Intertemporal Permit Trading for Stock Pollutants with Uncertainty*

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Abstract

This paper explores the efficiency of tradable permit markets for stock pollutants. With uncertainty about the future stock level or damages, a market with banking and borrowing is inferior, in terms of efficiency, compared to a market without banking and borrowing if the regulator commits to an initial allocation of permits. This result occurs because, with banking and borrowing and commitment, the regulator needs to specify the total allowable amount of emission over time at the initial time period before the uncertainty with the pollution stock is resolved. An alternative banking and borrowing scheme is proposed, where the regulator can update the allocation of permits to firms over time and achieve the efficient pollution accumulation.

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1 Introduction

As in the case of CO_2 emissions, for some pollutants it is the stock, or the accumulation of the emission over the past, rather than the current flow that determines current damages. This paper studies the efficiency of tradable permit markets for a stock pollutant in the presence of uncertainty about the accumulation of the stock of pollution. In particular, the focus is on the welfare property of intertemporal trade of permits (banking and borrowing, or B&B). It will be shown that, if the regulator allows the polluting firms to bank or borrow permits, then the regulator needs to change the firms' permit endowments at each point in time in order to achieve socially optimal emission of the pollutant.

For stock pollutants there have been several prior studies on the optimal emission path (Falk and Mendelsohn 1993, Tahvonen 1995) and on the efficiency of tradable permit markets (Fischer, et al. 1998, Newell and Pizer 1998, Kling and Rubin 1997, Leiby and Rubin 2001). For tradable permits, a banking and borrowing scheme has been considered to be a useful policy option since it may allow firms to reduce the total present value of pollution abatement cost by reallocating their emissions across time. Kling and Rubin (1997) analyzed whether allowing for B&B can result in socially optimal emission. They made clear that the regulator needs to correctly specify the rate at which a firm in the market can exchange a permit for a unit of current emission for the same amount of future emissions. In particular, with stationary abatement and damage functions over time, a simple one-to-one exchange ratio induces firms to postpone emission reduction to the future and therefore is not necessarily consistent with efficient emission reduction. Leiby and Rubin (2001) conducted a similar analysis for stock pollutants in a deterministic model, and derived the optimal trading ratio for banking and borrowing. Yates and Cronshaw (2001) describe a situation where a banking and borrowing scheme is efficiency-enhancing compared to a scheme that does not allow intertemporal trade of permits. They consider the case for flow pollutants in the presence of asymmetric information between the regulator and the firms. They showed that, if firms have better information on their abatement costs than the regulator does, then a banking and borrowing scheme can result in a more efficient outcome than a scheme which does not allow banking and borrowing since it allows firms more flexibility in choosing emission paths.

This paper considers another sort of uncertainty; instead of asymmetric information regarding the pollution abatement cost, the focus is on uncertainty in the pollution stock accumulation or the future damages. For stock pollution problems such as climate change and soil or water pollution, we do not have a complete knowledge about the dynamics of the pollution accumulation. The stock of a pollutant next period depends not only on controllable emissions due to human activities but also on uncontrollable emissions, climate conditions and so on. This study shows that, if there is such uncertainty in the pollution stock accumulation, then allowing firms for B&B may result in inefficiency. In particular, it is crucial whether or not the regulator commits to the amount of bankable permits to polluters which the regulator announces in the initial period. With deterministic stock accumulation, it suffices for the regulator to specify the total allowable amount of emissions over time and a trading ratio for banking and borrowing of permits at the initial period (Yates and Cronshaw 2001). With uncertainty about pollution accumulation, however, the regulator may not be able to achieve socially optimal accumulation of the pollutant if the regulator commits to the initial allocation of permits to firms. A market equilibrium without B&B is in fact superior to the one with B&B. Before giving a formal description of this claim, let us see the intuition behind it. Suppose that a regulator must choose the total amount of emission to be allowed over time by each firm in the initial period. If banking and borrowing are allowed, firms decide how much emission to discharge, how much to trade in the permit market, and how much to bank for or borrow from the permits in the future periods. For a stock pollutant whose accumulation involves uncertainty, the regulator needs to update the allowable amount of emission over time. However, this is not possible if the regulator needs to commit to the amount of permits given to firms in the initial period. On the other hand, if the regulator updates the allowable amount of emissions by firms in each period, a permit becomes a risky asset: its price and the amount will not be deterministic from the firms' point of view, and they need to make permit trading decisions under uncertainty. This paper gives a formal explanation of this argument and discusses what a regulator should do, at each point in time, in order to achieve efficient stock pollution accumulation while allowing firms to engage in banking and borrowing of permits.

Section 2 introduces a two-period model of tradable permit markets for the emission of a stock pollutant. We compare the efficiency of the outcomes in two cases; a market without B&B and a market with B&B where the regulator commits to the initial allocation of permits to firms. It will be shown that (1) if there is no uncertainty, both schemes yield efficient pollution accumulation (Proposition 1); (2) with uncertainty about the pollution accumulation, a market without B&B can achieve efficient pollution accumulation (Proposition 2); and (3) a market with B&B does not yield an efficient outcome (Proposition 3). An alternative banking and borrowing scheme, where the regulator has flexibility in reallocating permits to firms over time is introduced and shown to be efficient with correctly specified policy parameters (Proposition 4). Section 3 discusses the main finding and the policy implications.

2 The model

2.1 Model Enviroment

We introduce a two-period model of a tradable permit market for the emission of a stock pollutant with N polluting firms. As in Montgomery (1972) and Kwerel (1977), the focus is on the partial equilibrium in a tradable permit market; the economic decisions by the market participants outside the permit market (other output and input decisions) are omitted. Here the assumptions of the model are listed, and the market and intertemporal trade (banking and borrowing scheme) are defined in the next subsection. After characterizing the efficient emission across periods (2.3), this section concludes with the comparison of the efficient emission and the market outcomes under different specifications on banking and borrowing (2.4).

Let $I \equiv \{1, \ldots, N\}$ be the set of the firms and $C_i : I\!R_+ \to I\!R$ be the periodwise emission reduction cost function of $i \in I$. Denote the emission by firm i in period t by $e_{it} \in I\!R_+$ and total (industrial) emissions in period t by $e_t = \sum_{i \in I} e_{it}$. $C_i(e_{it})$ is the cost of reducing (achieving) emissions e_{it} at time t. Vector $e_i = (e_{i1}, e_{i2})$ is the emissions path by firm i. Throughout the paper we assume

Assumption 1 *C_i is twice continuously differentiable with derivatives $-C_i', C_i'' > 0$ for all $i \in I$.*

For simplicity, suppose C_i is time-invariant. Changes in $\{C_i\}$ across time do not affect our result. Under Assumption 1, the periodwise industry cost function $C : I\!R_+ \to I\!R$ defined by

$$\text{(IC)} \qquad \forall E \in I\!R_+ \quad C(E) = \min_{\{e_i\}_{i \in I}} \sum_{i \in I} C_i(e_i) \quad \text{subject to } 0 \le \sum_{i \in I} e_i \le E$$

is continuously differentiable with $-C', C'' > 0$ and $C'(E) = C_i'(e_i)$ for all $i \in I$ where $\{e_i\}_{i \in I}$ solves (IC). This industry cost function is used later to characterize the efficient emission path.[1]

Let $D_t(S_t)$ be the damage in period t caused by the pollution stock S_t. Assume

Assumption 2 *$D_t : \mathbb{R}_+ \to \mathbb{R}$ is twice continuously differentiable with bounded derivatives $D_t' > 0, D_t'' \geq 0$ for $t = 1, 2$.*

Suppose that the equation of motion of the stock pollutant is given by

$$S_t = \gamma_t S_{t-1} + e_t \tag{1}$$

where $1 - \gamma_t \in [0, 1]$ represents the natural rate of decay of the pollution in the air. The decay rate γ_t is a random variable with probability measure P and support $\Gamma \subseteq [0, 1]$. Fluctuation in γ_t may be due to the effect of climatic conditions or non-point sources of emissions that are independent of the industry in consideration.[2] Without loss of generality let $S_0 = 0$. Then the pollution stock dynamics is given by

$$S_1 = e_1, S_2 = \gamma_2 S_1 + e_2.$$

It is assumed that the regulator observes the realization of γ_t at the beginning of period t. Since there is only one random variable, denote γ_t by γ in what follows. γ denotes a random variable, and a particular realization of γ will be denoted by, say, $\bar{\gamma}$.

In this model, the only source of uncertainty is in the accumulation of the stock pollutant. More generally, the conclusion of the analysis holds true if the future damages are uncertain. For simplicity this study focuses on stock accumulation uncertainty.

The following assumption on a discount factor is also maintained throughout the paper.

Assumption 3 *The regulator and N firms have a common discount factor $\delta \in (0, 1]$.*

[1] See Kwerel (1977) for the property and the use of industry cost function in analyzing environmental policies.

[2] Estimates of γ for CO_2 range from 0.99 to 0.995 (Hoel and Karp 2001).
Here the uncertainty is represented by a multiplicative shock in this model. Alternatively, one can assume an additive shock: $S_{t+1} = \bar{\gamma}(S_t + \varepsilon_t + e_t)$ where $\bar{\gamma}$ is deterministic and ε_t is a random variable. The main result of this paper holds with this alternative specification as well.

2.2 Bankable permit markets and non-bankable permit markets

Suppose the regulator knows the true cost functions of the firms, the damage function at each period, and the dynamics of the pollution stock accumulation. Given N firms' cost minimizing behaviors, a regulator seeks to minimize the sum of the abatement cost and the damage. We consider two market-based policy instruments to increase welfare. The description of these instruments are embedded in the following definitions of the corresponding market equilibria.

Definition 1 [Non-Bankable Permit Market Equilibrium]
Given the total number of permits for each period and each contingency $L_1 = \sum_{i\in I} l_{i1}$ *and* $L_2(S_1,\bar{\gamma}) = \sum_{i\in I} l_{i2}(S_1,\bar{\gamma})$ *for all* $\bar{\gamma}\in\Gamma$ *and all* $S_1 \geq 0$*, a non-bankable market equilibrium consists of permit prices* $q = (q_1, (q_2(S_1,\bar{\gamma}))_{S_1\geq 0,\bar{\gamma}\in\Gamma})$ *and an allocation of emissions* $e^* = \{e_i^*\}_{i\in I}$*, where* $e_i^* = (e_{i1}^*, \{(e_{i2}^*(S_1,\bar{\gamma}))_{S_1\geq 0,\bar{\gamma}\in\Gamma}\})$*, such that*
(i) (Firms' cost minimization) Given q *and* l_i*,* e_{i1}^* *solves the period-1 cost minimization problem*

$$\min_{e_{i1}\geq 0} C_i(e_{i1}) + q_1(e_{i1} - l_{i1})$$

and, for each $S_1 \geq 0$ *and* $\bar{\gamma}\in\Gamma$*,* $e_{i2}^*(S_1,\bar{\gamma})$ *solves the cost minimization problem under state* $(S_1,\bar{\gamma})$*:*

$$\min_{e_{i2}(S_1,\bar{\gamma})\geq 0} C_i(e_{i2}(S_1,\bar{\gamma})) + q_2(e_{i2}(S_1,\bar{\gamma}) - l_{i2}(S_1,\bar{\gamma}))$$

for all $i \in I$*,*
(ii) (Market clearing)
$L_1 = \sum_{i\in I} e_{i1}$ *and* $L_2(S_1,\bar{\gamma}) = \sum_{i\in I} e_{i2}(S_1,\bar{\gamma})$ *for all* $S_1 \geq 0$ *and all* $\bar{\gamma}\in\Gamma$*.*

$\mathbb{E}$ in the above definition denotes the expected value operator with respect to P. Definition 1 corresponds to a period by period tradable pollution permit scheme without banking and borrowing. The regulator specifies the total number of permits L_t for each period and each contingency (i.e. each realization $(S_1,\bar{\gamma})$).[3] Correspondingly, the market clearing condition is defined for each period and each contingency. The order of move is as follows: at the beginning of period 1 the regulator announces the allocation of period-1 emission permits $\{l_i\}_{i\in I}$, and firms decide how much emission to

[3]For our analysis, the initial distribution of the permits among firms does not matter.

discharge (and how many permits to purchase or sell) in period 1. At the beginning of period 2 the regulator realizes the period-1 emission S_1 and some $\bar{\gamma}$, and announces the period-2 emission permits $\{l_{i2}(S_1, \bar{\gamma})\}_{i \in I}$. Then the firms make the emission and trading decisions.

Definition 2 describes a permit market equilibrium where banking and borrowing of permits are allowed.

Definition 2 [Bankable Permit Market Equilibrium (with Fixed Permit Allocation)] *Given a conversion coefficient for banking and borrowing $\theta \in I\!R_+$ and θ-weighted sum of permits $L(\theta) \geq 0$ defined by*

$$L(\theta) = L_1 + \frac{1}{\theta} L_2 \tag{2}$$

for any arbitrary $L_1 = \sum_i l_{i1} \geq 0, L_2 = \sum_i l_{i2} \geq 0$ that satisfy (2), a bankable market equilibrium consists of permit prices $q \equiv (q_1, q_2) \in I\!R^2_+$, emissions $\{(e^*_{it})_{t=1,2}\}_{i \in I}$ and permit purchases $\{x^*_i\}_{i \in I}$ such that
(i) (Firms' cost minimization) Given θ, q and $\{l_{it}\}_{t=1,2}$, e^*_i and x^*_i solve

$$\min_{(e_i, x_i) \in R^3_+} C_i(e_{i1}) + q_1 x_i + \delta[C_i(e_{i2}) + q_2\{\theta(e_{i1} - l_{i1} - x_i) + e_{i2} - l_{i2}\}] \tag{3}$$

for all $i \in I$,
(ii) (Market clearing)

$$L(\theta) = \sum_{i \in I} e_{i1} + \frac{1}{\theta} \sum_{i \in I} e_{i2}.$$

With banking and borrowing, the regulator specifies the conversion rate for banked or borrowed permits (θ) and the weighted sum of the permits across time ($L(\theta)$). As Yates and Cronshaw (2001) argue, these two parameters are the regulator's choice variables for a two-period bankable market. Correspondingly, the market clearing condition is defined for the weighted sum of permits across time. With bankable permits, firm i can save or borrow the difference between the emission and the initial allocation plus the purchased permit $b_i \equiv (e_{i1} - l_{i1} - x_i)$ for the use in period 2. The effective amount of permit carried to period 2 is given by θb_i. Notice that each individual firm's problem is static without B&B and dynamic with B&B.[4] In the above

[4]The model here does not consider investment on pollution abatement capital (e.g. a gas turbine with less CO_2 emission per cycle) by firms. If we consider such investment, a firm's problem will be dynamic even without banking and borrowing of permits. See Conclusion for further discussion.

definition, the regulator announces the allocation of permits to each firm and the conversion rate for banking and borrowing at the beginning of period 1, and commits to the announced allocation until the end of period 2. Therefore, it is called a bankable market equilibrium with fixed permit allocation.

In what follows we characterize the efficient emission paths.

2.3 Efficient emission paths

Given the structure of the costs and the damages, we can solve for the efficient (least cost) emission path in the following manner.

(Step 1) Given the pollution stock at the end of period 1, S_1, and a realized rate of decay $\bar{\gamma} \in \Gamma$, solve for the optimal emission in period 2:

$$V_2(S_1, \bar{\gamma}) = \min_{e_2 \geq 0} \{C(e_2) + D_2(\bar{\gamma} S_1 + e_2)\}$$

Given the convexity of C and D_2, the following first order condition is necessary and sufficient for an interior solution:

$$C'(e_2) + D_2'(\bar{\gamma} S_1 + e_2) = 0. \tag{4}$$

Denote the solution by $e_2^*(S_1, \bar{\gamma})$).

(Step 2) Solve for the optimal emission in period 1:

$$V_1 = \min_{e_1 \geq 0} \mathbb{E}\{C(e_1) + D_1(e_1) + \delta V_2(S_1, \gamma)\}.$$

where $e_1 = S_1$. The first order necessary and sufficient condition for an interior sulution is [5]

$$C'(e_1) + D_1'(e_1) + \delta \frac{d}{de_1} \mathbb{E}\{V_2(e_1, \gamma)\} = 0$$

or

$$C'(e_1) + D_1'(e_1) + \delta \mathbb{E}\{\gamma D_2'(\gamma e_1 + e_2^*(e_1, \gamma))\} = 0^6 \tag{5}$$

The solution to the above dynamic problem gives us the optimal emission rule

$$(e_1^*, e_2^*(S_1, \bar{\gamma})_{S_1 \geq 0, \bar{\gamma} \in \Gamma}),$$

[5]Given the convexity of C and D_1, V_2 is convex.

[6]Since D_2' is bounded, we have $\frac{d}{de} \mathbb{E} D(\gamma e) \equiv \frac{d}{de} \int D(\gamma e) dP = \int \gamma D'(\gamma e) dP$.

which is in a feedback form. Note that an optimal emission path is defined for each realization of S_1, γ and pollution stock at the end of period 1; depending on those values, the optimal emission in period 2 will be different.A particular optimal emission path, given $\bar{\gamma} \in \Gamma$, will be $(e_1^*, e_2^*(e_1^*, \bar{\gamma}))$. In what follows we will see whether the two market schemes defined in the previous subsection yield the efficient outcome.

2.4 Welfare properties of tradable permits with and without B &B

Now we examine whether these market-based instruments result in different outcomes. First we have

Proposition 1 [7] *If the emissions are deterministic (i.e. $\gamma \equiv \bar{\gamma}$ almost surely for some $\bar{\gamma} \in [0,1]$), then a bankable market equilibrium with fixed permit allocation where*

$$\theta = \frac{D_1'(e_1^*) + \delta\bar{\gamma}D_2'(\bar{\gamma}e_1^* + e_2^*)}{\delta D_2'(\bar{\gamma}e_1^* + e_2^*)} \tag{6}$$

and

$$L(\theta) = e_1^* + \frac{1}{\theta}e_2^* \tag{7}$$

(where (e_1^*, e_2^*) is the efficient emission path) is efficient.

Proof. Here we sketch the proof for an interior solution. With the pollution stock being deterministic, the efficient emission path (e_1^*, e_2^*) is unique and given by a solution to the following system of equations:

$$C'(e_1^*) + D'(e_1^*) + \delta\bar{\gamma}D'(\bar{\gamma}e_1^* + e_2^*) = 0,$$

$$C'(e_2^*) + D'(\bar{\gamma}e_1^* + e_2^*) = 0,$$

$$e_t^* = \sum_{i \in I} e_{it}^* \quad \text{for all } t = 1, 2.$$

The necessary and sufficient conditions to firm i's problem are

$$C_i'(e_1^*) + \delta q_2 \theta = 0,$$

[7] This result is a corollary to a theorem proved by Montgomery (1972); the theorem implies that the market prices for pollutants with distinct pollution effects should be different in general. The claim and the proof in the context of our particular set up was suggested by Stephen Polasky.

$$C_i'(e_2^*) + q_2 = 0$$

and

$$q_1 = \delta q_2 \theta. \tag{8}$$

Set θ as in equation (6), q_1 as in equation (8) and

$$q_2 = D'(\bar{\gamma} e_1^* + e_2^*).$$

Then any emission vector $\{e_{it}\}$ that satisfies the market equilibrium conditions satisfy the conditions for efficient emissions.

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Therefore, in a deterministic world the market with banking and borrowing results in efficiency. The following proposition asserts that a non-bankable permit market equilibrium allocation is efficient with correctly specified state-contingent permit allocation and, conversely, an efficient pollution can be supported as an equilibrium pollution regardless of uncertainty in the pollution.

Proposition 2 *A non-bankable permit market equilibrium allocation is efficient if* $L_1^* = e_1^*$ *and* $L_2^*(S_1, \gamma) = e_2^*(S_1, \gamma)$ *for all* $\gamma \in \Gamma$. *With such* L_1^* *and* $(L_2^*(S_1, \gamma))_{\gamma \in \Gamma}$, *an efficient level of emissions can be supported as an equilibrium outcome.*

Proof. This follows immediately from the definition of a non-bankable permit market equilibrium. ■

Note that, without banking and borrowing, the regulator can choose the amount of permits for period 2 after observing the realization of the stock at the end of period 1. Therefore, the two market schemes are equivalent in the deterministic case. Given $p_1 = \delta \theta p_2$, firms face the same cost minimization problem under the two schemes and the regulator's problem is also the same. Since the firms choose the same emission path in both cases, the argument that B&B gives firms more flexibility and lowers firms' cost burden does not apply in this case.

With uncertainty in the pollution stock, the two schemes may result in different outcomes. In particular, as the following proposition states, a bankable market with fixed permit allocation may have a suboptimal pollution outcome.

Proposition 3 *If the pollution stock is uncertain, then the expected welfare under a bankable market equilibrium with fixed permit allocation is less than or equal to the welfare under an efficient non-bankable market equilibrium.*

Proof. Under B&B with a fixed permit allocation, the regulator faces a version of Ramsey's optimal policy problem where the regulator chooses the permit conversion coefficient θ and total allowable emission $L(\theta)$ subject to the resulting emission being an equilibrium emission:

$$\begin{aligned} \min_{(\theta,L,e_1,e_2)\geq 0} \quad & C(e_1) + D_1(e_1) + \delta \mathbb{E}\{C(e_2) + D_2(S_2)\} \\ \text{s.t.} \quad & S_1 = e_1, \\ & S_2 = \gamma S_1 + e_2 \quad \text{for all} \quad \gamma \in \Gamma, \\ & C'(e_1) - \delta\theta C'(e_2) = 0, \qquad (9) \\ & L = e_1 + \frac{1}{\theta} e_2. \qquad (10) \end{aligned}$$

Equations (9) and (10) represent the necessary and sufficient conditions that the emission satisfies in a bankable market equilibrium (the 'implementability constraint' in the optimal taxation literature). Denote the solution to this problem by $(\theta^{**}, L^{**}, e_1^{**}, e_2^{**})$. Solving the above constrained minimization problem, one obtains the following necessary and sufficient conditions:

$$C'(e_1) + D_1'(e_1) + \delta \mathbb{E}\{\gamma \mathrm{D}_2'(\gamma \mathrm{e}_1 + \mathrm{e}_2)\} = 0 \qquad (11)$$

and

$$C'(e_2) + \mathbb{E}\{\mathrm{D}_2'(\gamma \mathrm{e}_1 + \mathrm{e}_2)\} = 0 \qquad (12)$$

if (9) is not binding; if it is binding, then

$$C'(e_2) + \mathbb{E}\{\mathrm{D}_2'(\gamma \mathrm{e}_1 + \mathrm{e}_2)\} + \frac{\mathrm{C}''(\mathrm{e}_2)\mathrm{C}'(\mathrm{e}_1)[\mathrm{C}'(\mathrm{e}_1) + \mathrm{D}_1'(\mathrm{e}_1) + \mathbb{E}\{\gamma \mathrm{D}_2'(\gamma \mathrm{e}_1 + \mathrm{e}_2)\}]}{\delta \mathrm{C}'(\mathrm{e}_2)\mathrm{C}''(\mathrm{e}_2)} = 0. \qquad (13)$$

Therefore, $e_2^{**}(\gamma) = e_2^{**}(\gamma')$ for all all $\gamma, \gamma' \in \Gamma$: in a non-bankable permit market equilibrium, the period-2 emissions cannot be differentiated for different contingencies. By Proposition 2, the first-order conditions for a non-bankable market equilibrium emissions are necessary and sufficient for efficiency. In particular, we must have

$$C'(e_2) + D_2'(\bar{\gamma} e_1 + e_2(e_1, \bar{\gamma})) = 0 \qquad (14)$$

for each realization $\bar{\gamma} \in \Gamma$. Whether or not constraint (9) is binding, period-2 emission e_2^{**} does not satisfy (14). This concludes our proof.

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The intuition behind this result is as follows. The market clearing condition for a bankable permit market involves the weighted sum of the emissions in multiple periods. The regulator needs to set the total number of permits and the conversion factor prior to period 1, when γ is uncertain. With the non-bankable permit market scheme, on the other hand, the regulator is able to set the total number of permits in period 2 following the realization of γ.

Note that the market scheme without B&B is more consistent with Bellman's principle of optimality. It allows the regulator to set the total amount of permits at each point in time as a feedback strategy contingent on the realized beginning-of-period pollution stock and the rate of decay. On the other hand, a bankable permit market equilibrium with fixed permit allocation is analogous to an equilibrium with incomplete markets. The regulator cannot choose the period 2 total emission contingent on the information available at the beginning of period 2. The degree of freedom in the instrument choice is larger without banking and borrowing, and a non-bankable permit market scheme utilizes available information more fully than the bankable permit market scheme. This leads to a higher expected welfare with a non-bankable permit market.

The following definition describes an alternative banking and borrowing scheme where the regulator allows for intertemporal trade of permits but does not commit to the initial allocation of permits to firms.

Definition 3 [Bankable Permit Market Equilibrium with Flexible Permit Allocation] *Given a conversion coefficient for banking and borrowing* $\theta \in I\!R_+$ *and state-contingent* θ*-weighted sum of permits* $\{L(\theta, S_1, \bar{\gamma})\}_{\bar{\gamma}\in\Gamma}$ *defined by*

$$\forall S_1 \geq 0 \; \forall \bar{\gamma} \in \Gamma \quad L(\theta, S_1, \bar{\gamma}) = L_1 + \frac{1}{\theta} L_2(S_1, \bar{\gamma}) \tag{15}$$

for any arbitrary $L_1 = \sum_i l_{i1} \geq 0, L_2(S_1, \bar{\gamma}) = \sum_i l_{i2}(S_1, \bar{\gamma}) \geq 0$ *that satisfy (15), a bankable market equilibrium consists of permit prices* $q = (q_1, (q_2(S_1, \bar{\gamma}))_{S_1 \geq 0, \bar{\gamma}\in\Gamma})$*, emissions* $\{e^*_{i1}, (e^*_{i2}(S_1, \bar{\gamma}))_{S_1\geq 0,\bar{\gamma}\in\Gamma}\}_{i\in I}$ *and permit purchases* $\{x^*_i\}_{i\in I}$ *such that*
(i) (Firms' cost minimization) Given θ*,* q *and* $\{l_{i1},\}_{t=1,2}$*,* e^*_i *and* x^*_i *solve*

$$\min_{(e_i,x_i)} C_i(e_{i1}) + q_1 x_i + \delta \mathbb{E}[C_i(e_{i2}(S_1,\gamma)) + q_2(S_1,\gamma)\{\theta(e_{i1} - l_{i1} - x_i) + e_{i2}(S_1,\gamma) - l_{i2}(S_1,\gamma)\}] \tag{16}$$

for all $i \in I$,
(ii) (Market clearing) For all $S_1 \geq 0$ *and all* $\bar{\gamma} \in \Gamma$,

$$L(\theta, S_1, \bar{\gamma}) = \sum_{i \in I} e_{i1} + \frac{1}{\theta} \sum_{i \in I} e_{i2}(S_1, \bar{\gamma}).$$

In words, the regulator specifies the conversion coefficient θ prior to period 1; however, the regulator announces each firm's endowment of the period-2 emission permit at the beginning of period 2 given a realization of the period-1 emission and the rate of decay. As in the following proposition, the above banking and borrowing scheme yields an efficient pollution outcome if θ and L are specified correctly.

Proposition 4 *A bankable market equilibrium with flexible permit allocation is efficient if the conversion coefficient* θ *and the number of permits* $L(\theta, S_1, \gamma)$ *are given by*

$$\theta^* = \frac{D_1'(e_1^*) + \delta \mathbb{E}\{\gamma D_2'(\gamma e_1^* + e_2^*)\}}{\delta \mathbb{E}\{D_2'(\gamma e_1^* + e_2^*)\}} \tag{17}$$

and

$$\forall S_1 \geq 0 \ \forall \bar{\gamma} \in \Gamma \quad L^*(\theta^*, S_1, \bar{\gamma}) = e_1^* + \frac{1}{\theta} e_2^*(S_1, \bar{\gamma}) \tag{18}$$

where $(e_1^*, (e_2^*(S_1, \bar{\gamma}))_{S_1 \geq 0, \bar{\gamma} \in \Gamma})$ *is the efficient emission path.*

Proof. Here we sketch the proof for an interior solution. Given θ and $\{L(\theta, S_1, \bar{\gamma})\}_{\bar{\gamma} \in \Gamma}$, (q, e, x) is an equilibrium if and only if it satisfies the following system of equations.

$$C_i'(e_{i1}) + \delta \theta \mathbb{E} q_2(S_1, \gamma) = 0 \text{ for all } i \in I, \tag{19}$$

$$q_1 = \delta \theta \mathbb{E} q_2(S_1, \gamma), \tag{20}$$

$$C_i'(e_{i2}(S_1, \bar{\gamma})) + q_2(S_1, \bar{\gamma}) = 0 \text{ for all } \bar{\gamma} \in \Gamma \text{ and all } i \in I, \tag{21}$$

$$L(\theta, S_1, \bar{\gamma}) = \sum_{i \in I} e_{i1} + \frac{1}{\theta} \sum_{i \in I} e_{i2}(S_1, \bar{\gamma}) \text{ for all } \bar{\gamma} \in \Gamma. \tag{22}$$

Set θ^* and L^* as in (17) and (18), and let $q_2(S_1, \bar{\gamma}) = D_2'(\bar{\gamma} e_1^* + e_2^*(S_1, \bar{\gamma}))$ for all $\bar{\gamma} \in \Gamma$ and q_1 as in (20). Then any e that satisfies (19) - (22) satisfies (4) and (5), the necessary and sufficient conditions for the efficient emission.

■

The above proposition suggests that, if the regulator has flexibility in updating the total allowable emission over time, then allowing for banking and borrowing of permits can be consistent with efficiency. Together with Proposition 2 and 3, it reveals the importance of the regulatory flexibility when the intertemporal trading of permits is allowed.

By Proposition 2 and 4, we see that the efficient emission can be achieved by a tradable permit market with and without banking and borrowing. Two remarks are in order. First, as discussed in Leiby and Rubin (2001), the conversion coefficient θ must be correctly specified. The calculation of the optimal conversion coefficient involves the expected marginal costs and damages, that are evaluated at the optimal emission path $(e_1^*, (e_2^*(e_1^*, \gamma))_{\gamma \in \Gamma})$. Secondly, in order for Proposition 4 to be valid, firms need to have a correct belief on how the regulator updates $L_2(S_1, \gamma)$. In order to achieve the efficient emission in a bankable permit market, the regulator needs to know the correct distribution of γ. At the same time, firms must know how $q_2(S_1, \gamma)$ is distributed. Without banking and borrowing, on the other hand, firms' problems are time- and state-separable and hence deterministic in effect. In period 1, firms can solve for cost-minimizing period-1 emission without knowing what q_2 or L_2 will be in period 2. Similarly, the choice of e_2 in period 2 is not affected by the choice in period 1. In this sense, a permit market with banking and borrowing demands that firms have more information than a permit market without banking and borrowing.

3 Conclusion

We compared the efficiency of tradable pollution permits with and without banking and borrowing of the permits for discharging a stock pollutant. The result here suggests that a bankable permit market scheme is inferior to a non-bankable permit market scheme if, for any reason, (1) the revision of the total number of permits is needed for efficiency as uncertainty is resolved over time (2) while the regulator does not have flexibility to revise it. This study considered uncertainty in the pollution stock dynamics, but the inferiority of a bankable permit market with fixed permit allocation holds for a wider range of cases (e.g. when the regulator learns about the distribution of random pollution dynamics and future damages over time).

We considered a tradable market scheme without banking and borrowing where the regulator is able to set the total amount of permits in each period. The firms may react strategically given the regulator's periodic update of the policy. An interesting case is where firms make investment decisions to install some pollution-abatement equipment. Depending on the way the regulator updates its policy, firms may not choose the optimal amount of investment.[8] Examinations of such strategic moves await future research. We also assumed the absence of asymmetric information; the regulator is assumed to know the correct cost functions of the firms. The optimal policy for stock pollutants in the presence of asymmetric information was analyzed by Benford (1998) and Hoel and Karp (2001). Benford showed that a combined price and quantity (tax and tradable permit) scheme for a static pollution problem with asymmetric information, which was first analyzed by Kwerel (1977), results in efficiency for a limited class of cost functions. The results here suggest that the result by Yates and Cronshaw (2001) (superiority of banking and borrowing scheme under asymmetric information between the regulator and the polluters) will be modified against banking and borrowing in the case of a stock pollutant with uncertainty.

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