Beggar-Thy-Self Advertising: a Multi-market Model of Generic Promotion for Dairy Products

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We develop a multi-market equilibrium displacement model that allows for demand linkages (substitutes or complements) across downstream product markets, and supply linkages through the common use of a raw commodity as the key input. Applying the model to the dairy sector, we find that the effectiveness of producer-funded advertising, and thus optimal advertising intensities, depends on the demand relationships across dairy product markets (cross-price and cross-advertising elasticities), as well as the re-allocation of raw milk towards the advertised market. We argue that the previous literature, which ignores the horizontal linkages highlighted here, tends to overstate the effectiveness of generic commodity promotion for dairy, and thus results in too much advertising.

1. Introduction

Numerous studies have examined the effectiveness of producer-funded generic promotion for milk and for cheese (e.g., Blisard et al.; Kaiser 1997 and 1999; Kaiser and Chung; Liu and Forker; Schmit and Kaiser). The typical analysis estimates econometric models of fluid milk or cheese demand as a function of own prices, prices of related goods, demographic characteristics, and generic advertising expenditure. While empirical findings vary across studies and across products, promotion is typically found to generate positive and significant increases in demand, as well as large returns to producers’ investment.

However, the typical approach, which models the market for the advertised product in isolation, is incapable of capturing the effects of commodity promotion on horizontally related markets (Alston, Freebairn and James; Piggott, Piggott, and Wright; Kinnucan; Kinnucan and Miao). This omission is particularly crucial for analysis of dairy product promotion for two reasons. First, individual dairy products are linked on the supply side through their common
use of milk components as key inputs. Thus, an increase in demand for any given product will result not only in increased sales of raw milk, but also a reallocation of milk components across product markets. Second, dairy product markets are arguably related on the demand side, so that prices and advertising for one product affect demand for other products.

This paper develops an analytical, multi-market model of the dairy industry that captures these horizontal linkages across dairy product markets. We apply the model to trace the economic effects of generic commodity promotion on markets for manufactured dairy products, the implicit markets for milk components, and the market for raw milk. Comparative statics show that the effect of advertising on the price of raw milk depends on the horizontal supply and demand linkages across dairy product markets. Further, we derive an expression for the optimal advertising expenditures for alternative dairy products. A key result is that ignoring the horizontal relationships that link dairy product markets leads to an inflated estimate of the effectiveness of advertising, and thus too much advertising. This is due to a “Beggar-Thy-Self” effect of advertising—since some dairy products are substitutes, increased demand for the advertised product comes, in part, at the expense of reduced demand for other dairy products.

A key contribution of this paper is the extension of work by Alston, Freebairn, and James to link the markets for advertised products through supply, as well as demand. This concept is applicable to other industries where a single commodity is allocated to multiple downstream
markets. Examples may include the allocation of a farm commodity in alternative processed markets, processed vs. fresh markets, or foreign vs. domestic markets.

In the next section, we present the model of dairy markets, and analyze the comparative static results. In Section 3, we develop an optimal advertising rule, and illustrate the implications of ignoring the horizontal linkages across dairy product markets. Section 4 concludes and discusses future directions for research on this topic.

2. A Multi-market Model of the U.S. Diary Industry

2.1. A 2-component x 2-product model of the dairy industry

We develop a Muth model of the U.S. dairy industry for the purpose of demonstrating analytically the role of linkages between related markets for determining the effects of generic promotion (Muth; Gardner; Alston, Norton, and Pardey). To keep the exposition relatively simple, we specify a model with 2 components of raw milk (e.g., fat, solids-not-fat) used in the manufacture of 2 distinct dairy products (e.g., fluid milk and manufactured products) (see Alston et al. for a related, milk-components model of dairy markets). Extension of the model to the n-component-m-product model is straightforward.

The model is written algebraically as follows:

(1) \[ \text{Raw milk supply} \quad M = f(W_m) \]
(2) \[ \text{Milk fat supply} \quad a = AM \]
(3) \[ \text{Milk snf supply} \quad b = BM \]
(4) Fluid product production  \[ X_1 = g_1(a_1, b_1) \]

(5) Manufactured product production  \[ X_2 = g_2(a_2, b_2) \]

(6) Fluid product demand  \[ X_1 = D_1(P_1, P_2, Z_1, Z_2) \]

(7) Manufactured product demand  \[ X_2 = D_2(P_1, P_2, Z_1, Z_2) \]

(8) Milk fat pricing  \[ g_{1a}P_1 = W_a \]

(9) Milk fat pricing  \[ g_{2a}P_2 = W_a \]

(10) Milk snf pricing  \[ g_{1b}P_1 = W_b \]

(11) Milk snf pricing  \[ g_{2b}P_2 = W_b \]

(12) Milk pricing  \[ W_m = AW_a + BW_b \]

(13) Milk fat adding up condition  \[ a = a_1 + a_2 \]

(14) Milk snf adding up condition  \[ b = b_1 + b_2 \]

Equation (1) expresses the supply of raw milk, \( M \), as a function of the pool price of raw milk. Equations (2) and (3) express the (fixed proportions) production relation between milk components and raw milk, where \( A \) and \( B \) are the quantities of milk components per unit of milk. Equations (4) and (5) are the production functions for dairy products, \( X_i \), for which fat and snf are inputs, and equations (6) and (7) are dairy product demand. Demand for each dairy product is a function of prices for both products, \( P_1 \) and \( P_2 \), as well as advertising for both products, \( Z_1 \) and \( Z_2 \). Equations (8)-(11) express the competitive market equilibrium condition for milk components, that the price of each component is the equal to the value marginal product of that component in its alternative uses (subscript \( a \) or \( b \) on the production functions indicate the marginal products of \( a \) or \( b \)). In equation (12) the raw milk price is calculated as the value of
milk components. Equations (13) and (14) are the market clearing conditions that the supply equals demand for each milk component.

Totally differentiating equations (1)-(14), and converting to elasticity form yields a system of equations linear in percentage changes (we use the symbol $E$ to denote percentage change):

\begin{align}
(15) & \quad EM = \varepsilon EW_m \\
(16) & \quad Ea = EM \\
(17) & \quad Eb = EM \\
(18) & \quad EX_1 = k_{1a} Ea + k_{1b} Eb \\
(19) & \quad EX_2 = k_{2a} Ea + k_{2b} Eb \\
(20) & \quad EX_1 = \eta_{i1}EP_1 + \eta_{i2}EP_2 + a_{i1}EZ_1 + a_{i2}EZ_2 \\
(21) & \quad EX_2 = \eta_{21}EP_1 + \eta_{22}EP_2 + a_{21}EZ_1 + a_{22}EZ_2 \\
(22) & \quad EW_a = \frac{k_{1b}}{\sigma_1} Ea + \frac{k_{1b}}{\sigma_1} Eb + EP_1 \\
(23) & \quad EW_a = \frac{k_{2b}}{\sigma_2} Ea + \frac{k_{2b}}{\sigma_2} Eb + EP_2 \\
(24) & \quad EW_b = \frac{k_{ia}}{\sigma_1} Ea - \frac{k_{ia}}{\sigma_1} Eb + EP_1 \\
(25) & \quad EW_b = \frac{k_{2a}}{\sigma_2} Ea - \frac{k_{2a}}{\sigma_2} Eb + EP_2 \\
(26) & \quad EW_m = v_aEW_a + v_bEW_b \\
(27) & \quad Ea = s_{ai}Ea + (1-s_{ai})Ea_2 \\
(28) & \quad Eb = s_{bi}Eb + (1-s_{bi})Eb_2
\end{align}

In the system, $\varepsilon$ is the supply elasticity for raw milk; $k_{ia}$ and $k_{ib}$ ($i = 1, 2$) are the cost shares of $a$ and $b$ in total costs for product $i$; $\eta_{ij}$ is the elasticity of demand for product $i$ with respect to the price of product $j$; $a_{ij}$ is the elasticity of demand for product $i$ with respect to advertising.
expenditure for product \( j \); \( \sigma_1 \) and \( \sigma_2 \) are the Allen elasticities of substitution between fat and snf in the production of fluid and manufactured dairy products; \( v_a \) and \( v_b \) are the value shares of fat and snf in raw milk; and \( s_{a1} \) and \( s_{b1} \) are the shares of fat and snf, respectively, allocated to fluid milk products. Under the assumption that fluid products and manufactured dairy products are substitutes in consumption, which we maintain throughout this paper, the signs of cross-price elasticities of demand are positive, and the cross-advertising elasticities of demand are negative.\(^1,2\)

The model can be expressed in matrix form as follows:

\[
\begin{bmatrix}
1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\xi & 0 & 0 & 0 \\
-2 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
-1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & -K_{a1} & 0 & -K_{a2} & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & -K_{a1} & 0 & -K_{a2} & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & -\eta_{11} & -\eta_{12} & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & -\eta_{21} & -\eta_{22} & 0 & 0 \\
0 & 0 & 0 & K_{a1} & 0 & -K_{a2} & 0 & 0 & 0 & -1 & 0 & 1 & 0 \\
0 & 0 & 0 & K_{a1} & 0 & -K_{a2} & 0 & 0 & 0 & -1 & 0 & 1 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 1 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 1 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\
0 & 1 & 0 & -s_{a1} & -1 + s_{a1} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 1 & 0 & 0 & 0 & -s_{b1} & -1 + s_{b1} & 0 & 0 & 0 & 0 & 0
\end{bmatrix}
\begin{bmatrix}
EM \\
Ea \\
Eb \\
Ea_1 \\
Ea_2 \\
Eb_1 \\
Eb_2 \\
EX_1 \\
EX_2 \\
EW_m \\
EP_1 \\
EP_2 \\
EW_a \\
EW_{b1}
\end{bmatrix}
= \begin{bmatrix}
0 \\
0 \\
0 \\
a_{12}EZ_1 + a_{12}EZ_2 \\
\alpha_{21}EZ_1 + \alpha_{22}EZ_2 \\
0 \\
0 \\
0 \\
0 \\
0 \\
0 \\
0 \\
0
\end{bmatrix}
\]

\((29)\)

\(^1\) Huang finds that some dairy products are substitutes, while others are complements. The recent “Three-A-Day Dairy” advertising campaign seems to support the idea that some dairy products are substitutes.

\(^2\) With two products, Basmann’s adding up condition implies that the cross-advertising elasticities are negative. However, Basmann’s condition only holds for a weakly separable group of products. To the extent that advertising for a dairy product increases total expenditure on the group of all dairy products, the magnitudes and even signs of the cross-advertising elasticities will be affected.
or, more compactly as

$$R Y = D$$

where $R$ is a matrix of model parameters, $Y$ a column vector of endogenous variables, and $D$ a column vector of zeros, advertising intensities, and advertising elasticities of demand.

### 2.2. Comparative Statics

The model defines equilibrium proportional changes in dairy prices and quantities in response to exogenous changes in advertising expenditure (or intensity) for the two dairy products. Setting $EZ_2 = 0$ and dividing all equations by $EZ_1$, the system can be solved for the elasticities of dairy sector prices and quantities with respect to advertising expenditure for fluid milk products. Of particular interest is the effect of advertising on the price of raw milk. To simplify the analytical expressions of the elasticities, we make two assumptions that can easily be relaxed without affecting our main conclusions:

- **Assumption 1:** Fat and snf are the only inputs into the manufacture of dairy products, so that the sum of the cost shares of the components in each product equals one ($k_{ia} + k_{ib} = 1, i = 1, 2$).
- **Assumption 2:** the share of fat and snf used in fluid milk products are initially the same ($s_{a1} = s_{b1} = s_1$).

Under these assumptions, the elasticity of the raw milk price with respect to advertising for fluid milk is

$$\frac{EW_m}{EZ_1} = \frac{s_1 \alpha_{11} + s_2 \alpha_{21}}{\epsilon - s_1 (\eta_{11} + \eta_{12}) - s_2 (\eta_{21} + \eta_{22})},$$

where $s_2 = 1 - s_1$ is the share of fat and snf initially allocated for use in manufactured dairy products.
products.

Similarly, setting $EZ_I = 0$ and dividing all equations by $EZ_2$ yields the effect of advertising for manufactured dairy products on dairy sector prices and quantities. The elasticity of the raw milk price with respect to advertising for manufactured dairy products is

$$\frac{EW_m}{EZ_2} = \frac{s_1a_{12} + s_2a_{22}}{\varepsilon - s_1(\eta_{11} + \eta_{12}) - s_2(\eta_{21} + \eta_{22})}.$$  

These elasticities take into account the direct effects of advertising on the demand for the advertised and non-advertised products (own- and cross-advertising effects on demand), and the subsequent shift in the derived demand for raw milk. In addition, these expressions take into account the reallocation of milk components towards the advertised product, and the equilibrium adjustments in dairy product prices.

It follows from equation (15) that the elasticities of the quantity of raw milk sold with respect to advertising for fluid and manufactured dairy products are:

$$\frac{EM}{EZ_1} = \frac{(s_1a_{11} + s_2a_{21})\varepsilon}{\varepsilon - s_1(\eta_{11} + \eta_{12}) - s_2(\eta_{21} + \eta_{22})}, \text{and}$$

$$\frac{EM}{EZ_2} = \frac{(s_1a_{12} + s_2a_{22})\varepsilon}{\varepsilon - s_1(\eta_{11} + \eta_{12}) - s_2(\eta_{21} + \eta_{22})}.$$  

Further, the elasticities of raw milk revenue with respect to advertising for fluid and manufactured dairy products are:

$$\frac{ER_m}{EZ_1} = \frac{(s_1a_{11} + s_2a_{21})(1 + \varepsilon)}{\varepsilon - s_1(\eta_{11} + \eta_{12}) - s_2(\eta_{21} + \eta_{22})}, \text{and}$$

$$\frac{ER_m}{EZ_2} = \frac{(s_1a_{12} + s_2a_{22})(1 + \varepsilon)}{\varepsilon - s_1(\eta_{11} + \eta_{12}) - s_2(\eta_{21} + \eta_{22})}.$$
where $R_m = MW_m$.

Because dairy product markets are linked through the common use of milk components, the effect of advertising on the milk price is dependent on related markets even under the restrictive assumptions that there are no cross-price or cross-advertising effects on demand. To illustrate, we set $\eta_{12} = \eta_{12} = 0$ and $\alpha_{12} = \alpha_{21} = 0$ in equations (31) and (32). The resulting elasticities are

$$
\frac{EW_m}{EZ_1} \bigg|_{\eta_{12} = \eta_{21} = 0, \alpha_{12} = \alpha_{21} = 0} = \frac{s_1 \alpha_{11}}{e - s_1 \eta_{11} - s_2 \eta_{22}}
$$

(37)

$$
\frac{EW_m}{EZ_2} \bigg|_{\eta_{12} = \eta_{21} = 0, \alpha_{12} = \alpha_{21} = 0} = \frac{s_2 \alpha_{22}}{e - s_1 \eta_{11} - s_2 \eta_{22}}
$$

(38)

In equation (37), the third term in the denominator, $-s_2 \eta_{22} > 0$ reduces the effect of fluid milk advertising on the price of raw milk. As milk is re-allocated towards fluid milk products in response to increased demand for fluid milk, less milk is sold to the manufacturing market, thereby decreasing the milk revenue generated on the manufacturing market and decreasing the average price of milk. The larger the share of milk initially allocated to manufacturing uses (the larger $s_2$), or the more elastic is demand (the larger is $\eta_{22}$ in absolute value), the less effective is fluid milk advertising at raising the price of raw milk. Similarly, the second term in the denominator of equation (38), $-s_1 \eta_{11} > 0$, reduces the effectiveness of manufactured product advertising in raising the price of milk.
3. Optimal Advertising with Lump-sum Funding

The comparative statics can be used to develop a rule for allocating dairy advertising expenditure. Following Alston, Freebairn, and James, we define optimal advertising expenditure for each dairy product as that which maximizes producer surplus:

\[ PS = TR - TVC - Z \]
\[ = W_m(Z_1, Z_2)M^d(Z_1, Z_2) - TVC(M'(Z_1, Z_2)) - Z_1 - Z_2 \]

where \( PS \) is the net producer surplus for dairy farmers, \( TR \) is the total milk revenue, and \( TVC \) is the total variable cost of producing milk. The first order condition for optimal advertising intensity under lump-sum funding is

\[ \frac{\partial W_m}{\partial Z_i} M = 1, \quad i = 1, 2. \]

This first order condition can be restated in proportional change form:

\[ \frac{E W_m}{E Z_i} M = \frac{Z_i}{W_m M}, \quad i = 1, 2 \]

or

\[ Z_i = \frac{E W_m}{E Z_i} M, \quad i = 1, 2. \]

Substituting equations (31) and (32) into (42) yields the optimal advertising expenditures for fluid and manufactured milk products:

\[ Z_1^* = W_m M \left[ \frac{s_1 \alpha_{11} + s_2 \alpha_{21}}{e - s_1 (\eta_{11} + \eta_{12}) - s_2 (\eta_{21} + \eta_{22})} \right] \]
\[ Z_2^* = W_m M \left[ \frac{s_1 \alpha_{12} + s_2 \alpha_{22}}{e - s_1 (\eta_{11} + \eta_{12}) - s_2 (\eta_{21} + \eta_{22})} \right] \]
In order to illustrate the importance of the multi-market equilibrium effects of dairy product promotion, we also calculate what we term naïve advertising expenditures, which ignore the cross-market effects. Setting the cross-price and cross-advertising elasticities of demand equal to zero ($\eta_{12} = \eta_{12} = 0$ and $\alpha_{12} = \alpha_{21} = 0$), and setting the own-price demand elasticity for the non-advertised product equal to zero ($\eta_{22} = 0$ in equation (43), and $\eta_{11} = 0$ in equation (44)), the naïve advertising expenditures are

\[
Z_1^n = W_m M \left[ \frac{s_1 \alpha_{11}}{e - s_1 \eta_{11}} \right], \quad \text{and}
\]

\[
Z_2^n = W_m M \left[ \frac{s_2 \alpha_{22}}{e - s_2 \eta_{22}} \right].
\]

The naïve advertising expenditures ignore that dairy product markets are linked through demand and supply.

Subtracting the optimal advertising expenditures from the naïve advertising expenditures yields the error that dairy farmers would make by choosing advertising intensities without regard for the multi-market equilibrium effects. These are

\[
Z_1^n - Z_1^* = W_m M \left[ \frac{s_1 \alpha_{11}}{e - s_1 \eta_{11}} - \frac{s_1 \alpha_{11} + s_2 \alpha_{21}}{e - s_1 (\eta_{11} + \eta_{12}) - s_2 (\eta_{21} + \eta_{22})} \right], \quad \text{and}
\]

\[
Z_2^n - Z_2^* = W_m M \left[ \frac{s_2 \alpha_{22}}{e - s_2 \eta_{22}} - \frac{s_1 \alpha_{12} + s_2 \alpha_{22}}{e - s_1 (\eta_{11} + \eta_{12}) - s_2 (\eta_{21} + \eta_{22})} \right].
\]

Under reasonable conditions ($|\eta_{12}| < |\eta_{11}|$ and $|\eta_{21}| < |\eta_{22}|$), equations (47) and (48) are
unambiguously positive. That is, by ignoring the equilibrium multi-market effects of advertising, milk producers would tend to overspend on advertising. The supply and demand linkages between dairy product markets tend to reduce the effectiveness of advertising as a tool to increase milk prices. On the demand side, under the (reasonable) assumption that dairy products are substitutes, increased demand for one product comes at the expense of decreased demand for the other. This is the “Beggar-Thy-Neighbor” effect of advertising that Alston, Freebairn, and James analyzed in the context of meat demand. However, in the present context, where milk producers have an interest in both dairy product markets, advertising in one market has a “Beggar-Thy-Self” effect. On the supply side, increased derived demand for milk is satisfied, in part, by a re-allocation of milk components from the non-advertised products. Thus, the increase in the quantity of milk sold is smaller than the shift in demand for milk derived from the shift in demand for the advertised product.

4. Conclusion and Directions for Further Research

This paper develops a model of the dairy industry that incorporates the horizontal supply and demand relationships across dairy product markets, and links the product markets vertically to the implicit markets for milk components and the market for raw milk. We use the model to derive analytical expressions for the effectiveness of dairy product advertising, and develop an advertising rule that maximizes milk producer surplus.
A key result is that producers would overspend on advertising if they ignore the horizontal linkages across dairy product markets. This is particularly important given that the extant literature typically considers only the direct (partial equilibrium) impact of advertising on the advertised market. The econometric analyses that measure the shift in fluid milk and cheese demand as a result of fluid milk and cheese advertising tend to omit the cross-price and cross-advertising effects in the estimated demand equations (Blisard et al.; Kaiser 1997 and 1999; Kaiser and Chung; Liu and Forker; Schmit and Kaiser). Further, both the academic literature and dairy industry reports seem to equate increased demand for the advertised product into increased sales of the equivalent quantity of raw milk without recognizing the re-allocation of raw milk components, equilibrium adjustments in markets for non-advertised dairy products, and the implications for the price of raw milk.

On-going work is extending the model and results presented here in several directions. Extension of the model to multiple products, while presenting problems for exposition, will allow for a more detailed picture of the interaction between product markets. Further, we are revisiting the optimal advertising expenditure rule under per unit and ad valorem funding mechanisms (the current dairy check-off operates under a per unit tax). Also interesting is the implication of non-competitive behavior along dairy market channels (Zhang and Sexton). Next, numerical simulation of the model will allow for a quantitative evaluation of the

Beggar-Thy-Self Advertising
importance of the horizontal linkages across product markets for accurate measurement of the effects of advertising. Moreover, further econometric work is needed to estimate the cross-price and cross-advertising demand relationships across dairy product markets.

Finally, while this paper explores an application to dairy markets, the economic concepts and implications are much more general. Our model extends work by Alston, Freebairn, and James by linking markets for advertised products through supply as well as demand. Other potential applications include commodities that are allocated to multiple downstream markets.

References


