Economies of Scope and Scale of Multi-Product U.S. Cash Grain Farms:
A Flexible Fixed-Cost Quadratic (FFCQ) Model Analysis

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Abstract

The evidence about the magnitude of scope and scale economies in U.S. cash grain farming is revealed from the empirical estimation of the flexible fixed cost quadratic (FFCQ) model. This framework explicitly disaggregates the crop output vector to take the heterogeneity of output and gives insight to farmers to answer interesting questions such as: Are three-crop farms more cost efficient than two-crop or single-crop farms? How important are economies of scope (fixed-cost and variable-cost components) in two-crop farms and three-crop farms? Two-crop farms as well as three-crop farms exhibit overall economies of scale and scope in all four-size categories that increase with the farm size. They are able to lower the cost of producing crops in the same farm by spreading fixed costs over two or three crops and/or by exploiting product cost complementarity, or diversifying risks.

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1. Introduction

Measuring scope economies allows for an assessment of the benefits from output diversification for multi-product firms in the agricultural industry. Increase in farm size may lead to cost economies, but the presence of scope economies in diversified versus specialized farms may tend to lower costs in terms of comparable level of output. In summarizing the major studies focusing on this issue, Hallam (1993) discussed the diversity in the approaches adopted in the literature in measuring economies of scale and scope in agricultural production.

Ray (1982), in exploring the presence of overall cost economies in crop and livestock enterprises, found jointness in the production of crops and livestock, implying that the marginal cost of producing crop is negatively related the livestock output. Hertel and McKenzie (1986) found corn and wheat, and soybeans and wheat net complements. These are due to the timing of these production activities to take advantage of crop diversification. Corn and soybeans were strong net substitutes. Ojemakinde, Lange and Zacharias (1989) showed that soybean-specific economies of scale were larger than those of rice. Leathers (1992) concluded that for high (low) levels of output of crops or milk, there were economies (diseconomies) of scope between milk and crops. Fernandez-Cornejo et al. (1992) identified substantial dynamic scope economies between cattle and others products (crops, hogs and milk) in German agriculture. Anderson and Helgeson (1974) found that sharing of labor and capital resources was the main source for cost savings from product diversification. Each of the four product lines (grain, feed, petroleum, and fertilizer) exhibited economies of size whose magnitude varied with the relative importance of fixed costs.

Baumol, Panzar and Willig (1982) suggested two conditions that may lead to economies of scope in multi-product farms: the cost complementarity between two crops and/or the sharing or joint utilization of quasi-fixed inputs by crops. But early studies did not isolate the effects of quasi-fixed costs to overall scope economies. They computed scope economies measures that do not provide information on significant benefits from sharing fixed costs. These studies also aggregated diverse outputs into a single measure of outputs and determine whether there are scale economies. The presence of scale economies for an aggregate measure of outputs does not imply the presence of scale economies for any of components of the aggregate measure of output.

This study uses a framework that explicitly disaggregates the crop output vector to take the heterogeneity of output and gives insight to farmers to answer interesting questions such as: Are three-product firms more productive and efficient than two-product or single-product firms? How important are economies of scope (fixed-cost and variable-cost components) in two-product firms and three-product firms? How are marginal and average costs of producing corn affected by changes in the acreage of soybeans or wheat? But despite the theoretical importance of these questions, the quantitative evidence of the presence and sources of farm size advantages in multi-product farm firms is mixed. Lack of sufficiently detailed data has made it difficult to control for firm-specific effects and to distinguish between product-specific fixed cost and variable cost economies of scope. The following section presents the theoretical framework of the study. The next section discusses estimation procedures. The fourth section describes the construction of farm-level panel data. The fifth section presents the empirical results. The last section provides a conclusion and directions for further research.

2. Conceptual Framework

Product-Specific Scope Economies

In this study, there are three products that lead to five possible configurations of production: (1) all three produced in one farm; (2-4) one product produced separately and the other two jointly; (5) all three produced separately (Leathers, 1992). If there is joint production of all three products at levels Q, then configuration (1) is the least cost configuration of producing Q.

Let define the set S as a subset of farms that produce all three products, T represents a subset of farms that produces only one product and S-T represents another subset of farms that produces two products. Overall scope economies exist or a multi-product cost function is subbaditive, if the cost of producing jointly three products, $C(Q_S)$ is less than the sum of costs of producing separately individual products, $C(Q_T) + C(Q_{S-T})$:

(1) $$C(Q_S) < C(Q_T) + C(Q_{S-T})$$

The degree of SCOPE, the percentage of cost savings from producing all products jointly as opposed to producing products in two subsets of farms separately is

(2) $$SCOPE = \frac{C(Q_T) + C(Q_{S-T}) - C(Q_S)}{C(Q_S)}$$

If SCOPE greater than 0, then scope economies exist and farms can be more cost efficient by diversifying production activities. That is, there exists something inherent in the production technology that makes it cheaper to produce a subset of products jointly.

Cost complementarity and Product-Specific Quasi-Fixed Costs

Dividing the total joint cost function, $C(Q_S)$, into quasi-fixed input costs, F(S), and variable input costs, $C^{var}(Q_S)$, two conditions leading to scope economies or subadditivity of a cost function are identified (Gorman, 1985). The first condition is the existence of cost complementarity ($COMP_{ij}$) or jointness in variable input between two products Q_{fit} and Q_{fjt}. That is, the marginal costs of producing two products are dependent. Since $C(Q_S) = FC_S + C^{var}(Q_S)$, the cost complementarity may be defined as

$$(3) \qquad COMP_{ij} = \frac{\partial^2 C(Q_S)}{\partial Q_{fit} \partial Q_{fjt}} = \frac{\partial^2 C^{var}(Q_S)}{\partial Q_{fit} \partial Q_{fjt}} < 0 \iff \frac{\partial MC_{fit}}{\partial Q_{fjt}} = \frac{\partial AIC_{fit}}{\partial Q_{fjt}} < 0.$$

If $COMP_{ij}$ is less than zero, there are gains in diversification or economies of scope.

The second condition is the presence of product-specific fixed costs that can overcome cost anti-complementarities. It is expressed as

$$(4) \qquad FC_S < FC_T + FC_{S-T}$$

As long as the fixed cost of producing all or a subset of products jointly (FC_S) is less than the sum of the fixed cost of producing two subsets of products (FC_T + FC_{S-T}) in different farms, two disjoint subsets of products share quasi-fixed inputs cost function that is subadditive.

Relationships between Scope, Product-Specific Scale and Overall Scale Economies

Multi-product scale economies (SCALE) measure the cost implications of varying all products simultaneously while holding the mix of products constant. It is defined as:

$$(5) \qquad SCALE = \frac{C(Q_S)}{\sum_i \frac{\partial C}{\partial Q_{fit}} Q_{fit}} = \frac{C(Q_S)}{\sum_i MC_{fit} * Q_{fit}}.$$

Multi-product scale economies (diseconomies) exist if SCALE is greater (less) than unity. Baumol, Panzar and Willig (1982) showed that overall scale economies result from product-specific scale and/or scope economies. That is, strong scope economies may lead to overall scale economies that can be greater than one even if there are constant or decreasing product-specific economies of scale:

$$(6) \qquad SCALE = \frac{\sum_i SH_i * SCALE_{fit}}{1 - SCOPE}$$

(7) $$SH_i = \frac{Q_{fit}\frac{\partial C}{Q_{fit}}}{\sum_i Q_{fit}\frac{\partial C}{\partial Q_{fit}}} = \frac{Q_{fit}*MC_{fit}}{\sum_i Q_{fit}*MC_{fit}}$$

where SH_i represents the share of variable cost of production incurred by Q_{fit}. The product-specific scale economies ($SCALE_{fit}$) gives information about changes in cost as the output of any product expands and are computed via

(8) $$SCALE_{fit} = \frac{C(Q_S) - C(Q_{S-T})}{Q_{fit}\frac{\partial C}{\partial Q_{fit}}} = AIC_{fit}*(MC_{fit})^{-1}.$$

AIC_{fit}, MC_{fit}, $C(Q_S)$ and $C(Q_{S-T})$ are, respectively, product-specific average incremental cost, marginal cost, joint cost of producing all products and cost of producing all products except one of them such as corn, soybeans or wheat. When $SCALE_{fit}$ is greater than 1 ($AIC_{fit} > MC_{fit}$), the average cost of producing Q_{fit} falls as Q_{fit} level increases reflecting economies of scale for the specific product. Notice that the average incremental cost of producing Q_{fit} includes any product-specific fixed costs associated with the production of Q_{fit} and depends on the assumed production of Q_{fjt}. Declining average incremental or marginal costs and cost complementarities are conditions needed for overall multi-product scale economies.

Flexible Fixed Cost Quadratic Model

The basic specification of the empirical model is a flexible fixed cost quadratic (hereafter FFCQ) function suggested by Lau (1974) and embellished by Baumol, Panzar and Willig (1982). In the case of three products, the FFCQ model may be written as follows:

(9) $$C_{ft} = \alpha_T DUM_T + \alpha_{S-T} DUM_{S-T} + \alpha_S DUM_S + \sum_i \beta_i Q_{fit} + 1/2\sum_i \sum_j \beta_{ij} Q_{fit} Q_{fjt}$$

where

C_{ft} = Total cost of farm f in year t;

DUM_T = Dummy variable for farm f that produces only one product;

$DUM_{S\text{-}T}$ = Dummy variable for farm f that produces two products;

DUM_S = Dummy variable for farm f that produces three products;

Q_{fit} = Quantity of product i produced by farm f in year t;

e_{ft} = Residual error term for farm f in year t.

Input prices are important in estimating cost functions. Due to the homogeneity of the farms location and little variation in input markets, input prices are not be included in this estimation (Hornbaker, Dixon and Sonka, 1989; Grosskopf, Hayes and Yaisawarng, 1992; Mafoua, Hornbaker and Sherrick, 1996). Although the use of inputs differs across Illinois cash-grain farms, the sets of variable inputs (fertilizer, pesticides and seed) used by farmers are quite homogeneous. Assuming constant input prices for all firms in the sample, the FFCQ model has been applied in many industries: energy (Mayo, 1984), education (Cohn et al., 1989; Llyod et al., 1993), mutual funds (Dermine and Roller, 1990), banking (Pulley and Humphrey, 1993), savings and loans (Gropper, 1995).

The standard translog cost (TLC) function and its hybrid (Box-Cox transformation) have been applied in many farms cost structure analysis (Akridge and Hertel, 1986; Moschini, 1988;Schroeder, 1992; Gallagher, Thraen, and Schnitkey, 1993). Their well-known disadvantage is its inability of modeling accurately the effects of specialization (Roller, 1990). They have yielded quite different scope economy results depending on how close the substituted positive values are to zero. Since the sample of this study contains farms that did not produce continuously soybeans or wheat from 1984 to 1994, the use of the TLC or its hybrid may provide biased estimates and lead to different policy conclusions if these farms are not accounted for in the empirical analysis.

Derived Product-Specific Cost Measures

The FFCQ function has the ability to provide information on the decomposition of scope economies into fixed-cost ($SCOPE_{FC}$) and variable-cost ($SCOPE_{VC}$) components. Economies of scope between

(10) $$SCOPE = \frac{[\alpha_T + \alpha_{S-T} - \alpha_S]}{C(Q_S)} + \frac{[-1/2\sum_i \sum_{j \neq i} \beta_{ij} Q_{fit} Q_{fjt}]}{C(Q_S)}$$

or

(11) $$SCOPE = \frac{[FC_T + FC_{S-T} - FC_S]}{C(Q_S)} + \frac{[-1/2\sum_i \sum_{j \neq i} \beta_{ij} Q_{fit} Q_{fjt}]}{C(Q_S)} = SCOPE_{FC} + SCOPE_{VC}$$

(12) $$SCOPE > 0 \Leftrightarrow SCOPE_{FC} > -SCOPE_{VC} \Leftrightarrow \alpha_T + \alpha_{S-T} - \alpha_S > 1/2\sum_i \sum_{j \neq i} \beta_{ij} Q_{fit} Q_{fjt}$$

two product sets is expressed as (Pulley and Humphrey, 1993):

(13) $$C(Q_S) = FC_S + \sum_i \beta_i Q_{fit} + 1/2\sum_i \sum_j \beta_{ij} Q_{fit} Q_{fjt}$$

Estimates of parameters α_T, α_{S-T} and α_S represent, respectively, the fixed costs of producing one product separately, another product separately or two products jointly, and two products or three products jointly. The expression $\alpha_T + \alpha_{S-T} - \alpha_S$ measures the saving in quasi-fixed costs associated with producing the two or three products jointly, and $-0.5\Sigma\Sigma\beta_{ij} Q_{fit} Q_{fjt}$ is the cost complementarities component of the overall scope economies. $SCOPE_{FC}$ measures the contribution to the economies of scope from spreading fixed costs across products i and j.

Extending the work of Baumol, Panzar, and Willig (1982), Gorman (1985) shows that, even

(14) $$SCOPE > 0 \Leftrightarrow SCOPE_{FC} > 0 \Leftrightarrow \alpha_T + \alpha_{S-T} > \alpha_S \Leftrightarrow FC_T + FC_{(S-T)} > FC_S$$

when $SCOPE_{VC}$ is equal to 0, the existence of subadditive product-specific fixed costs is a sufficient condition for presence of economies of scope:

where

(15) $$SCOPE_{VC} = 0 \Leftrightarrow COMP_{ij} = \frac{\partial MC_{fit}}{\partial Q_{fjt}} = \frac{\partial AIVC_{fit}}{\partial Q_{fjt}} = 0 \Leftrightarrow \beta_{ij} = 0.$$

If different products share variable inputs, the β_{ij} coefficients would be expected to be negative. Therefore, there will either be economies or diseconomies of scope depending on the signs of β_{ij}, the sizes of Q_{fit} and Q_{fjt}, and on the product-specific fixed costs. There is no reason to believe that farm's costs of producing soybeans are unaffected by the nature and scale of corn or wheat. Shumway, Pope and Nash (1984) stated that allocatable quasi-fixed inputs cause economies of scope when the marginal allocation of variable inputs depends upon the allocation of the fixed input and generate product-specific fixed costs. Using the FFCQ model, the overall scale and product-specific scale economies are computed as follows:

(16) $$SCALE = \frac{FC_T + FC_{S-T} - FC_S + \sum_i \beta_i Q_{fit} + 1/2 \sum_i \sum_j \beta_{ij} Q_{fit} Q_{fjt}}{\sum_i (\beta_i Q_{fit} + \sum_j \beta_{ij} Q_{fit} Q_{fjt})}$$

and

(17) $$SCALE_{fit} = \frac{\beta_i Q_{fit} + 1/2 \beta_{ii} Q_{fit}^2 + \sum_{j \neq i} \beta_{ij} Q_{fit} Q_{fjt}}{\beta_i Q_{fit} + \beta_{ii} Q_{fit}^2 + \sum_{j \neq i} \beta_{ij} Q_{fit} Q_{fjt}}$$

3. Estimation Procedures

Two estimators are discussed: the between-firm and the least-squares dummy variable (within-firm) estimators. Since farm-level panel data are used, unobserved heterogeneity among farms will be accounted when using the OLS regression model (Hsiao, 1986; Mafoua and Hossain, 2001). Consider the following linear regression model:

(18) $$TC_{ft} = G_f(Q_{fit}, \beta) + \varepsilon_{ft}.$$

For the fth farm at year t, TC_{ft} is the total cost; G_f is the production technology; Q_{fit} is the vector of product outputs; β is a vector of k unknown production parameters; ε_{ft} is the error term which represents the effects of the omitted variables that are specific to n farm and T years.

Between-Firm (BF) Estimator

The between-firm estimator uses only the variation among the farm means. The standard approach to obtaining the between-firms results is to regress the firm-specific means of the dependent variable on the firm-specific means of independent variables. For this study, this amounts to regressing the 1984-1994-farm average of total cost on the 1984-1994 farm average of crop outputs. The between-firm estimator is generally expressed as:

(19) $$\overline{TC}_f = \overline{G}_f(Q_{fit}, \beta) + \alpha + u_f + \bar{e}_f$$

There is a gain in efficiency that results from the utilization of the between-firm estimator in addition to the within-firm estimator. It is interpreted as a long-run estimator while the within-firm estimator is interpreted as a short-run estimator. An alternative explanation for the difference between the within- and between-firm estimators is attributed to measurement error .

Least-Squares Dummy (LSDV) Variable Estimator

The Least-square dummy variable (fixed-effect, FE or within-firm) estimator, used to estimate the FFCQ function, is generally expressed as:

(20) $$TC_{ft} = G_f(Q_{fit}, \beta) + \alpha_f + e_{ft}$$

where ε_{ft} is decomposed into α_f and e_{ft}. The following assumptions are made: α_f is the farm-specific fixed-effect representing the cost of an unmeasured input (e.g. fixed capital) that is quasi-fixed over time; e_{ft}, the stochastic costs of inputs that can not be controlled by any farm (e.g., weather, diseases). They are independently and identically distributed (i.i.d.) across farms and years and uncorrelated with the crop outputs. Farm- or crop-specific quasi-fixed costs of machinery capture differences in capital fixed-cost or technology between farms or farm groups which produce different crop mixes. They are assumed to be correlated with the crop outputs and their mixes. The LSDV model may also be written as

(21) $$TC_{ft} = \sum_{f=1}^{n} \alpha_f D_{ft} + G_f(Q_{fit}, \beta) + e_{ft}$$

where D_{ft} is the farm-effect dummy variable that takes the value 1 for farm f and zero otherwise. When the number of farms n is small, the estimation of the above model may be achieved (using OLS) by suppressing the constant term and adding a dummy variable for each of the n farms, or equivalently, by keeping the constant term and adding n-1 dummies (Hsiao, 1986).

4. Data Specification

To estimate short-run and long-run total cost functions, cash-grain farms that participated in the Illinois Farm Business Farm Management (IFBFM) Association from 1984 to 1994 are used. The IBFM Association farms are highly representative of commercial agriculture. Their records are primarily year-end financial statements for individual farms. They are reliable and consistent across farms. They contain cross-sectional and time-series data on acreage, yields, prices, and on aggregate expenditures on variable and quasi-fixed inputs. In this study, variable inputs expenses include expenses on fertilizer, pesticide, seed, drying and storage and miscellaneous expenses. Quasi-fixed expenses involve machinery depreciation and repair expenses, and insurance expenses. Any econometric model, with total cost as dependent variable, that includes time series data, involves the problem of how to deal with the general level of cost. In this study, this difficulty is handled by deflating the total cost of producing crops, with an indicator of the price level such as the consumer price index, CPI.

The sampled farms have soil productivity rating and tillable acreage greater than or equal to 60 and 50, respectively. Three farm groups are observed in panel data: (1) corn-soybean-wheat farms; and (2) corn-soybean farms, and (3) corn farms. For the average three-product firm, the sample data results show a mean tillable acreage of 691.30 acres. Farm size ranged from 102 to 2450 tillable acres. Table 1 presents summary statistics over eleven years for the three-product firms (corn, corn-soybean or corn-soybean-wheat farms). The average tillable acres of corn, soybeans, wheat, and set-aside for the 1984-1994 are, respectively, 334.15, 310.42, 2.55 and 42.49. From 1984 to 1994, on average, 231 farms had acreage in corn while soybeans and wheat were harvested on an average of 228 and 10 farms each year.

Removing farms that produced wheat during the observed period results in 185 two-product firms (corn or corn-soybean farms). They have a mean tillable acreage of 671.37 acres, which is close to the mean of the entire farms acreage. Farm size ranges from 102 to 2450 tillable acres. The average tillable acres of corn, soybean and set-aside for the observed period are, respectively, 326.10, 301.63, and 42.05. In addition, the IFBFM data set reports the total variable costs according to input but does not allocate costs to the individual crops. As opposed to considering all quasi-fixed expenses, the econometric model focuses on machinery fixed costs. Also set-aside and crop acreage, and yields are included in the IFBFM data. The number of set-aside acres for any farmer is a function of the corn and/or wheat base acreage. There is a strong linear dependence between corn acreage and set-aside acreage (Hornbaker, Dixon, and Sonka, 1989).

5. Empirical Results

Short-Run versus Long-Run Cost Estimates

Table 2 presents long-run and short-run parameter estimates of the FFCQ multi-product models using farm-level panel data. The between-firm (BF) estimates are interpreted as long-run estimates while the within-firm (LSDV) estimates are interpreted as short-run estimates. Total costs are estimated as dependent variables in two-product (corn and soybeans) and three-product (corn, soybeans and wheat) models. Two-product and three-product models are estimated using 185 and 231 farms observed during 11 years, respectively.

Overall, results of the cost estimations for alternative model specifications are consistent with our prediction. The obtained range of R-squares shows that these models explain at least 94% of the variation in the farm-level data. The F-statistics for model regressions reject the hypothesis that all parameters are zero at 0.01 level of significance. This indicates that goodness of fit of cost models is reasonably strong and that the independent variables have significant power in capturing variations in total cost. With a few exceptions, the majority of the parameters is highly and statistically significant at least at 0.10 level and carries the expected signs.

Results also show positive parameters on linear (β_i) and quadratic (β_{ii}) output terms except for wheat parameters. Positive β_i coefficients mean that the cost surface appears to satisfy monotonicity in output quantities. Parameter estimates of corn and soybeans are significant at 0.01, thus implying that total costs rise with increases in the production of corn or soybeans. Positive estimates for the quadratic corn and soybeans terms indicate that for farms that produce corn or soybeans the quadratic cost function is convex. These positive parameters (β_{cc} and β_{bb}) give rise to U-shaped average total costs for corn and/or soybeans producing farms along each output axis, which is consistent with classical theory. They also indicate that the marginal cost of corn is an increasing function of the quantity of corn produced and decreasing function of the quantity of soybeans produced. Similarly, the marginal cost of soybeans is an increasing function of the quantity of soybeans produced and decreasing function of the quantity of corn produced. This result is robust across models and estimators. It confirms the advantages of joint production of corn and soybeans, a feature that characterizes crop production in Illinois.

For wheat-producing farms, the coefficient estimates (β_{ww}) are negative and indicate that for wheat producing farms the quadratic cost function is concave. Most coefficient estimates related to wheat (except the cross-product of wheat with corn, β_{cw}) have large standard errors and do not differ statistically from zero.

Scope and Scale Economies at Three-Product versus Two-Product Firms

Four farm size categories are considered in this study: very small farms with no more than 300 tillable acres, small farms with between 300 and 600 tillable acres, medium farms with between 600 and 900 tillable acres, and large farms with more than 900 tillable acres. Table 3 reports long-run measures of scale, scope and product-specific scale economies that were evaluated at the mean values of the exogenous variables within each firm size range. These cost statistics correspond to the average farm of each size class. Since output mix varies among the farm size classes, a comparison across size classes is a comparison of changes in scale and composition. Two-product firms as well as three-product firms exhibit overall economies of scale in all four-size categories that increase with the firm size. This finding suggests that large farms possess a cost advantage compared to small and medium farms. For the three-product small and medium farms, the degree of overall scale economies is less than equal 1.008. This figure implies that there are mild economies of scale for small and medium farms. Increasing scale and scope economies mean that large, diversified corn-soybean or corn-soybean-wheat farms are more cost efficient than small, diversified or single-product firms. Three-product firms may experience economies of scope from their ability to produce corn, soybeans and wheat using inputs more efficiently than they would if production were performed separately.

Economies of scale and scope are detected for the same time span for both farm models using between-firm estimator (Table 4). Results suggest that corn-soybeans-wheat farm as well as corn-soybean farms are cost efficient from joint production. They are able to lower the cost of producing crops in the same farm by spreading fixed costs over two or three products and/or by exploiting product cost complementarity (corn-soybean farms), or diversifying risks (corn-soybean-wheat farms). Scale economies and scope economies of corn-soybean farms are larger than those of corn-soybean-wheat farms. This means that large corn-soybean farms are more cost efficient than corn-soybean-wheat farms. Note that all values of scope economies are positive, implying the presence of economies of scope. If the average farm combines the production of corn, soybeans and wheat, it can have a cost saving of 14.8 percent in total cost as given by degrees of 0.148 for overall scope economies.

There are product-specific diseconomies of scale of corn and soybeans and there are larger for the three-product firms. The product-specific economies of scale of corn and soybeans show that increasing production of either corn or soybeans will lead to an increase in total cost of production. Finally, the existence of multi-product economies of scale suggests a proportional increase in corn and soybean production would entail a less than proportional increase in total costs. The decrease in product-specific economies of scale of both corn and soybeans suggests that individual crops are not subject to returns to scale in the long run (Tables 3 and 4). Therefore, it is less expensive to produce both crops in the same farm than in separate farms. Thus, combining soybeans or wheat into a farm firm that currently does not produce soybeans or wheat will lead to economies of diversification. Since there are economies of diversification, a policy that remove restrictions on the number of acres of corn or soybeans that can be produced could lead to greater efficiency in the agricultural industry by allowing farms to select the most efficient mix of crops. But on the other hand, wheat has product-specific economies nearly equal to 1, indicating constant returns to scale at the mean output. That is wheat should not have a cost disadvantage or advantage in a corn-soybean-wheat farm.

Product-Specific Quasi-Fixed Costs

Farm-level panel data show that the underlying technology facing each farm group may be different because it has statistically been shown by the high significance of farm-group specific dummy variables (Tables 2 and 5). They are included to allow the intercept to vary between different farm groups observed in the sample (corn farms, corn-soybean farms, and corn-soybean-wheat farms). They have impacts on the level of total cost in both firm models. This is consistent with the theory that states that when a farm incurs joint quasi-fixed costs in producing a multiplicity of products, traditional measures of scale economies used in the single-product case are no longer legitimate. The specification of the FFCQ function allows computation of product-specific quasi-fixed and incremental fixed cost of producing individual product or product portfolios (Baumol, Panzar and Willig 1982). This further suggests that the FFCQ model is useful for estimating unobserved annual total level of quasi-fixed costs of machinery, which may vary for different crop output configurations for any cash-grain representative farms. Table 5 shows that the product-specific quasi-fixed cost of producing corn is $13,684 and $14,022, using the two-product and the three-product models, respectively. That of producing corn and soybeans together is $19,557 and $18,859, respectively. The quasi-fixed cost of producing corn, soybeans and wheat together is $22,699. The computed average fixed costs of specialized corn farm and diversified corn-soybean farm are respectively, $13,853 and $19,208. The derived incremental fixed cost of adding soybeans in a corn farm is $5,355. The incremental fixed cost of adding wheat in a diversified corn-soybean farm is $3,491.

Cost Complementarities and Scope Economies

Table 6 provides estimated derivatives of marginal cost of producing corn, soybeans or wheat in two-product or three-product average firm. The diagonal elements of the Hessian submatrix give information on the curvature of marginal cost curves for corn, soybeans and wheat, respectively.

Negative values of diagonal elements suggest that marginal cost curves are decreasing (wheat). Decreasing marginal costs are consistent with increasing returns to scale. Statistically significant negative off-diagonal elements provide evidence of a cost complementarity between any of two products (corn and soybeans). The robustness of this finding across models offers confidence in the estimation results and also indicates that corn-soybean farms are technically more efficient than the ones supplying only corn. There is also a cost complementarity between soybeans and wheat but this statistic is not significant for any estimator results. The remaining pair, corn-wheat has positive coefficient, which is consistent with anti-cost complementarity leading to diseconomies of scope. This coefficient value is statistically significant at 10 %. This result suggests that non-cost motivations may be important in explaining the joint production of corn and wheat as well as soybeans and wheat in some Illinois cash grain farms.

Effects of Fixed-Cost and Variable-Cost Components on Scope and Scale Economies

A correct procedure for measuring the fixed-cost savings from joint production ($Scope_{FC}$) requires a rich data set containing numerous observations from farms that specialize in only one of the possible outputs as well as from farms that specialize in all possible subsets of outputs. Since our data show three possible configurations of production: (1) corn, soybeans and wheat produced in one farm; (2) corn produced separately and (3) corn and soybeans produced jointly. The best we can do is to derive with existing information in Table 3 an approximate (upper bound estimate of the true, but unknowable) measure of product-specific quasi-fixed cost of producing soybeans or wheat. Estimate of quasi-fixed cost of corn was used as an upper bound estimate of product-specific quasi-fixed cost of soybeans (FC_{BN} = FC_{CO} = \$13,684) or wheat (FC_{WH} = FC_{CO} = \$14,022). We are implicitly assuming that specialized production of soybeans or wheat would require the same level of fixed costs. Table 7 reports the magnitude of savings in fixed costs ($Scope_{FC}$) and in variable costs ($Scope_{VC}$) associated with producing two or three products jointly. For the two-product firm as well as the three-product firm, variable cost scope economies is greater than fixed-cost scope economies. Two-product firms exhibit larger variable-cost scope economies than three-product firms. Three-product firms exhibit larger fixed-cost scope economies than two-product firms. Overall, three-product firms exhibit larger scope economies than two-product firms. This scope economies increase with time.

There is two main explanation of a preference for crop diversification among farmers. First, a preference for some degree of crop diversification among risk-neutral farmers can be explained by the existence of complementarity between crops (two-product firms). Second, uncertainty of net returns explains a preference among risk-averse farmers for crop diversification (three-product firms). The inclusion of wheat in diversified corn-soybean farms is due to non-cost motivations such as diversification to reduce risk. Farmers may be willing to incur additional operating costs (anti-complementarity between corn and wheat) in order to reduce risk in their income streams. Illinois farms are not single-crop firms since they produce at least two crops in part for customer convenience and also to reduce risk through farm portfolio diversification to take advantage of low correlation between corn and wheat or soybeans and wheat. This study confirms that risk avoidance is another cause of corn-wheat or soybeans and wheat jointness in a multi-output technology (Mafoua, Hornbaker and Sherrick, 1996). This crop diversification represents a potential means of overcoming some of the negative side effects of monoculture of corn such as pest problems and soil erosion.

Conclusion and Directions for Further Research

Empirical results suggest that scale economies are significant for large farms in Illinois, making it impossible to identify the most efficient farm size. But these production economies do not arise from the production of a specific crop (corn, soybeans or wheat). The primary advantage of larger farms appears to be their ability to exploit economies of scope and sustain a diverse portfolio of two or three crops. Second, they are able to spread the quasi-fixed costs of machinery and equipment over these crops. This analysis also supports the notion that the quadratic cost function examined is output-specific subadditive. Decreasing product-specific scale economies for corn or soybeans, and/or constant returns to scale for wheat along with strong scope (fixed-cost and variable cost) economies between products are sufficient for subaddivity. These production economies can be exploited by farms specialized only in single-crop production. The significance of returns to scope and scale for large farms implies that smaller farms or entering farms that operate at a small scale are at a cost disadvantage compared with larger, established farms. This also suggests that the long-run configuration of Illinois agricultural industry is characterized by a sharp reduction in the number of farms.

For further research, there are several issues that can be analyzed: First, the effects of livestock production on scope measures in cash-grain farms need to be addressed. Since the static framework provides satisfactory answers to many economic problems, it ignores central information when inter-temporal interdependencies are present. In the dynamic approach, a firm considers a multi-period horizon and inter-temporal allocation of resources to be an integral part of the cost structure analysis (Fernandez-Cornejo et al., 1992). Third, the ability of different functional forms such as generalized translog, hybrid (Box-Cox transformation) translog and generalized Leontief functions to reveal the cost efficiency may be analyzed using the same body of panel data. Using either of these functional forms may substantially alter conclusions about scope economies (Zhu, Ellinger and Shumway, 1995).

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Table 1: Summary Statistics over Eleven Years (1984-1994) for 231 Sample Farms

	Units	Minimum	Mean	Maximum	Std. Dev.
Acreage					
Tillable	acres	102	691.30	2,450.00	347.57
Corn	acres	48	334.15	1,372.00	176.57
Soybean	acres	0	310.42	1,084.00	169.45
Wheat	acres	0	2.55	269.00	18.48
Diverted	acres	0	42.49	355.00	42.99
Yield					
Corn	bu/acre	23.15	143.52	234.76	34.77
Soybean	bu/acre	10.30	45.53	71.04	8.92
Wheat	bu/acre	12.08	59.91	103.57	18.39
Expenses[a]					
Total Costs	dollars	58,971.15	8,477.36	224,768.09	30,915.85
Fixed Costs	dollars	7,955.08	433.66	55,370.21	5,897.90
Variable Costs	dollars	51,016.07	6,141.11	210,065.96	27,581.43

[a] All values are deflated to 1991 dollars by the consumer price index.

Table 2: Long-Run (Between-Firm) and Short-Run (LSDV) Parameter Estimates

		Two-Product Model		Three-Product Model	
Variable	Symbols	Between-Firm	LSDV	Between-Firm	LSDV
Co_dum	α_T	-	13,684	-	14,022
		-	(3.28)	-	(3.31)
CB_dum	$\alpha_{S/S\text{-}T}$	-	19,557	-	18,859
		-	(13.26)	-	(13.60)
CBW_dum	α_S	-	-	-	22,699
		-	-	-	(6.40)
Corn	β_C	1.1459	0.7267	1.2596	0.7173
		(6.25)	(10.80)	(7.63)	(11.61)
Soybeans	β_B	1.6458	1.3818	1.3430	1.6012
		(2.68)	(5.37)	(2.41)	(6.81)
Wheat	β_W	-	-	2.3432	1.5572
		-	-	(0.41)	(0.80)
Corn*Corn	β_{CC}	7.2E-06	4.6E-06	3.2E-06	4.5E-06
		(2.02)	(4.09)	(1.12)	(4.84)
Beans*Beans	β_{BB}	0.0001	8.2E-05	6.9E-05	7.3E-05
		(2.76)	(6.25)	(2.01)	(6.51)
Wheat*Wheat	β_{WW}	-	-	-1.5E-04	-3.8E-05
		-	-	(-0.77)	(-0.47)
Corn*Beans	β_{CB}	-6.2E-05	-3.8E-04	-3.6E-05	-3.7E-05
		(-2.74)	(-5.32)	(-1.96)	(-6.25)
Corn*Wheat	β_{CW}	-	-	2.4E-04	7.9E-05
		-	-	(0.95)	(1.77)
Beans*Wheat	β_{BW}	-	-	-3.8E-04	-1.3E-04
		-	-	(-0.39)	(-0.79)
Observations:		185	2035	231	2541
RMSE:		10635.14	20705	10616.11	21222.01
Adj. R-Square:		0.98	0.94	0.99	0.94
F-Value:		2314.78	4743.54	1694.69	3457.48

Note: T statistics in parentheses.

Table 3. Long-Run Measures of Scale, Scope and Product-Specific Scale Economies

		Two-Product Model				Three-Product Model				
Firm Size	Firm	Scale	Scope	Product-Specific		Scale	Scope	Product-Specific		
Class (acres)	Number			Corn	Soybeans			Corn	Soybeans	Wheat
< 300	(14;17)[b]	1.006	0.079	0.948	0.860	1.007	0.046	0.978	0.893	1.000
300-600	(76;95)[b]	1.012	0.177	0.898	0.699	1.008	0.094	0.958	0.772	1.002
600-900	(59;65)[b]	1.020	0.291	0.826	0.522	1.008	0.153	0.932	0.632	1.004
> 900	(36;54)[b]	1.042	0.567	0.635	0.126	1.032	0.275	0.876	0.440	1.001

[b] The first and second numbers are, respectively, firm numbers of two- and three-product firms.

Table 4. Measures of Scale, Scope and Product-Specific Scale Economies by Time Period

	Two-Product Model				Three-Product Model				
Year	Scale	Scope	Product-Specific		Scale	Scope	Product-Specific		
			Corn	Soybeans			Corn	Soybeans	Wheat
1984	1.017	0.211	0.866	0.612	1.007	0.106	0.945	0.710	1.004
1986	1.018	0.254	0.848	0.574	1.014	0.142	0.937	0.673	1.002
1988	1.002	0.149	0.922	0.749	1.000	0.084	0.969	0.799	1.002
1990	1.020	0.282	0.831	0.531	1.012	0.155	0.931	0.638	1.003
1992	1.027	0.321	0.798	0.412	1.024	0.181	0.915	0.573	1.001
1994	1.030	0.362	0.773	0.344	1.029	0.206	0.904	0.527	1.001
Mean	1.019	0.267	0.840	0.554	1.014	0.148	0.935	0.658	1.002

Table 5: Product-Specific Quasi-Fixed Costs

Farm Types	Two-Product Model	Three-Product Model
Corn	$13,684***	$14,022***
Soybean	na	na
Wheat	-	na
Corn-Soybean	$ 19,557***	$18,859***
Corn-Soybean-Wheat	-	$22,699***

Note: *, **, and *** denote significance at 10%, 5%, and 1% level.

Table 6: Product Cost Complementarity and Cost Function Convexity

Product	Two-Product Model		Three-Product Model		
	Corn	Soybeans	Corn	Soybeans	Wheat
Corn	4.6E-06***	-3.8E-05***	4.5E-06*	-3.7E-05*	7.9E-05*
Soybeans	-3.8E-05***	8.2E-05***	-3.7E-05*	7.9E-05**	-1.3E-04
Wheat	-	-	7.9E-05*	-1.3E-04	-3.8E-05

Note: *, **, and *** denote significance at 10%, 5%, and 1% level.

Table 7. Short-Run Measures of Fixed- and Variable-Cost Components of Scope Economies

	Two-Product Model			Three-Product Model		
Year	$Scope_{FC}$	$Scope_{VC}$	Scope	$Scope_{FC}$	$Scope_{VC}$	Scope
1984	0.117	0.134	0.251	0.140	0.117	0.257
1986	0.100	0.178	0.278	0.119	0.163	0.282
1988	0.149	0.089	0.238	0.174	0.081	0.255
1990	0.098	0.191	0.289	0.117	0.173	0.290
1992	0.088	0.222	0.310	0.106	0.204	0.310
1994	0.082	0.249	0.331	0.099	0.231	0.330
Mean	0.101	0.181	0.282	0.121	0.164	0.285