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Estimating The Opportunity Cost of Recreation Time in An Integrable 2-Constraint Count Demand Model

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Abstract

How researchers treat the opportunity cost of time substantially influences recreation demand parameter and welfare estimates. This paper presents a utility-theoretic and implementable approach, estimating the shadow value of time jointly with recreation demands for coastal activities, using a generalization of the semilog demand system in a two-constraint model.

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Introduction

The value of natural assets is often assessed, in part, using models of consumer behavior relating to the asset that reflect an individual's constraints on choice and opportunities for consumption. When the behavior of interest is recreational use, often the substitution between sites is important to measuring the value of the asset and any given site. A common approach used is the random utility model, which predicts the probability of a site being chosen on a given choice occasion. As an alternative, the demand systems popularized in the literature on demands for market goods have been recently been applied to the recreation demand and nonmarket valuation setting (e.g., Fugii *et al.*; Shaikh and Larson).

While the flexible functional forms often used in market demand analysis are attractive for their ease of use and familiarity to economists working with market goods, some interesting nuances arise in their application to the nonmarket setting. One of these is in the measurement of the total worth, or "access value," of the activity being consumed. It is not uncommon for recreation demands to be price-inelastic at the observed levels of consumption. Depending on the demand system being used, this can lead to problems with measuring access value.

For example, in the Almost Ideal Demand System (Deaton and Muellbauer), whose focus is explaining budget shares and elasticities, some ranges of parameter values imply that budget share increases with price, which leads to to an infinite Hicksian choke price (not, by itself, necessarily a problem) and an infinite willingness to pay for access.

In the Linear Expenditure System (Stone) applied to the nonmarket goods setting, the parameter interpreted as a "subsistence quantity" of each good may be negative, and in fact must be negative for access value to be finite (Kling). Another, more commonly used functional form in empirical practice, the Cobb-Douglas demand system (LaFrance 1986), implies that goods are necessities, with infinite access values, when they are own price-inelastic.

In each of these demand systems, the findings of infinite access value for some parameter ranges are artifacts of the convergence properties of the demand systems as own price for a good rises and quantity consumed goes to zero. This problem diminishes their appeal for empirical nonmarket valuation where determining the total value of resource-based activities is the goal.

In contrast, the "semilog" demand system, which relates log of quantity consumed to the levels of the independent variables, has finite access values, even though the Hicksian choke price is infinite and quantity consumed goes to zero only in the limit. This makes the semilog model a more attractive option for empirical recreation demand analysis, and it is often used in single equation models. However, LaFrance (1990) has shown that demand systems based on this functional form are quite restrictive, with cross-price effects that are either zero or the same across all goods, and income effects that are also either zero or the same for all goods.

This paper proposes a variation of the semilog demand system, the "Double Semilog" (DS) system, which retains its attractive features with respect to measuring access values, while achieving somewhat greater flexibility with respect to cross-price and income elasticities. The key differences between the DS and semilog systems are (a) each good can have a different income elasticity in the DS system, whereas all goods have the same income elasticity in the semilog system; and (b) elasticities for price and quality in the DS system are essentially the elasticities in the semilog system with the addition of an income elasticity adjustment.

The first part of the paper develops the basic demand system and its properties, then the its implementation in situations where both time and money are important constraints on demand (as is usually the case with recreation demand) is discussed. Finally, the DS demand system is illustrated using data on whalewatching in northern California. The empirical model jointly estimates the shadow value of leisure time and the 2-constraint whalewatching demand system for three sites in proximity to one another.

The demand model estimates are in conformity with the integrability conditions, and are highly significant for two of the three sites, with expected signs on quality effects and on the price-income relationships for all three. The marginal value of time implied by the model estimates is about \$6/hr, with a range in the sample from about \$0.50 per hour to \$13/hour. The demand parameters imply finite access values in spite of demands being price-inelastic at baseline prices and quantities, which illustrates a potential advantage of the DS system relative to some of the other flexible forms.

The Model

The DS model begins with an ***expenditure function*** of the form

$$e(\mathbf{p}^n, u) = \theta(\mathbf{p},\mathrm{M}) \cdot \left[-\,\mathrm{e}^{\gamma_0 + \sum \gamma_i \mathrm{p}_i^n} + u\mathrm{e}^{\sum \beta_j \mathrm{p}_j^n} \right] \tag{1}$$

where $\mathrm{p}_i^n = \mathrm{p}_i/\theta(\mathbf{p},\mathrm{M})$ are normalized prices, with $\theta(\mathbf{p},\mathrm{M})$ being any function of prices and income that is homogeneous of degree 1 in ($\mathbf{p}$,M). The use of normalized prices and income imposes the desired homogeneity properties on demands, expenditure, and indirect utility (LaFrance and Hanemann).

One can also define the ***normalized expenditure function*** as

$$e^n(\mathbf{p}^n,u) = e(\mathbf{p}^n,u)/\theta(\mathbf{p},\mathrm{M})$$

$$= \Big[- \mathrm{e}^{\gamma_0+\sum\gamma_i \mathrm{p}_i^n} + u\mathrm{e}^{\sum\beta_j \mathrm{p}_j^n}\Big]. \tag{2}$$

Equation (2) can be rewritten to solve for the ***indirect utility function***

$$\mathrm{V} = \Big[\mathrm{M}^n + \mathrm{e}^{\gamma_0+\sum\gamma_i \mathrm{p}_i^n}\Big]\mathrm{e}^{-\sum\beta_j \mathrm{p}_j^n} \tag{3}$$

$$= \mathrm{M}^n\mathrm{e}^{-\sum\beta_j \mathrm{p}_j^n} + \mathrm{e}^{\gamma_0+\sum(\gamma_i-\beta_i)\mathrm{p}_i^n}$$

where $\mathrm{M}^n = \mathrm{M}/\theta(\mathbf{p},\mathrm{M})$ is normalized income. From equation (3), it can be seen that in this model, the utility index is strictly positive.

Differentiating (2) with respect to p_i^n, the Hicksian demands are

$$\mathrm{x}_i^h(\mathbf{p}^n,u) \;=\; -\gamma_i\mathrm{e}^{\gamma_0+\sum\gamma_k \mathrm{p}_k^n} + \beta_i u\mathrm{e}^{\sum\beta_j \mathrm{p}_j^n}, \tag{4}$$

and the corresponding Marshallian demands, obtained by substituting in the indirect utility function (3), are

$$\mathrm{x}_i(\mathbf{p}^n,\mathrm{M}^n) = (\beta_i - \gamma_i)\mathrm{e}^{\gamma_0+\sum\gamma_j \mathrm{p}_j^n} + \beta_i\mathrm{M}^n. \tag{5}$$

These Marshallian demands have a functional form that is a hybrid of the semilog and linear demand functions: the price effects are similar to those of the semilog system while the income effects are linear. Notably, the income effects β_i in (5) are not restricted as they are in the semilog demand system, where they must all take on a single value.

In the DS system, the Marshallian income slope is $\partial \mathrm{x}_i(\mathbf{p}^n,\mathrm{M}^n)/\partial \mathrm{M}^n = \beta_i$, so that each good has a separate income effect (β_i), unlike the semilog demand system, where all income effects must be the same. The ***income elasticity*** for good i is, then,

$$\epsilon_{iM} \equiv \frac{\partial \mathrm{x}_i(\mathbf{p}^n,\mathrm{M}^n)}{\partial \mathrm{M}^n} \cdot \frac{\mathrm{M}^n}{\mathrm{x}_i}$$

$$= \frac{\beta_i \mathrm{M}^n}{\mathrm{x}_i}$$

$$= \frac{\beta_i \mathrm{p}_i^n}{\alpha_i}, \tag{6}$$

where $\alpha_i \equiv \mathrm{p}_i \mathrm{x}_i/\mathrm{M}$ is the Marshallian budget share of good i. Each good has an independent income effect, unlike the semilog system, where all income effects must be equal.

The Marshallian ***own-*** and ***cross-price elasticities*** ϵ_{ii} and ϵ_{ij} are, respectively,

$$\epsilon_{ii} \equiv \frac{\partial \mathrm{x}_i(\mathbf{p}^n \mathrm{M}^n)}{\partial \mathrm{p}_i^n} \cdot \frac{\mathrm{p}_i^n}{\mathrm{x}_i}$$

$$= \gamma_i \mathrm{p}_i^n \left[1 - \frac{\beta_i \mathrm{p}_i^n}{\alpha_i}\right] \tag{7}$$

and $$\epsilon_{ij} \equiv \frac{\partial \mathrm{x}_i(\mathbf{p}^n,\mathrm{M}^n)}{\partial \mathrm{p}_j^n} \cdot \frac{\mathrm{p}_j^n}{\mathrm{x}_i}$$

$$= \gamma_j \mathrm{p}_j^n \left[1 - \frac{\beta_i \mathrm{p}_i^n}{\alpha_i}\right], \tag{8}$$

where $\alpha_i \equiv \mathrm{p}_i^n \mathrm{x}_i/\mathrm{M}^n$ is the budget share of good i. Noting, from (6), that $\beta_i \mathrm{p}_i^n/\alpha_i$ is the income elasticity for good i, (7) and (8) can also be written as

$$\epsilon_{ii} = \gamma_i \mathrm{p}_i^n \Big[1 - \epsilon_{iM}\Big] \tag{9}$$

$$\epsilon_{ij} = \gamma_j \mathrm{p}_j^n \Big[1 - \epsilon_{iM}\Big]. \tag{10}$$

In comparing these to the own- and cross-price elasticities of the standard semilog model (Table 1), both have an extra term involving own income elasticity $(1 - \epsilon_{iM})$ which allows more flexibility in the values the elasticities can take.

As with the semilog system, in the DS system the own- and cross- price elasticities have the relative relationship *within* a given Marshalian demand,

$$\epsilon_{ij}/\epsilon_{ik} = \gamma_j \mathrm{p}_j^n / \gamma_k \mathrm{p}_k^n,$$

though it has greater flexibility in the elasticity of a given price in own demand relative to other demands,

$$\epsilon_{ij}/\epsilon_{kj} = \Big[1 - \epsilon_{iM}\Big] / \Big[1 - \epsilon_{kM}\Big]$$

which depends on the income elasticities of both goods. In the semilog system, by contrast, $\epsilon_{ij}/\epsilon_{kj} = 1$.

While (6)-(8) indicate that the DS system has a greater flexibility in representation of Marshallian elasticities, it still embodies some restrictions, due to its relatively simple functional forms for estimation and relatively small number of parameters to be estimated. From (9) and (10), it can be seen that the own- and cross-price elasticities of demand for good i are related to the income elasticity; this relationship is

$$\frac{\epsilon_{ij}}{\gamma_j \mathrm{p}_j^n} = \Big[1 - \epsilon_{iM}\Big] = \frac{\epsilon_{ii}}{\gamma_i \mathrm{p}_i^n}. \tag{11}$$

As always in specifying empirical demand and valuation systems, the tradeoff is between flexibility and relative ease of use and estimation. The DS system largely preserves the convenience and usefulness for measuring access values of the semilog system, while increasing its flexibility to represent price and income effects on demand.

Adding Quality Effects on Demand

A convenient way to represent quality effects is to allow the price coefficients to vary with quality. In (5), one can define $\gamma_j = \gamma_{j0} + \gamma_{jz} \cdot \mathrm{z}_j$, and substituting these into (5), each site demand function is a function of own- and substitute site quality levels. With this addition, the own-quality slopes are

$$\partial \mathrm{x}_i / \partial \mathrm{z}_i = \gamma_{iz} \mathrm{p}_i^n (\beta_i - \gamma_i) \mathrm{e}^{\gamma_0 + \sum \gamma_j \mathrm{p}_j^n} - \gamma_{iz} \mathrm{e}^{\gamma_0 + \sum \gamma_j \mathrm{p}_j^n}$$

$$= \gamma_{iz} \mathrm{e}^{\gamma_0 + \sum \gamma_j \mathrm{p}_j^n} \Big(\mathrm{p}_i^n (\beta_i - \gamma_i) - 1 \Big).$$

The sign of the Marshallian own-quality slope of demand, which is expected to be positive, depends not only on the quality parameter γ_{iz} but also the magnitude of normalized price p_i^n relative to $(\beta_i - \gamma_i)$.

The ***Marshallian own-quality elasticities***,

$$\epsilon_{iz_i} \equiv \frac{\partial \mathrm{x}_i}{\partial \mathrm{z}_i} \cdot \frac{\mathrm{z}_i}{\mathrm{x}_i}$$

can be written as

$$\epsilon_{iz_i} = \gamma_{iz}\mathrm{z}_i \cdot (1 - \beta_i\mathrm{p}_i^n/\alpha_i)[\mathrm{p}_i^n - 1/(\beta_i\text{-}\gamma_i)],$$

$$= \gamma_{iz}\mathrm{z}_i \cdot (1 - \epsilon_{iM})[\mathrm{p}_i^n - 1/(\beta_i\text{-}\gamma_i)], \tag{12}$$

where α_i is the budget share of good i. Again, in comparison with the semilog demand system where quality enters in a similar way (Table 1), the semilog own quality elasticity has additional terms involving ϵ_{iM} and (β_i-γ_i), which gives increased flexibility.

The Marshallian cross-quality slopes are given by

$$\partial\mathrm{x}_i/\partial\mathrm{z}_j = \gamma_{jz}\mathrm{p}_j^n(\beta_i - \gamma_i)\mathrm{e}^{\gamma_0+\sum\gamma_j\mathrm{p}_j^n}$$

$$= \gamma_{jz}\mathrm{p}_j^n(\mathrm{x}_i - \beta_i\mathrm{M}),$$

and the ***Marshallian cross-quality elasticities*** are

$$\epsilon_{iz_j} \equiv \frac{\partial\mathrm{x}_i}{\partial\mathrm{z}_j} \cdot \frac{\mathrm{z}_j}{\mathrm{x}_i}$$

$$= \gamma_{jz}\mathrm{z}_j\mathrm{p}_j^n \cdot (1 - \beta_i\mathrm{p}_i^n/\alpha_i).$$

$$= \gamma_{jz}\mathrm{z}_j\mathrm{p}_j^n \cdot (1 - \epsilon_{iM}). \tag{13}$$

Similarly to the price effects, the cross-quality effect in the DS system has an extra term, $(1 - \epsilon_{im})$, relative to the semilog system (Table 1). Combining (12) and (13) with (11), the full set of relationships between quality, price, and income effects within a given demand function are

$$\frac{\epsilon_{iz_j}}{\gamma_{jz}\mathrm{z}_j\mathrm{p}_j^n} = \frac{\epsilon_{ij}}{\gamma_j\mathrm{p}_j^n} = \left[1 - \epsilon_{iM}\right] = \frac{\epsilon_{ii}}{\gamma_i\mathrm{p}_i^n} = \frac{\epsilon_{iz_i}}{\gamma_{iz}\mathrm{z}_i[\mathrm{p}_i^n - 1/(\beta_i\text{-}\gamma_i)]}. \quad (14)$$

Welfare Measurement

As noted in the Introduction, a principal purpose of introducing the DS model is to evaluate its use for the purposes of measuring access value, the take-it-or-leave-it measure of the worth of recreational opportunities. This welfare measure, when applied to the value of a particular site, is defined as a change in price from a reference level p_i^0 to infinity, which causes quantity consumed to change from the baseline level x_i^0 to zero.

Welfare measures for smaller changes in price that leave the individual consuming the good before and after the price change are also often of interest. However, because they are straightforward to calculate in the DS model, as with other models, so they are not pursued further in this paper. Instead, price elasticities of whalewatching demand at the observed price and quality levels are presented. A similar approach is taken for quality effects, since they too are straightforward to evaluate in the DS and other models.

In general the integrability conditions for the model are satisfied for the following ranges of the income (β_i) and own-price (γ_i) parameters:

(a) $\beta_i < 0,\ \gamma_i < 0$

(b) $\beta_i = 0,\ \gamma_i < 0$

(c) $\beta_i > 0,\ \gamma_i < 0$

(d) $\beta_i > 0, \gamma_i > \beta_i$

For the purpose of measuring access values, in the different parameter ranges the DS model has characteristics similar to those of the other common demand systems. For parameter ranges (a) and (d), where $\mathrm{sgn}(\gamma_i) = \mathrm{sgn}(\beta_i)$, the model has finite "choke"[1] prices and access values, similar to the linear demand system or the LES system with negative subsistence quantities. For parameter range (b), it resembles the semilog demand system and the AIDS or Constant Elasticity systems with own price-elastic demands, in that the "choke" price is infinite but access value is always finite. For range (c), the model resembles the LES system with positive subsistence quantities in that demand converges to a positive quantity as own price goes infinite.[2]

Choke Prices

When finite [i.e., when $\mathrm{sgn}(\beta_i) = \mathrm{sgn}(\gamma_i)$], the normalized Hicksian choke price $\widehat{\mathrm{p}}_i^n$ is defined implicitly as

$$\mathrm{x}_i^h(\widehat{\mathrm{p}}_i^n, \mathbf{p}_{-i}^n, \mathbf{z}, u) = -\gamma_i \mathrm{e}^{\gamma_i(\widehat{\mathrm{p}}_i^n - \mathrm{p}_i^{n0})} \mathrm{e}^{\gamma_0 + \sum_k \gamma_k \mathrm{p}_k^{n0}} + \beta_i u \mathrm{e}^{\beta_i(\widehat{\mathrm{p}}_i^n - \mathrm{p}_i^{n0})} \mathrm{e}^{\sum_j \beta_j \mathrm{p}_j^n} \equiv 0, \tag{15}$$

where $\mathrm{sgn}(\beta_i) = \mathrm{sgn}(\gamma_i)$. The Hicksian demand now depends explicitly on the vector of qualities $\mathbf{z} = (\mathrm{z}_1,...,\mathrm{z}_n)$ at different sites since the price coefficients $\gamma_j = \gamma_{j0} + \gamma_{jz} \cdot \mathrm{z}_j$ depend on quality. Using the indirect utility function (3) evaluated at initial prices $\mathbf{p}^{n0}$ and M^n to identify the utility index u, the choke price $\widehat{\mathrm{p}}_i$ can be written explicitly in terms of observables as

$$\widehat{\mathrm{p}}_i^n = \mathrm{p}_i^{n0} + \frac{1}{\gamma_i - \beta_i} \ln\left\{ \frac{\mathrm{M}^n - \mathrm{x}_i^0/\gamma_i}{\mathrm{M}^n - \mathrm{x}_i^0/\beta_i} \right\}. \tag{16}$$

where $\mathrm{x}_i^0 = (\beta_i - \gamma_i)\mathrm{e}^{\gamma_0 + \sum \gamma_j \mathrm{p}_j^{n0}} + \beta_i \mathrm{M}^n$ is the Marshallian demand at initial prices.

In contrast, where it exists and is finite [i.e., for $(\beta_i - \gamma_i)\beta_i < 0$], the normalized Marshallian choke price $\mathrm{p}_i'^{\,n}$ sets Marshallian demand to zero, so is defined implicitly as

$$\mathrm{x}_i(\mathrm{p}_i'^{\,n}, \mathbf{p}_{-i}^n, \mathbf{z}, u) = (\beta_i - \gamma_i)\mathrm{e}^{\gamma \mathrm{p}_i'^{\,n}}\mathrm{e}^{\gamma_0 + \sum\limits_{k \neq i} \gamma_k \mathrm{p}_k^n} + \beta_i \mathrm{M}^n \equiv 0,$$

and simplifies to a form similar to (16),

$$\mathrm{p}_i'^{\,n} = \mathrm{p}_i'^{\,n0} + \tfrac{1}{\gamma_i}\ln\left\{\tfrac{\mathrm{M}^n}{\mathrm{M}^n - \mathrm{x}_i^0/\beta_i}\right\}. \tag{17}$$

Access Value and Consumer's Surplus

Access value for good i is defined as the change in expenditure resulting from the price change $\widehat{\mathrm{p}}_i^n \rightarrow \mathrm{p}_i^{n0}$; i.e.,

$$\mathrm{AV} \equiv e(\widehat{\mathrm{p}}_i^n, \mathbf{p}_{\boldsymbol{-i}}^n, \mathbf{z}, u) - e(\mathrm{p}_i^{n0}, \mathbf{p}_{\boldsymbol{-i}}^n, \mathbf{z}, u)$$

$$\equiv e(\widehat{\mathrm{p}}_i^n, \mathbf{p}_{\boldsymbol{-i}}^n, \mathbf{z}, u) - \mathrm{M}. \tag{18}$$

Using the indirect utility function (3) evaluated at initial prices $\mathbf{p}^{n0}$ and M^n to identify the utility index u, the expenditure function evaluated at the choke price for good i is

$$e(\widehat{\mathrm{p}}_i^n, \mathbf{p}_{\boldsymbol{-i}}^n, \mathbf{z}, u) = \theta(\mathbf{p}, \mathrm{M}) \cdot \Big[- \mathrm{e}^{\gamma_i(\widehat{\mathrm{p}}_i^n - \mathrm{p}_i^{n0})}\mathrm{e}^{\gamma_0 + \sum \gamma_j \mathrm{p}_j^{n0}}$$

$$+ \left(\mathrm{M}^n + \mathrm{e}^{\gamma_0 + \sum \gamma_k \mathrm{p}_k^{n0}}\right)\mathrm{e}^{\beta_i(\widehat{\mathrm{p}}_i^n - \mathrm{p}_i^{n0})}\Big]. \tag{19}$$

Using (19) in (18) and simplifying, access value can be written as

$$\mathrm{AV} = \tfrac{\beta_i+\gamma_i}{\gamma_i}\mathrm{M}^n - \left[\tfrac{\beta_i+\gamma_i}{\gamma_i}\mathrm{M}^n - \mathrm{x}_i^0/\gamma_i\right]\left\{\tfrac{\mathrm{M}^n-\mathrm{x}_i^0/\gamma_i}{\mathrm{M}^n-\mathrm{x}_i^0/\beta_i}\right\}^{\frac{\beta_i}{\gamma_i-\beta_i}} \tag{20}$$

The Marshallian consumer's surplus approximation to access value is the integral of the Marshallian demand over the interval $(\widehat{\mathrm{p}}_i^n, \mathrm{p}_i^{n0})$,

$$\mathrm{AV}^M = \int_{\mathrm{p}_i^{n0}}^{\widehat{\mathrm{p}}_i^n}\left[(\beta_i-\gamma_i)\mathrm{e}^{\gamma_0+\sum\gamma_j \mathrm{p}_j^n} + \beta_i\mathrm{M}^n\right]d\mathrm{p}_i$$

which, when integrated and simplified, can be expressed as

$$\mathrm{AV}^M = \mathrm{x}_i^0/\gamma_i - \tfrac{\beta_i}{\gamma_i}\mathrm{M}^n \cdot \ln\left\{\tfrac{\mathrm{M}^n}{\mathrm{M}^n-\mathrm{x}_i^0/\beta_i}\right\}. \tag{21}$$

The DS Model with Two Constraints on Choice

The foregoing discussion developed the new DS system in terms of a money expenditure function only, which is appropriate for standard money-constrained choice problems that are used in most areas of demand analysis. When choice is constrained by time in addition to money, as is likely with most recreational activities, a two-constraint version of the model is needed. The properties of two-constraint choice models have been discussed elsewhere (Bockstael, Hanemann, and Strand; Larson and Shaikh 2001). In particular, Larson and Shaikh (2001) have identified the parameter restrictions on demand systems that follow from the assumption that time is costly. It is straightforward to show that the Marshallian demand system in (5) satisfies these conditions.

Two-constraint demand systems have two expenditure functions dual to indirect utility: one is the money expenditure function given the time budget and utility level, and the other is the time expenditure function given money budget and utility. In the DS system with two constraints on choice, the money expenditure function is

$$e(\mathbf{p}^n,\mathbf{z},u) = \theta(\mathbf{p},\mathrm{M}) \cdot \Big[-\mathrm{e}^{\gamma_0+\sum\gamma_i \mathrm{p}_i^f} + u\mathrm{e}^{\sum\beta_j \mathrm{p}_j^f} - \rho^n \cdot \mathrm{T}^n \Big] \qquad (22)$$

which is similar to (5), with two major differences:

(a) the normalized prices p_i^n in (5) are replaced by "full" prices $\mathrm{p}_i^f = \mathrm{p}_i^n + \rho^n \cdot \mathrm{t}_i^n$, ρ^n is the normalized value of time,[3] and $\mathrm{t}_i^n \equiv \mathrm{t}_i/\psi(\mathbf{t},\mathrm{T})$ and $\mathrm{T}^n \equiv \mathrm{T}/\psi(\mathbf{t},\mathrm{T})$ are time price and time budget normalized by the deflator $\psi(\mathbf{t},\mathrm{T})$, which is homogeneous of degree 1 in $(\mathbf{t},\mathrm{T})$;

(b) it has an additional term involving the normalized value of time and time budget, $-\rho^n \cdot \mathrm{T}^n$.

The Hicksian and Marshallian demands are obtained from the two-constraint money expenditure function (22) in the usual way, *viz.*, by differentiating with respect to money price and initializing the utility term in terms of full budget and full prices. The functional form of the Marshallian demand system in (5) is unaffected, though the money prices p_j^n and money budget M^n are replaced by full prices p_j^f and full budget M^f. Similarly, if the normalized shadow value of time is independent of budget arguments (which satisfies the homogeneity requirements for it), the Hicksian and Marshallian access values have the same functional form as (20) and (21), with M^f replacing M^n.[4]

Empirically, the marginal value of time can be treated in at least three ways. If the individual is jointly choosing labor supply and recreation demands, the marginal value of time is equated to an observable parameter (the marginal wage) which can be used in its place (Becker; Bockstael *et al.*) The second is to identify it through auxiliary choices, such as the labor supply decision if that is predetermined with respect to the recreation choices (Heckman; Feather and Shaw). The third is to treat it as endogenous to the recreation choices and to estimate it jointly with recreation demands (McConnell and Strand; Larson and Shaikh 2002). In this case, the marginal value of time function must satisfy the requirements of choice subject to two constraints (Larson and Shaikh 2001).

The strategy here is to use a simple version of the latter approach, where the normalized marginal value of time is constant, which satisfies the homogeneity requirements with respect to money and time budget arguments. This also implies that the "absolute" marginal value of time, scaled to the levels of actual budgets and prices, varies across people if they have different prices or budget levels. The reason is that the relationship between the relative and absolute marginal values of time is

$$\rho(\mathbf{p},\mathbf{t},\mathrm{M},\mathrm{T}) = \rho^n \cdot \theta(\mathbf{p},\mathrm{M})/\psi(\mathbf{t},\mathrm{T}); \tag{23}$$

that is, the absolute marginal value of time is the relative marginal value of time scaled by the ratio of the deflators used to normalize the money and time budgets (Larson and Shaikh 2001). The end result is an estimate of the marginal value of time for each person that is a constant dollar hour per hour, similar to the approach taken in Hausman *et al*., with the per-hour value varying across the sample according to each person's time and money budgets.

Data

The data used to illustrate the model are from on-site intercepts of whale-watchers at three sites in Northern California during the winter of 1991-92. Whalewatching is an increasingly-popular form of winter recreation in California and along much of the rest of the western coasts of the United States and Canada.

The annual migration of gray whales along the coast, from summer feeding grounds in the Bering Sea off Alaska to the Gulf of Mexico for calving, is well-documented and publicized in the popular media. The southward migration runs closer to shore and may last for a period of 1-4 weeks, peaking in mid-December in central and Northern California. In the northward migration, whales travel farther offshore and its peak occurs in March. In many ports along the coast, offering whalewatching cruises is an important supplement to the winter incomes of fishing guides, party boat operators, and other boat owners. In addition to regularly-scheduled boat cruises and tours in ports up and down the coast, there are many opportunities for shore-based viewing of the migration from major headlands and promontories.

Two sites, Point Reyes and Half Moon Bay, are in the San Francisco area, with Point Reyes to the north of the Golden Gate Bridge and Half Moon Bay on the Pacific coast south of San Francisco. The third site, Monterey, is further to the south, some 110 miles from San Francisco. As these data are discussed in some detail elsewhere (Loomis and Larson), a relatively brief description is provided here.

Gray whale migration occurs on the Pacific coast in the winter months. The southward migration from the Bering Sea to Mexico generally occurs from November to January followed by several months of the return trip north. The whales travel very close to the shore and swim at about 3-5 miles per hour, making them very visible from the shore or a boat. Whales are viewed from the shore at Point Reyes, and predominantly from boats in Half Moon Bay and Monterey. The boat trips normally consist of a 2-4 hour excursion to view whales. Since the survey took place during the whale migration, which is in the winter months, most people were on the coast for the primary purpose of whale watching and not summer beach activities.

Each site visit has both a money price (p_j) and a time price (t_j). The money travel costs include round trip vehicle cost per mile, plus other travel expenses. On-site time is considered largely exogenous because most of the whalewatching at two of the three sites, Monterey and Half Moon Bay, occurs on boat trips of fixed length. Variations in onsite time are relatively small at the third site, Point Reyes, and in all cases whalewatching was a day trip activity. Household income before taxes was the money budget variable, and the respondent's time spent not working is the leisure time budget; this is obtained from the average hours worked per week and the number of days of paid vacation per year. The money and time budget levels for each individual were used as the deflators, so normalized money price of site j is $p_j^n = p_j/M$, normalized time price of site j is $t_j^n = t_j/T$, normalized money and time prices are $M^n = 1 = T^n$, full prices are $p_j^f = p_j^n + \rho^n \cdot t_j^n$, and full budget is $M^f = 1 + \rho^n$, with ρ^n estimated as a constant. The quality variable, z_j, is the number of whales vistors to each site expect to see. Table 2 provides a summary description of these variables for each of the three sites.

The system of Marshallian demands in (5), with full prices and full budget variables, was estimated for the three Northern California whalewatching sites (Point Reyes, Half Moon Bay, and Monterey) via maximum likelihood, using Gauss MAXLIK Version 4.0.22. Because the data represented visitors intercepted at the sites (i.e., those with positive quantities), the demand errors are likely to be truncated and this must be taken account of in estimation. If one writes the latent demand for site i as

$$x_i^*(\mathbf{p}^f, M^f) = (\beta_i - \gamma_i)e^{\gamma_0 + \sum \gamma_j p_j^f} + \beta_i M^f + \epsilon_i, \tag{23}$$

then a positive quantity $x_i(\mathbf{p}^f, M^f)$ is observed when $x_i^*(\mathbf{p}^f, M^f) > 0$, or when

$$\epsilon_i > -[(\beta_i - \gamma_i)e^{\gamma_0 + \sum \gamma_j p_j^f} + \beta_i M^f].$$

Due to the truncation, the expectation of ϵ_i is not zero and must be accounted for in estimation (Heckman; Greene). The inverse mills ratio

$$\mathrm{E}\{\epsilon_i|\epsilon_i > 0\} = \phi(\mathrm{w}_i)/\Phi(\text{-}\mathrm{w}_i)$$

with $\mathrm{w}_i \equiv -[(\beta_i - \gamma_i)\mathrm{e}^{\gamma_0+\sum\gamma_j \mathrm{p}_j^f} + \beta_i \mathrm{M}^f]$, was included in an additional regressor in estimating the demand systems (5) to assure that the estimation error has expectation zero.[5]

Results

The estimation results are presented in Table 3. The estimates for all three sites satisfy the integrability conditions for the parameters to represent a valid demand model, and the price, quality, and budget parameters are highly significant for the Point Reyes and Monterey trips, though not so for Half Moon. The model predicts the actual mean trips at each site relatively well: the predicted (actual) trips for Point Reyes was 2.16 (2.25), for Half Moon it was 1.15 (1.43), and for Monterey it was 1.64 (1.78). The Half Moon results are not too surprising in light of the relatively small number of people intercepted there (72) relative to the other sites, and the fact that there is less variation in the number of trips taken there.[6] The Pt. Reyes and Monterey results, though, illustrate some of the interesting features of the DS model.

First, the budget parameters β_j are the only ones whose sign directly indicates the direction of impact of the corresponding demand slope. The significant coefficients (β_1 and β_3) indicate that demand at Point Reyes has a positive income effect, while at Monterey it has a negative income effect. The finding of negative income effects is relatively common in recreation demand, and probably reflects the cross-sectional pattern of usage by different income groups at a point in time more than the changes in an individual's consumption as his or her income increases.

For the quality and price parameters, the signs do not indicate the direction of impact, since the own- and cross-elasticities with respect to quality depend not only on the γ_{jz} but also on the income elasticities [equations (12) and (13)]. The own- and cross-elasticities with respect to price depend on both the income and quality effects in addition to the γ_{j0}[equations (9) and (10)], since $\gamma_j = \gamma_{j0} + \gamma_{jz} \cdot$ z.

The sample means of elasticities at observed price, quality, and budget levels are presented in Table 4.[7] Because these are Marshallian elasticities, the price elasticities are not perfectly symmetric, though their signs are.[8] All three demands are own-price inelastic, with elasticities ranging from -.1 at Point Reyes to -.55 at Monterey. As noted in the Introduction, it is this own-price inelasticity that invalidates the use of several common and/or flexible functional forms for measuring access values. The pattern of cross-price elasticities suggests that Point Reyes and Monterey are substitutes; the insignificant Half Moon price coefficient means its substitution relationship with the other sites cannot be determined.

The income elasticity estimates, interestingly, suggest that demand is highly income elastic at all sites. As noted above, this is likely reflecting the relative patterns of visitation by income groups in the different areas: in Point Reyes, those with higher budgets for leisure activities (higher income, more leisure time, or both) go more frequently, while in Monterey, those with lower leisure budgets go less often.

The own-quality elasticities for each site (Table 4) are all positive, as one would expect, and are larger in magnitude than the cross-site quality elasticities. Magnitudes of the own-quality elasticity for Point Reyes and Monterey, the two sites with significant quality effects, are large relative to the cross-effects. The elasticities of .06 and .10, respectively, mean that a doubling of expected sightings would yield 6% and 10% increases in trips taken to Point Reyes and Monterey, respectively.

A final point about the estimation results concerns the marginal value of time, which is significant at the 10% level (1-tailed test) in Table 3. This parameter was estimated with a squared transformation to impose the requirement that the marginal value of time is nonnegative, and the estimate of -0.7778 implies that the relative marginal value of time is .605 for everyone (Table 5). Rescaling by the ratio of deflators M/T, the absolute marginal value of time is, on average, \$5.87 per hour, and varies from a low of \$0.45/hr. to a high of \$13.60/hr. in the sample.

Access value estimates are presented in Table 6. The consumer's surplus estimates of willingness to pay for access at prevailing price conditions are \$779 for Point Reyes and \$129 for Monterey, while the compensating variation estimates are \$834 and \$126, respectively. The magnitudes of the Hicksian and Marshallian measures are close, reflecting a small overall income effect at each site, despite the fact that demands are income elastic; also, the compensating variation measure is larger at Point Reyes, since it is a normal good, while consumer's surplus is larger at Monterey, because of its negative income effect. Measured relative to the mean number of trips, the access value on a per trip basis is approximately $\$779/2.16 \approx \$360$/trip at Point Reyes, and about $\$129/1.64 \approx \$79$/trip at Monterey. While the range in per-trip values may seem a bit large, in fact it is consistent with the difference in prices of whalewatching and in income elasticities at the two sites. Because most trips in Monterey are taken on boats, the price of a whalewatching trip is higher than at Point Reyes; because of this, access value will be lower at Monterey, all else equal. Similarly, the pattern of visitation being heavier among those with lower leisure budgets at Monterey suggests willingness to pay is lower.

Conclusions

This paper has introduced and illustrated a new empirical demand system that may be of some use in measuring access values for recreation activities that are commonly price-inelastic. Like the standard "semilog" demand system which relates demand covariates to log-quantities, the "double semilog" or DS system generates finite access values, or total consumer's surplus, estimates for own-price inelastic demands. This does not occur with several other common and/or flexible demand forms, including the Almost Ideal Demand System, the Linear Expenditure System, and the Cobb-Douglas demand models. In addition, the DS model has somewhat greater flexibility than does the semilog system to represent price, quality and income elasticities. Each demand has a separate income coefficient in the DS model, while all income coefficients are the same in the semilog model. Similarly, the price and quality elasticities in the DS model involve more parameters, including the income elasticity in every case and, for own-quality effects, additional parameters beyond that.

The model was developed initially for the standard single-constraint setting, then extended to the case of two binding constraints on choice, as is often expected with consumption of time-intensive goods such as recreation. The marginal value of time is a parameter or function that can be estimated jointly within the model, provided it meets certain homogeneity requirements implied by the two-constraint choice theory, or it can be assumed to be predetermined as is common in many other recreation demand studies.

An illustration of the model is provided, using data on whalewatching in Northern California at a system of 3 sites in relatively close proximity that one might expect act as substitutes in consumption. The demand model satisfies the integrability conditions, and estimates for two of the three sites, Point Reyes and Monterey, are highly significant with the expected signs. The estimated marginal value of time is approximately \$5.90 per hour, with a range from \$0.45/hr to \$14/hr. Despite the fact that demands are highly price-inelastic, the model readily produces access value estimates of approximately $360 per trip for Point Reyes and $79 per trip for Monterey. Several characteristics of demand that differ between the two sites suggest that this difference in per-trip values is plausible.

Several directions for further work are suggested by these results. It may be possible to estimate a more flexible individual-specific normalized value of time within the model, consistent with the two-constraint choice requirements. Using a count rather than continuous demand error may improve the estimates further, though the available count data estimators for systems of more than two goods are somewhat inflexible with respect to the cross-equation covariances. Finally, it may also be possible to further improve the flexibility of the demand model itself through the introduction of additional parameters, though this may come at the cost of greater difficulty in using the model to evaluate access values analytically or in finding global maxima of the likelihood function.

Footnotes

1. "Choke" prices are the minimum prices that choke off demand to zero; thus they are the price on the demand curve (whether Hicksian or Marshallian) for which quantity equals zero. In measuring access values, Hicksian choke prices are used; they are infinite for models where quantity consumed approaches zero asymptotically with price.
2. This latter case is the problemmatic one, for all demand systems, as it implies the good is a necessity, which is implausible for specific recreation activities; thus one would not expect to see this case in practice.
3. The normalized marginal value of time, $\rho^n(\mathbf{p},\mathbf{t},M,T)$, is the ratio of the marginal utility of time and the marginal utility of money in the normalized choice model and, as such, is potentially a function of all variables in the choice problem. Larson and Shaikh (2001) show that $\rho^n(\mathbf{p},\mathbf{t},M,T,\mathbf{s})$ is homogeneous of degree zero in ($\mathbf{p}$,M), ($\mathbf{t}$,T), and ($\mathbf{p}$,$\mathbf{t}$,M,T).
4. With ρ^n independent of the budget arguments, one can measure the money compensating variation of welfare change either as a difference in the money expenditure function or as a difference in full expenditure, since the term $\rho^n \cdot T^n$ does not change with money prices.
5. Because the truncation was at the same threshold, 0, it is not possible to estimate a scale coefficient so it is normalized to 1.
6. This was typically the case for other demand models explored using these data as well.
7. To give a sense for variation in these elasticities due to differences in demand covariates, the standard errors of the sample means are also provided in Table 4.
8. The corresponding Hicksian elasticities (not shown) are symmetric as expected and required by theory.

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Table 1. A Comparison of Marshallian Elasticities in the Semilog and Double Semilog Models

Elasticity	Semilog	Double Semilog
Income(ϵ_{im})	βM^n	β_iM^n/x$_i$
Own Price (i)	γ_ip$_i^n$	γ_ip$_i^n(1-\epsilon_{im})$
Cross Price (j)	γ_jp$_j^n$	γ_jp$_j^n(1-\epsilon_{im})$
Own Quality (i)	γ_{iz}z$_i$p$_i^n$	γ_{iz}z$_i \cdot (1-\epsilon_{im})[p_i^n - 1/(\beta_i$-$\gamma_i)]$
Cross Quality (j)	γ_{jz}z$_j$p$_j^n$	γ_{jz}z$_j$p$_j^n \cdot (1-\epsilon_{im})$

Table 2. Quantities, Prices, and Qualities by Site

Variable	Mean	Std Dev	Minimum	Maximum
Point Reyes (N=258)				
Actual Trips	2.2519	2.8230	1.0000	40.0000
Normalized Money Price	0.0010	0.0036	0.0000	0.0488
Normalized Time Price	0.0005	0.0005	0.0000	0.0049
Expected Sightings	4.1938	6.8532	0.0000	50.0000
Predicted Trips	2.1616	0.5450	-0.7509	5.0324
Half Moon Bay (N=72)				
Actual Trips	1.4306	1.0322	1.0000	8.0000
Normalized Money Price	0.0015	0.0019	0.0003	0.0136
Normalized Time Price	0.0008	0.0002	0.0004	0.0015
Expected Sightings	9.6944	9.7730	0.0000	50.0000
Predicted Trips	1.1572	0.0484	1.0942	1.4350
Monterey (N=102)				
Actual Trips	1.7843	2.4439	1.0000	24.0000
Normalized Money Price	0.0022	0.0042	0.0001	0.0402
Normalized Time Price	0.0009	0.0006	0.0001	0.0028
Expected Sightings	13.0588	10.6006	0.0000	50.0000
Predicted Trips	1.6444	0.6305	-1.1666	3.4718

Table 3. Estimation Results

Variable	Parameter	Estimate	Asymptotic Std. Error
Pt. Reyes Price	γ_{10}	16.3725	5.464
Pt. Reyes Sightings	γ_{1z}	-0.0637	-9.664
Pt. Reyes Budget Slope	β_1	6.8902	2.214
Half Moon Price	γ_{20}	3.8157	1.227
Half Moon Sightings	γ_{2z}	-0.0026	-0.297
Half Moon Budget Slope	β_2	1.7373	1.208
Monterey Price	γ_{30}	-17.4486	-5.936
Monterey Sightings	γ_{3z}	-0.0350	-4.968
Monterey Budget Slope	β_3	-6.3731	-2.125
Value of Time Constant	$-\sqrt{\rho^n}$	-0.7778	-1.518

Pseudo R^2	0.421
Mean log-likelihood	-6.05
Number of cases	432

Table 4. Price, Income, and Quality Elasticity Estimates

Elasticity of Trips to	With Respect to Price at Point Reyes	Half Moon	Monterey	With Respect to Income
Point Reyes	-0.1009	-0.0336	0.1960	6.1263
	(0.0230)[a]	(0.0071)	(0.0284)	(0.104)
Half Moon	-0.0884	-0.1193	0.0767	3.7578
	(0.0275)	(0.0294)	(0.0155)	(0.032)
Monterey	0.2095	0.1499	-0.5571	-14.1127
	(0.0448)	(0.0265)	(0.1503)	(1.262)

Elasticity of Trips to	With Respect to Expected Sightings at Point Reyes	Half Moon	Monterey
Point Reyes	0.0612	0.0000	0.0015
	(0.0053)	(9.62E-06)	(0.0002)
Half Moon	0.0004	0.0052	0.0007
	(7.22E-05)	(0.0008)	(0.0001)
Monterey	-0.0021	-0.0001	0.0961
	(0.0005)	(3.37E-05)	(0.0216)

[a] Standard errors of the means in parentheses

Table 5. Normalized and Absolute Shadow Values of Time (N=432)

Shadow Value of Time	Mean	Std Dev	Minimum	Maximum
Normalized	0.6050	0.0000	0.6050	0.6050
Absolute ($/hr)	5.8698	3.2893	0.4507	13.6010

Table 6. Hicksian and Marshallian estimates of Access Value

	Welfare Measure of Access Value	
Site[a]	Consumer's Surplus	Compensating Variation
Point Reyes	779.09	833.98
	(36.74)[b]	(42.56)
Monterey	128.71	125.58
	(6.96)	(6.72)

[a]Estimates not provided for Half Moon as demand coeffients are insignificant.
[b]Standard errors of the mean in parentheses.