The Dynamic Impact of Technical Progress on Common-pool Groundwater Use and Depletion.

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The Dynamic Impact of Technical Progress on Common-pool Groundwater Use and Depletion.

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WORKING PAPER

Abstract
A class of dynamic models is developed to assess potential gains from management of groundwater in arid and semi-arid regions in the presence of technical change. The aquifer is a common-pool resource (CPR) because users typically hold the right to pump groundwater but do not own the water contained in the aquifer and since groundwater is a fugitive resource, the stock available to an irrigator depends dynamically on the levels of extraction by neighboring irrigators. Allocations result from profit maximizing behavior of irrigating farmers. Competitive outcomes are obtained from periodic profit maximization while planning, or management, outcomes result from maximization of the net present value of the stream of profits over the lifetime of the aquifer. The divergence between these two outcomes over time indicates the magnitude of the common-pool resource externality. Dynamical systems govern the evolution of the aquifer, the climate, and the rate of technical progress. These dynamical systems may be dependent upon periodic allocations, as in the case of the aquifer, or independent of the periodic allocations, as in the case of precipitation. A simplified example of the model incorporating only deterministic aquifer and technical change dynamics is presented as a linear-quadratic optimal control problem.

1 Introduction
Irrigated agriculture is among the largest users of groundwater in the world. Irrigators extract and apply groundwater to satisfy their crop water requirement in order to achieve yield levels so as to maximize farm profits. Groundwater extraction occurs as long as the value marginal product of groundwater equals or exceeds the marginal cost of applying it. Since groundwater is a fugitive resource, the stock available to an irrigator depends dynamically on the levels of extraction by neighboring irrigators. This constitutes the common resource property of groundwater.
Population and economic growth along with climate change intensify the stress on already depleting groundwater resources. Philosophers, thinkers, and rulers have raised the issue of providing for a growing population with finite resources millenia before the term “economy” acquired its modern meaning. Malthus posited in 1798 that food production would not be able to keep pace with the rate of population growth without imposing controls or limits on population and that, consequently, food production would be the ultimate limiting factor. Technical progress in agriculture and the expansion of the global agricultural frontier has so far allowed food production to grow much more rapidly than population growth. However, agricultural output growth is fueled by the decline in the stocks of - essentially depletable - resources. Forrester (1971) suggested that depleting resources along with other environmental aspects play a similarly limiting role.

The question of depleting essential resources arouse the interest of a number of prominent economists, most notably Hotelling (1931), in determining optimal paths of exploitation and possible policy instruments to induce agents to allocate the depleting resources as closely to optimal as possible. Although essential resources may be depleted in principle, it is not necessarily the case that output produced with the resource would decline to zero or even that the resource would be completely depleted (Dasgupta and Heal, 1974). Indeed, economists argue that market forces will drive the depleting resource’s price up setting the incentives for economic agents to (i) develop or improve technologies to economize the resource and (ii) develop and utilize alternative inputs (Maddox, 1972; Nordhaus, 1973; and Kamien and Schwartz, 1978).

This paper explores the effect of technical progress on groundwater use and depletion using a model of inter-temporal common pool groundwater use, where pumping cost and stock externalities arise from the common property problem. Dynamic coefficients in a water-yield production function represent technical change, which is assumed to be such that similar irrigated yield levels can be achieved with lower levels of water being applied as time passes. Although technical progress makes it possible to produce at the same level with less water use, it also poses the incentive to expand irrigated acreage and intensify water application per acre, so the net effect is unclear in general.

The general framework to incorporate technical change dynamics in the analysis is described in the next section.
2 Conceptual Framework

A dynamic framework is essential to study groundwater use and management under changing conditions. This paper assumes that irrigated agriculture is the primary user of groundwater and the main driver of aquifer depletion. Given production technology and aquifer conditions, farmers decide how much groundwater to pump and apply to fields. A model based on the single-cell or “bathtub” aquifer framework is employed. The single-cell model considers an aquifer with a flat bottom, vertical sides, and water that flows laterally at an instantaneous rate so that withdrawals affect the water table height equally in all locations regardless of where it is pumped. A large number of users of water are assumed to be distributed across the (flat) overlying land surface, with identical technology and exogenous prices so that a representative, competitive user can be aggregated to reflect basin-level outcomes. The next three subsections detail the models of technical change, hydrology, and allocation decisions employed in this paper.

2.1 Technical Change

The economics profession narrative with respect to depleting “irreplaceable natural resources, or of natural resources replaceable only with difficulty and long delay,” (Hotelling, 1931) is that as a resource becomes more scarce it also becomes more valuable and, consequently, its price would increase providing the proper incentives for entrepreneurs to develop and supply substitutes (Maddox, 1971 and Nordhaus, 1972). There is an extensive literature suggesting this type of technical progress may emerge endogenously in economic systems. Induced innovation theory in agricultural development (Hayami and Ruttan, 1971) suggests that technical progress is determined by relative input and output relative prices. Specifically, increases in relative input prices induce input saving technologies while increases in output prices are related to research focus on the most valuable commodities (Quintana-Ashwell and Featherstone, 2014).

Although advances in agricultural biotechnology, equipment, and machinery may well occur in response to market signals, these occur at aggregation levels that are distant from the relevant decision unit: the irrigator. Consequently, these technical changes are exogenous to farmers. This paper assumes a composite irrigated crop that is sensitive to the volume of irrigation water applied to it. The crop yield response to irrigation is modeled following the agronomy-based model and production functions by Martin et al. (1984). Figure ?? illustrates the agronomic model from which the concept of technical change in this paper is derived.

Yield is a monotonic function of evapotranspiration (ET). There is a wilting point or
Figure 1: Yield response to water.

threshold, $W_m$, under which no yield is possible and once an amount $ET_r$ is evapotranspirated, the crop achieves its maximum “fully-watered yield”, $Y_{fw}$. Irrigation, $I_r$, plays a supplemental role to precipitation, $Pre$. If no irrigation is applied, a minimum yield level, $Y_{dry}$, is achieved given precipitation levels, $Pre$, and effectiveness of precipitation, $\eta_p$. The net irrigation requirements ($NIR$) is the additional amount of evapotranspiration required in addition to the effective precipitation to achieve $Y_{fw}$, i.e. $NIR = ET_r - \eta_p Pre$. However, irrigation water is not applied with perfect efficiency and not all applied groundwater is available for evapotranspiration. The gross irrigation requirement is then an amount greater or equal to the net irrigation requirement depending on the degree of application efficiency of the irrigation systems employed. The relationship between net and gross irrigation requirements is $GIR = \frac{NIR}{\eta_w}$, where $\eta_w$ is the irrigation application efficiency.

Technical change may occur in several ways. Advances in biotechnology may result in one or several of the following changes: (i) the wilting point, $W_m$, may be reduced; (ii) the fully-watered yield, $Y_{fw}$, may be increased; (iii) the required evapotranspiration, $ET_r$, to achieve $Y_{fw}$ may decrease or increase; and (iv) the shape of the yield water response function between $W_m$ and $ET_r$ may change. Advances in equipment, machinery, and
farming practices may result in improved precipitation effectiveness, $\eta_p$, or improved application efficiency, $\eta_w$. All these changes modify the incentives of farmers to pump groundwater.

A very relevant aspect in this line of thought is the impact of climate variables. In this formulation, the impact of precipitation is explicit and that of temperature implicit. Furthermore, these variables also present a dynamic characteristic under climate change. Quintana and Peterson (2014) show that the omission of climate change variables in the modeling tend to underestimate potential gains from groundwater extraction management. This paper omits the effects of climate change to concentrate on the impact of technical change on the common-pool resource externality. Results from this research will help to start answering the question of whether technical progress can mitigate the effects of climate change in agriculture.

For the purpose of this paper, climate is assumed stable so that both $\eta_p$ and $Pre$ are constant. This allows for the simplification of the agronomic model. Assuming a constant precipitation level that could be thought of as constant expected precipitation over time, the composite crop’s yield at time $t$ (subscripts omitted) responds to evapotranspiration as follows

$$
Y = \begin{cases} 
Y_d + aET - bET^2 & ET \leq NIR \\
Y_f & ET \geq NIR 
\end{cases}
$$

where $Y_d$ is “dryland” variety yields, $W$ is evapotranspiration, and $Y_f$ is fully watered yield. However, not all groundwater pumped and applied is available for evapotranspiration. Irrigation water application efficiency depends on several factors, most notably irrigation technology. Consequently, the yield response function to irrigation water becomes

$$
Y_{ir} = \begin{cases} 
Y_d + a\eta_w w - b(\eta_w w)^2 & w \leq NIR \\
Y_f & w \geq NIR 
\end{cases}
$$

where $\eta_w$ is the application efficiency of irrigation water and $w(t)$ is the rate of groundwater extraction at time $t$.

Figure ?? illustrates the adapted agronomical model where the solid black line represents the yield response function to evapotranspiration and the dashed line represents

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1Martin et al. (1984) compared Cobb-Douglas, Quadratic, and Mitscherlich-Spillman production functions and obtained similar results in the relevant range. The quadratic functional form is advantageous in an optimal control problem setting allowing for some analytical results.
the actual yield response function to irrigated water. The red lines represent a change in minimum yield, $Y'_d$, without a change in maximum yield. The blue line represent a change in application efficiency without any change in yields. Notice that any time there is a change in the difference between minimum and maximum yields, $\Delta Y = Y_f - Y_d$, this will require a change in the coefficients of the yield response function, $a$ and $b$. This paper will examine the effects of improvements in crop varieties that result in increasing “dryland” and “fully watered” yields which are controlled by parameters $a$ and $b$ in this particular formulation.

Technical change may be modeled as the evolution of parameter $a$ over time. A natural extension of this work would be to also model parameters $Y_d$ and $b$ dynamically. The equation of motion determine the rate of technical progress:

$$\dot{a} = \gamma_a$$ (3)

Although holding curvature constant, by fixing the parameter $b$ we are implicitly accounting for the relationship between dryland yield, represented by $Y_d$, and maximum irrigated yield. If $0 > \gamma_a$, the gap between dryland and irrigated yields close over time. Conversely, if $0 < \gamma_a$, the gap between dryland and irrigated yields expand over time. In
the next subsection, the hydrologic model is described. Different crop varieties show different patterns of yield increase between dryland and irrigated varieties. In the case of a composite crop as in this paper, it will depend on the mix of crops being considered.

2.2 Hydrology

The single cell framework has been a workhorse of the groundwater management literature since its inception (Gisser and Sanchez, 1980; Fienerman and Knapp, 1983). It can be criticized for its strong assumptions, which do not accord with the spatial heterogeneity and the slow rates of lateral flow observed in many aquifers (Saak and Peterson, 2012). Recent literature has relaxed the assumptions of instantaneous lateral flow and spatial uniformity (Gaudet et al. 2001, Xabadia et al. 2004, Saak and Peterson 2007, Brozovic et al. 2010, Pfeiffer and Lin 2012, Suter et al. 2012, Guilfoos et al. 2013, Peterson and Saak 2013,). However, the analysis in this paper is based on the single cell framework for several reasons. First, in spite of its limiting assumptions, it has been a productive research tool, which underlies a number of key insights on the nature of the common pool externalities. Second, results will be directly comparable with earlier insights obtained from the single-cell studies. Finally, it allows to study the research question - the effects of time-varying parameters on common pool externalities - in isolation from the effects brought about by spatial heterogeneity and varying lateral flows.

The aquifer is represented by the pumping lift from the aquifer at time $t$, $L(t)$. As the aquifer depletes, groundwater is pumped from deeper underground. The equation of motion for pumping lift is

$$\frac{dL}{dt} = \frac{1}{A_S} [(1 - \alpha) w - r]$$

(4)

where $A_S$ is the number of acres overlying the aquifer times the specific yield, $r$ is the rate of natural recharge of the aquifer and $\alpha$ is the return flow, the portion of water applied that returns to the aquifer.
2.3 Groundwater allocation

In this framework, farmers decide how much groundwater to allocate (pump) each period. The main periodic trade-off faced by farmers is between current versus future irrigation for food production. In this context, net farm benefits are a good approximation for social welfare (Quintana and Peterson, 2013). Let \( F(w, s; \beta) \) be a composite crop production function that follows the yield response function as described in subsection ?? such that

\[
F(w, L; \beta) = y_d + \beta_1 \eta_w w - \frac{1}{2} \beta_2 (\eta_w w)^2
\]  
(5)

where \( y_d \) is total dryland composite crop output, \( \eta_w \) is groundwater application efficiency, and \( w \) is total volume of groundwater pumped in the season. Farm revenues are then

\[
R(w, L; \beta) = pF(w, L; \beta) = \beta_1 \eta_w w - \frac{1}{2} \beta_2 (\eta_w w)^2
\]  
(6)

where \( p \) is a composite price index that accounts for non-water input costs and the coefficients \( \beta_i \) are the modified coefficients from the yield and production functions times the composite price. Revenue is represented as the area under the value marginal product of water. The omission of a constant term (the constant of integration, i.e. a \( \beta_0 \) term) is justified because no benefits from irrigation are derived with no irrigation, i.e. if \( w = 0 \).

The modified equations of motion parallel to equation ( ??) is

\[
\dot{\beta}_1 = \gamma_0 - \gamma_1 \beta_1; \gamma_0, \gamma_1 \in \mathbb{R}
\]  
(7)

The choice of functional form for the equations of motion is somewhat ad hoc to allow for a steady state of level of the parameter into the future.

The cost of pumping depends on the stock of groundwater available in the aquifer. As the stock of water in the aquifer decreases it becomes more costly to extract because pumping lift distances, \( L \), increase and well yields decrease, causing pumps to work harder and command larger operating costs, most notably pump energy consumption. The pumping costs as a function of aquifer conditions and water pumped is

\[
C(w, L) = (c_0 + c_1 L) w + c_2 L^2
\]  
(8)

where \( L \) is pumping lift and \( (c_0, c_1) \) are parameters calibrated to reflect the marginal cost

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\( ^2 \)This paper incurs in many simplifications to focus on the effect of technical change on the common pool resource externality. However, this framework can be easily extended to incorporate a land use perspective, in which case a constant term \( \beta_0 \) would represent dryland crop profits or cash rental income for farmers, for example.
Figure 3: Changing VMP and Marginal Cost of irrigation groundwater.

of pumping in the initial period when the initial pumping lift is $L_0$. The parameter $c_2$ captures accelerated cost increases derived from groundwater pumping from deeper wells and allows the Hessian of the reward function to conform with economic theory (Tomini, 2014).

The myopic farmers decide how much groundwater to pump based on profit maximization for each season. The periodic net benefits function is

$$B(w, L; \beta) = R(w, L; \beta) - C(w, L) = \beta_1 \eta_w w - \frac{1}{2} \beta_2 (\eta_w w)^2 - (c_0 + c_1 L) w - c_2 L^2. \quad (9)$$

Given the state of technology and the aquifer $(\beta, L)$, first order condition to maximize (9) determine periodic groundwater pumping:

$$\frac{\partial B(w, L; \beta)}{\partial w} = \beta_1 \eta_w - \beta_2 (\eta_w)^2 w - c_0 - c_1 L = 0 \quad (10)$$

As groundwater stock and technical change occur, the incentives for the farmer also change. Figure ?? illustrates changes in incentives that result in declining groundwater pumping as the states of technology and the aquifer change. The stock of water decreases whenever pumping net of return flows exceeds the natural rate of recharge of the aquifer, i.e. $w > \frac{r}{1 - \alpha}$. Consequently, the costs of pumping groundwater increases due to increased pumping lift and decreased well yields. Holding technology (and all else) constant, decreases in groundwater stock will cause a decrease in the rate of pumping in the subsequent period, as in all panels in Fig. ?? where myopic groundwater pumping decreases from $w_0^* \text{ to } w^{*'}$.

\[\text{For instance, ever increasing pumping lift commands upgrading of pumps and other equipment, resulting in additional expenses over time indirectly due to the decreasing groundwater stock level. This term is also added to ensure that the Hessian of the net benefit function is negative semi-definite, which was a shortcoming in the seminal work of Gisser and Sanchez (1980).}\]
However, when technical change occurs, the value marginal physical product of irrigation groundwater changes and modifies the underlying incentives to extract groundwater. Figure ?? presents three cases were technological change results in differing patterns of groundwater pumping over time. In this paper, we hold curvature of the benefit function constant so that technical change results in shifts of the VMP curve. Panel ?? illustrates the case in which technical change results in pumping reduced to a larger degree than induced by higher pumping costs only; panel ?? shows the case in which technical change induces higher pumping than suggested by increased pumping costs; and panel ?? shows the case where technical change results in a higher level of groundwater pumping even when compared to the increased cost of pumping effect alone so that technical change results in accelerated rates of extraction over time, i.e. \( w^*_o < w^*_t \).

In contrast to the myopic solution, the planning solution consists in maximizing the net present value of the stream of periodic net benefits over a planning horizon (infinity, in this case). A Social Planner would then maximize

\[
NPV = \int_0^{\infty} e^{-B(w_t, L_t, t; \beta_t)} dt = \int_0^{\infty} e^{-\left[R(w, L; \beta) - C(w, L)\right]} dt
\] (11)

subject to the equations of motion ( ??) and ( ??). The current value Hamiltonian is

\[
\tilde{H} = R(w, s; \beta) - C(w, s) + \mu_1 \left[(1 - \alpha)w - r\right] + \mu_2 \left(\gamma_0 + \beta_1 \gamma_1\right)
\] (12)

and optimality conditions

\[
\frac{\partial \tilde{H}}{\partial w} = \beta_1 \eta_w - \beta_2 \eta_w^2 w - c_0 - c_1 L + \mu_1 (1 - \alpha) = 0
\] (13)

\[
\dot{\mu}_1 - \rho \mu_1 = -\frac{\partial \tilde{H}}{\partial L} = c_1 w + 2c_2 L
\] (14)

\[
\dot{\mu}_2 - \rho \mu_2 = -\frac{\partial \tilde{H}}{\partial \beta_1} = -\eta_w w + \mu_2 \gamma_1
\] (15)

where \( \mu_1 \) and \( \mu_2 \) are the costate variables for ( ??) and ( ??) respectively. In the next section some analytic results are presented.
3 Analytical Results

The difference in allocations between a myopic farmer and a forward-looking manager is easy to see comparing (??) and (??). Most notably, the manager accounts for future value of the aquifer, through the scarcity rent \( \mu_1 \). From (??), the myopic farmer determines periodic groundwater pumping as

\[
 w^{myo} = \frac{1}{\beta_2 \eta_w^2} [\beta_1 \eta_w - c_0 - c_1 L] \geq 0, \tag{16}
\]

which, when differentiated with respect to time, evolves according to

\[
 \dot{w}^{myo} = \frac{1}{\beta_2 \eta_w^2} [\beta_1 \eta_w - c_1 \dot{L}] = \frac{1}{\beta_2 \eta_w^2} [c_1 r + \eta_w \gamma_0 + \eta_w \gamma_1 \beta_1 - c_1 (1 - \alpha) w +] \tag{17}
\]

For the planning solution, the first order conditions (??) indicate the value and evolution of the costate variable for the aquifer are

\[
 \mu_1 = \frac{1}{1 - \alpha} [c_0 + c_1 L + \beta_2 \eta_w^2 w - \eta_w \beta_1] \tag{18}
\]

which implies

\[
 \dot{\mu}_1 = \frac{1}{1 - \alpha} [c_1 \dot{L} + \beta_2 \eta_w^2 \dot{w} - \eta_w \dot{\beta}_1] \tag{19}
\]

and yields

\[
 \dot{w} = \phi_0 + \phi_1 L - \phi_2 \beta_1 + \rho w \tag{20}
\]

where

\[
 \phi_0 = \frac{\eta_w \gamma_0 + c_1 r + \rho c_0}{\beta_2 \eta_w^2}, \quad \phi_1 = \frac{2 c_2 (1 - \alpha) + \rho c_1}{\beta_2 \eta_w^2}, \quad \phi_2 = \frac{\eta_w (\gamma_1 + \rho)}{\beta_2 \eta_w^2}
\]

The optimal steady state is found by finding the solution to the 3x3 dynamical system formed by (??), (??), and (??). Since the Hessian of the reward function is negative definite, this optimal steady state is saddle point stable. The next two subsections present the steady states under the planning and the myopic scenarios. Such is a comparison of two futures is useful in terms of thinking the legacy to future generations which is a type of analysis typically trumped by the net present value type of analysis in which the largest
benefits occur in the long term and are discounted to insignificance in the present.

3.1 Planning Steady State

The system formed by (??), (??), and (??) may be expressed in matrix form as

\[ \dot{x} = Ax + d \]  (21)

where

\[
\begin{pmatrix}
  w \\
  L \\
  \beta_1
\end{pmatrix}
\]

\[
\begin{pmatrix}
  \rho & \phi_1 & -\phi_2 \\
  1 - \alpha & 0 & 0 \\
  0 & 0 & -\gamma_1
\end{pmatrix}
\]

\[
\begin{pmatrix}
  \phi_0 \\
  -r
\end{pmatrix}
\]

and, provided \( A \) is invertible, the steady state levels are found by setting \( \dot{x} = 0 \) and solving (??) for \( x \):

\[
x^{ss} = \begin{pmatrix} w^*_\infty \\ L^*_\infty \\ \beta_1^*\infty \end{pmatrix} = -A^{-1}d = \begin{pmatrix} \frac{\phi_0}{\phi_1} - \frac{r}{\phi_1(1-\alpha)} - \frac{\phi_0}{\phi_1} \\ \frac{\phi_0}{\phi_1} - \frac{\phi_0}{\phi_1} \end{pmatrix} \]  (22)

Notice that \( \gamma_0 \gamma_1 \) is the steady state level of the technological parameter \( \beta_1 \) and that the steady level of the aquifer is negatively affected by positive technical change; i.e. the greater \( \beta_1^*\infty \), the greater \( L^*_\infty \) which equates to the lower the amount of water in the aquifer (at the steady state)\(^4\). The form of the time path may be expressed as

\[
x^*(t) = x^{ss} + k\vec{v}e^{\lambda_1 t} \]  (23)

where \( \lambda_1 \) is a negative eigenvalue from \( A \) and \( \vec{v} \) is the eigenvector associated with \( \lambda_1 \). The constant \( k \) is a coefficient that can be determined using (??) for the aquifer initial conditions at \( t = 0 \). Consequently the path is established as:

\[
x^*(t) = \begin{pmatrix} w^*_t \\ L^*_t \\ \beta_1^*\_t \end{pmatrix} = \begin{pmatrix} w^*_\infty + (L^*_\infty - L_0) \frac{\gamma_1}{1-\alpha} e^{-\gamma_1 t} \\ L^*_\infty - (L^*_\infty - L_0) e^{-\gamma_1 t} \end{pmatrix} \]  (24)

and from (??) we can also recover the trajectory of the costate variable for the aquifer and its behavior over time.

\(^4\) Although one may say that this means that a greater degree of technical progress induces greater degrees of depletion of the aquifer, it is more intuitive to rather think that greater technical progress allows to exploit the resource to a greater degree.
3.2 Myopic Steady State

The myopic steady state for groundwater pumping is determined by \( \dot{L} = 0 \) which results in the same levels of groundwater pumping as in the planning case and similarly for \( \beta_{1\infty}^{myo} \). The steady state for the aquifer can be calculated from \( \dot{L} = 0 \). The myopic steady state is then

\[
x_{\infty}^{myo} = \left( \begin{array}{c} u_{\infty}^{myo} \\ L_{\infty}^{myo} \\ \beta_{1\infty}^{myo} \end{array} \right) = \left( \begin{array}{c} \frac{r}{1-\alpha} \\ \frac{\gamma_0}{\gamma_1} \eta_w - c_0 - \frac{r \beta_{2\infty}^2}{1-\alpha} \\ \frac{\gamma_0}{\gamma_1} \end{array} \right)
\]

(25)

3.3 Myopic versus Planned Scenarios

Because groundwater extraction and the state of technology are identical in the steady state, the difference in welfare between the myopic and planning solution is determined by the difference between the aquifer levels under each scenario:

\[
Z_{\infty} = L^*_\infty - L^{myo}_{\infty} = \frac{2c_2r\beta_{2\infty}^2\gamma_{1\infty}^2 - r\gamma_{1\infty}c_1^2 - 2c_2(\gamma_0\eta_w - \gamma_1c_0)}{\gamma_1c_1(2c_2(1-\alpha) + \rho c_1)}
\]

(26)

where the sign depends on parameter values. However, we can take the derivative of the difference with respect to the technical progress parameters \( \gamma_0 \) and \( \gamma_1 \). A way to look at this parameters is that \( \gamma_0 \) controls the positive progress of technology while \( \gamma_1 \) determines the decreasing gains in said progress.

\[
\nabla_\gamma Z_{\infty} = \left( \begin{array}{c} \frac{\partial Z}{\partial \gamma_0} \\ \frac{\partial Z}{\partial \gamma_1} \end{array} \right) = \left( \begin{array}{c} -\frac{2c_2\gamma_{1\infty}^2}{\gamma_1c_1(1+2c_2(1-\alpha))} < 0 \\ -\frac{2r\beta_{2\infty}^2\eta_w^2 - rc_1^2 + 2c_0c_2 + \gamma_1(2c_2(\gamma_0\eta_w - \gamma_1c_0) + \gamma_1c_1 - 2r\beta_{2\infty}^2\gamma_{1\infty}^2c_2)}{\gamma_1c_1(1+2c_2(1-\alpha))} \end{array} \right)
\]

(27)

where the sign of the second component depends on parameter values. Since the sign of \( Z_{\infty} \) can not be unambiguously determined, the general effect of the technical change parameters can not be unambiguously determined. However, it is to be expected that the planning solution will yield a smaller lift distance than the myopic case, in which case \( Z_{\infty} < 0 \) and the effect of positive technological progress results in an increase in the magnitude of the common-pool externality.
4 Case Study: Sheridan County, Kansas.

The framework employed in this paper is best applied to arid and semi-arid regions where the primary use of groundwater is agricultural and where little urban and industrial demand for water exists. In this setting, farm net benefits are good proxies for regional welfare. Furthermore, the common-pool resource externality becomes apparent in localities where high density of wells exists, inducing myopic groundwater use behavior. This setting largely describes the agricultural regions in western Kansas. Figure ?? presents a map of Kansas where counties and the High Plains aquifer are outlined along with the registered points of diversion. The high density of wells in the region induces the perception that groundwater saved by one farmer may be extracted by another in a neighboring well. Another important factor in selecting a county in Kansas is the network of institutions collecting agricultural, agronomical, and hydrological data in Kansas, including the Kansas Geological Survey (KGS), the Water Rights Information System (WRIS), and the Water Management and Analysis System (WIMAS) along with an large number of extension bulletins produced at the different Kansas Board of Regents universities.

Figure 4: Location of High Plains aquifer and points of diversion in Kansas. Source: USGS Kansas Water Science Center.

Figures ?? and ?? illustrate levels of saturated thickness and aquifer depletion for the Ogallala aquifer in Kansas. Although aquifer conditions and dynamics are not identical across the region, when considering smaller areas, i.e. at the county level, these become
more uniform. Consequently, even though there is an evident heterogeneity and asymmetry at a basin-wide scale, these vanish at more localized scales making the notion of a representative farmer more palatable at a county level scale. Amongst the different counties overlying the Ogallala aquifer in western Kansas, Sheridan county is particularly attractive because (i) it is relatively uniform in terms of its agricultural and hydrological conditions, (ii) the aquifer is depleting but there is still enough saturated thickness for groundwater management to make a difference in welfare, and (iii) recent policy innovations\textsuperscript{5} indicate that farmers in the area may welcome the introduction of groundwater management policies.

(a) Mean saturated thickness.  
(b) Change in water levels.

Figure 5: Ogallala Aquifer conditions in Kansas. Source: Kansas Geological Survey.

A numerical analysis based on agricultural and hydrological conditions in Sheridan county, KS, is presented in the next section.

\textsuperscript{5}One such policy is the “Sheridan 6 Local Enhanced Management Area (LEMA6)” initiative which was designated in April, 2013, and caps the amount of groundwater extraction to a maximum of 55 acre-inches per irrigated acre for the five-year period between 2013 and 2017, inclusive.
4.1 Model parameterization and initial values.

Initial conditions and various parameter values for Sheridan County, KS, are presented in table ?? . Aquifer initial conditions and parameters were obtained from the Kansas Geological Survey (KGS), the Water Rights Information System (WRIS), and the Water Information Management and Analysis System (WIMAS). The discount rate considered is the average interest rate on farm loans as reported from the Kansas City Federal Reserve Bank (November, 2011).

The parameters for the reward from irrigation function are constructed based on data and results from Hendricks and Peterson (2012). The mix of crops considered and the weighed average crop irrigation requirement, $C_R$, are presented in table ?? . The parameter $C_R$ is useful to approximate the number of irrigated acres in the area of study at any given time via the water equation: $\eta w = C_R A$, where $A$ is the number of irrigated acres. The parameters in the reward function are calculated from a linear demand function for groundwater considering an elasticity of 10 percent and the average pumping cost and water use reported in Hendricks and Peterson (2012). For an application efficiency $\eta w$ the (inverse) demand function for groundwater is $p^w(w, \eta w) = 286.19\eta w - 0.00377\eta_w^2 w$.

The parameters in the cost function are calculated in two stages from the nonlinear marginal pumping cost function calculated in Quintana and Peterson (2013) following Rogers and Alam (2006). The periodic marginal cost function is linearly increasing in groundwater pumping, i.e. $c_1 > 0$, and quadratically increasing in lift distance, $c_1, c_2 > 0$. The non-linearity of the cost function with respect to aquifer levels is due to declining well yields as the aquifer is depleted and the quadratic form is merely a tractable approximation.

In the next section, parameter values are assigned to the analytical results presented earlier.
Table 1: Parameters and aquifer initial values for Sheridan Co., KS

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Aquifer</td>
<td></td>
</tr>
<tr>
<td>$A_S$</td>
<td>716,844.5363</td>
</tr>
<tr>
<td>Irrigated area</td>
<td>77,745 acres</td>
</tr>
<tr>
<td>Irrigation efficiency ($\eta_w$)</td>
<td>0.879</td>
</tr>
<tr>
<td>Return flow ($\alpha$)</td>
<td>0.086795</td>
</tr>
<tr>
<td>Initial lift ($L_0$)</td>
<td>111.5 ft.</td>
</tr>
<tr>
<td>Rate of natural recharge ($r$)</td>
<td>28,747.08 AF/yr</td>
</tr>
<tr>
<td>Discount rate ($\rho$)</td>
<td>0.0389</td>
</tr>
<tr>
<td>Irrigated crop coverage</td>
<td></td>
</tr>
<tr>
<td>Corn</td>
<td>86.9%</td>
</tr>
<tr>
<td>Soybeans</td>
<td>4.8%</td>
</tr>
<tr>
<td>Alfalfa</td>
<td>4.8%</td>
</tr>
<tr>
<td>Wheat</td>
<td>2.8%</td>
</tr>
<tr>
<td>Sorghum</td>
<td>0.7%</td>
</tr>
<tr>
<td>Average Irrigation requirement ($C_R$)</td>
<td>0.897 AF/Acre</td>
</tr>
<tr>
<td>Reward function</td>
<td></td>
</tr>
<tr>
<td>$\beta_{10}$</td>
<td>286.19</td>
</tr>
<tr>
<td>$\beta_2$</td>
<td>0.00377</td>
</tr>
<tr>
<td>Cost function</td>
<td></td>
</tr>
<tr>
<td>$c_0$</td>
<td>-51.70</td>
</tr>
<tr>
<td>$c_1$</td>
<td>0.6689</td>
</tr>
<tr>
<td>$c_2$</td>
<td>64.324</td>
</tr>
<tr>
<td>Technical change</td>
<td></td>
</tr>
<tr>
<td>$\gamma_0$</td>
<td>open</td>
</tr>
<tr>
<td>$\gamma_1$</td>
<td>open</td>
</tr>
</tbody>
</table>

4.2 Simulation Results

Previous literature offers little help in determining the values for parameters $\gamma_0$ and $\gamma_1$. Applying the parameter and initial conditions according to table ?? but conditional on technical change parameter values allows to sign the derivatives with respect to the technical change parameters. The steady state levels for the planning and myopic solutions, respectively, are

$$x^{ss} = \begin{pmatrix} w^*_\infty \\ L^*_\infty \\ \beta^*_1 \infty \end{pmatrix} = \begin{pmatrix} 31,479 \text{ AF/yr} \\ 1.3059 \frac{\gamma_n}{\gamma_1} - 60.441 \text{ ft.} \end{pmatrix}$$ (28)
\[ x^{myo} = \begin{pmatrix} u^{myo}_\infty \\ L^{myo}_\infty \\ \beta^{myo}_\infty \end{pmatrix} = \begin{pmatrix} 31,479 \text{ AF/yr} \\ 1.3141 \frac{\gamma_0}{\gamma_1} - 59.79109913 \text{ ft.} \end{pmatrix} \] (29)

and the effects of technical change on the common pool resource externality can be obtained from

\[ Z_\infty = L^*_\infty - L^{myo}_\infty = -0.0082 \left( \frac{\gamma_0}{\gamma_1} \right) - 0.6499 < 0 \] (30)

\[ \Rightarrow \frac{dZ_\infty}{d\gamma_0} = -0.0082 < 0 \] (31)

The larger the productivity in the steady state, the larger the gap between the myopic and the planning solutions.

\[ \nabla_\gamma Z_\infty = \left( \begin{array}{c} \frac{\partial Z}{\partial \gamma_0} \\ \frac{\partial Z}{\partial \gamma_1} \end{array} \right) = \left( \begin{array}{c} -0.0082 \frac{\gamma_1}{\gamma_0} < 0 \\ 0.0082 \left( \frac{\gamma_0}{\gamma_1} \right) > 0 \end{array} \right) \] (32)

Once the steady state is reached, the difference in periodic welfare between the myopic and planning scenarios is

\[ B^*_\infty - B^{myo}_\infty = -c_1 w_\infty (L^*_\infty - L^{myo}_\infty) - c_2 (L^*_\infty)^2 - L^{myo2}_\infty \] (33)

\[ = 173.19 \left( \frac{\gamma_0}{\gamma_1} \right) + 13,684.5 - 64.3 \left( -0.0215 \left( \frac{\gamma_0}{\gamma_1} \right)^2 - 0.727 \left( \frac{\gamma_0}{\gamma_1} \right) + 79.11 \right) \]

\[ = 1.3825 \left( \frac{\gamma_0}{\gamma_1} \right)^2 + 219.94 \left( \frac{\gamma_0}{\gamma_1} \right) + 8,597.73 \]

and the difference in their net present values is

\[ NPV^*_\infty - NPV^{myo}_\infty = \frac{1}{\rho} (B^*_\infty - B^{myo}_\infty) \] (34)

\[ = 35.53 \left( \frac{\gamma_0}{\gamma_1} \right)^2 + 5,652.36 \left( \frac{\gamma_0}{\gamma_1} \right) + 220,961.58 \]

The parameterization also allows us to establish a clearer time path for the variables of
interest

\[
x^*(t) = \begin{pmatrix} w^*_t \\ L^*_t \\ \beta^*_t \\ t \end{pmatrix} = \begin{pmatrix} 31.479 - \left( \frac{20}{\gamma_1} \right) - 171.941 - \frac{70}{\gamma_1} + \left( 286.16 - \frac{20}{\gamma_1} \right) e^{-\gamma_1 t} \\ 1.30670 - 60.441 - \left( \frac{20}{\gamma_1} \right) - 171.941 e^{-\gamma_1 t} \end{pmatrix} (35)
\]

Figure ?? shows the time path for the control, state, and costate variables considering parameter values calculated so that if the steady state value for \( \beta_1 \) is used in the initial period, it would have resulted in about twice the initial revenue (yield from applied water) if compared to the starting value of \( \beta_1 \); and assuming that \( \beta_1 \) grew 1.2 percent in the first period.

Figure 6: Time path of groundwater extraction \((w_t)\), lift \((L_t)\), and costate variables\((\mu_1, \mu_2)\).
Notice in the top left of figure ?? that, in this formulation, the planning solution prescribes increased rates of groundwater pumping initially with an eventual decline. This result is driven by the costate value of technical progress, $\mu_2$ (see top right in fig. ??). Lift increases almost linearly over time (see bottom left in fig. ??) and the value of its costate variable $\mu_1$ (see bottom right in fig. ??) declines over time. Since lift is a state variable that increases as the aquifer depletes, this is interpreted as the scarcity rents of stock of groundwater in the aquifer increasing over time, i.e. there is a negative relationship between groundwater stock and pumping lift.

As expected, when the planner has perfect foresight with respect to technical change, the optimal paths can be quite different. Figure ?? illustrates how dramatically different the paths of control, state, and costate variables can be in scenarios where technical progress is present versus models of fixed technology. With respect to the groundwater resource, this ad hoc model indicates that positive technical progress allows higher levels of lift in the long run which equates to lower sustainable levels of groundwater stock. Also noticeable is the larger gap between planning and myopic solutions when technical change is included in the model.

5 Conclusions

This paper presents a simple framework to study the effects of technical change on common-pool resources. The problem is presented as a tractable linear quadratic optimal control problem where groundwater extraction is the control variables and the aquifer and a technological parameter are the state variables. In a numerical exercise with parameters drawn from Sheridan county, KS indicate that (positive and deterministic) technical change induces larger benefits from resource management than in the case where no technical change is accounted for. Under positive technical change the incentive to conserve water arise from the ability to use more water later, when higher yields (revenues) can be achieved; however, an incentive to increase consumption also appears as groundwater use in subsequent periods may be more profitable than in the past when the yield increase trumps the pumping cost increases due to increasing pumping lift distances over time. In the case study, the optimal groundwater extraction path shows both periods of increasing groundwater use as well as decreasing pumping over time. This last effect is relevant in policy analysis because invariant policy instruments may be inadequate to capture the potential gains from management and aquifer- or state of technology-indexed policy instruments may be more appropriate.
Figure 7: Difference in optimal paths under “Technical change” vs. “No Technical change” scenarios.

References


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