Discrete and Continuous Time Models for Farm Credit Migration Analysis

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Xiaohui Deng, Cesar L. Escalante, Peter J. Barry and Yingzhuo Yu

Author Affiliations:
Xiaohui Deng is Ph.D. student and graduate research assistant, Department of Agricultural and Applied Economics, University of Georgia, Athens, GA;
Cesar L. Escalante is assistant professor, Department of Agricultural and Applied Economics, University of Georgia, Athens, GA;
Peter J. Barry is professor, University of Illinois at Urbana-Champaign - Department of Agricultural and Consumer Economics, Champaign, IL;
Yingzhuo Yu is Ph.D. student and graduate research assistant, Department of Agricultural and Applied Economics, University of Georgia, Athens, GA.

Contact:
Xiaohui Deng
Department of Agricultural and Applied Economics, University of Georgia
Conner Hall 306
Athens, GA 30602
Phone: (706)542-0856 Fax: (706)542-0739
E-mail: sdeng@agecon.uga.edu

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Abstract: This paper introduces two continuous time models, i.e. time homogenous and non-homogenous Markov chain models, for analyzing farm credit migration as alternatives to the traditional discrete time model cohort method. Results illustrate that the two continuous time models provide more detailed, accurate and reliable estimates of farm credit migration rates than the discrete time model. Metric comparisons among the three transition matrices show that the imposition of the potentially unrealistic assumption of time homogeneity still produces more accurate estimates of farm credit migration rates, although the equally reliable figures under the non-homogenous time model seem more plausible given the greater relevance and applicability of the latter model to farm business conditions.

Keyword: Credit migration; Transition matrix; Markov chain; Cohort method; Time homogeneous; Time nonhomogeneous

* Xiaohui Deng and Yingzhuo Yu are Ph.D. students and graduate research assistants, University of Georgia-Department of Agricultural and Applied Economics, Athens, GA; Cesar L. Escalante is assistant professor, University of Georgia-Department of Agricultural and Applied Economics, Athens, GA; Peter J. Barry is professor, University of Illinois at Urbana-Champaign - Department of Agricultural and Consumer Economics, Champaign, IL.

Introduction

Credit migration or transition probability matrices are analytical tools than can be used to assess the quality of lenders' loan portfolios. They are cardinal inputs for many risk management applications. For example, under the New Basel Capital Accord, the setting of minimum economic capital requirements have increased the reliance of some lending institutions on the credit migration framework to methodically derive these required information ((BIS (2001)).

There are two primary elements that comprise the credit migration analysis. First is the choice of classification variables which are criteria measures used to classify the financial or credit risk quality of the lenders' portfolio. The variables could be single financial indicators, such as measures of profitability (ROE) or repayment capacity, or a composite index comprised of many useful financial factors, such as a borrower's credit score. The second element is the time horizon measurement or the length of the time to construct one transition matrix (Barry, Escalante, Ellinger). Normally the shorter the horizon or time measurement interval, the fewer rating changes are omitted. However, shorter durations also result in less extreme movements, as greater ratings volatility would normally result across wider horizons characterized by more diverse business operating conditions. In addition, short duration is prone to be affected by "noise" which could be cancelled out in the long term (Bangia, Diebold, and Schuermann (00-26)). In practice, a common time horizon is one year, which would be an "absolute" one-year measurement or a "pseudo" one year, which is actually an "average" of several years' data into a single measurement (Barry, Escalante, Ellinger).

The application of the migration analytical framework has been extensively used in corporate finance (Bangia, Diebold, and Schuermann (00-26); Schuermann and Jafry (03-08); Jafry and Schuermann (03-09); Lando, Torben, and Skodeberg; Israel, Rosenthal, and Wei). Most of these studies focus their analyses on the intertemporal changes in the quality of corporate stocks usually using S& P databases as well as corporate bonds and other publicly traded securities, which are reported and published quarterly.

Credit migration analysis, however, is a relatively new concept in the farm industry. There is a dearth of empirical works in agricultural economics literature that discuss the application of the migration framework to analyzing farm credit risk-related issues or replicate the much richer theoretical models in migration that have been tested and richly applied in corporate finance. Among the few existing empirical works on farm credit migration is a study by Barry, Escalante and Ellinger which introduced the measurement of transition probability matrices for farm business using several time horizons and classification variables. Their study produced estimates of transition rates, overall credit portfolio upgrades and downgrades, and financial stress rates of grain farms in Illinois over a fourteen-year period. Another study by Escalante, et al. identified the determinants of farm credit migration rates. They found that the farm-level factors did not have adequate explanatory influence on the probability of credit risk transition. Transition probabilities are instead more significantly affected by changes in macroeconomic conditions.

The study of farm credit transition probabilities can lead to a greater understanding and more reliable determination of farm credit risk. For this model to be a more effective analytical tool, it is crucial to adopt a more accurate estimation for the migration matrices. Notably, current estimates of farm credit migration rates presented in the literature have been calculated using a discrete time model. In corporate finance, however, the adoption of more sophisticated techniques for transition probability estimation using the duration continuous time model based on survival analysis has been explored. A number of studies have focused on demonstrating the relative strengths of the continuous time models over the conventional discrete time model.

In this study, we introduce the application of continuous time models to farm finance. Specifically, we will develop farm credit migration matrices under three approaches, namely, the traditional cohort method for discrete time model and two duration continuous time model variants —— time homogeneous Markov chain and time non-homogeneous Markov chain. As a precondition to the adoption of the continuous time models, we establish the conformity through eigen analysis of our farm credit migration data to the Markovian transition process, which is a basic assumption under these models. We expect this study to establish the practical relevance of using one of the two continuous time models in the better understanding of changing credit risk attributes of farm borrowers over a significant period of time.

The Ratings Data

The annual farm record data used in this study are obtained from farms that maintained certified usable financial records under the Farm Business Farm Management (FBFM) system between 1992 and 2001. The FBFM system has an annual membership of about 7,000 farmers but stringent certification procedures lead to much fewer farms with certified usable financial records. So the number of farm observations that can be used in this analysis vary from year to year throughout the 10-year period. Specifically, there are 3,867 certified farms rated at least once in the 10-year period. However, only 117 farms are rated constantly over the whole period. Figure 1 shows the number of farm observations in each year over the 10-year period. Each year less than 10% of the farms were constantly certificated by FBFM. Constraining our data set only to the constant sample comprising of farms having certified records over the whole period will significantly reduce our sample size. Thus, we allowed the sample composition to vary over time, which incorporate new farms that received their credit rating in that specific year and discard those that were not certified in that specific year. This procedure helps ensure that the sample size is always large enough to derive reliable statistical inferences.

Annual farm record data are subsequently classified into 5 different credit categories based on the farm's credit score. For this measure, we adopted a uniform credit-scoring model for term loans reported by Splett et al., which has been used in previous studies (Barry, Escalante, Ellinger; Escalante, et al.)

Analysis of Eigenvalues and Eigenvectors

Before we explore the application of the continuous time models, we initially need to verify the validity of the markov chain process assumption, which is a necessary condition for the construction of such time models.

A Markov process is a sequence of random variables $\{X_t \mid t = 0, 1, 2, ...\}$ with common space S whose distribution satisfy

$$(1) \quad \mathbf{Pr}\{X_{t+1} \in A \mid X_t, X_{t-1}, X_{t-2, ...}\} = \mathbf{Pr}\{X_{t+1} \in A \mid X_t\} \quad A \subset S.$$

It indicates that the Markov process is memory-less because the distribution of X_{t+1} conditional on the history of the process through time t is completely determined by X_t and is independent of the realization of the process prior to time t.

A Markov chain is a Markov process with a finite state-space $S = \{1, 2, 3, \ldots ,n\}$. A Markov chain is completely characterized by its transition probabilities

(2) $P_{ij} = \mathbf{Pr}\{X_{t+1} = j \mid X_t = i\} \quad i, j \in S$

There has been a long time debate pertaining to whether the credit migration follows a Markov chain or not. In many literature and practical analyses (Jarrow, Lando, and Turnbull; Lando, Torben, and Skodeberg; Schuermann and Jafry (03-08)), first-order Markov process has been merely assumed as true without any tests or justification provided by the analysts.

One of the more widely used approaches to test the Markovian property of a matrix is through the analysis of eigenvalues and eigenvectors (Bangia, Diehold, and Schuermann). The information of any transitional matrix could be broken apart into its eigenvalues and eigenvectors, written as $\mathrm{P} = \mathrm{U}_{n*n}\Lambda_{n*n}\mathrm{U}_{n*n}{}^{T}$, where $\boldsymbol{P}$ is the transitional matrix; $\boldsymbol{\Lambda}$ is a diagonal matrix, each element on the diagonal representing one eigenvalue of $\boldsymbol{P}$; $\boldsymbol{U}$ is a matrix with columns $\mathbf{u}_1, \mathbf{u}_2, \cdots, \mathbf{u}_n$ representing $\boldsymbol{P}$'s eigenvectors corresponding to each element of $\boldsymbol{\Lambda}$. In addition, any transition matrix can be taken to k^{th} power by simply increasing its eigenvalues to its k^{th} power while leaving its eigenvectors unchanged, written as $\boldsymbol{P}^{\boldsymbol{k}} = \boldsymbol{U}_{n*n}\boldsymbol{\Lambda}^{\boldsymbol{k}}_{\boldsymbol{n*n}}\boldsymbol{U}_{n*n}{}^{T}$. Thus for transition matrices to follow a Markov chain, two conditions have to be met.

Condition 1: The eigenvalues of transition matrices for increasing time horizons need to decay exponentially; and

Condition 2: The set of eigenvectors for each transition matrix need to be identical for all transitional horizons.

All transition matrices have a trivial eigenvalue of *unity*, which is of the highest magnitude and stems from the nature of transition matrices of row sum equal to one. The remaining eigenvalues have magnitudes smaller than *unity*. Those eigenvalues are what we focus on in the analysis.

Using such an eigen analysis, we find it very difficult to reject the Markov chain process assumption. Figure 2 presents a plot of the second to the fifth eigenvalues of transition matrices with transition horizons varying from one year, two years to four years. The calculated eigenvalues show a strong log-linear relationship over the increasing transition horizons, thus providing some evidence that farm credit migration rates tend to follow the Markov chain process. The results of the eigenvector analysis are presented in Figure 3. The three plots in Figure 3 represent the trends in the values of the 2^{nd} eigenvectors for the transition matrices over different horizons. These plots all seem to follow an identical path, which actually suggests that the assumption of a Markov chain process cannot be rejected

Developing Transition Probability Matrices

The results of the eigen analysis which failed to reject the Markov chain process assumption then allow us to explore the development of the farm credit migration matrices under the two continuous time models, in addition to the discrete time model.

The following sections describe the theoretical frameworks of these models and discuss the construction of these different matrices.

Cohort Method

Cohort method is the current standard method to estimate the obligor's credit migration rate under the discrete-time framework. The basic idea is as follows: considering a specific time horizon Δt, given N_i obligors being in rating category i at the beginning of the time horizon, there are N_{ij} obligors that migrate to rating category j at the end of the time horizon, then $P_{ij}^{\Delta t}$, the probability estimate of migrating from category i to category j over Δt is

$$(3) \quad \hat{P}_{ij}^{\Delta t} = \frac{N_{ij}}{N_i}$$

The probability estimate is the simple proportion of obligors in category j at the end of the time horizon out of the obligors in category i at the beginning of the time horizon. Typically obligors whose ratings are withdrawn are excluded from the sample.

The major problem associated with cohort method is the incomplete information it provides. It only concerns the rating categories at both ends of the time horizon. Any rating change activity occurring in-between the endpoints or within the period is ignored. In addition, the discrete time (cohort) model only considers direct migration, for instance from category 1 to category 2, but ignores the effect of indirect migration, which, in this case would be from category 1 to category 3 via category 2. In other words, if there are, for example, two direct migrations from category 1 to category 2 and from category 2 to category 3 but no direct migration from category 1 to category 3, the cohort method will yield a zero migration rate from category 1 to category 3. But in reality this specific transition could happen via successive downgrades within the continuous time period. Stated in another way, if in a time horizon there is no transition from category 1 to category 3, but there is at least a transition from category 1 to category 2 and another transition from category 2 to category 3, then the maximum-likelihood estimator for the transition from category 1 to category 3 should be non-zero, since evidently there is a chance, though it might be quite small, of such migration within the time horizon via successive downgrades, even if it did not happen on a single particular obligor in the sample. Notably, the cohort method, due to discrete time restriction, could not capture this probability measure, whereas the continuous time methods could capitalize on it.

Time Homogeneous Markov Chain

Under the time homogeneous case, only the length of the time interval matters, while the specific time state will not affect the migration rate at all. For example, under the time homogeneous case, a one-year period migration rate from 1992 to 1993 is the same as that from 1994 to 1995. We can see that this is a really strong assumption which will be revisited and refuted later in another continuous time model, the time non-homogeneous framework.

Following Lando and Skodeberg (2002), we define $\boldsymbol{P}(t)$ as a $K \times K$ transition matrix of Markov chain for a given time horizon, whose ij^{th} element is the probability of migrating from state i to state j in a time period of t. The generator matrix $\boldsymbol{\Lambda}$ is a $K \times K$ matrix for which

$$\text{(4)}\quad \mathbf{P}(t) = \mathbf{exp}(\mathbf{\Lambda}t) \cong \sum_{k=0}^{\infty} \frac{\mathbf{\Lambda}^k t^k}{k!} \qquad\qquad t \geq 0$$

where the exponential function is a matrix exponential, which would be approximated by the infinite summation defined as the most right-hand side.

The entries of the generator $\mathbf{\Lambda}$ satisfy

$$\text{(5)}\quad \begin{aligned} &\lambda_{ij} \geq 0 \quad \text{for } i \neq j \\ &\lambda_{ij} = -\sum_{i \neq j} \lambda_{ij} \end{aligned}$$

The second equation merely guarantees that the row sum of the matrix is equal to one.

Then the problem of estimating the transition matrix is transformed to estimating the generator matrix $\mathbf{\Lambda}$. We are left with obtaining the estimates of the entries of $\boldsymbol{\mathit{\Lambda}}$. The maximum likelihood estimator of λ_{ij} is given by

$$\text{(6)}\quad \hat{\lambda}_{ij} = \frac{N_{ij}(T)}{\int_0^T Y_i(s)ds}$$

where $N_{ij}(T)$ is the total number of transitions over the period T from credit category i to j, $Y_i(s)$ is the number of obligors assigned credit category i at time s. The numerator counts the number of observed transition from i to j. The denominator, the integral of $Y_i(s)$, effectively collects all obligators assigned with category i over the period T. Thus within T, any period an obligator spends in a category will be picked up through the denominator. To illustrate, suppose a farm spent only some of the time period T in transit from category 1 to 2 before landing in 3 at the end of T, that portion of time spent in category 2 will be counted in estimating the transition probability from category

1 to 2. In the cohort method this information has been overlooked. In addition, any indirect transition activity could be captured so that that there is always a positive, though possibly very small, transition rate for extreme migration movement.

Non-Homogeneous Markov Chain

Although the homogeneous markov chain transition matrix could provide richer migration information than the cohort method, it is actually very hard to convince that the specific time date is unimportant. In fact, in reality (and most especially when considering the more volatile farm business conditions) period-specific and heterogenous time conditions suggest that the intertemporal placement and sequence of a particular observation actually do matter in the analysis of credit migration trends A plausible justification is the economic cycle in which the obligator is involved. It is reasonable to believe that the migration from i to j over the expansion cycle would be significantly different from the same migration over the contraction cycle.

When we relax the assumption of time homogeneity, we direct to the less restriction case of non-homogeneous Markov chain. Again following Lando and Skodeberg (2002), let $\mathbf{P}(s,t)$ be the transition matrix from time s to t. Then the ij^{th} element indicates the transition probability from category i in time s to category j in time t. Given a sample of m transitions over the period from s to t, the maximum likelihood estimator of $\boldsymbol{P}(s,t)$ could be derived using the nonparametric product-limit estimator (Klein and Moeschberger)

$$(7) \qquad \hat{\boldsymbol{P}}(s,t) = \prod_{k=1}^{m} (\boldsymbol{I} + \Delta\hat{\boldsymbol{A}}(T_k))$$

where T_k is a jump in the time interval from s to t.

$$(8) \qquad \Delta \mathbf{A}(T_k) = \begin{bmatrix} -\frac{\Delta N_{1\bullet}(T_k)}{Y_1(T_k)} & \frac{\Delta N_{12}(T_k)}{Y_1(T_k)} & \frac{\Delta N_{13}(T_k)}{Y_1(T_k)} & \cdots & \frac{\Delta N_{1p}(T_k)}{Y_1(T_k)} \\ \frac{\Delta N_{21}(T_k)}{Y_2(T_k)} & -\frac{\Delta N_{2\bullet}(T_k)}{Y_2(T_k)} & \frac{\Delta N_{23}(T_k)}{Y_2(T_k)} & \cdots & \frac{\Delta N_{2p}(T_k)}{Y_2(T_k)} \\ \vdots & \vdots & \vdots & \cdots & \vdots \\ \frac{\Delta N_{p1}(T_k)}{Y_p(T_k)} & \frac{\Delta N_{p2}(T_k)}{Y_p(T_k)} & \frac{\Delta N_{p3}(T_k)}{Y_p(T_k)} & \cdots & -\frac{\Delta N_{p\bullet}(T_k)}{Y_p(T_k)} \end{bmatrix}$$

where the numerator of each off-diagonal entry, $\Delta N_{ij}(T_k)$, donates the number of transitions away from rating i to rating j at time T_k; the numerator of the diagonal entry, $\Delta N_{i\bullet}(T_k)$, counts the total number of transitions away from i at time T_k; the denominator of each entry, $Y_i(T_k)$, is the number of the exposed farms or farms at risk, that is, the number of farms at rating i right before time T_k.

So the diagonal entry counts, at any time T_k, the fraction of the exposed farms at rating i migrating away from that rating. And the off-diagonal entry counts the fraction of exposed farms at rating i migrating away from that rating to another specific rating j at the particular time $T_{k.}$ Note that the row sum of the matrix $\boldsymbol{I} + \Delta\hat{\boldsymbol{A}}(T_k)$ is equal to one, which conforms to the transition property. Also note that when there is only one transition between time s to t, $m = 1$, the non- homogeneous product-limit estimator reduces to the cohort method. Or in other words, the non-homogeneous method could be viewed as a cohort method applied to extreme short time intervals.

Comparison of Matrices under the Three Time Models

There are various ways of comparing matrices including L^1 and L^2 (Euclidean) distance metrics, and eigenvalue and eigenvector analysis, which are extensively introduced and discussed by Jafry and Schuermann (03-09).

In our study, we use the L^1 norm, which is simple but without less power in comparing the distance between two matrices. This evaluation criterion is derived as:

$$(9) \quad L^1 \text{ Norm} = \sum_{i=1}^{N}\sum_{j=1}^{N} | P_{A,i,j} - P_{B,i,j} |$$

L^1 norm gives the sum of the absolute value of difference between each corresponding entry of any two transition matrices.

Results

We have presented three different methods for estimating the farm transition matrix. In corporate finance studies, credit migration estimation is based on the widely used S&P database. These types of data are recorded on a quarterly basis so the three time models considered in this study could be applied to annual transition matrices and do some matrices comparisons. However, since our farm data are recorded annually, we cannot replicate here the approach used in corporate finance to derive the annual transition matrices. To force this method using the farm financial data in this study will produce identical matrices under the cohort method and non-homogeneous method. To resolve this issue, we have opted to derive biannual transition matrices, i.e. instead of one-year horizon in any two-year period, we used a two-year horizon. In this case, for the cohort method, there is only one discrete transition from the first year to the third year, while the continuous time models will produce two transitions within the two-year horizon.

Tables 1-3 present the average transition matrices of the eight biannual transitions from 1992 to 2001 for the three different methods. More specifically, for the cohort method, transition matrix 1 corresponds for the subset of observations for years 1992-1994 with transition rate calculated as the change from 1992 rating to 1994 rating. The rest of the transition matrices (numbers 2 to 8) are derived in a similar fashion with transition rates calculated based on the rating in the two endpoints of every three-year period. We use equation (3) to calculate the transition matrices. The average of these 8 matrices are calculated and reported in Table 1.

The two continuous methods use a similar procedure. The only difference is that, instead of grouping only two boundary years within a three-year period, we group all the three years together for realizing two continuous transitions to capitalize on equation (6) and (8) to derive the generator matrix for homogeneous method and $\Delta\hat{\boldsymbol{A}}(T_k)$ matrix for nonhomogeneous method. Then the two matrices are converted to transition matrices using equations (4) and (7). The same procedure is repeated 8 times and we present the average results in Tables 2 and 3 for the last two methods.

The results presented in Tables 1-3 reveal striking differences between transition probabilities reported in four decimal places obtained under the discrete and continuous time models. Firstly, there is measurable extreme migration from the top rating category to the lowest or the opposite extreme migration in the two continuous time models. As expected, the equivalent/counterpart measures of these entries in the cohort matrix are zeroes. Secondly, the retention rates using cohort method, except for category 1, are notably smaller than their corresponding measures in the other two continuous methods. This could be due to the fact that under the cohort method, we only count a migration as retention when the ratings at both ends of the time interval are the same. Any migration that starts from one category and ends up in another category is treated either as an upward movement or a downward movement. However, it could be possible that a migration starts from category *i* to category *j* somewhere within the time interval and retains there from then on to the end. Cohort method will not capture this probability for retention while the other two methods will capture it and count the latter part of the migration as retention.

Moreover, based on the continuous time model matrices, the estimator based on exponential of the generator and the non-parametric product-limit estimator are slightly different. This difference, however, is apparently much less compared to the difference between cohort and either of the two continuous time methods.

Using the L^1 norm, the differences between the average transition matrices presented in Tables 1-3 are quite distinct. The L^1 norms for the difference between the cohort methods and the duration method are 2.1086 and 2.0725 for the time-homogenous and time non-homogenous methods, respectively. Both of them are fairly larger than the L^1 norm for the difference between the duration methods, which only yields a difference of 0.3672 at the same scale level. For illustration purposes, Figure 4 compares L^1 norm difference between the pairs of methods among the cohort and the two duration methods in each bi-annual period.

In reality, it is hard to believe the plausibility and relevance of the time homogeneous model to farm businesses, especially considering the amount of uncertainty and risk involved in agricultural operations. Farm business performance could easily fluctuate from year to year due to influence of weather, technological change, and pests, among other things, on productivity. Farm businesses could also be more susceptible to swings in macroeconomic conditions that modify market environments that ultimately results in high price risks. Given these considerations, it is definitely convincing that transition probabilities for farm credit risk could be affected not just by the length or duration of migration, but also by the specific placement in time when the migration actually occurs. However, from our results we can see that imposition of the potentially unrealistic time homogeneity assumption does not significantly affect the result comparing to the one of relaxing the time-homogeneous assumption.

Conclusion

The increasing importance of the migration framework in the determination of the quality of farm credit portfolio creates the need to explore for alternative methods to develop more accurate measures of farm transition probability rates.   In this paper, we revisit the cohort discrete time method that has been conventionally used in the few empirical works, but we also introduce two new approaches based on a continuous time framework.   The application of two duration variants, i.e. the continuous time homogeneous and nonhomogeneous Markov chains, are introduced in this analysis

Our results in this study indicate that the Markov chain assumption, which is an important condition for the two duration variants, cannot be rejected, which therefore warrants the use of these two alternative time models. The resulting matrices developed both duration continuous time models provide richer, more detailed credit migration information that are usually undetected under the traditional cohort method. In addition, although the assumption of time homogeneity seems implausible, there is relatively little deviation between matrices developed using the two duration continuous time methods. In farm credit migration, however, we feel strongly that the non-homogeneous Markov Chain approach would be a more realistic and relevant model vis-à-vis the time-homogenous model that could provide more accurate, reliable and plausible estimates of the rate of change in credit risk ratings among farm borrowers across heterogeneous time periods.

Appendix

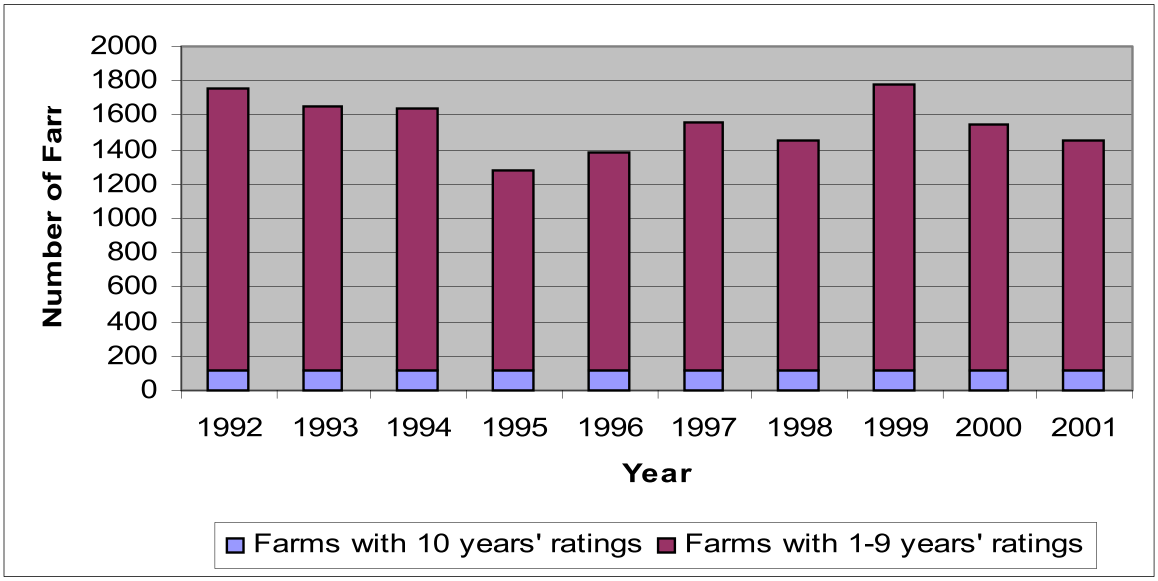

Figure 1: Evolution of Number of Farm Observations with Rating over Time

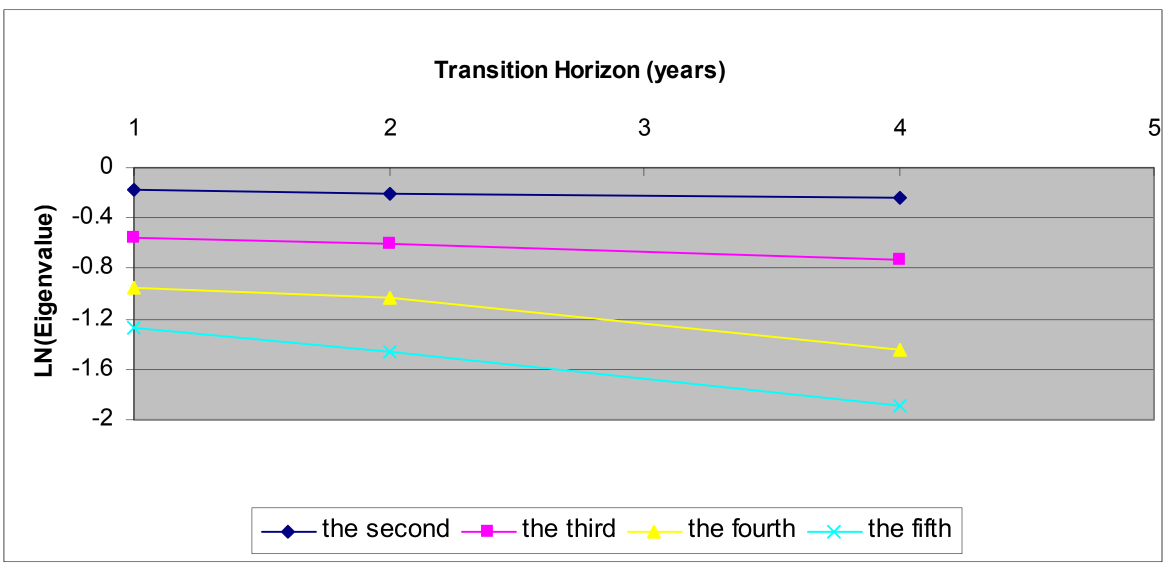

Figure 2: Decay of Eigenvalues with Transition Horizon

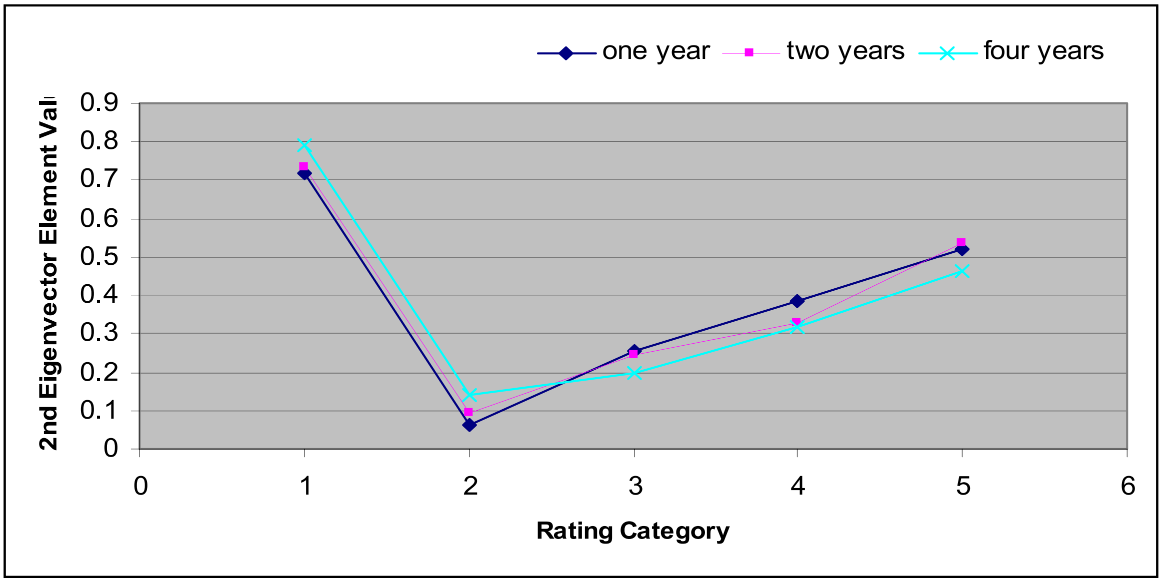

Figure 3: the 2^{nd} Eigenvector of Matrices with Transition Horizon

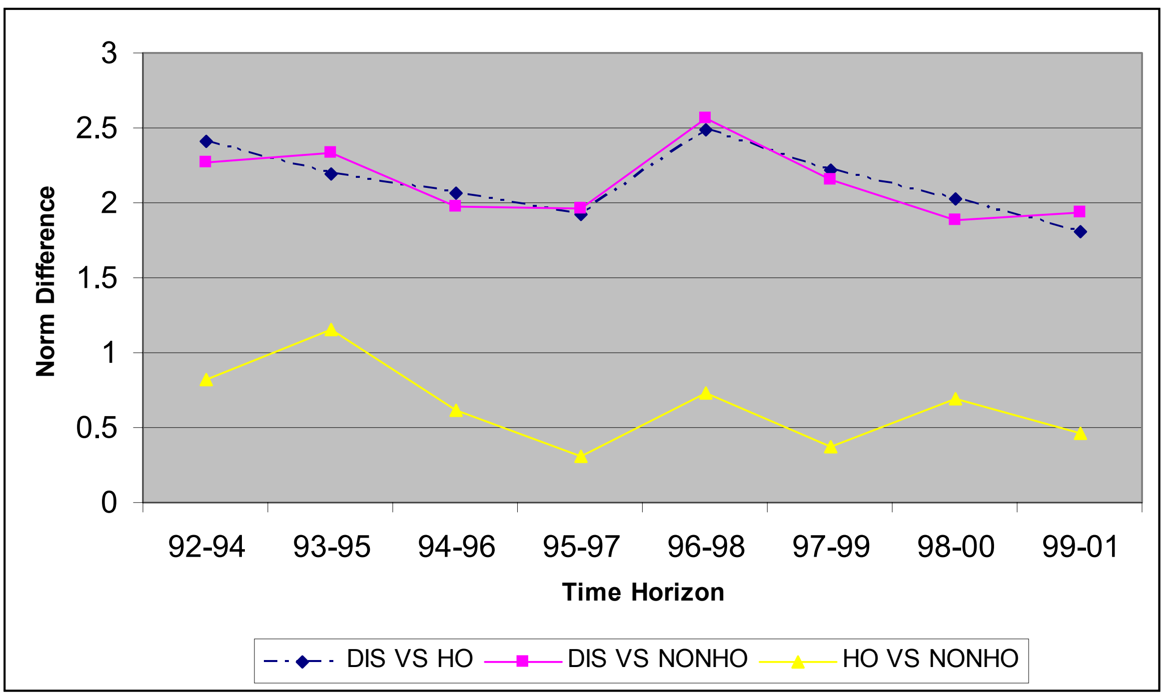

Figure 4: L^1 Norm Differences between Pairs of The Cohort and Two Duration Methods in Each Biannual period, 1992-2001.

Table 1: Average of 8 Biannual Transition Matrices, Each Estimated Using a Cohort Method in the Period 1992-2001.

	1	2	3	4	5
1	0.6381	0.2359	0.1136	0.0124	0.0000
2	0.1787	0.3825	0.3123	0.1107	0.0158
3	0.0597	0.2097	0.4557	0.2115	0.0634
4	0.0226	0.2007	0.4058	0.2629	0.1079
5	0.0000	0.0434	0.3883	0.3019	0.2664

Table 2: Average of 8 Biannual Transition Matrices, Each Estimated Using the Homogeneous Method in the Period 1992-2001.

	1	2	3	4	5
1	0.6984	0.1612	0.1086	0.0270	0.0049
2	0.1215	0.5681	0.2087	0.0831	0.0187
3	0.0528	0.1320	0.6465	0.1269	0.0417
4	0.0308	0.1191	0.2338	0.5576	0.0586
5	0.0128	0.0560	0.2329	0.1650	0.5333

Table 3: Average of 8 Biannual Transition Matrices, Each Estimated Using a Non-Homogeneous Method in the Period 1992-2001.

	1	2	3	4	5
1	0.7439	0.1485	0.0837	0.0199	0.0039
2	0.1310	0.5904	0.1930	0.0712	0.0144
3	0.0560	0.1515	0.6307	0.1198	0.0421
4	0.0316	0.1205	0.2628	0.5266	0.0585
5	0.0160	0.0741	0.2636	0.1631	0.4832

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