Product Quality and Grower Reputation: Dynamic Contracts With Adverse Selection

by

Yanguo Wang
Pennsylvania State University

and

Edward C. Jaenicke
Department of Agricultural Economics and Rural Sociology
Pennsylvania State University
208A Armsby Building
University Park, PA 16802
Tel.: (814) 865-5282
Email: tjaenicke@psu.edu

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Abstract: We investigate the design of a two-period contract between an agricultural processor and growers whose quality-ability types are not observable to the processor. After characterizing the optimal contracts and establishing conditions for a separating equilibrium, we investigate how a payment based on first-period reputation may induce more first-period effort. We show that this reputation-based payment can improve both the processor’s and the grower’s welfare, resulting in a dominant equilibrium.

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Due to a long list of contributing factors – including asset specificity, holdup, imperfectly observable product quality or grower effort, transaction costs, and risk reduction, the use of agricultural contracts has grown to now govern at least 36 percent of the value of U.S. agricultural production (MacDonald et al. 2004). While most contracts cover only relatively short time periods such as a single growing season or less, some contract-inducing factors – such as long-term capital investments and unobservable quality – tend to favor even longer-term contracts. For example, MacDonald et al. note several agricultural sectors where long-term contracting is already occurring, and Martinez and Zering (2004) list numerous hog contracts with durations as long as ten years. In these situations, a grower’s reputation for product quality may play an important role in maintaining strong long-term relationships with processors.

A number of papers, both within and outside the agricultural sector, investigate reputation and quality issues associated with production and marketing. Beyond specific agricultural examples, hidden information is the major force in two separate types of studies: hidden producer type is central in game-theoretic models (Selten 1978; Kreps and Wilson 1982; Milgrom and Roberts 1982; Kreps et al. 1982; and Fudenberg and Tirole, 1983), while hidden product quality is central for many market models (Shapiro 1982 and 1983; Allen 1984). For the most part, papers set within agriculture investigate market linkages between price premiums and reputation without specifically addressing contracting (e.g., Worth 1999; Quagrainie, McCluskey, and Loureiro 2001; Goodhue et al. 2000; Schamel 2002). An exception is Goodhue et al. (2000), which tests hypotheses regarding long-term relationships between contracting and reputation of grape quality in the California wine grape industry.
While none of these reputation studies is formulated in the principal-agent framework, repeated moral hazard models (Rubinstein and Yaari 1983; Radner 1985; Rogerson 1985; Lambert 1983) and repeated adverse selection models (Freixas, Guesnerie, and Tirole 1985; Laffont and Tirole 1988; Hosios and Peters 1989) can serve as good starting points. Generally, these studies suggest that inefficiencies arising from moral hazard can be completely overcome in a dynamic context; however, inefficiencies from adverse selection may be more persistent in a dynamic context. In some cases of repeated adverse selection, a fully separating equilibrium that induces the truthful revelation of producer types cannot be sustained.

This paper’s objectives are to determine feasible conditions for separating equilibrium in a two-period contracting model where a grower’s ability type is hidden from a processor, to characterize equilibrium conditions, and to investigate the impacts of allowing grower compensation in the second period to be based on reputation achieved at the end the first period. Taken together, a number of factors including the absence of full commitment to second-period contract terms by both parties, the existence of hidden information over time, and sequential grower decisions, all enable grower reputation to have an important effect in the two-period dynamic contract. As a baseline, we first develop a two-period, full-commitment model, which requires that both parties be committed to the contract terms and the contract cannot be breached or renegotiated during the contracting period. Then we apply the two-period contract model to the case where commitment is not guaranteed. Specifically, a no-commitment contract assumes that neither the processor nor the grower can commit to an two-period scheme, i.e., the processor can revise the contract in the second period conditional on the grower’s first-period performance and the grower can quit the relationship at the end of each period. Finally, we incorporate a
reputation reward contingent on the grower’s past observed performance into the model. To simplify the analysis, we assume the reputation reward takes the form of a lump sum payment.

For the no-commitment contract, we establish optimal conditions for a fully separating equilibrium, a semi-separating equilibrium, and a pooling equilibrium. In this case, reputation effects are embodied in the posterior probability assessment (Bayes’ rule) of the grower’s types by the processor at the end of the first period. Anticipating the processor’s strategies, the high-quality grower type chooses to build up some reputation by either imitating the low-quality type or revealing his true type, whichever is favorable. In fact, imitating the dominant behavior of a low-quality type yields future information rents to the high-quality type by sustaining the processor’s belief that the grower might be of low-quality type. After incorporating a reputation reward into the model, we demonstrate that the reputation reward contingent on the grower’s history of performance can provide incentives for the grower to invest efforts in building a reputation for high quality, and thereby improve both the processor’s and the grower’s welfare, resulting in a dominant equilibrium.

**Two-Period Contracts with Full Commitment**

The following model starts with a standard a short-term contract (Salanie 1997) and, in a straightforward way, extends it to two periods. In terms of their capability to produce high-quality products, growers fall into one of two types – high-ability and low-ability type growers. For example, growers may differ in their production technology, management skills, soil conditions, or other factors that can be sustained over time as long as grower types are not fully revealed to the processor. Let \( \theta \in \Theta = \{\underline{\theta}, \bar{\theta}\} \) denote the two possible quality-ability types of the growers with \( \underline{\theta} < \bar{\theta} \). The processor cannot observe \( \theta \), but she has some prior belief \( f(\theta) \) that the proportion of low-quality type \( \underline{\theta} \) is \( 1 - r \) and that of high-quality type \( \bar{\theta} \) is \( r \). At the beginning
of each period \( t \), the grower privately chooses an action, \( e_t \), to improve the quality of his products, which is only observable to the grower. For example, in the production of wine grapes, this action may include pruning, irrigating, and/or pest management. Thus, the observed or realized quality \( q_t \) of the grower’s products is determined by \( q_t = q_t(\theta, e_t) \). For simplicity, we assume that no uncertainty is involved in the production process. In particular, the quality structure is governed by the following

\[
P_t = q_t(\theta, e_t) = \theta e_t.
\]

The processor can observe the realized quality of the finished products produced by the grower, but she cannot distinguish the effects of the grower’s type \( \theta \) and his effort \( e_t \) on improving quality. For simplicity, we assume that the processor can sell the product at price, \( P_t = q_t \).

Since the quality structure is deterministic, in each period, each grower type can set a specific target of realized quality level for his product that corresponds to an optimally chosen effort level.

The processor is risk neutral and has a profit function, \( \pi_t(P_t, w_t) = P_t - w_t \), where \( w_t \) is the reward to the grower at period \( t \). Each grower type \( \theta \) has a time-separable utility function

\[
U_t(w_t, e_t, \theta) = u(w_t) - g(e_t, \theta), \text{ where } g(e_t, \theta) = v(e_t) / \theta.
\]

From (3.1), the utility function is equivalent to

\[
U_t(w_t, e_t, \theta) = u(w_t) - v(P_t / \theta) / \theta.
\]

It is assumed that \( u \) is strictly concave in \( w_t \) with

\[
u'(w_t) > 0 \text{ and } u''(w_t) < 0; \text{ and } v \text{ is strictly convex in } e_t: v_e > 0, v_{ee} > 0 \text{ and } v(0) = 0.
\]

Hence, we know that \( g_e(e_t, \theta) > 0, g_\theta(e_t, \theta) < 0 \), and \( g_{ee}(e_t, \theta) < 0 \). Note that growers differ in their disutility of efforts and marginal contribution of efforts to realized quality. The low-quality type incurs higher costs relative to the high-quality type for a same level of effort. In addition, the marginal disutility of efforts decreases with \( \theta \), i.e., decreases with grower abilities.
Case 1a, Two-Period Full Commitment Contract with Perfect Information: In this case, both parties are committed to a two-period contract and the contract cannot be breached or renegotiated during the contracting period. An alternative interpretation of full commitment is that the processor promises at date one to a multi-period incentive scheme and commits not to use the information revealed by the grower in the first period during the second period. Hence, reputation has no effect on the optimal incentives to revealing the grower’s private information.

Under perfect information, the processor can perfectly observe the grower’s type and the incentive problem is absent. Since the two periods are independent, the processor would solve, in each period \( t \), for each type \( \theta \in \Theta = \{ \underline{\theta}, \overline{\theta} \} \),

\[
\max_{w(\theta),e(\theta)} Z_t(\theta) = P_t(\theta) - w_t(\theta) = \theta e_t - w_t
\]

s.t. \( U_t = u(w_t) - v(e_t) / \theta \geq u_0 \).

We arrive at the standard results for optimal effort for each \( \theta \):

\[
\frac{v'(e_t^*)}{u'(w_t)} = \theta^2.
\]

Thus, given the assumptions of the utility function and disutility of efforts, condition (3.8) states that the optimal level of effort for each type \( \theta, e^*(\theta) \), increases with \( \theta \). In other words, the optimal contract requires that less efforts be demanded from the low-quality type. In addition, the grower of each quality type obtains the reservation utility \( u_0 \) in both periods.

After adding some notation, Figure 3.1 depicts processor profits and grower utility, thereby illustrating the results. Let \( Z^*(\theta) = \max_{w(\theta),e(\theta)} Z_t(\theta) \). Then in each period, the processor can obtain net profit \( Z^*(\overline{\theta}) = \overline{\theta} e^*(\overline{\theta}) - w^*(\overline{\theta}) \) from the grower type \( \overline{\theta} \), and \( Z^*(\underline{\theta}) = \underline{\theta} e^*(\underline{\theta}) - w^*(\underline{\theta}) \) from the grower type \( \underline{\theta} \). Note in Figure 3.1 that since \( P(\theta) = \theta e = 0 \) when \( e = 0 \), the processor’s net
profits for each $\theta \in \{\theta, \overline{\theta}\}$ are exactly the distances from the origin O to the point A and point B on the vertical axis respectively. Given strict concavity of $u(w)$ and convexity of $v(e)$ and the processor’s profit function, clearly, $OB > OA$, or, $Z^*(\overline{\theta}) > Z^*(\theta)$.

Thus, in a full commitment contract under perfect information, the optimal contract will mimic a sequence of repeated static contracts. The static contract in every period is exactly same and independent over time. For future reference, denote this perfect information contract $C^* = \{C^*_L, C^*_H\}$, where $C^*_L = \{w^*(\theta), e^*(\theta)\}$ and $C^*_H = \{w^*(\overline{\theta}), e^*(\overline{\theta})\}$. To simplify the notation further, let $w^* = w^*(\theta)$, $e^* = e^*(\theta)$ and $\overline{w}^* = w^*(\overline{\theta})$, $\overline{e}^* = e^*(\overline{\theta})$. Thus, the optimal contract under perfect information can also be written as $C^*_L = \{w^*, e^*\}$ and $C^*_H = \{\overline{w}^*, \overline{e}^*\}$.

Case 1b, Two-Period Full Commitment Contract with Imperfect Information: With asymmetric information, incentive constraints must be imposed to truthfully reveal the grower’s type. Since the processor commits not to use the first-period information during the second period, the optimal allocations in the two periods are independent. Thus, in each period $t$, the processor maximizes its expected net profit subject to the participation constraint and the incentive constraints.

Given the two distinct types, the optimal contract requires that the grower type $\theta$ produces at quality level, $P(\theta) = \theta e(\theta)$ (recall that market price is set equal to the observed quality) and receives $w(\theta)$, while the grower type $\overline{\theta}$ produces at quality level $P(\overline{\theta}) = \overline{\theta} e(\overline{\theta})$ and receives $w(\overline{\theta})$. Denote this full commitment contract with asymmetric information $C^F = \{C^F_L, C^F_H\}$, where $C^F_L = \{w^F(\theta), e^F(\theta)\}$ and $C^F_H = \{w^F(\overline{\theta}), e^F(\overline{\theta})\}$. To simplify the notations, let $w^F = w^F(\theta)$, $e^F = e^F(\theta)$, $\overline{w}^F = w^F(\overline{\theta})$, $\overline{e}^F = e^F(\overline{\theta})$, $P^F = P^F(\theta)$, and $\overline{P}^F = P^F(\overline{\theta})$. (The
superscript “F” is omitted when it is unnecessary.) Therefore, the processor solves, in each period $t$,

$$\max_{c(\theta), w(\theta)} \Pi_t = r_i Z_t(\bar{\theta}) + (1 - r_i) Z_t(\theta) = r_i [\bar{\theta} \bar{e}_t - \bar{w}_t] + (1 - r_i) [\theta e_t - w_t],$$

s.t.

$$U_t = u(w_t) - v(e_t)/\theta \geq u_0, \quad \forall \theta \in \Theta = \{\theta, \bar{\theta}\}$$

$$u(w) - v(e)/\theta \geq u(\bar{w}) - v(\bar{P}/\bar{\theta})/\bar{\theta}$$

$$u(\bar{w}) - v(\bar{e})/\bar{\theta} \geq u(w) - v(P/\theta)/\theta$$

To help solve the above problem, the following standard results are employed to simplify the problem and make a separating equilibrium possible:¹ First, an optimal contract must be such that $P(\theta)$ increases with $\theta$ and $w(\theta)$ increases with $\theta$. That is, the optimal reward is lower for low-quality type $\theta$ and is higher for high-quality type $\bar{\theta}$. Second, the participation constraint for type $\theta$ (7) is not binding, while the participation constraint for type $\bar{\theta}$ is binding, i.e.,

$$u(w) - v(e)/\theta = u_0.$$ 

We can now derive the optimal choices of efforts by each grower type. Ignoring the participation constraint for the high-quality type $\bar{\theta}$, Let $\lambda, \mu_L$, and $\mu_H$ denote the Lagrangian multipliers for (5), (6), and (7). Then the Lagrangian for the above problem (4) is

$$L = (1 - r_i)[e\theta - w] + r_i[\bar{e} \bar{\theta} - \bar{w}] + \lambda(u(w) - v(e)/\theta - u_0)$$

$$+ \mu_L(u(w) - v(e)/\theta - u(\bar{w}) + v(P/\theta)/\bar{\theta})$$

$$+ \mu_H(u(\bar{w}) - v(\bar{e})/\bar{\theta} \geq u(w) - v(P/\theta)/\bar{\theta})$$

The solution has grower type $\bar{\theta}$ earning strictly positive information rents from the optimal contract under asymmetric information. Thus, this optimal full commitment contract under

¹ Contact the authors directly for a thorough derivation of these results and all results that follow.
asymmetric information can be written as $C^F = \{C_L^F, C_H^F\}$ with $C_L^F = \{w^F, \varepsilon^F\}$ and $C_H^F = \{\overline{w}^F, \overline{\varepsilon}^F\}$, where $\{w^F, \varepsilon^F\}$ and $\{\overline{w}^F, \overline{\varepsilon}^F\}$ are given by the following conditions:

(8) $\varepsilon \in \arg\{(1 - r_i)\theta[1 - v'(\varepsilon)u'(w)/\theta^2] + \frac{r_i}{\theta v'(\varepsilon)}[\theta^2v'(P/\theta) - \theta^2v'(\varepsilon)] = 0\}$,

(9) $u(w) - v(\varepsilon)/\theta = u_0$,

(10) $\overline{\varepsilon} \in \arg[\theta^2u'(\overline{w}) = v'(\overline{\varepsilon})]$, and

(11) $u(\overline{w}) - v(\overline{\varepsilon})/\theta = u(w) - v(P/\theta)/\theta = u(w) - v(e)/\theta + v(e)/\theta - v(P/\theta)/\theta

= u_0 + v(e)/\theta - v(P/\theta)/\theta$.

To summarize, if the processor commits herself in the first period not to use the information revealed by the grower in the following period and the grower commits to the two-period contract and cannot breach the relationship, the optimal incentives with full commitment mimic the same static contract in both periods. Further, commitment by the processor eliminates the possibility of incorporating reputation effects into the optimal incentives. Therefore, under the assumption of full commitment, a grower’s reputation for quality does not affect the dynamics of the optimal contract with asymmetric information.

**Two-Period Equilibrium Contracts with No Commitment**

In this case, it is assumed that neither the processor nor the grower can commit to an two-period incentive scheme. Thus, the processor chooses the optimal incentive scheme in the second period conditional on the grower’s first-period performance. Likewise, the grower can quit the relationship at the end of the first period. If he does quit, we assume that the grower can obtain his reservation utility.
In this case, there is some motivation for grower types to deviate. If the grower type $\bar{\theta}$ deviates and pretends to be the grower type $\theta$, i.e., choose the target market price $P(\theta)$, he would earn relatively less profit in the first period and enjoy positive information rent in the second period. In the following analysis, we exclude the possibility that the low-quality type would mimic the high-quality ability because it would lead to a loss.

The processor’s strategy consists of incentives schemes $\{w_1(P_1), w_2(P_1, P_2, w_1)\}$, and the grower’s strategy is a sequence of decisions of the effort levels $\{e_1(w_1, \theta), e_2(w_1, w_2, \theta, e_1)\}$. Denote the set of feasible contract as $\{C_1(L, C_{1H}) \}$ and $\{C_2(L, C_{2H}) \}$ where $C_{1j} = \{w_{1j}, e_{1j}\}$ and $C_{2j} = \{w_{2j}, e_{2j}\}$ for $j \in \{L, H\}$. The optimal strategies must form a perfect Bayesian equilibrium that meets the following five conditions: (i) $e_2$ is optimal for the grower given $w_2$, (ii) $w_2$ maximizes the processor’s expected profit given her belief about $\theta$, $f_2(\theta | w_2, e_2)$, in the second period, (iii) $e_1$ is optimal for the grower given $w_1$ and the second-period incentive schemes, (iv) $w_1$ maximizes the processor’s expected profit given his belief about $\theta$, $f_1(\theta)$, in the first period and the second-period strategies, and (v) the processor’s second-period belief $f_2(\theta | w_1, e_1)$ is derived from the first-period belief $f_1(\theta)$ and the grower’s first-period strategy using Bayes’ rule.

Without commitment, three types of continuation equilibria could potentially be sustained: (a) a separating equilibrium where both grower types are revealed after the first period; (b) a pooling equilibrium where both grower types choose low quality in the first period; and (c) a semi-separating equilibrium where the high-quality growers randomize in the first period. More specifically, in this last equilibrium, the high-quality type randomizes over $q(\theta)$ and $q(\bar{\theta})$, which correspond to the market prices $P_1(\theta)$ and $P_1(\bar{\theta})$, and the processor updates her belief
using Bayes’ rule. Letting \( \pi \) be the probability that the grower type \( \bar{\theta} \) chooses the contract designed for the grower type \( \theta \), the processor’s second-period belief becomes

\[
\hat{\pi}_2(P_1(\theta), \pi) = \frac{r_1 \pi}{r_1 \pi + 1 - r_1} < r_1 \text{ and } r_2(P_1(\bar{\theta}), \pi) = 1.
\]

In words, (12) says that if the processor observes \( P_1(\bar{\theta}) \) in the first period, she believes with certainty that the grower is a high-quality type. Alternatively, if she observes \( P_1(\theta) \), she believes the grower is a high-quality type with probability \( \hat{\pi}_2 \), after using Bayes’ rule.

Thus, for a given belief of grower types in the second period, we denote the processor’s second-period net profit as \( W_2(r_2) \) and the first-period net profit as \( W_1(r_1, C_{1L}, C_{1H}) \). Similarly, we define the grower’s second-period utility as \( U_2(C_2, r_2) / \theta \) and the first-period utility \( U_1(C_i, \theta) \), for \( i \in \{L, H\} \).

The processor’s problem is solved with backwards induction, starting with the second-period incentives schemes. Because there is no third period, the optimal second-period incentives can be derived following the same procedure as described in the full commitment case. In this case, without loss of generality, we normalize the reservation utility for all grower types \( u_\theta \) to be zero. Given the processor’s belief about the grower types \( r_2 \) in the second period, the processor solves the following problem:

\[
\text{max}_{e_2, w_2} W_2(r_2) = r_2 Z_2(\bar{\theta}) + (1 - r_2) Z_2(\theta) = r_2 [\bar{\theta}e_{2\bar{H}} - w_{2\bar{H}}] + (1 - r_2)[\theta e_{2L} - w_{2L}] \text{ s.t.}
\]

\[
U_2 = u(w_2) - v(e_2) / \theta \geq 0, \quad \forall \theta \in \Theta = \{\theta, \bar{\theta}\}.
\]

\[
u(w_2L) - v(e_2L) / \theta \geq u(w_{2H}) - v(e_{2H} \bar{\theta} / \theta).
\]

Note that we use \( \theta_i = \theta \) and \( \theta_H = \bar{\theta} \) here. From now on, these notations might be used interchangeably for notational simplification.
\[ u(w_{2H}) - v(e_{2H})/\bar{\theta} \geq u(w_{2L}) - v(e_{2L}\bar{\theta}/\bar{\theta}). \]

The optimal contract for the second period are derived contingent on the processor’s belief about \( r_2 \) and the possible existence of three different types of equilibria given different values of \( r_2 \):

**Case 2a, Second-Period Strategies Following a Separating Equilibrium:** If the first-period equilibrium is separating, i.e., \( r_2(w_i(\theta), P(\bar{\theta})) = 1 \) and \( r_2(w_i(\theta), P(\bar{\theta})) = 0 \), then second-period equilibrium is exactly same as the optimal contract under perfect information. In other words, once the grower’s true type is revealed in the first period, the grower’s private information concerning their quality becomes public. Hence, the processor can offer a contract that provides the reservation utility to the grower of each type and extract all surplus from the grower. Thus, in a dynamic two-period contract, if the first-period contract is separating, the optimal contract for the second period is \( \{C_{2L}, C_{2H}\} \), where \( C_{2L} = (\bar{w}^*, \bar{e}^*) \) and \( C_{2H} = (\bar{w}^*, \bar{e}^*) \), as in the perfect information contract (i.e., Case 1a). Recall that the optimal contract \( C_2 \) requires

\[ U_2(C_{2L} | \theta_L) = U_2(C_{2H} | \theta_H) = 0. \]

**Case 2b, Second-Period Strategies Following a Pooling Equilibrium:** If both growers types pool in the first period and choose \( P_1(\bar{\theta}) \), the processor adopts the same distribution of grower types as in the prior distribution, i.e., \( r_2 = r_1 \). As a result, the optimal contract for the second period is same as that in the full commitment contract (i.e., case 1b). That is,

\[ C_2^P = \{C_{2L}^P, C_{2H}^P\}, \text{ where } C_{2L}^P = (\bar{w}^F, \bar{e}^F) \text{ and } C_{2H}^P = (\bar{w}^F, \bar{e}^F). \]

---

3 The superscript “\( P \)” stands for a “pooling” continuation equilibrium when the first-period contract is fully concealing.
Case 2c, Second-Period Strategies Following a Semi-Separating Equilibrium: If the high-quality type randomizes over \( P_t(\theta) \) and \( P_t(\tilde{\theta}) \), then the processor updates her belief using Bayes’ rule and solves the problem (13) given \( r_2 \) is specified by (12).

Given any value of \( r_2 \), the optimal contract can be solved using the similar procedure described in the full commitment contract. Precisely, the optimal contract \( C_2 \) must satisfy the following conditions:

\[
\begin{align*}
(17) & \quad e_{2H} \in \arg \{ \tilde{\theta}^2 u'(w_{2H}) = v'(e_{2H}) \}. \\
(18) & \quad e_{2L} \in \arg \{ (1 - r_2)\tilde{\theta} [1 - \frac{v'(e_{2L})}{u'(w_{2L})\tilde{\theta}^2}] + \frac{r_2}{\tilde{\theta}^2} v'(e_{2H}) - \tilde{\theta}^2 v'(e_{2L}) = 0 \}. \\
(19) & \quad u(w_{2L}) - v(e_{2L})/\tilde{\theta} = 0. \\
& \quad u(w_{2H}) - v(e_{2H})/\tilde{\theta} = u(w_{2L}) - v(e_{2L} \theta / \tilde{\theta})/\tilde{\theta} \\
& \quad = u(w_{2L}) - v(e_{2L})/\tilde{\theta} + v(e_{2L})/\tilde{\theta} - v(e_{2L} \theta / \tilde{\theta})/\tilde{\theta} \\
& \quad = v(e_{2L})/\tilde{\theta} - v(e_{2L} \theta / \tilde{\theta})/\tilde{\theta} > 0. \\
(20) & \quad I_{2H}(r_2) = v(e_{2L} \theta / \tilde{\theta}) - v(e_{2L} \theta / \tilde{\theta})/\tilde{\theta} > 0. 
\end{align*}
\]

Hence, if the grower type \( \tilde{\theta} \) deviates in the first period and pools with the grower type \( \theta \) or randomizes, he obtain positive information rents in the second period:

Note that the low-quality type always obtains his reservation utility in the second period regardless of the processor’s belief of grower types. Therefore, there is no incentive for the low-quality type to deviate in the first period and the low-quality type always chooses his own contract in the first period. On the other hand, the high-quality grower can obtain greater payoff in the second period by mimicking a low-quality type in the first period. In addition, the more likely the processor believes that the grower is of low-quality type (smaller value of \( r_2 \), the
greater payoff the high-quality grower could obtain in the second period. This discussion leads
to the following lemma:

**Lemma 1**: \( U_2(C_{2H}(r_2) | \bar{\theta}) = I_{2H}(r_2) \) decreases in \( r_2 \).

Lemma 1 can be better understood in Figure 3.2. Since condition (16) is binding, that is, \( U_2(C_{2H}(r_2) | \bar{\theta}) = U_2(C_{2L}(r_2) | \bar{\theta}) \) for any given \( r_2 \), the optimal contract for the high-quality type must be on the indifference curve that intersects with the indifference curve \( U_2(C_{2L} | \bar{\theta}) = 0 \) through point \( (w_{2L}(r_2), e_{2L}(r_2)) \). Since \( v(0) = 0 \), the information rent \( I_{2H}(r_2) \) is exactly the distance from the origin to the point C on the vertical axis. Hence, if \( r_2 \) decreases, the contract for the low-quality type \( (w_{2L}(r_2), e_{2L}(r_2)) \) moves along the indifference curve \( U_2(C_{2L} | \bar{\theta}) = u(w) - v(e) / \bar{\theta} = 0 \) toward point \( (w^*, e^*) \), which represents the optimal contract for the low-quality type under perfect information. That is, in the limit, when \( r_2 = 0 \), the contract \( C_{2L} \) converges to the perfect information contract \( C^*_L = (w^*, e^*) \) at which the information rent for the high-quality type is maximized. Therefore, \( U_2(C_{2H}(r_2) | \bar{\theta}) = I_{2H}(r_2) \) decreases in \( r_2 \).

Intuitively, this lemma states that the information rent for the high-quality grower type increases as the processor believes that the grower is more likely to be a low-quality type.

Because the processor’s second-period net profit depends on her belief about growers types in the second period, denote \( Z_2(\bar{\theta}) = \bar{\theta}e_{2H}(r_2) - w_{2H}(r_2) \) and \( Z_2(\theta) = \theta e_{2L}(r_2) - w_{2L}(r_2) \), then
\[
W^*_2(r_2) = \max_{e_2(\theta), w_2(\theta)} W_2(r_2) = r_2 Z_2(\bar{\theta}) + (1 - r_2) Z_2(\bar{\theta}) \]
as the maximum second-period net profit contingent on \( r_2 \). It can be shown that the processor’s second-period net profit increases in \( r_2 \).

In other words, we have demonstrated the following Lemma:

**Lemma 2**: \( W^*_2(r_2) \) increases in \( r_2 \) and is convex in \( r_2 \).
We now turn to the first-period incentive schemes. In the first period, the processor maximizes her expected payoff subject to the participation constraints and incentive compatibility constraints. Since the processor cannot commit not to use the first-period information revealed by the growers, the incentive compatibility constraints must take into account the effects of first-period decisions on the second-period payoff.

For any first-period contract, \( C_1 = \{C_{1L}, C_{1H}\} \), let \( V_H(C_{1H} | \overline{\theta}) \) denote the two-period payoff to the grower type \( \theta \) if \( P_1(\overline{\theta}) \) is observed, i.e., the high-quality grower chooses his own contract \( C_{1H} \) in the first period. Recall that if \( P_1(\overline{\theta}) \) is observed, the processor updates her belief such that \( r_2(w_1(\overline{\theta}), P_1(\overline{\theta})) = 1 \). Thus, \( V_H(C_{1H} | \overline{\theta}) = U_1(C_{1H} | \overline{\theta}) + \delta U_2(C_H^* | \overline{\theta}) = U_1(C_{1H} | \overline{\theta}) \).

Note that if the first-period contract is fully revealing, the second-period contract is same as the perfect information contract in which the high-quality grower obtains his reservation utility zero (i.e., \( U_2(C_H^* | \overline{\theta}) = 0 \)). Similarly, denote \( V_H(C_{1L}, \pi | \overline{\theta}) \) as the two-period payoff to the grower type \( \overline{\theta} \) if \( P_1(\overline{\theta}) \) is observed and the high-quality grower type chooses the contract designed for the low-quality grower type with probability \( \pi \). Thus, \( V_H(C_{1L}, \pi | \overline{\theta}) = U_1(C_{1L} | \overline{\theta}) + \delta U_2(\hat{C}_2) \),

where, from (12), \( \hat{r}_2(P_1(\overline{\theta}), \pi) = \frac{r_1 \pi}{r_1 \pi + 1 - r_1} \).

Now the grower’s equilibrium strategy \( \hat{\pi} \) must be optimal for him given the processor’s belief. In other words, the grower must be indifferent between revealing his true type and mimicking the other type at the equilibrium given the optimal \( \hat{\pi} \). Therefore, an equilibrium strategy must satisfy the following condition:

\[(22) \quad V_H(C_{1H} | \overline{\theta}) = V_H(C_{1L}, \hat{\pi} | \overline{\theta}).\]
Recall that in the full commitment contract (also the static contract), the incentive constraint for the high-quality type must be binding, i.e., \( U_1(C_H^F \mid \overline{\theta}) = U_1(C_L^F \mid \overline{\theta}) \). However, in the dynamic setting, the contract \( C^F = \{C_L^F, C_H^F\} \) can only result in
\[
V_H(C_{1H} \mid \overline{\theta}) < V_H(C_{1L}, \hat{\pi} \mid \overline{\theta})
\]
because \( I_{2H}(\hat{r}_2) > 0 \) for \( 0 \leq r_2 < 1 \). That is, the high-quality type always gains from mimicking the low-quality type if the optimal static contract is offered in the dynamic setting. Therefore, the static contract \( C^F \) cannot be an optimal separating equilibrium in a dynamic context.

Given continuity of \( V_H(C_{1L}, \pi \mid \overline{\theta}) \) in \( \pi \) and condition (22), three types of equilibrium could be sustained:

(23) Separating equilibrium if: \( V_H(C_{1H} \mid \overline{\theta}) \geq V_H(C_{1L}, 0 \mid \overline{\theta}) \),

(24) Pooling equilibrium if: \( V_H(C_{1H} \mid \overline{\theta}) \leq V_H(C_{1L}, 1 \mid \overline{\theta}) \), and

(25) Semi-separating equilibrium if \( V_H(C_{1H} \mid \overline{\theta}) = V_H(C_{1L}, \hat{\pi} \mid \overline{\theta}) \) for some \( \hat{\pi} \).

For each type of equilibrium, the processor maximizes her discounted expected two-period payoff subject to the participation constraints and incentive compatibility constraints for both grower types. Let \( \psi \) denote the probability of a grower choosing contract \( C_{1L} \) given the contract \( C_1 = \{C_{1L}, C_{1H}\} \). Since only the high-quality type has incentive to deviate, then for any \( \pi \), \( \psi = r_1 \pi(C_1) + 1 - r_1 \). Thus, the processor’s two-period net profit is

\[
W_i(r_i, C_{1L}, C_{1H}) = (1 - \psi)[\overline{\theta} e_{1H} - w_{1H} + \delta W_2(1)] + \psi[\overline{\theta} e_{1L} - w_{1L} + \delta W_2(\hat{r}_2)]
\]

Case 2a, First-Period Strategies Inducing a Separating Equilibrium: In a separating equilibrium, the high-quality grower chooses his own contract with probability 1, or \( \pi = 0 \).

Thus, to induce a separating equilibrium, the processor solves the following problem:
\[
\max_{\theta_i, \theta_{i+1}, s_{iL}, s_{iH}} W_i(r_i, C_{iL}, C_{iH}) = r_i[\theta_{iH} - w_{iL} + \delta W_2(1)] + (1 - r_i)[\theta_{iL} - w_{iL} + \delta W_2(0)], \quad \text{s.t.}
\]

\[
U_1 = u(w_{iL}) - v(e_{iL}) / \theta_i \geq 0, \quad \forall i \in \{L, H\}
\]

\[
U_1(C_{iH} | \bar{\theta}) \geq U_1(C_{iL} | \bar{\theta}) + \delta \theta_{2H}(0)
\]

\[
U_1(C_{iL} | \theta) \geq U_1(C_{iH} | \theta)
\]

Conditions (29) and (30) state that the each grower type prefers his own contract to the contract designed for the other type. Note that from (30) the low-quality quality type always chooses the contracts that he most prefers in the short run because if he mimics the high-quality type in the first period, the processor will only offer the contract \(C_{iL}^*\) under which the low-quality type makes loss.

Following the same procedure described in the previous sections, the optimal separating contract must satisfy the following three conditions: The participation constraint for the low-quality grower type must be binding; the incentive compatibility constraint for the high-quality type is binding; and the low-quality type strictly prefers his own contract to the contract designed for the high-quality type, i.e.,

\[
u(w_{iL}) - v(e_{iL}) / \theta = 0,
\]

\[
U_1(C_{iH} | \bar{\theta}) = U_1(C_{iL} | \bar{\theta}) + \delta \theta_{2H}(0), \quad \text{and}
\]

\[
U_1(C_{iL} | \theta) > U_1(C_{iH} | \theta).
\]

The feasible set of contracts can be demonstrated in Figure 3.3. First, since the low-quality type always chooses his own contract and obtains the reservation utility, the feasible set of contracts for the low-quality type must be the segment from the origin to the point \((w^*, e^*)\) on his indifference curve \(u(w) - v(e) / \theta = 0\). Denote \(S_{iL}\) as this set. Given any contract for the low-quality type \(C_{iL} = (w_{iL}, e_{iL})\) in the set \(S_{iL}\), properties of the optimal contract (31)-(33) require that
the feasible contract for the high-quality type must be in the region below the low-quality indifference curve \( u(w) - v(e)/\theta = 0 \) and above the high-quality indifference curve \( H_2 \) in Figure 3.3. More specifically, from condition (32), the optimal contract for the high-quality type must be located on the indifference curve \( H_2 \). Note that the indifference curve \( H_1 \) intersects with the low-quality indifference curve \( u(w) - v(e)/\theta = 0 \) through the point \((w_{1L}, e_{1L})\) and the distance between \( H_1 \) and \( H_2 \) is exactly \( \delta I_{2H}(0) \). Hence, for any given contract \( C_{1L} = (w_{1L}, e_{1L}) \), the contract for the high-quality type \( C_{1H} = (w_{1H}, e_{1H}) \) always satisfies condition (28), (29), (30) and properties (31)-(33) at the equilibrium. Thus, the problem boils down to solving the optimal contract for the low-quality type. Once the optimal contract for the low-quality type is determined, it is straightforward to find the optimal contract for the high-quality type.

The optimal first-period contract can be solved in the similar manner as in the previous sections. Ignoring the participation constraint for the high-quality type and the incentive compatibility constraint (30), let \( \lambda \) and \( \mu_H \) denote the Lagrangian multipliers for conditions (29) and (28) respectively, thus, the Lagrangian to the problem (27) is:

\[
L = r_1[\bar{\theta} e_{1H} - w_{1H} + \delta W_2(1)] + (1 - r_1)[\bar{\theta} e_{1L} - w_{1L} + \delta W_2(0)] + \lambda [u(w_{1L}) - v(e_{1L})/\theta] \\
+ \mu_H [u(w_{1H}) - v(e_{1H})/\bar{\theta} - u(w_{1L}) + v(e_{1L}/\bar{\theta})/\bar{\theta} - \delta I_{2H}(0)]
\]

Thus, the optimal contract \( \hat{C}_1 = \{\hat{C}_{1L}, \hat{C}_{1H}\} \) is given by the following conditions:

\[
e_{1H} \in \arg \{\bar{\theta}^2 u'(w_{1H}) = v'(e_{1H})\}, \tag{34}
\]

\[
e_{1L} \in \arg \{(1 - r_1)\bar{\theta}[1 - \frac{v'(e_{1L})}{u'(w_{1L})}] + \frac{r_1}{\bar{\theta}v'(e_{1H})} [\bar{\theta}^2 v'(e_{1L}/\bar{\theta}) - \bar{\theta}^2 v'(e_{1L})] = 0\} \tag{35}
\]

\[
u(w_{1L}) - v(e_{1L})/\theta = 0 \tag{36}
\]

\[
u(w_{1H}) - v(e_{1H})/\bar{\theta} = u(w_{1L}) - v(e_{1L}/\bar{\theta})/\bar{\theta} + \delta I_{2H}(0) = u(w_{1L}) - v(e_{1L})/\bar{\theta} + \delta I_{2H}(0) > 0 \tag{37}
\]

\[
\delta I_{2H}(0) > 0
\]
The optimal contract is demonstrated with the help of Figure 3.3. Denote this contract as 
\( \hat{C}_1 = \{\hat{C}_{1L}, \hat{C}_{1H}\} \) where \( \hat{C}_{1L} = (\hat{w}_{1L}, \hat{e}_{1L}) \) and \( \hat{C}_{1H} = (\hat{w}_{1H}, \hat{e}_{1H}) \). Given the first order conditions (34)-(37), the contract \( \hat{C}_{1L} = (\hat{w}_{1L}, \hat{e}_{1L}) \) and \( \hat{C}_{1H} = (\hat{w}_{1H}, \hat{e}_{1H}) \) illustrated in Figure 3.3 constitutes a separating equilibrium. In fact, assuming that both grower types participate in the first period, the optimal separating equilibrium contract \( \hat{C}_{1L} = (\hat{w}_{1L}, \hat{e}_{1L}) \) and \( \hat{C}_{1H} = (\hat{w}_{1H}, \hat{e}_{1H}) \) is unique for a given prior belief \( \theta_r \). Given this contract, the low-quality type strictly prefers the contract \( \hat{C}_{1L} = (\hat{w}_{1L}, \hat{e}_{1L}) \) to \( \hat{C}_{1H} = (\hat{w}_{1H}, \hat{e}_{1H}) \) both in a one-period static contract and in a two-period dynamic contract, while the high-quality type strictly prefers the contract \( \hat{C}_{1H} = (\hat{w}_{1H}, \hat{e}_{1H}) \) to \( \hat{C}_{1L} = (\hat{w}_{1L}, \hat{e}_{1L}) \) in the short run (i.e., in a one-period contract) and is indifferent in the two-period dynamic context. Thus, at the separating equilibrium, the low-quality type chooses his own contract \( \hat{C}_{1L} = (\hat{w}_{1L}, \hat{e}_{1L}) \) in the first period and will be offered \( \hat{C}_{2L} = (\bar{w}^*, \bar{e}^*) \) in the second period, and earns payoff zero in two periods. Similarly, the high-quality type chooses \( \hat{C}_{1H} = (\hat{w}_{1H}, \hat{e}_{1H}) \) in the first period and obtains positive payoff

\[ v(\hat{e}_{1L})/\theta - v(\hat{e}_{1L}, \hat{e}_{1L})/\theta + \delta L_{2H}(0) > 0, \]

and will be offered \( \hat{C}_{2H} = (\bar{w}^*, \bar{e}^*) \) in the second period. If the high-quality type deviates and chooses \( \hat{C}_{1L} = (\hat{w}_{1L}, \hat{e}_{1L}) \) in the first period, he will earn

\[ v(\hat{e}_{1L})/\theta - v(\hat{e}_{1L}, \hat{e}_{1L})/\theta \]

in the first period and information rents \( I_{2H}(0) \) in the second period, which makes him indifferent between \( \hat{C}_{1L} = (\hat{w}_{1L}, \hat{e}_{1L}) \) and \( \hat{C}_{1H} = (\hat{w}_{1H}, \hat{e}_{1H}) \). Note that the contract \( \hat{C}_{1H} = (\hat{w}_{1H}, \hat{e}_{1H}) \) is the tangent point between the processor’s iso-profit line and the high-quality type’s indifference curve H2 in Figure 3.3, therefore, the optimal contract for the high-quality type is efficient. Additionally, as in the full commitment contract with asymmetric
information, the processor offers the low-quality type a contract that is suboptimal in order to reduce the information rents paid to the high-quality type.

However, besides this separating equilibrium, other separating equilibrium might also exist. In particular, define \( C^0 = (w, e) = (0,0) \) as the null contract (i.e., a grower type does not sign the contract if a null contract is offered), the contract \( \tilde{C}_1 = \{\tilde{C}_{1L}, \tilde{C}_{1H}\} = \{C^0, \tilde{C}_{1H}\} \) establishes another separating equilibrium, where \( \tilde{C}_{1H} = (\tilde{w}_{1H}, \tilde{e}_{1H}) \) in Figure 3.4 is the tangent point between the high-quality type’s indifference curve \( H_1 \) and the processor’s iso-profit line for the high-quality type. This contract is actually the limit of the separating contract \( \hat{C}_1 = \{\hat{C}_{1L}, \hat{C}_{1H}\} \) as \( r_i \) approaches 1. Because the low-quality type will not participate in the first period given this contract, sometimes we call it the “cream-skimming separating equilibrium”. More specifically, the contract \( \tilde{C}_{1H} = (\tilde{w}_{1H}, \tilde{e}_{1H}) \) must satisfy the following conditions:

\[
\begin{align*}
\tilde{e}_{1H} &\in \arg \{\tilde{\theta}^2 u'(\tilde{w}_{1H}) = v'(\tilde{e}_{1H})\} \\
\frac{u(\tilde{w}_{1H}) - v(\tilde{e}_{1H})}{\tilde{\theta}} &= \partial I_{2H}(0)
\end{align*}
\]

Under this contract, the low-quality type strictly prefers the null contract \( C^0 \) to the contract \( \tilde{C}_{1H} \) because he could earn losses in both periods if he chooses \( \tilde{C}_{1H} \) in the first period. Note that the low-quality type also strictly prefers \( C^0 \) to \( \tilde{C}_{1H} \) in a one-period static contract. Similarly, by choosing \( \tilde{C}_{1H} \) in the first period, the two-period payoff to the high-quality type is

\[
\begin{align*}
U_1(\tilde{C}_{1H} | \bar{\theta}) + \delta U_2(C^*_H | \bar{\theta}) &= U_1(\tilde{C}_{1H} | \tilde{\theta}) = \partial I_{2H}(0), \text{ while by choosing } C^0, \text{ he obtains} \\
U_1(C^0 | \bar{\theta}) + \delta U_2(C^*_L | \bar{\theta}) &= \delta U_2(C^*_L | \tilde{\theta}) = \partial I_{2H}(0). \text{ Hence, the high-quality type is indifferent between the contract } \tilde{C}_{1H} \text{ and } C^0. \text{ Therefore, } \hat{C}_1 = \{\hat{C}_{1L}, \hat{C}_{1H}\} = \{C^0, \tilde{C}_{1H}\} \text{ constitutes another separating equilibrium.}
\end{align*}
\]
Given the two separating equilibria, the processor must offer the one that maximizes her net profit. That is, the optimal contract maximizes the maximum of 

\[ W_1(r_1, \hat{C}_{1L}, \hat{C}_{1H}) \] and \[ W_1(r_1, C^0, \tilde{C}_{1H}) \]. The results are summarized in the following proposition.

**Proposition 1.** There exists two possible separating equilibria to the two-period problem \((27): \tilde{C}_i = \{C^0, \tilde{C}_{1H}\} \) and \(\hat{C}_i = \{\hat{C}_{1L}, \hat{C}_{1H}\}\). In addition, there exists a \(r^*_1\) such that for \(r_1 < r^*_1\), the optimal separating equilibrium is \(\hat{C}_i = \{\hat{C}_{1L}, \hat{C}_{1H}\}\), while for \(r_1 > r^*_1\), the optimal separating equilibrium is \(\tilde{C}_i = \{C^0, \tilde{C}_{1H}\}\).

Intuitively, Proposition 1 states that the separating contract \(\tilde{C}_i\) would dominate \(\hat{C}_i\) when the processor believes that a large proportion of the growers are of high-quality type. Thus, it is less costly for the processor if she only offers a contract to the high-quality type and handicaps the low-quality type. On the contrary, if the processor believes that the proportion of high-quality type is sufficiently small, then she would be better off by offering the separating contract \(\hat{C}_i\). In addition, we could show that the processor’s two-period profit increases with the proportion of the high-quality type. This result is summarized in the following corollary.

**Corollary 1.1:** In a separating equilibrium, \(W_1(r_1, C_{1L}, C_{1H})\) increases with \(r_1\).

Up to now, we have assumed that for any contract \((w, P)\), it is always true that \(P - w \geq 0\), i.e., on the right side of the 45 degree line in the \((w, P)\) space. However, restricting positive profit reduces the set of feasible contracts. Specifically, given the optimal contract \(\hat{C}_{1L}\), the separating contract \(\hat{C}_{1H} = (\hat{w}_{1H}, \hat{P}_{1H})\) in \((w, P)\) space may become infeasible for \(\delta\) sufficiently large because it would lie to the left side of the zero-profit line \(w = P\). A similar argument could

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4 Contact the authors directly for proofs for Proposition 1 and all corollaries and propositions that follow.
5 Here, we use the contract space \((w, P)\) instead of \((w, e)\). Recall that \(P = q = b\epsilon\). From now on, we may use these two alternative spaces interchangeably.
be made for the cream-skimming separating equilibrium \( \{ C^0, \hat{C}_{1H} \} \). An example of an infeasible separating contract is illustrated in Figure 3.5. These arguments are provided in the following corollary without further proof.

**Corollary 1.2:** There exists a \( \delta^* \) such that for \( \delta > \delta^* \), the separating equilibrium \( \hat{C}_1 = \{ \hat{C}_{1H}, \hat{C}_{1L} \} \) becomes infeasible.

Although the value of \( \delta^* \) cannot be precisely determined, the intuition behind this corollary is that if the grower are patient (i.e., \( \delta \) is large), it becomes too costly for the processor to induce a separating equilibrium in the first period. When growers are patient, the processor is better off by providing a pooling contract or a semi-separating contract instead of a fully separating contract.

For the separating equilibrium to be stable, these contracts must not be dominated by other contracts. This notion and Corollary 1.2 yields the following corollary.

**Corollary 1.3:** For the difference between \( \overline{\theta} \) and \( \underline{\theta} \) sufficiently large and \( \delta \) sufficiently small, there exists a fully separating equilibrium.

The intuition behind this corollary can be described as follows: For large values of \( \delta \) (close to 1), the high-quality grower type is very patient and it is prohibitively costly for the processor to distinguish the grower types. For \( \delta \) sufficiently small and the difference between \( \overline{\theta} \) and \( \underline{\theta} \) sufficiently large, not only is it less costly for the processor to distinguish the grower type, but also the high-quality grower type would intend to distinguish himself from the low-quality type.

**Case 2c, First-Period Strategies With a Semi-Separating Equilibrium:** Using the similar procedures as described in the previous section, a semi-separating equilibrium can be
established. Let \( C^*_i = \{C^*_{iL}, C^*_{iH}\} \) be the semi-separating contract, where \( C^*_{ii} = (w^*_{ii}, e^*_{ii}) \) for \( i \in \{L, H\} \). Recall that for the contract to be semi-separating, the condition (3.39) must be satisfied. Specifically, the condition (3.39) is equivalent to

\[
(40) \quad U_1(C^*_i | \bar{\theta}) = U_1(C^*_{iL} | \bar{\theta}) + \delta U_2(C^*_i(\hat{r}_2) | \bar{\theta})) = U_1(C^*_i | \bar{\theta}) + \delta \mathcal{I}_{2H}(\hat{r}_2),
\]

where \( \hat{r}_2(P_1(\bar{\theta}), \pi) = \frac{r_i \pi}{r_i \pi + 1 - r_i} \), and \( \pi \) is the probability that the grower type \( \bar{\theta} \) chooses the contract designed for the grower type \( \theta \) in the first period. In addition, the semi-separating contract for the low-quality type must lie on his zero-utility indifference curve, \( u(w) - v(e) / \bar{\theta} = 0 \).

Thus, the processor must solve the following problem:

\[
(41) \quad \max_{w^*_{iL}, e^*_{iL}} W_1(r_i^*, \hat{\pi}, C^*_{iL}, C^*_{iH}) = (1 - \psi)[\bar{\theta} e^*_{iH} - w^*_{iH} + \delta W_2(1)] + \psi[e^*_{iL} - w^*_{iL} + \delta W_2(\hat{r}_2)], \text{ s.t.}
\]

\[
(42) \quad U_1(C^*_i | \bar{\theta}) = U_1(C^*_{iL} | \bar{\theta}) + \delta \mathcal{I}_{2H}(\hat{r}_2).
\]

\[
(43) \quad u(w^*_{iL}) - v(e^*_{iL}) / \bar{\theta} = 0,
\]

where \( \psi = r_i \pi + 1 - r_i \).

Let \( \lambda \) and \( \mu_{iL} \) denote the Lagrangian multipliers for (43) and (42). Then the Lagrangian for the above problem (41) is

\[
L = (1 - \psi)[\bar{\theta} e^*_{iH} - w^*_{iH} + \delta W_2(1)] + \psi[e^*_{iL} - w^*_{iL} + \delta W_2(\hat{r}_2)]
\]

\[
+ \lambda(u(w^*_{iL}) - v(e^*_{iL}) / \bar{\theta}) + \mu_{iL} [u(w^*_{iH}) - v(e^*_{iH}) / \bar{\theta} - u(w^*_{iL}) + v(P^*_{iL} / \bar{\theta}) / \bar{\theta} - \delta \mathcal{I}_{2H}(\hat{r}_2)].
\]

The first-order conditions lead we can get the optimal levels of effort for the high-quality and low-quality types:

\[
(44) \quad e^*_{iH} \in \text{arg} \left\{ \bar{\theta}^2 u'(w^*_{iH}) = v'(e^*_{iH}) \right\}, \text{ and}
\]

\[
(45) \quad e^*_{iL} \in \text{arg} \left\{ \psi \bar{\theta} [1 - \frac{v'(e^*_{iL})}{u'(w^*_{iL}) \bar{\theta}^2}] + \frac{1 - \psi}{\bar{\theta} v'(e^*_{iH}) / \bar{\theta} - \bar{\theta}^2 v'(e^*_{iL})] = 0 \right\}.
\]
This optimal semi-separating contract $C^*_i = \{C^*_{iL}, C^*_{iH}\}$ is illustrated in Figure 3.6. In this figure, H1-H4 are the high-quality type’s indifference curves, where the distances between the curve H1 and the curves H2, H3, and H4 are $\delta I_{2H}(r^*_1), \delta I_{2H}(r^*_2)$, and $\delta I_{2H}(0)$, respectively and $\delta I_{2H}(r^*_1) < \delta I_{2H}(r^*_2) < \delta I_{2H}(0)$. From conditions (23)-(25), a semi-separating contract $C^*_i = \{C^*_{iL}, C^*_{iH}\}$ must satisfy

$$\leq\leq\leq$$

(46) $$U(C^*_i | \overline{\theta}) + \delta I_{2H}(r^*_1) \leq U(C^*_i | \overline{\theta}) \leq U(C^*_i | \overline{\theta}) + \delta I_{2H}(0).$$

Thus, given that the optimal contract $C^*_{iL}$ is located on the low-quality type’s indifference curve $u(w^*_{iL}) - \nu(e^*_i) / \overline{\theta} = 0$, the optimal contract $C^*_{iH}$ must lie on a indifference curve, with the curve H3 as an illustration, which is above the indifference curve H2 and below H4. Therefore, there exists a $\hat{\pi}$ such that $U(C^*_{iH} | \overline{\theta}) = U(C^*_{iL} | \overline{\theta}) + \delta I_{2H}(\hat{r}_2)$, i.e., a semi-separating equilibrium.

To guarantee that the semi-separating equilibrium could be sustained, we need to compare the semi-separating equilibrium with other potential equilibria. However, it is not trivial to determine the relationship between the semi-separating equilibrium with other potential equilibria analytically without specifying the functional forms of $u()$ and $\nu()$.

**Reputation Rewards**

In the previous section, reputation effects are embodied in the posterior probability assessment (using Bayes’ rule) of the growers’ types by the processor at the end of the first period. Anticipating the processor’s strategies, the high-quality grower type chooses to build up some reputation by either imitating the low-quality type or revealing his true type whichever is favorable. Under this scheme, however, imitating the dominant behavior of a low-quality type can yield greater future information rents to the high-quality type. This sort of reputation effects, therefore, ends up reinforcing the potential ratchet effects.
In this section, we assume that at the beginning of each period, a reputation, \( R_t \), of the grower is formed from his past observed performance. More importantly, we now allow the processor to offer a direct reward to the grower contingent on his reputation. Accumulation of the grower’s reputation is assumed to be based on an exogenous rule 
\[
R_t = \beta q_{t-1} + (1 - \beta) R_{t-1}
\]
with \( 0 \leq \beta \leq 1 \), and the grower’s initial reputation equal to some \( R_0 \). For the most part, we demonstrate the effects of the reputation reward on the two-period contract after restricting the analysis to a special case where \( R_t = q_{t-1} \) and \( R_0 = 0 \).

Accumulation of reputation can be interpreted differently given different values of \( \beta \).

When \( \beta \) is small, i.e., very close to zero, the latest period quality does not provide much contribution to the grower’s reputation. This situation could occur under some circumstances such that the processor already has a long-term relationship before this contract and the grower’s reputation has almost converged to a constant as in the latest period. In this case, including reputation effects in the contract would not improve much on the optimal incentives. On the other hand, when \( \beta \) is large, the latest period quality is crucial for the grower’s reputation in the current period. Thus, stronger incentives can be provided by the processor when reputation of growers is incorporated into the contract. \( R_t = q_{t-1} \) is a special case of this example when setting \( \beta = 1 \). More specifically, we let the processor offer the grower a reputation rewards, \( s_t(R_t) \), when she observes \( R_t \) from the previous periods. If the processor observes \( P(\overline{\theta}) \) in the first period, the reward in the second period will be \( s(P(\overline{\theta})) \); alternatively, if the processor observes \( P(\theta) \) in the first period, the reward in the second period will be \( s(P(\theta)) \), which we normalize to zero without loss of generality. Moreover, we assume that the reputation reward take the form of a lump-sum payment.
Similarly to the previous section, the processor maximizes her expected profit subject to the participation constraints and incentive compatibility constraints for both grower types in both periods. However, to simplify the analysis, only a separating equilibrium will be discussed. First, let us investigate the second-period incentive scheme.

**Case 3, Second-period Strategies with Reputation Awards.** Because the reputation rewards do not affect the grower’s second-period participation constraints and incentive compatibility constraints, the second-period incentive schemes are identical to the previous section. In a separating equilibrium, the private information concerning the grower’s types becomes perfect information in the second period. Thus, given any \( s_2(P(\theta)) \), for each grower type, \( \theta \in \Theta = \{\theta, \overline{\theta}\} \), the processor offers the contract \( C_2 = \{C_{2L}, C_{2H}\} \), where \( C_{2L} = (w^*, e^*) \) and \( C_{2H} = (\overline{w}^*, \overline{e}^*) \).

**Case 3, First-period Strategies with Reputation Awards.** In the first period, the processor must maximize the two-period expected profit to find a separating equilibrium. Letting \( s_{2H} = s_2(P(\overline{\theta})) \), the processor maximizes

\[
\max_{e_{1H}, w_{1H}, C_{1L}, C_{1H}} W_1(r_i, C_{1L}, C_{1H}) = r_i [\overline{\delta} e_{1H} - w_{1H} + \delta W_2 (1 - \overline{\delta} 2H)] + (1 - r_i) [\overline{\delta} e_{1L} - w_{1L} + \delta W_2 (0)]
\]

The participation constraints take the following form:

\[
U_1 = u(w_{1i}) - v(e_{1i}) / \theta_i \geq 0, \quad \forall i \in \{L, H\}
\]

However, risk aversion brings about some complications to the formulation of incentive compatibility constraints. To induce a separating equilibrium in the first period, the incentive compatibility constraints for the high-quality grower type must satisfy

\[
U_1(C_{1H} | \overline{\theta}) + \delta S_{2H}(\overline{\theta}) \geq U_1(C_{1L} | \overline{\theta}) + \delta S_{2H}(0).
\]
This constraint states that, at the equilibrium, the high-quality grower type must prefer revealing his true type to mimicking the low-quality grower type. Note that the extra reward $s_{2H}$ in the processor’s profit $W_1(r_1, C_{1L}, C_{1H})$ is given in monetary units, while $\hat{s}_{2H}(\theta)$ is the equivalent amount in the units of the high-quality type’s utility. More specifically,

\[ \hat{s}_{2H}(\theta) = u(w_{2H} + s_{2H}) - u(w_{2H}) \]

If, instead, the growers are risk neutral, then $\hat{s}_{2H}(\theta) \equiv s_{2H}$.

Due to risk aversion, the same amount of monetary reward results in different utility measures for different grower type. Thus, for the low-quality type, the incentive compatibility constraint must satisfy

\[
U_1(C_{1L} | \theta) \geq U_1(C_{1H} | \theta) + \hat{s}_{2H}(\theta) + \delta I_{2L}(l).
\]

This constraint states that the low-quality type prefers revealing his true type than mimicking the high-quality type. Likewise, $\hat{s}_{2H}(\theta)$ represents the equivalent measure of the monetary reward in the units of the low-quality type’s utility, and $I_{2L}(l)$ denotes the loss the low-quality type would make if he mimics the high-quality type in the first period. However, there is a little relaxation of the notations here because the two terms on the right hand side, $\hat{s}_{2H}(\theta)$ and $\delta I_{2L}(l)$ cannot add together directly due to risk aversion. For the moment, we use the current formulation but modify it later. Recall that in the previous section, the low-quality type always chooses the contract that he prefers in the short run because he always makes loss if he deviates. From the condition (49), if $\hat{s}_{2H} < -\delta I_{2L}(l)$, or $\hat{s}_{2H}$ sufficiently small, then the low-quality type has no incentive to deviate in the first period. In other words, only when the extra reward is sufficient large would the low-quality type deviates. Therefore, for the moment, we assume $\hat{s}_{2H} < -\delta I_{2L}(l)$. Thus, the incentive compatibility constraints (49) is equivalent to

\[
U_1(C_{1L} | \theta) \geq U_1(C_{1H} | \theta).
\]
After ignoring the participation constraint for the high-quality type and the incentive condition (50), the Lagrangian for this problem is:

\[
L = r_i[\bar{\theta} e_{1H} - w_{1H} + \delta W_z (1) - \delta s_{2H}] + (1 - r_i) [\bar{\theta} e_{1L} - w_{1L} + \delta W_z (0)] \\
+ \lambda [u(w_{1L}) - v(e_{1L}) / \bar{\theta}] + \mu_{s_2} [u(w(1H)) - v(e_{1H}) / \bar{\theta} + \delta s_{2H} (\bar{\theta}) - u(w_{1L}) + v(e_{1L} / \bar{\theta}) / \bar{\theta} - \delta s_{2H} (0)]
\]

Following the similar procedures in the previous section, the optimal contract can be solved as the following:

\[
e_{1H} \in \arg\{v'(e_{1H}) = \bar{\theta}^2 u'(w_{1H})\}
\]

\[
e_{1L} \in \arg\{(1 - r_i) \bar{\theta} [1 - v'(e_{1L}) / \bar{\theta} u'(w_{1L})] + r_i \bar{\theta}^2 v'(e_{1H} / \bar{\theta}) - \bar{\theta}^2 v'(e_{1L}) = 0\}
\]

\[
u(\bar{w}_{1L}) - v(e_{1L}) / \bar{\theta} = 0
\]

\[
u(w_{1H}) - v(e_{1H}) / \bar{\theta} = v(e_{1L}) / \bar{\theta} - v(e_{1L} / \bar{\theta}) / \bar{\theta} + \delta s_{2H} (0) - \delta s_{2H} (\bar{\theta})
\]

Note that the condition (53) implies that \(1 - r_i\) for \(r_i > 0\), thereby implying that the optimal effort choice of the grower type \(\bar{\theta}\) is less than that under perfect information.

Comparing the optimal contract \(C^R_1\) and the optimal contract \(\hat{C}_1\) in the previous section, the optimal contract \(C^R_1\) simply requires that the processor takes a portion of the high-quality grower type’s wage from the first period to the grower in the second period if high quality is actually observed. However, due to risk aversion, the reputation reward also affects the optimal choices of efforts in the first period, and hence the optimal contract.

If the growers are risk neutral, i.e., \(u(w) = w\), then \(\hat{s}_{2H} (\bar{\theta}) = s_{2H}\). Thus, from condition (52), the change in \(w_{1H}\) does not affect the optimal choice of effort for the high-quality grower type. Hence, from condition (53), the optimal choice of effort for the low-quality grower type stays constant. In summary, given reputation rewards under the assumption of risk neutrality,
the optimal contract $C^R_i$ only changes the payoff to the high-quality type without affecting the optimal contract for low-quality grower type and the processor’s two-period expected profit. Note that, to guarantee that this contract is indeed fully revealing, the reputation rewards must satisfy (47) and (49) or, in words, the reputation rewards must be sufficiently small such that the high-quality type participates in the first period and the low-quality type has no incentive to deviate given the reputation reward.

If growers are risk averse, given any positive reputation reward $s_{2H}$ for observed high quality, decreases in the optimal wage $w_{1H}$ requires that the optimal effort $e_{1H}$ increases from the condition (52). Hence, from the conditions (53) and (54), both the optimal effort $e_{1L}$ and the optimal wage $w_{1L}$ for the low-quality type increase. This effect is illustrated in Figure 3.8.

Note that the effect of increases in $e_{1H}$ on the optimal contract $C^R_{1L}$ is similar to that of decreases in $r_1$. Because the positive reputation reward reduces the optimal $w_{1H}$ and raises the corresponding optimal effort $e_{1H}$, the optimal contract for the low-quality type, $C^R_{1L}$, must move upward along the low-quality type’s zero-utility indifference curve as illustrated in Figure 3.8. The effects of the reputation rewards are summarized in the following proposition.

**Proposition 2:** There exists some reputation reward, $s_{2H}^*$, such that the separating equilibrium $C^R_i$ would dominate the contract $\hat{C}_1$.

Proof: As discussed above, the introduction of the reputation reward for high quality reduces the optimal wage $w_{1H}$ and raises the optimal effort $e_{1H}$ for the high-quality grower type. Hence, the processor can obtain more profit from the high-quality grower type in the short run (because the profit from the high-quality type is $Z_i(\bar{\theta}) = \bar{\theta} e_{1H} - w_{1H}$). Because the optimal contract $C^R_{1L}$ moves upward along the low-quality indifference curve $u(w_{1L}) - v(e_{1L})/\bar{\theta} = 0$, the
processor makes more profit from the low-quality grower as well. In addition, using Envelop theorem, a small change in \((w_{1L}, e_{1L})\) that keeps the low-quality type’s utility constant only has a second-order effect on the processor’s profit, while a small change in \((w_{1H}, e_{1H})\) has a first-order effect on the processor’s profit. Thus, using the optimal revealing contract \(\hat{C}_{1H}\) as a reference point, for a sufficiently small reputation reward \(s_{2H}\), the processor’s gain in the first period exceeds the reward paid to the high-quality grower type in the second period. Hence, introduction of the reputation reward brings positive gains to the processor in the two-period contract duration.

On the other hand, from the grower’s perspective, the high-quality grower type also prefers the contract \(C^R_1\) to \(\hat{C}_1\) for a sufficiently small \(s_{2H}\). Again, using \(\hat{C}_{1H}\) as a reference point, since \(\hat{w}_{1H} > \bar{w}\) (recall that the second-period separating equilibrium offers the high-quality grower type the perfect information contract \(C^*_H\)), for a sufficiently small reputation reward \(s_{2H}\),

\[
u(\hat{w}_{1H}) - \nu(\hat{w}_{1H} - s_{2H}) < \nu(\bar{w}^* + s_{2H}) - \nu(\bar{w}^*).\]

In words, the high-quality grower type would value the reward more in the second period than in the first period due to risk aversion. Thus, the high-quality grower type gains from the reputation reward, while the low-quality grower type is indifferent between the two contracts. Therefore, there exists some reputation rewards such that the separating equilibrium with direct reputation rewards contingent on observed performance, \(C^R_1\), dominates the separating contract in the absence of the reputation rewards, \(\hat{C}_1\).

Note that Proposition 2 applies only when both the separating equilibrium \(\hat{C}_1\) and \(C^R_1\) are feasible and these contracts are feasible only when the difference between \(\bar{\theta}\) and \(\theta\) are sufficiently large and \(\delta\) sufficiently small. In addition, if \(r_1\) is large, a cream-skimming separating equilibrium which only offers a contract to the high-quality grower type may become
dominant. A similar statement to Proposition 2 could be made for the cream-skimming separating equilibrium, which is omitted here.

Recall that the reputation reward must be sufficiently small such that the low-quality grower type has no incentive to deviate. If the reputation reward is large, not only would the high-quality grower type prefer to reveal his true type, but also the low-quality type would prefer to mimic the high-quality type. Thus, large reputation rewards would bring another set of equilibria. However, these potential cases are beyond the scope of this essay.

Effects of the direct reputation rewards would be more significant if the model is extended to a longer-term context. In addition, the longer the contract duration, the more both the grower and the processor would benefit from the direct reputation rewards.

Taking the fully separating equilibrium $\hat{C}_1$ as a reference point, recall that, to induce a separating equilibrium, the optimal payment to the high-quality type in the first period must include the information rent he would obtain in the second period if he would deviate in the first period. As the contract duration increases, the optimal payment to the high-quality type in the first period would become prohibitively large and the processor would be reluctant to pay the grower to have his true type revealed. In contrast, with the reputation rewards contingent on the grower’s past performance, the potential large information rents in the first period under the contract $\hat{C}_1$ could be broken down and be distributed into the remaining contract periods. More precisely, as the number of contract periods approaches infinity, there would exists a reputation reward to the high-quality type such that the optimal first-period dynamic contract $C_1^r$ would converge to the optimal static contract $C_1^F$ if the processor promises to pay the reputation reward every period in which good performance is observed. In other words, if the processor promises to pay the reputation reward whenever good performance is observed, the optimal incentive
scheme in the static contract could result in a fully separating equilibrium in the dynamic context when the number of contract periods is large. Following the similar arguments used for the two-period case, for a sufficiently small reputation reward to the high-quality type, both the processor and the grower would be better off with the direct reputation reward in the long run.

**Conclusion and Discussion**

This essay investigates the implications of growers’ reputation when a processor designs a two-period dynamic contract with asymmetric information. The optimal strategies of the processor and the grower form a perfect Bayesian equilibrium. Under full commitment by both parties, growers’ reputation has no effect on the optimal incentives. Hence, the optimal two-period contract mimics a sequence of optimal static contracts in the contract period. However, with no commitment by both parties, the optimal dynamic contract is rather complex. Since grower types are assumed unobservable to the processor, a potential ratchet effect would occur in a dynamic context that would induce the grower from hide his true type in the first period. In other words, the grower would tend to conceal his true type in the first period due to concerns that the processor would extract more of his surplus in the second period if his true type were revealed in the first. Thus, to induce the grower to reveal his true type, the optimal contract must specify a payment for the first period such that it consists of information rents the grower could obtain in both periods. Moreover, the reputation effects embodied in the processor’s posterior probability assessment about grower types reinforce the potential ratchet effect when the processor updates her beliefs of the grower’s type based on the grower’s past performance using Bayes’ rule. More precisely, if the high-quality type grower conceals his type or randomizes in the first period, the processor would believe that it is less likely that the grower is a high-quality type. Consequently, the high-quality type obtains a greater payoff in the second period from
deviating in the first period. In the limit, the processor believes the grower is a low-quality type and only offers a contract to the low-quality type under which the high-quality type realizes the maximum information rent.

Further, the optimal contract that could be sustained depends on growers’ time preferences and differences between the two grower types. Proposition 1 and its corollaries establish that a separating equilibrium could be sustained only if the discount factor is sufficiently small and the difference between the grower quality types is sufficiently large. In addition, a cream-skimming separating equilibrium would dominate the fully separating equilibrium when probability of high-quality growers is large. For a sufficiently large discount factor (i.e., when the grower is patient) and sufficiently small difference between the grower types, it would become too costly for the processor to have the growers’ private information revealed. Hence, a pooling equilibrium would dominate the separating equilibrium. Unfortunately, the exact nature of the relationship among the separating equilibrium, the pooling equilibrium, and the semi-separating equilibrium could be not explicitly determined without making further assumptions about functional forms of the grower’s utility function and disutility function.

Based on the optimal dynamic contract with no commitment, the processor offers a direct reputation reward to the grower in the second period if high quality is observed at the end of the first period. Proposition 2 demonstrates that both the processor and the grower can gain from the direct reputation reward. Thus, the optimal dynamic contract with the reputation reward would dominate that in the absence of reputation rewards. Moreover, effects of the reputation reward would become more significant in the longer-term dynamic contract.

The results presented in the essay are in general consistent with the existing literature in dynamic contracts. However, several major differences exist. Firstly, past studies have found
mixed results about existence of a separating equilibrium under different assumptions. For example, Hosios and Peters (1989) show that no fully separating equilibrium exists in a dynamic insurance contract with two types. Laffont and Tirole (1988) conclude similar results with continuous agent types. On the other hand, Freixas, Guesnerie, and Tirole (1985) derive optimal conditions for a separating equilibrium in a linear dynamic contract. In this essay, we not only derive optimal conditions for a separating equilibrium, a semi-separating equilibrium, and a pooling equilibrium, but also discuss the optimality of a cream-skimming separating equilibrium. Secondly, this essay introduces a direct reputation reward contingent on past performance that has never been analyzed in a dynamic principal-agent framework. The analysis presented in the text demonstrates that introduction of a direct reputation reward would provide more effective incentive schemes, and thus, result in a dominant dynamic contract relative to that without the reputation reward.

However, the analysis presented in this essay is far from exhaustive. Several straightforward generalizations of the model would be interesting for future research. First, the two-period model could be extended to allow for more than two periods. Second, uncertainties of realized quality or the production process could be incorporated into the model, though this treatment would significantly complicate the processor’s updating process. Third, more complicated structures of reputation accumulation could be used in the model.
References


Figure 1: The optimal contract with perfect information

Figure 2: Illustration of effects of $r_2$ on high-quality type’s information rents
Figure 3: A separating equilibrium

Figure 4: A handicapped separating equilibrium
Figure 5: An illustration of an infeasible separating equilibrium

Figure 6: A semi-separating equilibrium
Figure 7: Illustration of the effects of the reputation reward

\[ u(w) - v(e) / \theta = 0 \]

Legend:
- H1
- H2
- H3
- H4

Axes:
- w
- e

Points:
- \( \hat{\theta} \)
- \( \hat{\theta} \)
- \( \hat{\theta} \)
- \( \hat{\theta} \)

Lines:
- \( \hat{\theta} \)
- \( \hat{\theta} \)
- \( \hat{\theta} \)
- \( \hat{\theta} \)