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**Optimal livestock diet formulation  
with farm environmental compliance consequences**

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**Selected Paper prepared for presentation at the American Agricultural  
Economics Association Annual Meeting, Providence, Rhode Island,  
July 24-27, 2005**

Subject Code: farm management

Short Abstract: The current method to derive livestock diets is to optimize cost performance subject to animal performance and resulting nutritional requirements via a linear programming model. In contrast, we pose the livestock diet formulation problem as a multi-criteria decision model with the criteria being cost performance, feed efficiency, and environmental compliance costs. We find that there are many situations where farm financial situations are improved by feeding products with higher costs per unit of protein but lower phosphorus levels.

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## **Optimal livestock diet formulation with farm environmental compliance consequences**

Historically, livestock diet formulation was determined with linear programming minimizing cost subject to animal nutrient requirements. This approach allowed the farm manager to make livestock diet decisions based on the prices of feed products. Potential adverse effects of choosing feeds that resulted in high levels of phosphorus and nitrogen on farmland were ignored. Growing concerns about environmental pollution, worker health, and the welfare of animals from intensified livestock production systems require re-evaluation of livestock diet formulation to account for disposal costs of phosphorus and nitrogen on farm land. With increasingly rigorous enforcement of the Clean Water Act and 1990 Clean Air Act farmers are now facing environmental compliance costs and nutrient prevention costs. This paper reconsiders livestock diet formulation to determine the cost effective feeding methods explicitly using environmental compliance consequences unique to the farm explicitly considered.

The traditional method to derive livestock diets is to minimize cost subject to animal performance and nutritional requirements that the specified performance level dictates. For example, dairy cows are fed to minimize cost subject to achieving a level of milk production, which, in turn, dictates protein, energy, and vitamin and mineral levels. This method ignores the cost of over-feeding specific nutrients that may accompany the lowest cost protein and energy sources. However, excess nutrients must be disposed of with animal waste. For example, many common sources of protein fed to cattle are by-products such as corn gluten meal resulting from corn used to make corn syrup. These products are often high in phosphorus and nitrogen, which may result in increased nutrient disposal costs.

The Environmental Protection Agency (EPA) has authority to regulate emissions into water bodies since the 1972 Clean Water Act and air emissions since the 1990 U.S Clean Air Act. In the past much of the focus has been directed on other industries, but recently more attention has been focused on agriculture and specifically livestock operations. Primarily attention has been directed at concentrated animal feeding operations (CAFO), which historically have been defined as 1,000 or more animal units (e.g., 700+ dairy cows). Manure and wastewater from these operations can contribute to excess phosphorus and nitrogen levels in soil and water, therefore indicating the need for nutrient loading limit levels to prevent pollution. Most states have defined generally acceptable management practices to prevent further pollution. In many states these practices are related to a “Right to Farm” law protection making the standards quite important for farms to avoid nuisance lawsuits. The management practices in accordance with the Michigan Right to Farm guidelines emphasize the use of pounds of phosphorus per acre as a criterion for the quantity of manure to be applied (Satyal, 2001). The requirement guidelines are:

- Less than 150 lbs/acre of phosphorus, manure may be applied such that the total nitrogen level does not exceed the crop removal rates.
- From 150 to 299 lbs/acre of phosphorus, manure may be applied at phosphorus removal rates, in addition two years worth of manure may be applied
- More than 300 lbs/acre of phosphorus, no manure may be applied

Actual compliance costs pertaining to phosphorus removal are individual to the farm situation and depend on land availability, animal density, waste management methods, and feeding practices, among other factors. Thus, the cost of handling excess phosphorus is farm-

specific and where one farm might find the cost prohibitive another would not. However, it is clear that environmental compliance costs are significant on many farms and the feeding decision is a major source of nutrient import onto the farm.

Past literature has recognized that the current input cost minimization is limiting in its ability to optimize a dairy ration and minimize costs subject to nutritional and environmental requirements due to the non-linearity of the constraints. Many methods have been suggested to solve this problem: weighted goal programming, goal programming with penalty functions, multiple objective programming, compromise programming, and multi-goal programming (Romero and Rehman, 2003). In an applied dairy setting multi-goal programming was evaluated by Lara and Romero. They argued producers were more interested in the optimal ration which achieves a compromise amongst several objectives versus the least cost ration, therefore utility functions must be incorporated in the model to account for the individual farmers' preferences (Lara and Romero, 1992). The model is difficult to evaluate due to the difficulties of defining utility functions. Therefore it may be more appropriate for research purposes, rather than a farm specific guidance tool. Stokes and Tozer implemented a ration formulation using distance functions in a compromise goal setting. Subjectivity was introduced to this model with the use of ideal and anti-ideal values. In addition, Stokes and Tozer identified three main limitations with the model: (1) deterministic nature of the model, (2) lack of consideration for time dimension, (3) lack of information regarding the appropriate metric and weights to use in the compromise programming. Another possible solution to this problem is the implementation of separable programming, which is a non-linear technique used to find a global or local optimum to a large number of non-linear problems allowing a linear approximation to a curve. This is

applicable to the model due to the non-linearities in the cost curve and nutritional requirements (Miller, 1963).

There are several areas that the previous research can be improved upon. These include capturing the dynamic aspects of the decisions and making the problem operational on an individual farm. The true complexity of this diet formulation problem is modifying the diet formulation decision through a derivation of new non-linear constraints pertaining to phosphorus excretion levels, which is defined as a nutrient penalty function. This cost of phosphorus removal is a function of land availability, animal density, waste management methods, feeding practices, and the ability to export manure. This constraint is built using an external simulation model, which models different farm scenarios pertaining to different amounts of phosphorus fed. The results from the simulation generate the single variable functions needed to implement the environmental consequences into the traditional diet formulation problem.

The objective of this research is to operationalize a programming method to be used on a farm for decision making processes involving optimum diet ration formulation and minimization of nutrient loading. In contrast to the previous cost minimization methods using linear programming, we pose the livestock diet formulation problem as a multi-criteria decision model with the criteria being non-linear functions of cost performance, feed efficiency, and environmental compliance goals, which are approximated linearly using separable programming. Our model allows the farm decision-maker to assess trade-offs between higher costs for livestock diets and the resulting environmental compliance costs.

### **Linear Programming for cost minimizing feed decisions**

Linear programming (LP) is used to find solutions to a constrained optimization problem. The components of a LP problem include an objective function, decision variables, constraints, and parameters. The main assumptions of an LP program include linearity, divisibility, certainty, and non-negativity (Stevenson, 1989). Linear programming has been widely used in the area of optimizing cost performance subject to animal performance and the nutritional requirements that a specified performance level dictates. France and Thornley specified the following simplified LP program that achieves this objective:

$$\begin{aligned}
 \text{Minimize } Z &= \sum_{j=1}^n c_j x_j && \text{[objective function]} \\
 \text{Subject to: } &\sum_{j=1}^n e_j x_j \geq E_{req} && \text{[energy constraint]} \\
 &\sum_{j=1}^n p_j x_j \geq P_{req} && \text{[protein constraint]} \\
 &\sum_{j=1}^n x_j \leq F_{\max} && \text{[intake constraint]} \\
 &x_j \geq 0 && \text{[non-negativity conditions]}
 \end{aligned}$$

where  $x_j$  are the quantities of feed ingredients with cost  $c_j$ ;  $e_j$  represents the per unit energy content;  $p_j$  the per unit protein content;  $E_{req}$  the minimum energy requirement;  $P_{req}$  the minimum protein level; and  $F_{\max}$  the total feed intake constraint. This LP program produces the least cost combination of feed ingredients that meet the nutrient requirements for the specified performance standard.

According to Romero and Rehman linear programming in the area of diet rations depends on the following fundamental assumptions:

1. There is a single objective, which is a mathematical function of decision variables.
2. The decision variables are the amounts of the available ingredients that will constitute the diet.

3. The nutritional requirements are convertible to mathematical functions of the decision variables and form the constraint set of the problems.
4. The optimum diet is the one that minimizes the single specified objective without any violation of the constraints imposed (Romero and Rehman, 2003).

Unfortunately these assumptions are difficult to conform to in this optimization problem since the farmer is attempting to optimize the ration and minimize costs and nutrient loading subject to a set of conflicting constraints based on nutritional requirements, animal performance, and environmental compliance issues. These conflicting constraints are reality for farmers, and therefore must be incorporated in an optimization framework.

### **Deriving the P Balance and Penalty Function**

Balancing phosphorus is a simple concept where phosphorus imports are set equal to phosphorus exports resulting in a zero farm balance. This balance is highly dependent on an accurate account of phosphorus since there is a direct relationship between phosphorus levels in dairy cow rations and the amount of phosphorus excreted. As phosphorus does not volatilize into the atmosphere or leach to ground water, tracking changes in the soil is fairly straightforward:

$$\text{Ending phosphorus} = \text{Initial capacity} - \text{crop removal} + \text{added fertilizers}.$$

Using the traditional feeding problem solved with a linear program, a nutrient penalty function was introduced to quantify farm specific compliance consequences. Farm characteristics that influence this penalty function are animal density, the amount of phosphorus fed to the animals, cropping program, application of commercial fertilizers, land availability for manure application, current soil phosphorus levels and distance manure is hauled. Based on



these characteristics the nutrient penalty function is created using livestock data, crop program information, and manure handling costs that create the phosphorus capacity cost calculator.

The livestock data generates the amount of manure and the phosphorus content of that manure on the farm. In particular, the livestock data was divided into heifer and cow results due to the difference in the amount of phosphorus excreted by the different animals. The most accurate method of quantifying phosphorus excretion for cows is phosphorus intake - phosphorus in milk = phosphorus excreted (Myers, 2003). Phosphorus excretion for heifers is more difficult to quantify since they are utilizing P to facilitate growth. Therefore, gallons of manure per day excreted were used as an exogenous variable to determine the manure excretion value for the heifers.

The cropping program uses the number of farm acres and current soil phosphorus content to determine the availability of phosphorus to be applied on the soil and the amount of phosphorus that is utilized by the cropping program on that specific farm. This helps calculate the average phosphorus removal rate for the particular soil type and the average net phosphorus application rate.

Once the livestock data and cropping program information are determined they are used to derive manure handling and disposal cost. Within this section the acres of cropland for manure application are quantified by the capacity in the soil for phosphorus and the distance the manure is hauled to determine phosphorus disposal cost.

1. Allowable phosphorus application from manure

- $(\text{lbs/acre phosphorus} * \text{manure application rate}) \leq \text{Total phosphorus for soil level}$

2. Allowable gallons of manure applied

- Values from step 1 are divided by the total farm average of phosphorus to convert values into lbs of phosphorus per 1000 gallons of manure
3. Application cost by distance
- Values from step 2 are multiplied by its respective distance and costs to result in an average disposal cost per gram of phosphorus

The final results of the phosphorus capacity cost calculator give us a cost to dispose of phosphorus in the ration for the given amount of phosphorus fed to the animals. This calculator then determined the cost to dispose of phosphorus for a range of values of phosphorus fed to a particular herd. These values were added to the LP model using separable programming to act as buying phosphorus capacity for the nutrient penalty function in the LP program. This allows for a new LP to include the nutrient disposal constraint (penalty function) which is compared to the previous LP results without the constraint for the farmer to assess the tradeoffs.

### **Example Dairy Farm**

The initial LP model for the model dairy farm was developed to minimize the cost of a ration subject to nutritional requirements. The ration formulated for this model was based on a standard Holstein lactating cow with a weight of 640 kg, 120 days in milk (DIM), 3.0 body condition score (BCS), 3.5% fat corrected milk with production of approximately 32 kg/milk per day and 0.58 kg weight gain per day. The nutrient requirements utilized for the model were specified in the Nutrient Requirements for Dairy Cattle (NRC, 2001) with the additional support of the Spartan Ration (MSU). Dry matter intake (DMI) has an upper limit due to the limitations of the cow size and is reported in kg/day. Net energy for lactation (NEL) is reported in megacalories (Mcal) and specifies the amount of energy a dairy cow needs for maintenance and

lactation. All other requirements are reported in grams rather than percentages. Neutral detergent fiber (NDF) determines the fiber requirements for the dairy cow. Effective neutral detergent fiber (efNDF) evaluates the actual amount of fiber that is utilized by the cow from the different feeds. This percentage was provided by Spartan ration. Rumen undegraded protein (RUP) is the amount of protein that is not degraded in the rumen, which can also be referred to as bypass protein. Rumen degraded protein (RDP) is the amount of protein degraded in the rumen. Crude protein (CP) is the total amount of protein in the diet, therefore  $RDP + RUP = CP$ . To control the mineral balances in the ration past literature has suggest using calcium (Ca) to P ratios (Black and Hlubik, 1980). The Ca to P ratio was constrained between 1.8 and 3.5.

The nutrient and mineral content of the feeds were taken from the NRC 2001 and Spartan Ration (MSU). Feeds for the model were categorized into 5 areas: silage, hay, energy, minerals, and protein. The ingredients were limited to those typically available to producers in Michigan and surrounding areas. All feed prices were calculated as \$/kg as a percent of dry matter and were compiled from the April 2005 feed price list (Ishler, 2004).

The LP for minimization of costs determined the least cost ration as \$5.89/lb with the animal consuming 57.0904 lb/day. This is an acceptable diet and was used to determine the whole farm P balance. In the example farm 107 g of phosphorus was fed to a cow per day, which exceeds the requirement of 64.9 g by 42.1 g. The amount of phosphorus in milk is recorded at 95 g per 100L of milk and  $P_2O_5=42.3\%$  phosphorus. Using a representative farm of 160 cows producing 70 lbs/day of milk, 140 heifers, and 300 acres of land base a phosphorus balance was evaluated. Land base and heifers structure are described in Tables 1 and 2.

Based on the land available, the example farm had the capacity for the disposal of 12,438 lbs of P. The amount of P leaving the cow in milk was calculated for the 160 cows assuming 70

lbs/day of milk to get a value of 3108 lbs of P leaving in the milk for the herd per year.

Phosphorus intake was calculated for the herd at the 107 g/day for the year to get the amount of 13,763 lbs of phosphorus intake per year, indicating a total phosphorus excretion for dairy cows of 10,656 lbs. In addition the farm has 140 youngstock, which generate 2,434 lbs of phosphorus in their manure. The crop removal of phosphorus is 6,219 lbs of phosphorus and phosphorus supplied by commercial fertilizer was 752 lbs as indicated in Table 2. Therefore the farm has a net reserve of phosphorus -7,623 lbs, indicating this farm does not have the correct land capacity for the amount of phosphorus in the manure, and will therefore need to export manure off farm.

After the phosphorus balance was generated for the farm we were able to apply the nutrient penalty function to the new LP program to get the results demonstrated in table 3. Reviewing the results it is apparent that with the nutrient constraint the soybean meal 44% CP was pulled from the ration. The phosphorus, energy, and protein supplied by the soybean meal 44% CP was reallocated between an increase in alfalfa of 7.45 lbs, corn silage of 2.21 lbs, soybean meal expellers of 1.28 lbs, and distillers dried grains of 0.34 lbs. With this new allocation of feed, the ration is feeding approximately 2.2 lbs less of by-product feeds. If the herd has 100 cows, this is a savings in feed of 220 lbs per day, which is approximately 3.3 tons of by-product feeds per month. The increase in the cost of the feed ration from \$5.89 to \$7.34 is due to the increased amount of alfalfa, which has a lower phosphorus content than alternative sources of protein previously in the ration.

## **Conclusions**

Based on the results from the example dairy farm, with the inclusion of the nutrient penalty function the ration formulation reallocates the ingredients to accommodate lower levels of

phosphorus in the ration. In the short run it may increase the ration costs, but simultaneously decreases the amount of by-product in the ration which may lead to cost savings when the total cost of nutrients such as phosphorus is considered.

With the increasing availability of by-product feeds, producers must be aware of the total cost rather than the input cost of feedstuffs, therefore a tool to be used at an operational level is vital. The new LP program with the nutrient penalty function presents the farmer with a decision making tool that can be adjusted to farm specific situations, therefore allowing the prevention of phosphorus loading and future environmental compliance costs/fines. This research is the beginning stages of a much larger project that will include such factors as nitrogen levels and alternative nutrient management strategies.

**Table 1: Heifer structure**

<i>Heifer</i>	<i>Number in herd</i>	<i>P<sub>2</sub>O<sub>5</sub> (lbs/animal)</i>	<i>P<sub>2</sub>O<sub>5</sub> (lbs/herd)</i>
Age, 1-2 years	67	61	4099
Age, 6-12 months	35	30	1050
<u>Age, 0-6 months</u>	<u>38</u>	<u>16</u>	<u>605</u>
Total	140		5754
Pounds of P			2434

Note: Manure nutrient values from Jacobs.

**Table 2: Cropping Program Data**

<i>Crop</i>	<i>Unit</i>	<i>Yield</i>	<i>Acres</i>	<i>Removal P<sub>2</sub>O<sub>5</sub> (lbs)</i>	<i>Applied P<sub>2</sub>O<sub>5</sub> (lbs)</i>
Corn	Bu	120	82	3455	823
Corn Silage	Ton	20	74	5320	739
Hay	Ton	4	100	4020	0
<u>Other crop</u>	<u>44</u>	<u>--</u>	<u>43</u>	<u>1907</u>	<u>217</u>
Total			300	14702	1778
Pounds of P				6219	752

**Table 3: Comparison of LP programs**

<i>Ingredient</i>	<i>Pre-penalty LP amount as fed (lbs/d)</i>	<i>Post-penalty amount as fed (lbs/d)</i>	<i>P content (grams)</i>	<i>Cost/Unit</i>
Corn silage, normal	13.55	15.76	2.6	\$20/ton
Alfalfa	14.95	22.4	2.8	\$125/ton
Ground corn	15.12	7.64	3	\$2.4/bu
Soybean meal, expellers	0	1.28	6.6	\$236/ton
Soybean meal, 44% CP	3.82	0	7.4	\$225/ton
Distillers dried grains	10.48	10.82	8.3	\$130/ton
<b>Total Ration Cost</b>	<b>\$5.89</b>	<b>\$7.34</b>		

## References

- Black, J.R. and J. Hlbuik. "Basics of Computerized Linear Programs for Ration Formulation." *Journal of Dairy Science* 63(1980):1366-1378.
- Boland, M.A., P.V. Preckel, and K.A. Foster. "Economic Analysis of Phosphorus Reducing Technologies in Pork Production." *Journal of Agricultural and Resource Economics* 23(1998):468-482.
- Fleming, R., B. Babcock, and E. Wang. "Resource or Waste? The economics of swine manure storage and management." *Review of Agricultural Economics* 20(Spring/Summer 1998):96-113.
- France, J. and J.H.M Thornley. *Mathematical Models in Agriculture*. Butterworth, London. 1984.
- Ishler, V., 2005. Feed Price List, 29 April 2005. Available at: <http://www.das.psu.edu/teamdairy/>.
- Jacobs, L.W., 1997. "Resource Value of Dairy Manure." 12 Feb 1997. Available at: <http://www.canr.msu.edu/flcrp/manure2.htm>
- Lara, P. and Romero, C. "An interactive multigoal programming model for determining livestock rations: an application to dairy cows in Andalusia, Spain." *Journal of the Operational Research Society* 43(1992):945-953.
- Miller, C.E. The simplex method for local separable programming. In *Recent Advance in Mathematical Programming*. (ed. R.L. Graves and P. Wolfe), pp311-317. New York: McGraw-Hill. 1963.
- Myers, Z.H. "Estimation of inevitable phosphorus losses and predicting phosphorus excretion of dairy cattle fed a range of dietary phosphorus concentrations." M.S. Thesis. Dept. Animal Science. Michigan State University, 2003.
- National Research Council. 2001. Nutrient requirements of Dairy Cattle, 7<sup>th</sup> edition. National Academy of Science, Washington, D.C.
- Romero, C. and T. Rehman. *Multiple Criteria Analysis for Agricultural Decisions*. 2<sup>nd</sup> edition. Amsterdam: Elsevier Science B.V. 2003.
- Spartan Ration Evaluator/Balancer, 1992. Michigan State University.
- Saytal, V.H. "Environmental and economic impacts of manure management strategies for representative Michigan dairy farms." Plan B Research Paper for the Degree of M.S. Dept. Agric. Econ.. Michigan State University, 2001.
- Stevenson, W.J. *Introduction to Management Science*. Richard D. Irwin, Inc: USA. 1989.

Stokes, J.R. and P.R. Tozer. "Cost Minimization and Managing Soil Nutrient Loading: Conflict or Compromise?" *Canadian Journal of Agricultural Economics* 50(2002):151-169.