"Transaction chain approach to the regulation of the nonpoint water pollution from farms- runoff."

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We offer a decentralized solution to the asymmetric information and hidden action problems in the nonpoint source (NPS) pollution case. Farmers in the same watershed generate homogeneous NPS pollution. The regulator, R, pays for (or represents a group of point-source, PS, polluters who pay for) pollution reduction credits earned by the group of the farmers. To resolve the asymmetric information problem, R is concerned with only the total level of the abatement achieved, while the group of farmers (called the Association, A), undertakes responsibility to distribute the payment so as to induce farmers to deliver abatement. We show that A can devise an optimal contract to deal with the farmers' hidden action problem. We identify the restrictions under which such a policy can be implemented, evaluate its effects on the product market, and show that in the NPS case information rents are higher than in the PS case.
I. Introduction.

During the last decade the attention from policy-makers to market-based environmental policy instruments were increased dramatically.

T. Tomasi, K. Segerson and J. Braden provide a careful review and analysis of the role of the asymmetric information in NPS pollution control (1994). Within the NPS literature papers that address the implications of the asymmetric information fall into two groups, those that deal with the adverse selection problem and those that deal with moral hazard.

Results from studying the adverse selection problem include the following. The full information allocation can be attained if and only if net benefits to consumers exceed the informational rent that must be paid to firms in the incentive scheme (Spulber, 1989). Truthful reporting of type occurs under a mechanism that takes form of a tax and a management practice requirement after the message is received (Dosi and Moretto 1990).

Authors who addressed the moral hazard issue, in which input choices are not observed, include Segerson (1988), Dosi and Moretto (1990, 1992), and Xepapadeas (1991, 1992). Segerson proposed an incentive scheme similar to that of Holmstrom (1982) for problems of moral hazard in teams. “Correct short run marginal incentives can be achieved when each firm that is active in equilibrium pays the full marginal damages, rather than just that firm’s share of the damages.” This scheme solves free riding, but it is not budget balancing at margin. Segerson’s proposal was further developed by R Cabe (1992), R Horan (1998), Dosi and Moretto (1990, 1992) and others. Xepapadeas (1991) presents a dynamic moral hazard model. He first solves for the optimal time path of abatement and pollution by solving a social planner problem in the absence of moral hazard. Then, he assumes that the planner cannot observe abatement, but can observe firms’ type, so he searches for optimal contracts between the social planner and polluters. One of those contracts is similar to Segerson’s, another includes the stochastic system of fines, based on those proposed by Rasmusen (1987) for moral hazard problem in the teams.

Each of those approaches has advantages and disadvantages, and each of them is more suitable for some situations, then others. Policymaker has to choose one specific tool out of variety of available. This paper presents the algorithm how the policy maker can act to design the regulatory policy, which is feasible and efficient, and evaluate restrictions under which the policy can be applied.
The nonpoint water pollution problem from farms’ runoff was considered as an example. Several farmers live in the same watershed and produce homogeneous nonpoint pollution as a by-product of their production process. The regulator knows only the distribution of parameters of cost of abatement functions, while farmers have full knowledge about each other’s cost functions. However they do not know each other’s realized costs.

Basic results are following. First, we argue that, given the long-standing political preference for subsidizing agriculture rather than regulating it, a permit trading system should be based (at least initially) on the recognition of that status quo: farmers would be invited to enter the market as sellers of pollution reduction credits, not buyers of pollution permits. Then we show how the asymmetric information and hidden action problems can be solved through decentralization. In our scheme, the regulator pays for, and is concerned with, only the total level of the abatement achieved by the group of the farmers. The responsibility to distribute the payment and encourage farmers to deliver abatement performance is vested in the group of farmers (called the Association), which can use the informational advantages that farmers in the same watershed enjoy. Given this assignment of responsibility, the Association needs to take care about the hidden action problem. It is shown in the paper that there exists an optimal contract, which farmers can use to deal with the hidden action problem. In the standard principal-agent setting Groves mechanisms are used to solve the hidden action problem. In general those mechanisms do not satisfy budget-balancing constraint. It is shown in this paper that, given restrictions on domain of farmers’ reports, it is possible to design the mechanism, which is not Groves, but incentive-compatible, efficient and budget balancing.

Presentation of the material in the paper will be based on the Transaction Chain Approach, which we developed, and is organized as following: in Section II, the idea of “transaction chain” approach is presented in short; in Section III, IV, and V, this approach is applied to the nonpoint pollution problem, in particular, the justification for the subsidy scheme, the contract between the Regulator and the group of farmers, and the contract among members of group are introduced; finally, in Section VI, it is evaluated how such regulation will affect market outcome, and under which condition it is reasonable to apply it.

II. Transaction chain approach.

The Transaction Chain Approach (TCA) will be used below to solve the nonpoint pollution problem. Here we present the approach itself.

This approach can be considered as a way to reformulate the problem so it will look clearer for the Regulator how to solve the problem.

TCA is based on two ideas. The first idea is that the typical market transaction consist of
three separate steps: (1) property rights or initial endowment assignment; (2) price/quantity determination; (3) optimal design of the exchange process. Figure 1 illustrates this idea.

For transaction to be completed it should go through all three steps in order, like through rings in the chain. This gave the name to the approach.

The second idea is that the policy maker can manipulate the outcome of the market by using specific policy tools on each step of the market transaction. For example, initial endowment can be fixed in the form of the property rights by specific law, which insures legal protection of those rights; or it can be supported just through the choice of the subsidies versus tax.

Figure 1. Transaction chain.

On the second step, when the price and quantity of the product are to be determined, the Regulator can simply pick some price and quantity; he can allow free trade in the market; or he can participate in the trade as a buyer, a seller, or a rule maker. On the third step the Regulator can introduce either restrictions on existing mechanisms of exchange or new forms of those mechanisms. Table 1 below summarizes possible policies, which can influence each step of the transaction chain.

Table 1. Policies.

<table>
<thead>
<tr>
<th>Transaction steps</th>
<th>Property rights/ initial endowment assignment</th>
<th>Price/quantity determination</th>
<th>Optimal design of the mechanism of exchange</th>
</tr>
</thead>
<tbody>
<tr>
<td>Kind of policy</td>
<td>Laws on property rights. Legal protection of property rights, etc.</td>
<td>Tax, subsidies, Government as a buyer/seller, restrictions on trade, etc.</td>
<td>Restrictions on contracts, government as a contractor, mechanism provision (like taxation of auctions, and etc.)</td>
</tr>
</tbody>
</table>
At the end it is necessary to perform a welfare analysis, to determine how related markets would be affected by the policy, and are there any restrictions for the policy implementation. Each time, when the Regulator uses a policy, he changes the market situation. Therefore, after he went through all transaction steps once, he has to go back to the beginning of the chain and check whether applied earlier policies changed the circumstances.

It is important to understand that the regulation process must be iterative – the market transaction is a continuous process, and changes in any step of this process can, and most probably will, affect other steps of the process. For example, it might be the case that optimal policy to the second step is conditional on optimal policy to the third step. So, the Regulator has to fix the third step and then go back to the second. Figure 2 illustrates what the policy maker is supposed to do if he uses the TCA.

**Figure 2. Algorithm for the policy maker.**

TCA does not offer something entirely original. However it helps to subdivide the complicated problem into smaller parts, and this makes it easier to find the solution, and the same time to always remember the general picture and not to be carried away with important but small details of the problem. Sometimes it is enough to fix just one step of the transaction chain in order to solve the whole problem. So the fact that the process is iterative will insure that the same problem would not be solved two times. Another advantage is that TCA gives the opportunity to watch how complication in the model for one step of the transaction will affect the whole picture, which helps to keep track of all consequences of new assumption of the model. Finally, it helps to combine the use of different tools of the economic analysis, from general market analysis to optimal contract theory and game theory, while moving from the aggregate market level of the problem to the individual level and back.

The next section illustrates how TCA works on the example of nonpoint pollution from farms’ runoff problem.
III. Application: Use TCA to solve the nonpoint pollution problem.

The nonpoint pollution from farms’ runoffs problem can be formulated as following.

There are $n$ farmers in the watershed, who produce some kind of output and homogeneous pollution as a by-product. Farmers are assumed to be profit maximizers. Water consumers receive negative utility from polluted water. However, farmers are not only practically unaffected by pollution produced by them, but also any reduction of pollution is costly for them. The problem the Regulator faces is how to decrease the pollution level from the watershed to the level A.

Assumptions are following: farmers know parameters of their cost functions. The Regulator knows the distribution of those parameters, but he does not know which farmer has which parameters. In this paper it is assumed that there is no weather uncertainty.

It is reasonable to assume that “real life” cost of abatement function satisfies the following conditions:

1. $C_i(0) = 0$; It costs nothing not to produce abatement;
2. $C_i(a_{\text{max}}) = \infty$, where $a_{\text{max}}$ is maximal possible abatement, which can be considered as a point of “zero pollution”. It is intuitive, that any production process will always produce some, may be very small level of pollution, and therefore cost of last unit of pollution is infinitely high;
3. if $a_i > a_j$, then $MC(a_i) > MC(a_j)$, it cost more to produce next unit of abatement then previous one.

In this model a cost of abatement function is approximated by quadratic function, hence the cost of abatement function for the farmer $i$ is

$$C_i = \frac{1}{2} \alpha_i a_i^2 + \beta_i a_i$$

where $C_i$ - cost of abatement function for farmer $i$ ;

$\alpha_i, \beta_i$ - parameters of farmer $i$’s cost of abatement function;

$a_i$ - farmer $i$th’s level of abatement.

Given the form of the cost function assumptions can be rewritten as:

1. farmer $i$ knows $\alpha_i, \beta_j \forall j = 1, ..., n$ ;
2. Regulator knows distribution of $\{\alpha_i\}_{i=1}^{n}$ & $\{\beta_i\}_{i=1}^{n}$

Below the TCA will be used to solve the nonpoint pollution from farms’ runoff problem.
**STEP 1. Property rights/initial endowment assignment.**

First, the kind of the product should be defined. Define the product as clean water. Whoever owns it can do with it whatever he wants, pollute or keep it clean.

In the case of the nonpoint pollution (or a least at the stage this problem exist now) only the initial endowment of the product (not property rights), i.e. the point from which the trade starts, will be assigned. It should be assigned in terms of proportion of clean water to polluted water. We choose that the trading starts from the status quo for farmers. If the water users want to reduce the level of the pollution, they have to buy the clean water from farmers. Assume for now that regulatory agency will act as a buyer on behalf of government, nonpoint polluters, or any other water user. Therefore farmers will be subsidized to reduce pollution.

This manner to assign the initial endowment seems to be reasonable in the initial stage of pollution regulation, since it does not use any punishment, and therefore does not create any incentives to hide true level of pollution, or fact that somebody is producing the pollution. In contrary, farmers will want to reveal themselves, since they get extra profit from producing abatement. On the other hand the Regulator does not need to know exact number of people, involved in the pollution process, which often is impossible to know. He is going to pay people, who identify themselves as polluters.

This is not the only way to assign the initial endowment, however once choice was made next steps of policy design will consider instruments, which can be used within class of subsidies.

**STEP 2. Market trade: quantity and price determination.**

In this model it is assumed that the Regulator picked some level of abatement A, which he wants to get from the watershed. (This assumption can be replaced by using the benefit of abatement function, which is the same as Regulator’s demand for abatement function.)

The assumption in the model is that the Regulator knows the distribution of the parameters of the cost function; therefore he knows the supply of abatement from the group of the farmers’ function, given that they choose the best effort.

Therefore, the optimal price/quantity can be determined directly from the equation $\text{Supply} = A$.

Since the regulator can measure the total level of abatement, then if he pays for this abatement to the group of farmers who decide to participate, then he does not face the hidden action problem. Farmers have to distribute this payment among themselves. However he should be concern with farmers’ decision to accept his offer. So it is his problem to create incentives for farmers to join the Association and to design the structure of the Association, so farmers would believe that this type of contract works.
The regulator has to announce payment as a function of total abatement and offer the sample mechanism of how Association can distribute total payment among its members. Then farmers, who choose to accept the offer have to announce themselves.

In this model it is proposed to use quadratic function for the total payment. Then price for abatement is upward sloping linear function: \( P = \gamma u + \eta \), where \( a \) – is a total abatement and \( \gamma \) & \( \eta \) are parameters. Therefore price for the abatement will be \( P^A = \gamma A + \eta \), and total payment is \( A(\gamma A + \eta) \).

Upward sloping price line creates additional incentives for farmers to stay in the Association, since payment per unit of abatement, and therefore total payment to each farmer depends on actions of others. Moreover, price increases in number of members of Association, so it is in best interests of every participant to act so nobody would want to leave the Association, but new would join the group.

Total payment depends on parameters \( \gamma \) & \( \eta \). Higher values of those parameters attract more farmers with higher abatement costs, lower values will decrease number of participants to those with lower costs. So the Regulator can control number of the farmers in the Association.

Farmers’ decisions on whether to produce abatement or not and how much, besides parameters of the contract offered by the Regulator, depends on how exchange abatement/money is designed. Given design of payment distribution inside the Association, the Regulator can choose the optimal number of participants – number which will produce desirable level of abatement \( A \) at lowest possible cost.

In mathematical terms the Regulator’s problem is:

\[
\text{Min} \ \ a(\gamma u + \eta) \\
\text{subject to} \quad (1) \ \ \text{design of exchange} \\
\quad (2) \ a = A
\]

**STEP 3. Optimal design of mechanism of exchange.**

Problem to be solved is the principal—multiple agents. Since the Association knows parameters of each member cost function, it can calculate and assign to each member the optimal level of abatement \( a_i^* \). However the hidden action problem remains. It can be solved by designing a mechanism which is efficient, individually rational, incentive compatible in dominant strategies, and budget balancing.

Efficiency condition guaranties that the first best would be achieved. Individual rationality insures that farmers will participate in the program, incentive compatibility in
dominant strategies secures against uncertainty, budget balancing should hold, since only money that are received from the Regulator can be distributed among members of the Association.

If all those conditions hold, then farmers will join the Association and produce optimal level of abatement under both certainty and uncertainty. However, since some administrative structure will be build there is a possibility for rent seeking behavior on behalf of the Association. If that would happen, then farmers might decide to leave the Association. Therefore impossibility of rent-seeking behavior is an additional condition on the mechanism.

There are also some informational restrictions. Since Association knows only parameters of the cost functions, total level of abatement, and parameters of Regulator’s offer, then distribution rule can depend only on those variables.

The same time farmers who decided to join the Association have an incentive to stay in the Association and keep additional members, since given upward sloping contract curve sum of payments to two farmers, produced abatement separately, is less then total payment those farmers will get, if they form the Association.

The Solution to this problem is given in the Section IV.

STEP 4. Determination of optimal parameters from Regulator’s problem.

Once the Regulator learns how the Association is going to respond to the contract he offers, he can set and solve his problem. From the solution to the Association problem the Regulator understands that parameters of the contract curve simultaneously determine number of farmers in the Association and level of abatement they are going to produce. Therefore, the Regulator’s problem can be rewritten as following:

$$\text{Min}_{\eta_\gamma} a(\gamma a + \eta)$$

s.t. (1) participation const., \(k\) farmers in the Association

(2) Association's response const., Association produces \(a\)

(3) \(a = A\)

The participation constraint insures that exactly \(k\) farmers decide to join the Association; the Association’s response constraint shows how much abatement those \(k\) farmers will produce.

The Regulator’s goal is to achieve \(a = A\) at minimum cost. The same time it does not matter for him, how many members are in the Association. So he can choose, by setting parameters, optimal number of farmers in the Association, i.e. the number, which will produce required level of abatement at minimal for the Regulator cost. As soon as parameters of the contract curve are determined, the optimal price for a abatement is determined. Section V solves this problem.
Hence, a new regulation policy for nonpoint pollution from farms’ runoff is designed. Now it is necessary to evaluate effect of this policy on related markets, and restrictions, which those effects may impose on the policy implementation.

**STEP 5. Analysis of effects of this policy on related markets.**

Farmers produce pollution as a by-product of production of good $Q$. Quantity of this good depends on pollution level. Reduction of the pollution will change quantity of the good produced, so the product market will be heavily affected. Therefore before the policy is implemented the Regulator needs to analyze those effects and determine if the product market impose any constraints.

Since the Regulator designed the structure of the Association so it would be stable, and he can measure abatement from the Association, then he can treat the Association as a point polluter. Therefore an analysis similar to the one in Spulber paper 1987 can be performed. This analysis is presented in the Section VI.

**IV. The Association’s problem.**

There are $n$ farmers in the watershed. They can produce abatement. The cost of abatement function for the farmer $i$ is $C_i = \frac{1}{2} \alpha_i a_i^2 + \beta_i a_i$, where $\alpha_i, \beta_i$ are parameters of the cost function and $a_i$ is abatement, produced by farmer $i$. Farmers form the Association. Association receives payment for the total level of abatement $a \cdot (\gamma a + \eta)$. It has to distribute payments $t_i$ among its members – farmers.

This problem solved in the Pushkarskaya, Randall 2002. Solution is the contract of the form:

$$t_i = (\alpha_i, a_i^*, \beta_i) \left\{ \text{Min} \left\{ a - \sum_{j \neq i} a_j; \left\| 2a^* \frac{k-1}{k} - \sum_{j \neq i} a_j^* - a \frac{k-2}{k} \right\| \right\} \right\} \frac{2a^*}{k};$$

With following notation:

- $k_i$ - number of members in the Association
- $t_i$ - $i^{th}$ Farmer payment
- $a$ - The total level of pollution reduction
- $a_i$ - $i^{th}$ Farmer level of pollution reduction
- $c_i$ - $i^{th}$ Farmer cost of pollution reduction function
- $\alpha_i, \beta_i$ - Parameters of the cost of abatement function $C_i(a_i)$
$\pi_i$ - $i^{th}$ Farmer profit function

$\gamma, \eta$ - Parameters of "contract"

$\{a_i^*\}_{i=1,...,k}$ - Is the solution of the system of equations:

$$2 \times \gamma \times a + \eta = \alpha_i \times a_i + \beta_i \quad \forall i = 1,...,k$$

Each farmer is assigned $a_i^*$ - level of abatement he has to produce in order for the outcome to be efficient. All $a_i^*$ are calculated by the Association according to the following rule $\alpha_i a_i + \beta_i = 2 \gamma a + \eta$, where $a = \sum_{i=1}^k a_i$ for all $i=1,...,k$. Each farmer chooses individually level of abatement $a_i$ he is going to produce, and $\sum_{i=1}^k a_i = a$. However, only total level of abatement can be detected, as a result individual transfers can be functions of the total level $a$. The same time farmers’ cost functions are functions of their individual $a_i$. So utilities are functions of both $a$ & $a_i$: $u_i(a, a_i, t_i, a_i^*)$.

Term $\text{Min}\left\{a - \sum_{j \neq i} a_j^* \left| 2 a_i \frac{k-1}{k} - \sum_{j \neq i} a_j^* - a \frac{k-2}{k}\right.\right\}$ secures BB around the optimum.

That is if $a^* \neq a$, then $\sum_{i=1}^k t_i(a_i) < a(\gamma a + \eta)$, with

$$\text{Min}\left\{a - \sum_{j \neq i} a_j\left| 2 a_i \frac{k-1}{k} - \sum_{j \neq i} a_j^* - a \frac{k-2}{k}\right.\right\} = a - \sum_{j \neq i} a_j^*, \text{ if } a < a^* - \text{ case of the underproduction}$$

and

$$\text{Min}\left\{a - \sum_{j \neq i} a_j\left| 2 a_i \frac{k-1}{k} - \sum_{j \neq i} a_j^* - a \frac{k-2}{k}\right.\right\} = 2 a_i \frac{k-1}{k} - \sum_{j \neq i} a_j^* - a \frac{k-2}{k}, \text{ if } a > a^* - \text{ case of the overproduction}.$$}

Note that for both underproduction and overproduction the difference between payment from the Regulator and sum of all transfers to farmers are equal, i.e.

$$(a^* + \Delta)(\gamma(a^* + \Delta) + \eta) - \sum_{i=1}^k t_i(a_i^* + \Delta_i) = (a^* - \Delta)(\gamma(a^* - \Delta) + \eta) - \sum_{i=1}^k t_i(a_i^* - \Delta_i),$$
where $\Delta_i = |a - a^*$| and $\sum_{i=1}^{k} \Delta_i = \Delta$. Those differences are represented on the picture by the shaded areas.

Figure 3. Optimal contract.

So, all other agents are also “punished for the $i^{th}$ agent underproduction by the $(\alpha, a^* + \beta_j) \Delta_i$. Note that $\alpha_i a^*_j + \beta_j = \alpha_i a^*_i + \beta_i = 2\gamma a^* + \eta \forall i \& j = 1,...,k$. That means that transfers to each farmer is reduced by equal amount. If agent i underproduceced, then his cost will be less, therefore he is punished by losing only $\alpha_i a^*_i \Delta_i$, and all other agents are punished equally, each will lose $(2\gamma a^* + \eta)\Delta_i$. This mechanism is similar to the one proposed by Segerson 1988. The difference is that in Segerson’s case polluters are punished by tax, and in this case polluters are punished by reduction in award. Given such an incentives scheme one will expect, that members of the Association can create some kind of the monitoring among members. But this issue is not addressed in this paper.

In the case of the overproduction by the one agent in the group again everybody will be punished equally.
If several members of the group will deviate from assigned level, then each member of the group will be underpaid by \( (2\gamma a^* + \eta)\Delta \), where \( \Delta = \sum_{i=1}^{k}\Delta_i \).

The difference between payment from the Regulator to the Association and sum of the transfers to agents is \( (2\gamma a^* + \eta)\Delta k \forall \Delta \). That means that BB holds for any allocation. The fact that constraints IC and IR also hold is proven in the appendix 1.

To understand how each farmer is paid in the case when all members of the Association produce optimal abatement we rewrite the payment \( t_i \) in the form:

\[
t_i = (\gamma a^* + \eta)a_i^* + \gamma a^* (a_i^* - \frac{a^*}{k}).
\]

Therefore, the contract, which the Association creates is such as: “total payment is the sum of “payment for absolute performance” - \( (\gamma a^* + \eta)a_i^* \) -price of level of pollution times amount by which \( i^{th} \) farmer reduce pollution, and of “payment for relative performance” - \( \gamma a^* (a_i^* - \frac{a^*}{k}) \) - marginal change in price times difference between farmer \( i^{th} \) performance and average performance in the group”; Each farmer benefits from decrease of the cost of any other member of the Association. Therefore they have incentive to share their cost-decreasing innovations.

V. The Regulator’s problem.

As it was stated in the Section III, the Regulator’s problem is:

\[
\text{Min } a(\gamma a + \eta)
\]

s.t. (1) participation const.

(2) Association’s response const.

(3) \( a = A \)

The Association’s respond constraint should show how much of abatement will be produced for each set of parameters \( \gamma \& \eta \). From the Section IV it is known that Association will choose \( a \) such that condition \( \alpha, a_i + \beta_i = 2\gamma a + \eta \), where \( a = \sum_{i=1}^{k}a_i \) holds \( \forall i \).
\[ \alpha a_i + \beta_i = 2\gamma a + \eta, \text{ where } a = \sum_{i=1}^{k} a_i \Rightarrow \]
\[ a_i = \frac{2\gamma a + \eta - \beta_i}{\alpha_i} \Rightarrow \]
\[ \sum_{i=1}^{k} a_i = \sum_{i=1}^{k} \frac{2\gamma a + \eta - \beta_i}{\alpha_i} \Rightarrow \]
\[ a = 2\gamma a \sum_{i=1}^{k} \frac{1}{\alpha_i} + \eta \sum_{i=1}^{k} \frac{1}{\alpha_i} - \sum_{i=1}^{k} \frac{\beta_i}{\alpha_i} \Rightarrow \]
\[ \left( \sum_{i=1}^{k} \frac{1}{\alpha_i} = \lambda, \text{ and } \sum_{i=1}^{k} \frac{\beta_i}{\alpha_i} = \mu \right) \Rightarrow \]
\[ a = 2\gamma a \lambda + \eta \lambda - \mu \Rightarrow \]
\[ a = \frac{\eta \lambda - \mu}{1 - 2\gamma \lambda} \]

Therefore, the Regulator knows that if he sets parameters \( \gamma \) & \( \eta \), then the Association will produce \( a = \frac{\eta \lambda - \mu}{1 - 2\gamma \lambda} \), where \( \lambda \) & \( \eta \) are parameters, which characterize the whole group of farmers in the watershed.

The participation constraint should ensure that exactly \( k \) farmers decide to join the Association. Assume that there are many farmers in the watershed. Let’s rank-order them by their cost functions. That means that \( \alpha_1 < \alpha_2 < \ldots < \alpha_k < \ldots < \alpha_n \) & \( \beta_1 < \beta_2 < \ldots < \beta_k < \ldots < \beta_n \). Each farmer is going to produce \( a_i^* \) such that \( \alpha_i a_i^* + \beta_i = 2\gamma a^* + \eta \). Then last farmer, who decide to join the Association is going to be marginal – for him it does not matter to be in the Association or not. That means that \( \pi_k (a^*_k) = t_k (a^*_k) - C_k (a^*_k) = 0 \). Therefore, the following condition must hold:
\[
\pi_k(a^*_k) = (\alpha_k a^*_k + \beta_k) \left( \min \left\{ a^* - \sum_{j \neq k} a^*_j ; 2a^* - a^* - \sum_{j \neq k} a^*_j \right\} \right) - \frac{\gamma a^*}{k} - \frac{1}{2} \alpha_k a^* - \beta_k a^*_k = 0 \Rightarrow
\]

\[
(\alpha_k a^*_k + \beta_k) a^*_k - \frac{\gamma a^*}{k} - \frac{1}{2} \alpha_k a^* - \beta_k a^*_k = 0 \Rightarrow
\]

\[
\Rightarrow \frac{1}{2} \alpha_k a^* - \frac{\gamma a^*}{k} = 0 \Rightarrow
\]

\[
\Rightarrow 2\gamma a^* = k\alpha_k a^* \Rightarrow \gamma = \frac{k\alpha_k a^*}{2a^*} \Rightarrow
\]

\[
\Rightarrow \left\{ \text{Since} \ \alpha_k a^*_k + \beta_k = 2a^* + \eta, \ \text{then} \ a^*_k = \frac{2\gamma a^* + \eta - \beta_k}{\alpha_k} \right\} \Rightarrow
\]

\[
\Rightarrow \gamma = \frac{k(2\gamma a^* + \eta - \beta_k)^2}{2a^* \alpha_k}
\]

Substitute the total level of abatement from the Association’s response constraint.

\[
a^* = \frac{\eta \lambda - \mu}{1 - 2\gamma \lambda}, \text{where} \ \lambda^k = \sum_{i=1}^{k} \frac{1}{\alpha_i} \ \& \ \mu^k = \sum_{i=1}^{k} \beta_i
\]

Then, if the following condition holds, then there are exactly \( k \) farmers in the Association:

\[
2\gamma \alpha_k \left( \frac{\eta \lambda - \mu}{1 - 2\gamma \lambda} \right)^2 = k(2\gamma \left( \frac{\eta \lambda - \mu}{1 - 2\gamma \lambda} \right) + \eta - \beta_k)^2.
\]

The Regulator’s problem can now be rewritten as following:

\[
\min_{\gamma, \eta} a(a \gamma a + \eta)
\]

s.t. (1) \( 2\gamma \alpha_k \left( \frac{\eta \lambda - \mu}{1 - 2\gamma \lambda} \right)^2 = k(2\gamma \left( \frac{\eta \lambda - \mu}{1 - 2\gamma \lambda} \right) + \eta - \beta_k)^2 - \text{participation const.} \)

(2) \( a = \frac{\eta \lambda - \mu}{1 - 2\gamma \lambda} - \text{Association’s response const.} \)

(3) \( a = A \)

The Regulator’s problem can be solved the following way. First, one needs to solve constraints. The solution to this system of equations is the optimal \( \gamma^k \) & \( \eta^k \) for each \( k \)-number of members in the Association. Therefore, \( \gamma^k \) & \( \eta^k \) can be considered as functions of \( k \). Exact solution for optimal \( \gamma^k \) & \( \eta^k \) is given in the appendix 1.

Second, one has to substitute optimal \( \gamma^k \) & \( \eta^k \) into objective function and solve for the
optimal number of members in the Association:

\[ \text{Min}_k \ A(\gamma^k A + \eta^k). \]

Solution to this part of the problem depends on the distribution of parameters of cost of abatement functions.

VI. Analysis of effects of the policy on product market.

This part of the paper analyzes effects of such a contract on product market and Social welfare. Analysis is similar to the one presented by Spulber (1987) as well as results: policy may be implemented if net benefits from the product market must exceed the cost of inducing truth-telling by firms. Since regulator has less information about polluters in NPS setting then in PS setting, information rents are higher in the NPS then in PS setting. So net gain from regulation of NPS polluters must be higher then net gain from regulation of PS pollution.

III.2. Formal model.

A competitive market for a good \( Q \) supplied by \( n \) non-point polluters. Each firm \( j \) produces output \( q^j \) and discharges effluents \( e^j \). Market inverse demand is given by \( P = P(Q) \), where \( Q = \sum_{j=1}^{m} q^j \) is total output. External damages are \( G(A) \), where \( A \) is total level of abatement produced by firms. Market inverse demand, \( P \), is continuously differentiable, positive, and downward sloping. The damage cost function, \( D \), is twice differentiable, positive, increasing and convex for \( X > 0 \).

A firm of type \( j \) has a cost function

\[ C(q^j, e^j, \theta^j) = \frac{\sigma}{2} (e^j)^2 + (q^j)^2 - \xi e^j q^j - \theta^j e^j \quad \text{for } j = 1, \ldots, m \]  

(1)

Before any regulation was used there is equilibrium in the market with equilibrium price \( P^e \), total equilibrium quantity \( Q^e \), and individual equilibrium quantities \( q^{ie} \). The regulator knows all of them.

Since the market is competitive, then \( P^e = MC^e = \sigma q^{ie} - \xi q^{ie} \). Therefore, regulator can calculate \( e^{ie} \)

Let \( n < m \) polluters work in the same watershed.

And let cost of abatement function (which is total lost profit due to abatement) be of the form \( C^a(a) = \frac{1}{2} \alpha_i a_i^2 + \beta_i a_i \).
If the regulator contracts with the polluters in the watershed, then he can get abatement at the level $A$ for the cost of $A(\gamma A + \eta)$, where $\gamma$ & $\eta$ are functions of the distribution of the parameters of the cost of abatement functions $\{\alpha, \beta\}$, $i = 1,\ldots,n$.

As a result of this policy, quantity of product supplied are reduced, so equilibrium total quantity is reduced and equilibrium price is increased.

New price is equal new MC. It is possible to express MC and so $P$ as a function of abatement.

$$ P = \sigma q^j - \xi (e^{\gamma e} - a_j) \quad \text{individual supply functions;} $$

$$ q^j = \frac{1}{\sigma} \left( P + \xi (e^{\gamma e} - a_j) \right) \quad \text{individual quantity supplied;} $$

$$ Q^A = \frac{1}{\sigma} \left( Pk + \xi \left( \sum_{j=1}^{k} e^{\gamma e} - A \right) \right) \quad \text{quantity, supplied by the Association;} $$

$$ P = \frac{1}{k} \left( \sigma Q^A - \xi \sum_{j=1}^{k} e^{\gamma e} + \xi A \right) \quad \text{supply from the Association.} $$

Therefore we can consider product market with demand $P(Q) = P(\sum_{j=k+1}^{m} q^j + Q^A)$, and supply functions: $P = \sigma q^j - \xi e^j$ for $j=k+1,\ldots,m$ and $P = \frac{1}{k} \left( \sigma Q^A - \xi \sum_{j=1}^{k} e^{\gamma e} + \xi A \right)$.

Combining the demand and supply, we can solve for market equilibrium outputs:

$$ \begin{align*}
q^j &= q^j (e^{k+1},\ldots,e^m, A) \text{ for } j = k+1,\ldots,m; \\
Q &= Q(e^{k+1},\ldots,e^m, A)
\end{align*} $$

We can calculate effects of effluents and $A$ – level of abatement, which the regulator bought from the Association.

$$ \begin{align*}
P'(\sum_{j=k+1}^{m} q^j + Q^A) \left( \sum \frac{\partial q^j}{\partial A} + \frac{\partial Q}{\partial A} \right) &= \sigma \frac{\partial q^j}{\partial A} \text{ for } j = k+1,\ldots,m; \\
P'(\sum_{j=k+1}^{m} q^j + Q^A) \left( \sum \frac{\partial q^j}{\partial A} + \frac{\partial Q}{\partial A} \right) &= \frac{\sigma}{k} \frac{\partial Q^A}{\partial A} + \frac{\xi}{k}
\end{align*} $$

And also conclude, that a firm’s market equilibrium output is increasing in the $A$, but output, produced by the Association is decreasing in $A$. Equilibrium price has increased due to abatement, and equilibrium quantity has decreased in pollution.
III.3 Effect of the policy on Social Welfare.

The change in the social welfare is equal to sum of the change of the Consumer Surplus \( \Delta CS \), change in Producer Surplus \( \Delta PS \), and environmental gain.

Denote new equilibrium price as \( P(e^{k+1},...,e^m,A) \), and new equilibrium output as \( Q(P(e^{k+1},...,e^m,A)) \).

Consumer surplus then will decrease by
\[
\Delta CS = \int_{p'} Q(P(e^{k+1},...,e^m,A)) \, dp'
\]

Producer surplus will increase, since members of the Association are compensated for any loss in profit, and all others enjoy increase in both price and quantity by \( \Delta PS \).

Regulator should implement the policy, if
\[
G(A) - \Delta CS(A) + \Delta PS - A(\gamma A + \eta) \geq 0.
\]
\[
G(A) \geq \Delta CS(A) - \Delta PS + A(\gamma A + \eta).
\]

Term \( A(\gamma A + \eta) \) plays the role of informational rents. Since regulator knows almost nothing about producers, then he has to compensate producers for all their losses. This rents are greater then those in PS setting.

Similar analysis will show that if a point polluter, who was taxed for exceeding some specific level of pollution is allowed to buy an abatement from nonpoint polluters in the same watershed, and he will use the same contract as was designed for the Regulator above then cost of abatement from the watershed will be reduced and Social Welfare will increase.

IV. Advantages/disadvantages.

IV.1 Advantages.

This paper presents both an approach to the nonpoint pollution, which as well can be used to any other regulatory problem, and a first-best solution within that approach.

TCA does not offer something entirely original. However it helps to subdivide the complicated problem into smaller parts, and this makes it easier to find the solution, and the same time to always remember the general picture and not to be carried away with important but small details of the problem. Sometimes it is enough to apply a policy to one step of the transaction chain and let the market solve the rest of the problem.

TCA requires that each proposed policy is accompanied by restriction on implementation of the policy by related markets. Therefore different policies can be compared by restrictions under which they can be applied and their effects on Social Welfare.
Another advantage is that TCA gives the opportunity to watch how complication in the model for one step of the transaction will affect the whole picture, which helps to keep track of all consequences of new assumption of the model.

Finally, it helps to combine the use of different tools of the economic analysis, from general market analysis to optimal contract theory and game theory, while moving from the aggregate market level of the problem to the individual level and back.

Solution to the nonpoint pollution problem obtained using TCA achieves first best in the presence of both asymmetric information and hidden action.

It is shown that regulated point polluters can use the contract designed in this paper to buy abatement from nonpoint polluters. This will increase the Social Welfare and expend the range of market situations, where such policy can be applied.

VI.2. Disadvantages.

First, since it is proposed in the model to use subsidies instead of tax, such a policy can be used only when expected environmental gain is big enough to cover cost of the project and losses in product market.

Second, informational requirement are still very high. In this model the Regulator needs to know distribution of parameters of the farmers’ cost functions, and polluters need to know their and each other’ costs of abatement functions However, if the same problem will be considered in the dynamic setting, then the Regulator and polluters can learn over time.

Appendix 1.

From appendix 2 it is known that
\[ a = \frac{\eta \lambda_j - \mu_j}{1 - 2\gamma \lambda_j} > 0 \]

\[ \eta \lambda_j - \mu_j = \eta \sum_{i=1}^{k} \frac{1}{\alpha_i} - \sum_{i=1}^{k} \frac{\beta_i}{\alpha_i} = \sum_{i=1}^{k} \frac{\eta - \beta_i}{\alpha_i} \]

\[ 1 - 2\gamma \lambda_j = 1 - 2\gamma \sum_{i=1}^{k} \frac{1}{\alpha_i} = 1 - \sum_{i=1}^{k} \frac{2\gamma}{\alpha_i} \]

If there is only one member in the Association, then
\[
\frac{\eta - \beta_i}{\alpha_i} = \frac{\eta - \beta_i}{1 - 2\gamma} > 0
\]
\[
\Rightarrow \begin{cases} 
\eta - \beta_i > 0 \quad \text{(case 1)} \quad \text{or} \quad \eta - \beta_i < 0 \quad \text{(case 2)} 
\end{cases}
\]
If the case 2 is true, then farmers will choose to produce infinite amount of abatement for infinitely high cost.
If the Regulator wants to induce some specific level of abatement, then he needs to choose case 1. Since
\[
\alpha_1 < \alpha_2 < ... < \alpha_k \quad \text{and} \quad \alpha_1 > 2\gamma,
\]
then \( \forall i : \alpha_i > 2\gamma \).

Appendix 2.
TheRegulator’s problem:

\[
\text{Min } a(\gamma u + \eta)
\]

s.t. \(1\) \(2\gamma \alpha_i \left( \frac{\eta \lambda - \mu}{1 - 2\gamma \lambda} \right)^2 = k(2\gamma \left( \frac{\eta \lambda - \mu}{1 - 2\gamma \lambda} \right) + \eta - \beta_k)^2 - \text{participation const.}\)

\(2\) \(a = \frac{\eta \lambda - \mu}{1 - 2\gamma \lambda} - \text{Association’s response const.}\)

\(3\) \(a = A\)

First, we need to solve constraints with respect to \( \gamma \& \eta \):
\[
\begin{align*}
2\gamma \alpha_k \left( \eta \lambda - \mu \right)^2 &= k \left( 2\gamma \left( \eta \lambda - \mu \right) + \eta \right)^2 \\
a &= \frac{\eta \lambda - \mu}{1 - 2\gamma \lambda} \\
\Rightarrow \quad a &= A \\
\Rightarrow \quad \eta &= \frac{A(1 - 2\gamma \lambda) + \mu}{\lambda} \\
\Rightarrow \quad \gamma &= \frac{k(A - \beta_k \lambda + \mu)^2}{2A^2 \alpha_k \lambda^2} \\
\Rightarrow \quad \eta &= \frac{1}{\lambda} \left[ \mu + A \left( 1 - \frac{k(A - \beta_k \lambda + \mu)^2}{A^2 \alpha_k \lambda} \right) \right]
\end{align*}
\]

Therefore, if the Regulator wants to have \( k \) farmers in the Association, then he needs to set

\[
\gamma^k = \frac{k(A - \beta_k \lambda + \mu)^2}{2A^2 \alpha_k \lambda^2} \quad \text{and} \quad \eta^k = \frac{1}{\lambda} \left[ \mu + A \left( 1 - \frac{k(A - \beta_k \lambda + \mu)^2}{A^2 \alpha_k \lambda} \right) \right].
\]

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