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# The Design and Pricing of Fixed and Moving Window Contracts: An Application of Asian-Basket Option Pricing Methods to the Hog Finishing Sector*

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*Abstract*: Asian-Basket type moving window contracts are an increasingly used risk management tool in US hog sector. The moving window contract is decomposed into a portfolio of a long Asian-Basket put and a short Asian-Basket call option. A projected breakeven price is used to determine the floor price, and then Monte Carlo simulation methods are used to price both a moving and a fixed window contract. These methods provide unbiased pricing of fixed and moving window hog finishing contracts of one-year duration.

*Keywords*: Derivative pricing, hog finishing, agricultural marketing contracts, Monte Carlo simulation methods, moving window contract

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# I. Introduction

During recent years the structure of the hog production industry has experienced profound changes. One obvious tendency is that more producers and packers are involved in long- or short-term marketing contracts, in which producers' revenues are partially or fully insolated from systematic price risk in the finished hog market.[1] Many producers engage in such contracts to enhance average hog price and to limit downside price risk (Lawrence and Grimes). In the 1980s, there were few contracts of any kind in US hog industry. In 1993, only 11 percent of hogs were transacted under a marketing or production contract while this figure increased to 83 percent by 2001 (Grimes and Meyer). Within this overall trend towards contracting, the prevalence of marketing contracts that transfer finished hog price risk from the producer to the contractor has also increased.[2] Such contracts include window contracts, in which the final price of finished hogs is constrained by a price floor and ceiling, and contracts in which the price of finished hogs is determined by a formula based on recent feed prices. In 1997 8.4 percent of all hogs were transacted under such contracts while in 2001 that figure increased to 22.8 percent (Grimes and Meyer).

Since input and output price risks are among the most important risk factors in the realization of profit, it is not surprising that hog producers have increased participation in risk-shifting marketing contracts. Such contracts can effectively reduce downside risk without requiring the producer to engage in continuous market price and basis recognizance as is often required for implementing forward, futures and options contracts. Among all types of contracts, window contracts are one popular contractual form; 6.6 percent of all hogs are transacted under such contracts (Grimes and Meyer) and more than 18 percent of all hogs marketed by firms with annual marketing of 10 thousand to 500 thousand hogs are transacted under such contracts

(Lawrence and Grimes). A window contract specifies a fixed price window with a floor and a ceiling for the duration of the contract. When the reference market price for hogs used in the contract (which might include a quality-based premium or discount) falls within the window, the producer receives the reference price and accepts all the price risk. When the reference price falls above (below) the window, the packer pays the ceiling (floor) price or pays a price that is some function of the reference price and the ceiling (floor) price, e.g., the producer receives the average of the two prices.

Window contracts are frequently used risk management tools and the effectiveness of these contracts rests upon their price and design, which must be desirable to both producers and packers. However, there is little published research concerning the structure of such contracts. Thus there are not many clear criteria for their pricing and design. Unterschultz *et al*. (1998) propose a promising approach for the pricing and design of window contracts, which is to view a window contract from the producer's perspective as an European option portfolio that is long in puts and short in calls. The long put strike price is the floor price and the short call strike price is the ceiling price of the window. Then, by using standard option pricing theory, one can easily calculate the premium the producer pays for buying the put option and receives for selling the call option. The difference of the premiums is the final price of this window contract. However, in real life a window contract is designed such that the cost to enter into it is assumed to be zero. Furthermore, window contracts typically specify a minimum delivery amount and, hence, likely provide efficiencies to the issuer (usually a processor) with respect to production scheduling. The question becomes, given the premiums a producer pays and receives are equal, how to design the window contract, namely, how to determine the floor and ceiling price of the window contract. Unterschultz *et al*. proposed two methods: a confidence interval approach, in which the floor price is determined by the lower bound of the confidence interval of the futures price distribution, and a break-even approach, in which the projected break even price for hog finishing is used to determine the floor price and then the width of the window was altered to equate the premiums paid and received.

However, Unterschultz *et al.*'s paper only examines short-term window contracts that involve a delivery of hogs at one point in time. Furthermore, in the real world, the floor and ceiling of the window contract must also be adjusted with a basis forecast to localize the contract. This study addresses both these issues by examining window contracts involving multiple deliveries of hogs over the period of one calendar year. While window contracts are often specified for durations longer than one year, the current effort can help shift the focus from short-term contract design to longer-term contract design.

Unterschultz *et al.*'s method forms the basis for the pricing and design of a window contract with fixed floors and ceilings (hereafter, fixed window contracts). But fixed window contracts are but one type of hog marketing contract. Many contracts specify a floor and ceiling price as a moving average of input prices. For example, the floor price may be a linear function of the 6-month moving average feed price and the ceiling price is the floor price plus a fixed number. Thus, this window contract can be viewed as a combination of a long European-type Asian-Basket put option and a short European-type Asian-Basket call option.

Thus, this study provides another contribution to the current literature by designing and pricing a multiple period moving window contract. This contract can be decomposed as a combination of multiple long European-type Asian-Basket put options and multiple short European-type Asian-Basket call options. Section II discusses the properties of Asian and Basket options and introduces a particular multiple period moving window contract to be designed. In section III, a Monte Carlo simulation model is used to price an Asian-Basket option applicable to the hog-finishing sector. In section IV, the simulation-pricing model is applied to design a moving window and a fixed window contracts for the hog finishing sector and the resulting contracts are evaluated for price bias. Section V concludes the study and provides further discussion.

## II. Asian-Basket Option Type Window Contracts

The payoff of an Asian option depends on the average price of the underlying asset during a specified period. The payoff from an Asian call is max[A-K, 0] and that from an Asian put is max[K-A, 0], where A is the arithmetic average value of the underlying asset calculated over a predetermined averaging period and K is the strike price defined prior to entry into the option contract. Another type of Asian option is an average strike, or floating strike option. The payoff from an average strike call is max [PT-A, 0] and that from an average strike put is max[A-PT, 0], where PT is the price of underlying asset at delivery. The advantages of Asian options are that, unlike regular European type options, they can protect the option holder from dramatic price changes at the delivery day and reduce the chance of market manipulation in a thinly traded market. Also, Asian options are usually less costly than European options because the volatility of average price is less than that of price at delivery (Kemna and Vorst, 1990). The regular European option is just a special case of an Asian option when there is only one set point at the expiration; hence the price of a European option gives an upper bound for the price of an Asian option.

The payoff of a basket option depends on the value of a portfolio (or basket) of assets. For a hog producer, the basket is mostly composed of three assets, which are live market hogs (the output) and corn and soybean meal (two key inputs). Basket options can reduce the cost for risk management compared to hedging each of the three assets separately because it can take advantage of the price co-movement of the three assets which, in the case of hogs and corn, often involve some elements of a natural hedge (i.e., high corn prices are often correlated with high hog prices).

The window contract to be designed will last for one year and requires the hog producer to deliver the same number of hogs each week to the contract issuer. Thus, the window contract can be decomposed into 52 sub-window contracts. For each sub-window contract, the floor price is defined as:

**(1)** $$floor_t = \alpha + \frac{\beta_1}{m}\sum_{j=t-m+1}^{t} p_{c,j} + \frac{\beta_2}{m}\sum_{j=t-m+1}^{t} p_{s,j} ,$$

where $floor_t$ is the floor price at *t*th week; $\alpha$, $\beta_1$ and $\beta_2$ are parameters to be determined; $P_{c,i}$ and $P_{s,i}$ are corn and soybean meal cash price respectively at time *i;* and *m* is the number of weeks used in calculating corn and soybean meal moving average price. A representative value of *m* observed in sample hog contracts is eight. Thus, the ceiling price is:

**(2)** $$ceiling_t = floor_t + \Delta$$

where $ceiling_t$ is the ceiling price at *t*th week and $\Delta$ is the width of the window. Note, $\Delta$ is a constant term to be calculated by the option pricing method. It does not change across the 52 weeks. The specification of this window contract involves average prices and multiple assets; hence the use of the term Asian-Basket type window contract.

Following Unterschultz *et al.*'s method, each of the 52 sub-window contracts can be decomposed as a long put option and a short call option. Then the entire window contract can be decomposed as 52 long Asian-Basket type put options and 52 short Asian-Basket type call options. If it is a zero cost contract at the entry point, the premiums paid for buying the 52 put options should be equal to the premiums received by selling the 52 call options. Equation (3) illustrates this relationship:

$$\textbf{(3)} \qquad \sum_{t=1}^{52} \exp(-rt/52)E[\max(floor_t - p_{h,t}, 0)] = \sum_{t=1}^{52} \exp(-rt/52)E[\max(p_{h,t} - ceiling_t, 0)]$$

where $r$ is the risk-free interest rate at the start of the contract ($r$ is assumed to be constant over the entire year); $E$ is the expectation operator; and $P_{h,t}$ denotes the hog cash price at time $t$. The left-hand side is the present value of premiums paid for buying the 52 put options and the right-hand side is that of the premiums received by selling the 52 call options.

Option pricing methods are needed to solve equation (3) for the width of the window, i.e., Δ. Before that, floor price must be specified; particularly, the parameters $\alpha$, $\beta_1$ and $\beta_2$ in equation (1). Unterschultz *et al*. proposed a confidence interval approach and a projected break-even approach to determine the floor price. Here the projected break-even method is followed. Though hog production technology and, hence, production parameters can vary considerable across hog producers, the benchmark production parameters taken from the 1999 Ohio State University Extension Livestock Budgets. These benchmarks assume that a hog is marketed at 250 pounds and that it consumes 8.5 bushels of corn and 84 pounds of soybean meal during its growth from a 50-pound feeder pig. $\beta_1$ and $\beta_2$ can be determined by this benchmark technology. For example, if the price of live hogs is expressed in \$/cwt, that of corn in \$/bushel and that of soybean meal in \$/ton, $\beta_1 = 8.5/250*100 = 3.4$ and $\beta_2 = 84/2000/250*100 = 0.0168$. $\alpha$ denotes the costs other than feed cost and it is decomposed into two parts. The first part is the average feeder pig price for next 52 weeks. A simple linear model is used to forecast this average price:

$$\textbf{(4)} \qquad \bar{p}_{f,t} = \gamma_0 + \gamma_1 \bar{f}_{h,t} + \gamma_2 \bar{f}_{c,t} + \gamma_3 \bar{f}_{s,t} + \gamma_4 \sigma_{h,t} + \gamma \sigma_{c,t} + \gamma \sigma_{s,t}$$

where $\overline{p}_{f,t}$ is the 52 week average feeder pig price from time $t$; $\bar{f}_{h,t}, \bar{f}_{c,t}$ and $\bar{f}_{s,t}$ are the average observed futures price for next year at time $t$ for hog, corn and soybean meal respectively; $\sigma_{h,t}$, $\sigma_{c,t}$ and $\sigma_{s,t}$ are annualized implied volatilities calculated by the Black-Sholes formula at time $t$ for hog, corn and soybean meal respectively; and $\gamma_0$ to $\gamma_6$ are parameters to be estimated by applying standard regression techniques to historical data (Table 1). The projected feeder pig price varies each week because the observed futures prices and calculated implied volatilities change each week. The second, less volatile part of $\alpha$ represents costs other than the feed and the feeder pig. This part is fixed at 25 dollars per head, which is representative of costs provided by several land grant university livestock budgets.

**Table 1. Regression Results: Feeder Pig Price Equation.**

| $\gamma_0$ | $\gamma_1$ | $\gamma_2$ | $\gamma_3$ | $\gamma_4$ | $\gamma_5$ | $\gamma_6$ | $R^2$ |
|---|---|---|---|---|---|---|---|
| -1.87 | 77.86* | -12.00* | 0.07* | 32.20* | 24.97* | -17.67* | 0.44 |
| (2.94) | (6.04) | (1.06) | (0.02) | (4.28) | (6.30) | (5.91) | |

*Significant at 5 percent level. Standard errors are in brackets.

Note that the regression results are only used to help determine the floor price to be used in the window contract and not to price the window contract directly. That is, rather than trying to predict the average cost of feeder pigs for the upcoming year, an arbitrary industry expected average could be put in its place with no impact on the efficiency of the contract pricing, which takes place in the next section. Hence, the results of the regression do not impact on the pricing efficiency achieved by the methods detailed in the next section.

## III. Asian-Basket Option Pricing: A Simulation Model

To price the Asian-Basket window contract described above, methods similar to those used by Hart, Babcock and Hayes (2001) in evaluating complex livestock revenue policies are used to solve for the value of the Asian-Basket option. A typical stochastic process assumed for commodity price is geometric Brownian motion, which implies the price follows a lognormal distribution. In the case of Asian options and basket options, however, it is impossible to find a closed-form solution for call or put premiums. This stems from the following statistical fact: when the price of the underlying asset is assumed to be log normally distributed, the arithmetic average will not have a lognormal distribution. Hence, closed-form solutions in the spirit of Black and Scholes are elusive.

To solve this problem, various methods have been used, including Monte Carlo simulation methods, partial differential equation methods and analytic approximations. Boyle (1977) introduced a Monte Carlo approach with a variance reduction technique to price options. Since then, Monte Carlo simulation has become a more popular approach to option pricing because of its flexibility to fit various complicated options and various assumed distributions and because of the recent improvement in computation speeds. Kemna and Vorst (1990) follow this way to price a geometric average option and almost any study proposing alternative solutions for arithmetic average option uses Monte Carlo simulation results as benchmark to evaluate the proposed method. Partial differential equation (PDE) techniques comprise another set of promising methods to price arithmetic average options. The PDE approach is a traditional way to understand options (Black and Scholes, 1973; Merton, 1973). Kemna and Vorst (1990) introduced a PDE with two state variables: the price of the underlying asset and the average price over the relevant time period. Alziary *et al*. (1997) propose a one-state-variable differential equation by using a change of numeraire; the solution of this PDE gives the price of an arithmetic average Asian option. The third method is to establish an analytic approximation for the price of arithmetic average Asian options. Although the sum of log normally distributed variables does not follow a lognormal distribution, it may be approximated by some known distributions. Turnbull and Wakeman (1991), Ritchken *et al.* (1993), Levy (1992), Curran (1992) and Milevsky and Posner (1998) follow this school of thoughts and each proposes approximations. Compared to the results of Monte Carlo simulation, their results perform very well under most cases.

Due to the complicated specification of the Asian-Basket window contract detailed in the previous section, and because of a desire to incorporate practical issues such as basis risk, Monte Carlo simulation methods are used to price these contracts. This first requires specification of the joint distribution of hog, corn and soybean meal cash and futures prices. Namely, the mean, volatility and correlation of the six price series must be articulated for a one-year time period.

According to the efficient market hypothesis (EMH) proposed by Fama (1970 and 1991), an efficient market is one that has incorporated all known information in determining price. Thus, the observed futures price of a commodity should be an unbiased forecast of the commodity spot price at the futures expiration date, provided that a valid reason for a risk premium does not exist, i.e.

**(5)** $$F_0 = E[P_T],$$

where $F_0$ is the observed futures price for a futures contract expiring at time $T$ and $P_T$ is the spot price at time $T$. An assumption that the futures market is efficient is a controversial one. There are large literatures on this issue, with diverse procedural approaches used to draw diverse sets of conclusions. Across all commodities, the evidence favors futures market efficiency. For example, Zulauf and Irwin (1998) indicate that the futures market can be used as a source of unbiased information for crops. However, there is a greater tendency to find inefficient livestock futures markets (Garcia, Hudson and Waller, 1988; Kolb 1992 and 1996). Zulauf (1999) found the hog futures market was downward biased to some extent though Wright found no significant statistical evidence of such a bias in the live or lean hog futures contract over the 1975 to 2001 time period. A 2000 survey shows hog producers were evenly divided in their agreement with the statement that the futures market price of hogs to be delivered in six months is an unbiased estimate of the cash price (Patrick *et al.*, 2000). Thus, it may be important that simple futures-based price forecasting procedures allow for possible underlying biases. However, at this stage, the EMH is adopted for the three commodities used in this model. Thus, currently, the observed futures price are used as the point estimate for the spot price and used to formulate forecasts of spot price movement over the one-year forecasting timeframe.

The covariance matrix of the joint distribution of the three price series is another key for the Monte Carlo simulation method. Three widely used volatility forecasts for the variance terms in the covariance matrix are: historical volatility, implied volatility and GARCH-based volatility. Their relative forecasting power has been compared in the fields of stock indices (Canina and Figlewski, 1993; Lamoureux and Lastrapes, 1993) and currency indices (Amin and Ng, 1997). Variance and covariance among these six series have been explored by the authors in previous research (Shao and Roe, 2001). These results showed that implied volatility usually provides the best forecasting performance for up to a 26-week horizon while historical volatility usually provided the best forecasts for longer horizons. Hence, the current study follows these guidelines and historical correlations among series to forecast covariance terms.

Specifically, assume the futures price of a commodity follows a random walk with drift in its natural logarithm level:

**(6)** $$\ln(f_{i,t}) = \ln(f_{i,t-1}) + \mu_i + \varepsilon_{i,t},$$

where ln denotes the natural logarithm; $f_{i,t}$ is the futures price of the commodity $i$ at time $t$; $\mu_i$ is the drift; and $\varepsilon_{i,t}$ is an innovation term at time $t$ for commodity $i$ that follows a normal distribution. The drift term, which represents the intrinsic force driving the price movement, is determined by linear interpolation of the observed futures prices with different maturities. Reasons for price changes could include the influence of the interest rate, storage cost, convenience yield and so on. The innovation term is the random shock outside the intrinsic factor. Thus, the joint distribution for the natural logarithm level futures prices can be modeled as:

**(7)** $$\ln(f_t) = \ln(f_{t-1}) + \mu + \varepsilon_t,$$

where $f_t$ is a price vector at time $t$; $\mu$ is a vector for drifts; $\varepsilon_t$ is an innovation vector at time $t$, i.e.,

$$f_t = \begin{bmatrix} f_{h,t} \\ f_{c,t} \\ f_{s,t} \end{bmatrix}, \quad \mu = \begin{bmatrix} \mu_h \\ \mu_c \\ \mu_s \end{bmatrix}, \quad \varepsilon_t = \begin{bmatrix} \varepsilon_{h,t} \\ \varepsilon_{c,t} \\ \varepsilon_{s,t} \end{bmatrix} \sim \text{N}\,(0, \Sigma) \text{ and } \Sigma = \begin{bmatrix} \sigma_h^2 & \sigma_{hc} & \sigma_{hs} \\ \sigma_{hc} & \sigma_c^2 & \sigma_{cs} \\ \sigma_{hs} & \sigma_{cs} & \sigma_s^2 \end{bmatrix},$$

where the subscript $h$, $c$, and $s$ denote hogs, corn, soybean meal, respectively, and $\Sigma$ is the symmetric covariance matrix. The variance terms in the matrix are weekly variance, which are derived by dividing the square of corresponding annualized volatilities by 52. The covariance terms are derived from the following equation:

**(8)** $$\hat{\sigma}_{t,i,j} = \rho_{i,j}\sigma_i\sigma_j$$

where $\hat{\sigma}_{t,i,j}$ is the forecasted weekly covariance for commodity *i* and *j* starting from time *t*; $\rho_{i,j}$ is the historical correlation of the return series for commodity *i* and *j* from the beginning of the dataset until the start of the forecasting period; $\sigma_i$ and $\sigma_j$ are the forecasted volatilities for commodity *i* and *j* respectively by using the best approach (implied or historical volatility depending on horizon).

Before simulation can proceed, simulated futures prices must be adjusted by expected basis and account for basis risk to yield realistic cash price paths because the reference prices of the contract are defined as cash prices rather than futures prices.

Basis is the difference between futures price and cash price. The variation of basis is called basis risk. Compared to price risk, basis risk is usually less influential. Some times an unexpected basis realization does cause a serious problem and can hurt producers' incomes and invalidate hedging efforts. Hence, an accurate out-of-sample forecast of basis is key for successful simulation and pricing. Many basis forecasting methods have been studied and their relative forecasting powers have been compared for agricultural commodities. These methods include using: current basis, last year's basis, historical average basis, the futures spread method, predictions from an economic structural model, predictions from a seasonal ARIMA (Autoregressive Integrated Moving Average) model, predictions from a neural network model and so on. Hauser *et al*. (1990) use the Theil coefficient as a comparison criteria for Illinois soybean basis forecasting and found the futures spread method outperforms alternatives for the post-harvest period. During other periods, a historical average seems to be the best. Garcia and Sanders (1996) compare an economic structural model, an ARIMA model and a 3-year historical average model based on root mean squared error and use a Henriksson-Merton test for sign prediction for Omaha and Illinois live hog basis and found the first two generally outperform the last one. Jiang and Hayenga (1997) examine the alternative forecasting methods for corn and soybeans in several different markets. Their conclusion is that, based on RMSE, a 3-year historical average, seasonal ARIMA (SARIMA) and 3-year historical average plus current market information model are among the best and can compete with each other for short term forecasting for corn basis. For long term forecasting of corn basis, however, no method outperformed the 3-year historical average. For soybean basis, a 3-year historical average plus current market information model and a SARIMA generally outperformed 3-year historical average. Dhuyvetter and Kastens (1998) found for Kansas crops such as corn and soybeans, a 5-7 year historical average may be generally optimal and the futures spread model incorporating current market information improves forecasting power for near term forecasting but not for long term horizons. Kastens *et al*. (1998) also indicate that, for Kansas crops and livestock, forecasting cash price by adding up deferred futures price and historical average basis has the greatest accuracy. Based on the results of this recent research and data availability, a 3- to 5-year historical average, futures spread model and seasonal ARIMA are considered as candidates for basis forecasting basis model.

The basis forecasting by n-year historical average can be defined as:

**(9)** $$\hat{B}_{i,T,n} = \frac{1}{n}\sum_{t=T-1}^{T-n} B_{i,t}$$

where $B$ is basis, $i$ is the week of year, $t$ and $T$ denote years and $n$ is the number of years in the historical average ($n$ = 3, 4, or 5).

The potential advantage of futures spread model is that it incorporates current market information because it is believed that the futures spread, which is the difference between the prices of two futures contracts with different expiration dates, reflects different market expectations for hog price at different dates in the future. The basis forecasting model is

**(10)** $$\hat{B}_{i+h} = (1 + \frac{F2_i - F1_i}{F1_1})^{h/w} C_i - F_{i+h}$$

where $\hat{B}_{i+h}$ is the basis to be forecasted at week $i$ and the forecasting horizon is $h$; $C_i$ is the cash price at week $i$; $F1_i$ is the nearby futures price to week $i$ and $F2_i$ is the first deferred futures price to week $i$; $w$ is the number of weeks between the nearby and first deferred futures contracts to week $i$; $F_{i+h}$ is the nearby futures price to week $i+h$. For example, assume the starting point of the forecasting period is the 1st week of a year and the forecasting horizon is 26 weeks, then the February and April hog futures contract are the nearby and first deferred hog futures contract to 1st week of this year, respectively, and the July futures contract is the nearby futures contact to the end of the forecasting period, namely, the 27th week of this year.

The general form of the seasonal ARIMA model SARIMA(p,d,q)(P,D,Q) is:

**(11)** $$\Phi_P(L^S)\phi_p(L)(1-L)^d B_t = \Theta_Q(L^S)\theta_q(L)$$

where $L$ is the lag factor; $S$ is the seasonal period (for weekly data it is 52); $p, d, q$ are the orders of autoregressive, differencing and moving average terms respectively; $P, D, Q$ are the orders of seasonal counterparts of $p, d, q$; $\Phi$, $\varphi$, $\Theta$, and $\theta$ are the coefficients for seasonal autoregressive, regular autoregressive, seasonal moving average and regular moving average terms.

These three basis forecasting methods are applied to the current situation and their forecasting power compared by Mean Squared Error (MSE), computational costs, a complementary method proposed by Kastens *et al*. (1998) and the Henriksson-Merton timing test. Results suggest that the futures spread model works best for near term basis point forecasting and the 5-year historical average will work best for longer term basis point forecasting. The switches in forecasting methods occur at 1-week for hogs, 8-weeks for corn and 12-weeks for soybean meal (Shao, 2001).

To account for the uncertainty of basis, historical data is examined and, under most cases, the basis for the same time period in different years follows a normal distribution. The standard deviation of the normal distribution varies with the time of the year examined, which is consistent with seasonality of basis. Thus, it is assumed that, at any time within a year, the basis follows a normal distribution with a mean forecasted by the appropriate method (futures spread or historical average) and variance forecasted by historical variance specific to the time of the year. Also, historical data show there is no statistically significant correlation among the level of basis for the three commodities; hence the basis correlation factors are set to zero.

Now that the method for determining all mean, variance, covariance and basis terms available, Monte Carlo simulation proceeds by repeatedly generating sequences of random numbers following the specified multivariate normal distribution to get the cash price paths for the three commodities. For each simulated time series, the payoff of the Asian-Basket call and put options are calculated and their respective means are denoted as the option premium. Thus, equation (3) is solved for $\Delta$, the width of the moving window, by searching over candidate values for $\Delta$ until the value that solves equation (3) for the given time period is found.

## IV. Contract Evaluation

The Monte Carlo simulation model is applied to historical data to test the performance of the model. Unterschultz *et al*. indicate that one problem of setting the floor for a window contract at the projected breakeven level is that it can cause the window to become inverted, i.e., the efficient value for $\Delta$ to be negative. Here an inverted window means the call strike price is below the put strike price. This is caused when many of the futures prices are below the projected breakeven price. Under this case, the hog producer is projected to lose money under the contract. Whenever the pricing algorithm yields an inverted window, the floor is re-specified as 90 percent of the projected breakeven price, which avoids all instances of the inversion problem.

The cash and futures prices and options data for these three commodities are all Wednesday prices from 1991 to 2001. The hog cash price ($/pound) is the price of eastern Corn Belt plant delivered US 1-2 51-52 percent live hogs. The corn cash price ($/bushel) is the price of Toledo No.2 yellow corn. The soybean meal price ($/ton) is the price of truck delivered Illinois 48 percent soybean meal. If a weighted average cash price is unavailable, the average of the reported low and high price is used. The risk-free interest rate is the 6-month t-bill rate from the Federal Reserve. The Wednesday settlement prices for futures and options for hog contracts from Chicago Mercantile Exchange and those for corn and soybean meal contracts from Chicago Board of Trade are used.

For each week from 1991 to 2000 the simulation model is used to price the moving window contract for the next 52 weeks using information that would have been available before the first week of the contract. In total 520 contracts are priced. The performance of each contract is then evaluated by comparing the issuer's expenditures if an equal number of hogs were purchased each week under the contract to the issuer's expenditures if the same hogs were purchased at prevailing cash prices (without the contract). In a risk neutral world with a correctly priced window contract average expenditures should be equal. Most contract issuers are meat packers. Compared to hog producers, they may have a greater ability to weather short-term price risk than individual hog producers. Thus, the assumption of risk-neutral contract issuer (or at least less risk averse than the hog producer) is a reasonable one.

To assist comparisons to the existing literature (Unterschultz *et al.*) a fixed window is also priced using these methods. The floor of this fixed window is determined by equation (1) except the moving average corn and soybean meal cash prices are replaced by average corn and soybean meal futures prices observed at the beginning of the contract. Thus, the floor and ceiling will be a constant number across all the 52 weeks. The width of the 520 moving and fixed window contracts are depicted by Figure 1 and some descriptive statistics are shown in table 2. The widths of the window contracts priced by the model vary dramatically over time, which is consistent with Unterschultz *et al.*'s results.

**Figure 1. Fixed and Moving Window Width Determined by Simulation Model, 1991-2000.**

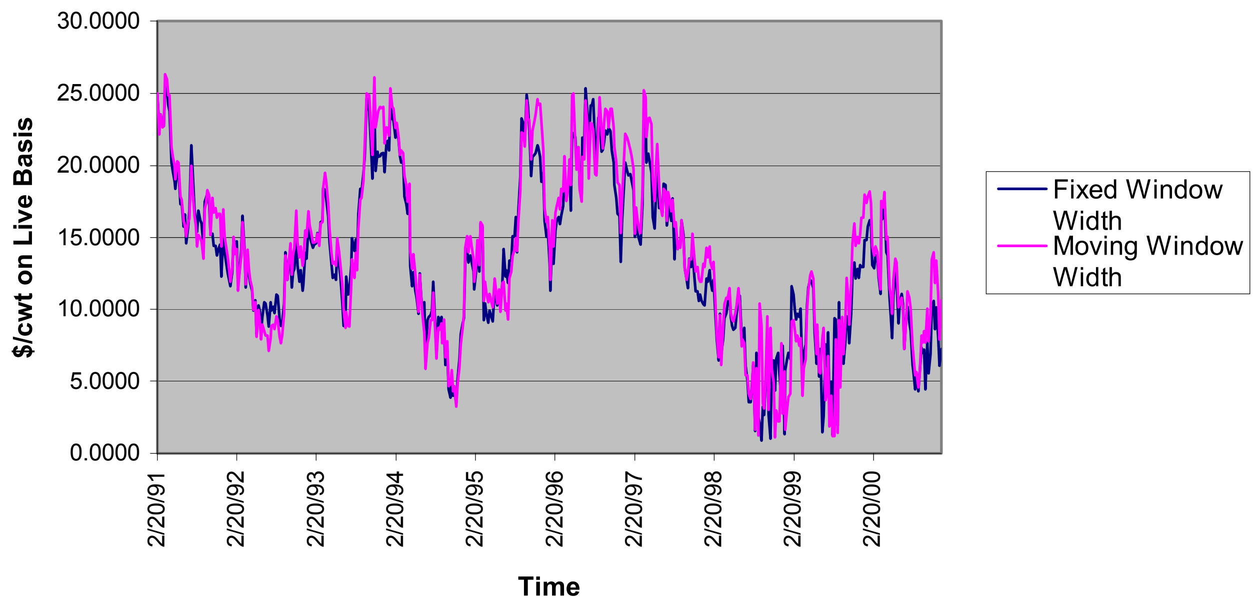

**Table 2. Descriptive Statistics for Fixed and Moving Window Widths.**

| | Moving Window | Fixed Window |
|---|---|---|
| Average ($/cwt*) | 13.40 | 13.91 |
| Std. Dev. ($/cwt) | 5.47 | 5.82 |
| Min ($/cwt) | 0.89 | 1.11 |
| Max ($/cwt) | 25.46 | 26.34 |
| Percentage of Window Wider Than $20 / cwt | 15.23 | 17.97 |
| Percentage of Window Narrower Than $5 / cwt | 6.05 | 6.84 |

*Cwt is hundredweight of live hog.

The prices paid by the contract issuer under the moving window and fixed window are formally compared to the price paid to procure hogs at the prevailing cash market price,[3] by testing if the expected value of the differences are zero. Specifically, define the difference as

**(12a)** $$e_{mw,t} = \sum_{i=t+1}^{t+52} p_{mw,i} - \sum_{i=t+1}^{t+52} p_{c,i} \text{ and}$$

**(12b)** $$e_{mw,t} = \sum_{i=t+1}^{t+52} p_{fw,i} - \sum_{i=t+1}^{t+52} p_{c,i}$$

where $P_{mw,i}$, $P_{fw,i}$ and $P_{c,i}$ are the prices paid for per hundred weight of live hog at time $i$ under the moving window, fixed window and cash market procurement strategies, respectively; $e_{mw,t}$ is the total difference of price paid between the moving window and the cash market procurement strategy for one entire year starting from time $t$ and $e_{fw,t}$ is the total difference of price paid between the fixed window and the cash market procurement strategy for one entire year starting from time $t$. If all the observations of $e_{mw,t}$ and $e_{fw,t}$ in the sample were independent, the central limit theorem would hold and a simple t-test could be applied. Note, however, the individual observations are autocorrelated because, within $e_{mw,t}$ and $e_{fw,t}$, there exists an overlap of $P_{mw,i}$,

$P_{fw,i}$ and $P_{c,i}$ up to 51 time periods. To solve this problem, the assumption is made that the autocorrelation disappears after 52 weeks and the Harvey, Leybourne and Newbold test (HLN test, 1997) is applied. The test statistic is defined as:

**(13)** $$S_1^* = \left[ \frac{n+1-2h+n^{-1}h(h-1)}{n} \right]^{1/2} S_1$$

where $S_1 = [V(\bar{e})]^{-1/2}\bar{e}$, $S_1^*$ is the HLN test statistic; $n$ is the number of observations; $h$ is the number of overlapping periods which is 51 here; $\bar{e}$ is the sample mean of the difference and $V(\bar{e})$ is the asymptotic variance of $\bar{e}$, which takes into account of the autocorrelation over 51 weeks. $S_1^*$ follows a Student's t-distribution with ($n$-1) degrees of freedom. The test results and descriptive statistic are shown in table 3.

**Table 3. The Descriptive Statistic and HLN Test Results for Difference between Price Paid under Moving Window, Fixed Window and Cash market procurement Strategy.**

| | $e_{mw,t}$ | $e_{fw,t}$ |
|---|---|---|
| Mean | \$ 0.17 / cwt* | \$ 0.09 / cwt |
| Std. Dev. | \$ 1.97 / cwt | \$ 2.49 / cwt |
| Max | \$ 5.36 / cwt | \$ 6.76 / cwt |
| Min | \$ -6.73 / cwt | \$ -8.32 / cwt |
| HLN Test | 1.66 (0.10) | 0.72 (0.47) |

*Cwt is hundredweight of live hog. P-values are in brackets.

Neither of the two HLN test statistics is significant though the HLN statistic for the moving window contract borders upon the significant region. Hence, neither the moving window nor the fixed window significantly outperforms or significantly under-performs the cash market procurement strategy. That is to say, in a risk-neutral world, the model "correctly" designs the moving and fixed window such that a risk-neutral issuers indifferent between the contract and a cash market procurement strategy, or that the Monte Carlo methods can in practice efficiently price complex window contracts. Note that, while the fixed window shows a smaller absolute level of bias than the moving window ($0.09/cwt vs. $0.17/cwt), the standard deviation of the realized bias is smaller for the moving window contract, which is not surprising as moving average terms exhibit less volatile movements than the individual terms that comprise the moving average. While the contracts appear unbiased, the pricing for any particular period can result in large losses or gains for the issuer. The maximum deviations from cash-only procurement for any given 52-week period are rather large and represent a magnitude that is approximately 20 percent as large as the average hog price over this time period.

## V. Conclusions and Extensions

Window contracts are an increasingly used risk management tool in US hog sector. Because the payoffs of Asian-Basket type moving window contracts depend upon moving average prices rather than single period prices, the distribution of payoffs exhibits less volatility than a fixed window contract and, hence may be more attractive. However, designing and pricing moving window contracts is quite challenging because of the complex nature of price relationships. A Monte Carlo simulation model is used to price and design both moving and fixed window contracts. These methods provide unbiased pricing of fixed and moving window hog finishing contracts of one-year duration over the 1991 to 2000 time period. From the risk-neutral contract issuer's perspective, the contract pricing is unbiased in that the average cost of hog procurement through the spot market was not significantly different than the cost of procurement via contracts. The contract issuer could benefit from these contracts for reasons other than price, however. For example, the contract may specify a quality of delivered hogs and a delivery schedule that might be unobtainable from the spot market on a regular basis. The hog producer, who may have greater risk aversion than contract issuers, may benefit from such contracts because they limit price risk and provide a guaranteed market outlet for finished hogs. These additional benefits may suggest that observed window contract prices will not match risk-neutral prices. However, deriving the exact pricing implications of these additional benefits would require comparing the risk-neutral price of window contracts to the contract prices revealed in real markets; such analysis is beyond the scope of the current study but should be the subject of future research.

The specification of the floor of the moving window is a critical issue. When hog futures prices are quite low and the floor is specified as the breakeven price of hog production, the risk-neutral window tends to be inverted. In this paper the floor is reduced to 90 percent of the projected breakeven in such cases and all instances of inverted windows are avoided. On the other hand, when hog futures prices are quite high, the floor seems to be too low, which causes the width of the window to be greater than window widths observed in real hog window contracts. For example, sample hog contracts mentioned in the agricultural press or posted on the Iowa Attorney General's web page seldom have windows with width above \$20/cwt on a live weight basis. However, about 15 percent of the moving windows and 18 percent of the fixed windows priced in this study have a width larger than \$ 20/cwt. This also causes the problem that the cash price seldom hits the floor or ceiling of the window. Thus, the window seems not to play its risk management role to hog producers. One likely reason for this departure from observed window widths is that most observed contracts have durations of several years while those priced in this paper are of a single year in duration. Hence, one-year contracts priced at the top of a hog cycle may feature a very wide window because hog futures prices are quite high. If the contract were written for a longer duration, and hence included years projected to be in the downturn phase of a hog cycle, the optimal window width would probably be narrower. This leads to area of great importance for future research: the pricing of complex contracts written for durations that include years for which no futures markets contracts are currently traded. While the futures-calibrated methods used in this study would still be applicable for simulating the early portion of longer contracts, other methods, e.g., forecasts from structural models calibrated with historical data or contingent window updating based on past market performance, appear to be the only alternative for pricing longer-run contracts.

Finally, to reduce the computational costs needed to design the Asian–Basket type moving window contract, analytical approximation might be an alternative to Monte Carlo simulation. Though the literature has proposed several approaches, it is not an easy task to price such a complicated contract solely by analytic approximation. Also, given the recent advances of the speed of computation and the lack of generalizability of many of the more complex contracts, the long-term efficacy of developing analytical pricing approximation techniques must be questioned.

## References


Alziary, B., Decamps, J., and Koehl, P., 1997. "A PDE Approach to Asian Options: Analytical and Numerical Evidence." *Journal of Banking and Finance,* 21, 613-640.

Amin, K., and Ng, V., 1997. "Inferring Future Volatility from the Information in Implied Volatility in Eurodollar Options: A New Approach." *The Review of Financial Studies*, 10, 333-367.

Black, F., and Scholes, M., 1973. "The Pricing of Options and Corporate Liabilities." *Journal of Political Economics,* 81, 637-654.

Boyle, P., 1977. "Options: a Monte Carlo Approach." *Journal of Financial Economics,* 4, 323-338.

Canina, L., and Figlewski, S., 1993. "The Informational Content of Implied Volatility." *The Review of Financial Studies*, 6, 659-681.

Curran, M., 1992. "Beyond Average Intelligence." *Risk Magazine,* 10.

Dhuyvetter, K., and Kasten, T., 1998. "Forecasting Crop Basis: Practical Alternatives." NCR-134 Conference on Applied Commodity Price Analysis, Forecasting, and Market Risk Management.

Fama, E., 1970. "Efficient Capital Market: A Review of Theory and Empirical Work." *Journal of Finance,* 25, 383-417.

Fama, E., 1991. "Efficient Capital Markets .2." *Journal of Finance*, 46, 1575-1617.

Garcia, P., Hudson, M., and Waller, M., 1988. "The Pricing Efficiency of Agricultural Futures Markets: An Analysis of Previous Research Results." *Southern Journal of Agricultural Economics*, 20, 119-130.

Garcia, P., and Sanders, D., 1996. "Ex Ante Basis Risk in the Live Hog Futures Contract: Has Hedgers' Risk Increased?" *Journal of Futures Markets,* 16, 421-440.

Grimes, G. and Meyers, S., 2001. "Hog Marketing Contract Study." University of Missouri and the National Pork Board. http://agebb.missouri.edu/mkt/vertstud.htm.

Hart, C., Babcock, B., and Hayes, D., 2001. "Livestock Revenue Insurance." *Journal of Futures Markets,* 21, 553-580.

Harvey, D., Leybourne, S., and Newbold, P., 1997. "Testing the Equality of Prediction Mean Squared Errors." *International Journal of Forecasting,* 13, 281-291.

Hauser, R., Garcia, P., and Tumblin, A., 1990. "Basis Expectation and Soybean Hedging Effectiveness." *North Central Journal of Agricultural Economics,* 12, 125-136.

Hayenga, M., Rhodes, J., Grimes, G. and Lawrence, J.,1996. "Vertical Coordination in Hog Production." *GIPSA-RR,* 96-5.

Jiang, B., and Hayenga, M, 1997. "Corn and Soybean Basis Behavior and Forecasting: Fundamental and Alternative approaches." *NCR-134 Conference on Applied Commodity Price Analysis, Forecasting, and Market Risk Management.*

Kastens, T., Jones, R., and Schroeder, T., 1998. "Futures-Based Price Forecasts for Agricultural Producers and Business." *Journal of Agricultural and Resource Economics,* 23, 294-307.

Kemna, A., and Vorst, A., 1990. "A Pricing Method for Options Based on Average Asset Values." *Journal of Banking and Finance,* 14, 113-129.

Kolb, R., 1992. "Is Normal Backwardation Normal?" *Journal of Futures Markets*, 12, 75-91.

Lamoureux, C., and Lastrapes, W., 1993. "Forecasting Stock-return Variance: Toward an Understanding of Stochastic Implied Volatility." *The Review of Financial Studies*, 6, 293-326.

Lawrence, J., 2000. "Understanding Hog Marketing Contracts." *Working Paper,* Iowa State University.

Lawrence, J., Grimes, G. and Hayenga, M., 1999. "Production and Marketing Characteristics of US Pork Producers, 1997-1998." *Working Paper,* Iowa State University and University of Missouri.

Levy, E., 1992. "Pricing European Average Rate Currency Options." *Journal of International Money and Finance,* 11, 474-491.

Levy, E. and Turnbull, S., 1992. "Average Intelligence." Risk Magazine, 5, 53-59.

Merton, R., 1973. "The Theory of Rational Option Pricing." *Bell Journal of Economics and Management Science,* 4, 141-183.

Milevsky, M., and Posner, S, 1998. "Asian Option, the Sum of Lognormals, and the Reciprocal Gamma Distribution." *Journal of Financial and Quantitative Analysis,* 33, 409-422.

Patrick, G., Baquet, A., Coble, K., and Knight, T., 2000. "Hog Risk Management Survey: Summary and Preliminary Analysis." *Staff Paper,* Purdue University.

Ritchken, P., Sankarasubramanian, L., and Vijh, A., 1993. "The Valuation of Path Dependent Contracts on the Average." *Management Science,* 39, 1202-1213.

Shao, R., 2001. "The Evaluation and Design of Price Risk Management Strategies in the U.S. Hog Industry." *Unpublished dissertation proposal.* Ohio State University.

Shao, R. and Roe, B., 2001. "Underpinnings for Prospective, Net Revenue Forecasting in Hog Finishing: Characterizing the Joint Distribution of Corn, Soybean Meal and Lean Hogs Time Series." *American Agricultural Economics Association Annual Conference.*

Turnbull, S., and Wakeman, L., 1991. "A Quick Algorithm for Pricing European Average Options." *Journal of Financial and Quantitative Analysis,* 26, 377-389.

Unterschultz, J., Novak, F., Bresee, D., and Koontz, S., 1998. "Design, Pricing, and Returns of Short-Term Hog Marketing Window Contracts." *Journal of Futures Markets,* 18, 723-742.

Wright, M., 2001. "Informational Efficiency and Routine Bias: Three Studies Involving Corn, Soybean, Hog and Cattle Futures Markets." Masters Thesis, Ohio State University, Columbus, OH.

Zulauf, C., 1999. "Are Hog Futures Prices Biased?" *National Pork Producers Council Newsletter, Pork Report,* Fall 1999, 10-11.

Zulauf, C., and Irwin, S., 1998. "Market Efficiency and Marketing to Enhance Income of Crop Producers." *Review of Agricultural Economics,* 20, 308-331.

**Endnotes**

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[1] Note that marketing contracts are distinct from production contracts. The former primarily focus on issues of guaranteed price-quality schedules for finished hogs while the latter often involve a shift of asset ownership from producer to contractor as well as a loss of autonomy concerning husbandry practices.

[2] Note that a large portion of hog marketing contracts do not insolate the producer from systematic risk in the price of finished hogs but, rather, are merely mechanisms to guarantee the producer access to a slaughter outlet. In such formula and basis contracts the price received by the producer is linearly related to either cash or futures prices for hogs and, as such, do little to transfer risk between parties.

[3] The quality of hogs procured under contract and in the spot market are assumed to be the same. While somewhat unrealistic, this assumption is made because hog quality-pricing issues are not the central theme of this paper.