8.5 by 11 inch color photographs.

Table 1 reports the quadratic and semi-log bid coefficients that were estimated using the air quality data. Variables used in the estimation were formulated in a manner consistent with equations (4) and (5). The dependent variable in all cases is the respondents' elicited valuation of the described changes in visual air quality. The variables in the quadratic are formulated as in equation (4). Those in the semi-log equation follow equation (5).

The independent variables are described in the first column of Table 1. The first two variables listed, Grand Canyon Air Quality (GCAQ) and Chicago Air Quality (CAQ), measure the changes in air quality brought about by policy. The initial air quality level was normalized to one and the post-policy level was measured as a proportional change (e.g. 1.83, 2.0). Coefficients across the first two rows of Table 1 measure elements of $\mathbf{\beta}$ for the quadratic and elements of $\mathbf{\gamma}$ for the semi-log form.

The third, fourth, and fifth rows in Table 1 give coefficients for the $\mathbf{B}$ and $\mathbf{D}$ matrices of, respectively, the quadratic and semi-log functions. Rows three and four give the estimated coefficients for the diagonal terms. The fifth row states the off-diagonal terms. The remaining eight rows list the estimated elements of the $\mathbf{A}$ and $\mathbf{C}$ matrices that represent the interaction of
Table 1. Estimated Quadratic and Translog Value Functions

<table>
<thead>
<tr>
<th>Variable^a or Statistic</th>
<th>Quadratic^b,d</th>
<th>Semi-Logarithmic^c,d</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>OLS</td>
<td>GLS</td>
</tr>
<tr>
<td>Grand Canyon Air Quality (GCAQ)</td>
<td>132**</td>
<td>121**</td>
</tr>
<tr>
<td>(26.9)</td>
<td>(21.7)</td>
<td>(45.7)</td>
</tr>
<tr>
<td>Chicago Air Quality (CAQ)</td>
<td>234**</td>
<td>238**</td>
</tr>
<tr>
<td>(49.9)</td>
<td>(38.0)</td>
<td>(100)</td>
</tr>
<tr>
<td>(GCAQ)^2</td>
<td>-30.9**</td>
<td>-28.3**</td>
</tr>
<tr>
<td>(10.3)</td>
<td>(6.74)</td>
<td>(38.8)</td>
</tr>
<tr>
<td>(CAQ)^2</td>
<td>-66.1**</td>
<td>-66.1**</td>
</tr>
<tr>
<td>(22.5)</td>
<td>(13.8)</td>
<td>(90.0)</td>
</tr>
<tr>
<td>GCAQ X CAQ</td>
<td>-46.8++</td>
<td>-42.6++</td>
</tr>
<tr>
<td>(25.6)</td>
<td>(13.9)</td>
<td>(50.9)</td>
</tr>
<tr>
<td>GCAQ X Household Income ($1000)</td>
<td>0.706</td>
<td>0.491</td>
</tr>
<tr>
<td>(0.547)</td>
<td>(0.372)</td>
<td>(23.9)</td>
</tr>
<tr>
<td>GCAQ X Years in School</td>
<td>-0.902</td>
<td>0.814</td>
</tr>
<tr>
<td>(3.27)</td>
<td>(2.26)</td>
<td>(86.5)</td>
</tr>
<tr>
<td>GCAQ X Age in Years</td>
<td>-1.24**</td>
<td>-0.910**</td>
</tr>
<tr>
<td>(0.454)</td>
<td>(0.317)</td>
<td>(33.0)</td>
</tr>
<tr>
<td>GCAQ X Gender</td>
<td>-12.2</td>
<td>-7.82</td>
</tr>
<tr>
<td>(14.5)</td>
<td>(11.2)</td>
<td>(27.6)</td>
</tr>
<tr>
<td>CAQ X Household Income ($1000)</td>
<td>-1.41**</td>
<td>-0.736</td>
</tr>
<tr>
<td>(0.541)</td>
<td>(0.502)</td>
<td>(21.7)</td>
</tr>
<tr>
<td>CAQ X Years in School</td>
<td>15.0**</td>
<td>6.63**</td>
</tr>
<tr>
<td>(5.39)</td>
<td>(3.33)</td>
<td>(87.3)</td>
</tr>
<tr>
<td>CAQ X Age in Years</td>
<td>1.48**</td>
<td>0.392</td>
</tr>
<tr>
<td>(0.646)</td>
<td>(0.661)</td>
<td>(49.0)</td>
</tr>
<tr>
<td>CAQ X Gender</td>
<td>-26.0</td>
<td>-19.8</td>
</tr>
<tr>
<td>(27.2)</td>
<td>(25.3)</td>
<td>(47.5)</td>
</tr>
<tr>
<td>Root Mean Square Error</td>
<td>233</td>
<td>112</td>
</tr>
<tr>
<td>R-squared</td>
<td>0.28</td>
<td>0.22</td>
</tr>
<tr>
<td>F Value</td>
<td>19.5</td>
<td>14.0</td>
</tr>
</tbody>
</table>

a. Variables are described in Table 1.
b. Variables in the quadratic were formulated as in equation 9.
c. Variables in the semi-logarithmic were formulated as in equation 10 except for the dummy variable gender. The dummy variable was entered without transforming it by a logarithm.
d. Columns labeled OLS were estimated using ordinary least squares. Columns headed GLS were estimated using the error components model described in Judge, et al. (1985, pp.521-525). A double pound sign indicates that a coefficient is significantly different from zero at the 95 percent level for a one way test. A single (double) asterisk indicates that a coefficient is significantly different from zero at the 90 (95) percent level for a two way test. Standard errors are in parentheses. Estimates have 640 degrees of freedom.
air quality and socioeconomic characteristics.

Initial coefficient estimates were obtained using ordinary least squares (OLS). These are given in the second and fourth columns of Table 1. Each of the air quality coefficients carries an intuitively consistent sign for environmental improvements that are amenities and substitutes. As air quality increases, a respondent's valuation increases but at a decreasing rate. The negative cross product terms (fifth row) indicates substitution effects in the valuation of the two air quality variables.

Contingent valuation studies have generally ignored the inefficiency of OLS estimates when multiple valuations are obtained from the same individual respondents. An alternative is to specify the error structure as an error component model. An error component model allows for a non-zero covariance between a respondent's bids. With this model, the error terms in equations (4) and (5) are decomposed as \( \epsilon_{rn} = \mu_r + \xi_{rn} \) in the quadratic and \( v_{rn} = \omega_r + \xi_{rn} \) in the semi-log where \( E(\mu_r^2) = \sigma^2_{\mu} \), \( E(\omega_r^2) = \sigma^2_{\omega} \), and both \( \xi_{rn} \) and \( \xi_{rn} \) are uncorrelated across \( r \) and \( n \). Generalized least squares (GLS) yields efficient estimates for the error component model (Judge, et al, 1985, pp. 521-525).

The GLS estimates used for hypothesis testing are given in the third and fifth columns of Table 1. The GLS data are identical to those used in the OLS equations. Since both the OLS and GLS estimators are unbiased, it is notable that the two sets of coefficients are indeed very similar. As with the OLS estimates, air quality coefficients indicate that values increase at a decreasing rate with increases in air quality. The cross-quality interaction terms again suggest that regional air quality conditions are substitutes in valuation. Finally, consistent with the theoretical efficiency of the GLS estimator, the GLS estimated root mean square error is about 50 percent smaller.
than that of OLS for both the quadratic and semi-log models.

Four sets of hypothesis tests were carried out with the GLS equations. First, theory implies a bid function (1) without an intercept and (2) where the socioeconomic variables enter only as interaction terms with the air quality variables. For the quadratic, an F test failed to reject the joint hypothesis of $\alpha_0$ and $\alpha_5$ equal to zero (F value of 1.0; 5 and 635 degrees of freedom). An F test also failed to reject the hypothesis of $\rho_0$ and $\rho_5$ equal to zero in the semi-log model (F value of 0.53; 5 and 635 degrees of freedom).

Second, the inequality restrictions imposed by concavity are testable using the likelihood ratio method. This method compares a set of restricted estimates with the unrestricted estimates given in Table 1. Examination of Table 1, however, shows that the concavity restrictions are non-binding. The diagonal elements of B and D are strictly negative and $|B|$ and $|D|$ are strictly positive. In this non-binding case, restricted least square estimates are identical to the unrestricted estimates (Judge, et al, 1988). The results therefore fail to reject a concave bid function.

Third, the significance of the substitution effect was examined. The coefficient of GCAQ X CAQ is negative and different from zero at any conventional level of significance in both the quadratic and semi-log equations. These results confirm the predictions of Theorem 1 and reject both independence and complementarity. Regional air quality conditions are substitutes in valuation.

Finally, a J test (MacKinnon, 1983) was used to test the quadratic and semi-log specifications. With the J test, the quadratic and translog specifications are tested against each other. To test the quadratic, the predicted values from the estimated semi-log equation are entered as an
additional explanatory variable in the quadratic equation and the equation is reestimated. If the coefficient on the predicted bid from the semi-log equation is significantly different from zero, the predicted value contributes to the quadratic's explanatory power and the quadratic specification is rejected as incomplete. The predicted bid from the estimated quadratic equation is used to test the semi-log in a similar manner.

J tests were conducted for both the quadratic and the semi-log. In the quadratic equation, the estimated coefficient for the predicted hc was 1.09 and had a t value of 1.9 with 639 degrees of freedom. This result rejects the quadratic as a valid specification at the 90 percent significance level. In the semi-log equation, the coefficient for the predicted hc was 0.0668 and had a t value of 0.11 with 639 degrees of freedom. This result fails to reject the semi-log specification at any conventional significance level. The semi-log is therefore the best fitting specification for these data.

IV. Concluding Comments

Environmental values are contextual—they are conditioned on the presence of multi-dimensional resource flows. A conventional benefit cost design ignores the contextual nature of value data and overstates the benefits of policy change. A valid benefit cost design avoids these errors.

This paper identified procedures for implementing a valid benefit cost design with contingent valuation. Contingent value data were used to estimate multi-dimensional bid functions based on the quadratic and semi-log functional forms. Estimated coefficients conform to the restrictions of theory and demonstrate the significant interaction effects that arise in valuing policy change. Consistent with theory, regional environmental conditions were shown to be strict substitutes in valuation.
References


