Economies of Size and Total Factor Productivity in Alberta Cow-calf Production

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Abstract

This paper applies both parametric and non-parametric approaches to evaluate Economies of Size and Total Factor Productivity (TFP) in Alberta cow-calf production based on the unbalanced panel data from 196 farmers during the time period from 1995 to 2002. Under the parametric approach, a random parameter translog cost system and a translog cost frontier are constructed and estimated respectively. The results from the cost system estimation suggest that on average, cow-calf production in Alberta exhibits Economies of Size, technical progress and positive TFP. However, exploitable Economies of Size decrease over time, technical change rate and TFP even become negative at 2002. The critical problem, therefore, is how to reverse the trend and maintain good growth pattern. The translog cost frontier is also estimated but the results are unreliable. Therefore, the Non-parametric approach (Malmquist TFP index) is adopted. The results suggest that inefficiency exists in Alberta cow-calf production.

1. Background

Alberta is the largest beef-producing province in Canada. The province leads the nation in cattle and calf inventories, accounting for 5.2 million head as of January 1,

2003. In the 2001 Census of Agriculture, 31,774 Alberta farms reported live cattle. The cattle industry is a significant contributor to Alberta's farm economy, accounting for nearly 77 per cent of cash receipts from livestock and livestock products sales in 2002. Roughly 61 per cent of Alberta's $8.3 billion in total farm cash receipts in 2002 came from livestock and livestock product sales. In 2002, Alberta was responsible for slightly over 66 per cent of the $5.9 billion in sales generated by western Canada in cattle and calf cash receipts and more than one-half of the national total of $7.6 billion (Government of Alberta, 2003).

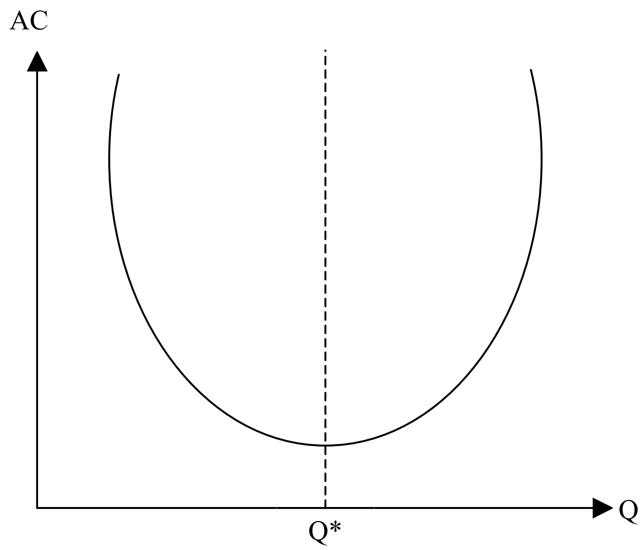
In recent years, the beef-cow industry has developed and expanded. The average herd size of farmers has increased gradually. However, the effect of increased output on the average cost of farmers is not clear. If in the long run, the average cost is decreasing with increased output, economies of size exist, under which farmers can generate more profit by expanding herd size. If the average cost is rising with the increased scale, it is diseconomies of size, under which farmers will lose if they expand herd size. Therefore, it is very import to evaluate the economy of scale in cow-calf production of Alberta in terms of providing useful information to producer organizations and policy makers.

2. Overview of previous research

2.1 The economic theory of economies of size

Economies of size (ES) is an important conception in economic theory. In the long run, when the average cost of production decreases with increased output, economies of size exist. At this stage, the farm or company can generate more profits by raising its output. Diseconomies of size (DES) is the opposite situation. At this stage, one more unit output becomes more costly and the profit of farm or company decreases. The long run average cost function can be shown at the following graph.

Figure 1 Economies of Size



In the long run, an optimal output Q* exists where the average cost reaches a minimum. To the left hand side of Q*, ES exists because the average cost decreases with output increases. To the right hand side of Q*, DES exists because the average cost increases with increased output.

ES may come from different sources, either internal or external. Within a farm or company, ES may result from (Young, 2003):

- Technical economies achieved in the actual production of the good. For example, large farms or companies can use expensive machines intensively.
- Managerial economies achieved in the administration of a large farm or

company by splitting up management jobs and employing specialists.

- Financial economies achieved by borrowing money at lower rates of interest than smaller farms or companies.
- Marketing economies achieved by spreading the high cost of advertising on television and in national newspapers, across a large level of output.
- Commercial economies achieved when buying supplies in bulk and therefore gaining a larger discount.
- Research and development economies achieved when developing new and better products.

Outside the farm or company, ES may occur from:

- Availability of a local skilled labor force is available.
- Specialized local firms that can supply parts or services.
- A good transport network..
- An area having an excellent reputation for producing a particular good.

To identify ES or DES, we need to know how many units of cost are added for one more unit output. However, the change of cost cannot fully attribute to ES or DES. It is also affected by technical change and efficiency change. Therefore, it's better to address ES, technical change and efficiency change simultaneously. This can be realized by evaluating Total Factor Productivity.

2.2 The economic theory of total factor productivity

Productivity is an important topic for economic development. It has different definitions for different people. A broad definition is that productivity measures the relationship between the quantity of goods and services produced during a period of time and the input of labor, capital, and natural resources used in the production process. Simply, it is a quantitative relationship between output and input (Iyaniwura and Osoba, 1983, Antle and Capalbo, 1988). This definition is prevalent because it isn't limited in a certain area or certain type of economy. There are two dimensions for productivity measurement. One is to relate the output with one type of input such as labor, capital and energy. The other relates the output with a combination of inputs, extending to a weighted aggregate of all associated inputs. The first one is called partial productivity and the latter total productivity. Partial productivity measures the joint effect (including the substitution effect of one factor for another) of a number of interrelated influences on the use of factor in production. It cannot reflect the effect of all factors changing on the productivity movement. Therefore, total productivity is more important when examining total effect of input factors, including material input, technology and institution transformation etc. The measurement of total factor productivity (TFP) shows that the technical efficiency with which all inputs are utilized in a production function. Whereas the partial productivity index measures the value of output per unit of input, the TFP index sums the partial productivities of all inputs in the production process. In this paper, we will focus on TFP analysis. Antle and Capalbo (l988) identified two major approaches to total factor productivity measurement; these are:

- The growth accounting (index number) approach.
- The econometric approach.

Growth accounting is a method for estimating the contribution of different factors to economic growth. Based on marginal productivity theory, growth accounting decomposes the growth of output into growth of labor, land, capital, education, technical knowledge and other sources. The residual growth in output not accounted for by the growth in factor inputs is associated with productivity growth. It is necessary in the growth accounting approach to obtain detailed data for inputs and outputs and use certain aggregate method to formulate the input and output index. By the input and output index, we can calculate a TFP index. Five kinds of indexes are usually used: Laspeyres exact index, geometric exact index, Tornqvist – Theil index that approximates the Divisia index, Fisher's Ideal index and Malmquvist index. The Fisher's Ideal index is the geometric mean of the Laspeyres and Paasche indexes, which implies the quadratic function form. The Tornqvist index is a discrete approximation to the more general Divisia index, implying a homogenous translog production function. The Malmquvist index is a more general productivity index. It is based on the distance function approach and therefore, can describe very general technology. In 1992 Färe et al. first provided the foundation to empirically estimate the Malmquist productivity index. Since then, this index has enjoyed an increased popularity. It has several advantages. First, it can be constructed from quantity data only; second, the index requires less restrictive assumptions than other traditional index numbers; Third, we does not need econometric estimation for its construction.

The distance function is first suggested by Malmquist. Shephard (1970) defined the distance function by the production function. As what is developed by Fare and Grosskopf (1992), the construction of distance function is as follows:

At period t, t=0,1, the production technology S^t models the transformation of inputs $x^t \in R_+^N$ into outputs $y^t \in R_+^N$, and $S^t = \{(x^t, y^t) : x^t can \quad produce \quad y^t\}$. The technology may also be modeled by the input correspondence $L^t(y^t) = \{x^t : (x^t y^t) \in S^t\}$ or the output correspondence $P^t(x^t) = \{y^t : (x^t y^t) \in S^t\}$

Here we assume that $L^t(y^t)$ is a closed convex set for all y^t, and that $0 \notin L^t(y^t)$. if $y^t \geq 0$, $y^t \neq 0$, and $L^t(o^t) = R_+^N$ .In addition, inputs are assumed to be strongly disposable, i.e. $L^t(y^t) = L^t(y^t) + R_+^N$

The input distance function is defined as:

$$D_i^t(y^t, x^t) = \sup(\lambda > 0 : (x^t / \lambda) \in L^t(y^t))$$

When we use (X^0, Y^0) and (X^1, Y^1) to express the input vectors at terms of 0 and 1, we can formulate the distance functions $D_i^0(y^0, x^0)$, $D_i^1(y^0, x^0)$, $D_i^0(y^1, x^1)$ and $D_i^1(y^1, x^1)$. By Caves et al.(1982) the input Malmquist productivity index is defined as:

$$M_i^t(y^1, x^1, y^0, x^0) = [\frac{D_i^0(y^1, x^1)}{D_i^0(y^0, x^0)} \frac{D_i^1(y^1, x^1)}{D_i^1(y^0, x^0)}]^{\frac{1}{2}}$$

This definition is the geometric mean of two distance functions with reference of technologies in terms 0 and 1 respectively (similar to the construction of Fisher Ideal Index).

Nishimizu and Page (1982) divided the TFP into two different parts, namely, technology progress and improvement of technical efficiency. Färe et.al.(1994) proved that the Malmquvist production index can also be divided into technology and technical efficiency changes. Further, they divided the technical efficiency into changes of pure technical efficiency and scale efficiency.

So the Mlamquist Productivity Index can be transformed into:

$$M_i^t(y^1,x^1,y^0,x^0)=\frac{D_i^1(y^1,x^1)}{D_i^0(y^0,x^0)}\left[\frac{D_i^0(y^1,x^1)}{D_i^1(y^1,x^1)}\frac{D_i^0(y^0,x^0)}{D_i^1(y^0,x^0)}\right]^{\frac{1}{2}}=Ech\bullet Tch$$

Where Ech is the efficiency change and Tch is the technological progress. When constant return of scale is assumed, we can divide Ech into the changes of pure technical efficiency and scale efficiency. The distance function approach can be realized by Data Envelopment Analysis Program (non-parametric method) software developed by Coelli (1996). While having the definite advantage of not requiring the specification of a particular parametric model, the distance function approach precludes hypothesis testing regarding certain features of the technology (e.g., returns to scale). To remedy this situation, Atkinson and Cornwell (1998) proposed an alternative econometric cost frontier framework to decompose productivity change into technical change and change in firm efficiency relative to the frontier. The Malmquvist index is a more general productivity index. Diewert (1976) and Caves et al.(1982) have demonstrated that the torquist index can be derived from Malmquist indexes. Färe and Grosskoff (1992) also show that Fisher ideal index can be derived from Malmquist indexes.

The econometric approach to productivity measurement is to estimate the specified production function or the dual (cost or profit) function so that productivity growth can be calculated by the parameters of the functions. Comparing to the non-parametric method, this approach can generate and test the parameter estimates of the production technology in the process of measuring productivity advancement. Generally, people use production, cost or profit functions to estimate TFP. Two types of functions, namely, average function and frontier function are used. In average function form, the growth can be separated into two parts: growth from input increase and growth from technical change. For a frontier cost function, however, the growth can be separated further into: growth from input change, growth from technical change and growth from efficiency change. Inefficiency is included in the frontier model. A generally used frontier function is stochastic frontier function. Take frontier production function as an example:

$$Y=XB+v-u=XB+W$$

Where Y is the N*1 vector to represent the N outputs. X is the N*K vector to represent the N observed values of K inputs. B is K*1 parameter vector. W is the total errors. V expresses the random variable due to the statistical or other random factors (overlooked in production). It can be positive, negative or zero. U is N*1 vector to express efficiency variable($u \geq 0$). Ui is the difference between the optimal output and actual output of unit i.

2.3 The research on economies of size (scale) in cow-calf production

McCoy and Olson (1970) reported that in the early 1960s, the net increase in cattle feeding in major producing states of the U.S occurred in herds of more than 1000 head. Heady and Gibbons (1968) compared the effects of different cattle feeding methods and systems on cost per steer fed, profit maximization and stability of returns. They reported when the cost of labor was considered, large cattle feeding enterprises might gain cost advantages by adapting more highly mechanized systems rather than intensive labor systems. The cost of farm labor rises as off-farm employment opportunities expend, large and more specialized cattle feeding operations can be expected.

Michanel R. Langemeier (1994) analyzed the effect of scale on the cost in beef-cow production. The results suggest that the higher scale, the less cost involved. However, because the limitation of the sample, he cannot tell us what happens if the further bigger farms are included in the study. John D. Lawrence et.al.(1999)sets up a cost function in which annual cost per cow is dependent variable determined by the amount of harvested forage fed, number of pasture days, operating cost, fixed costs, hours of labor, herd size, percent calf crop, and weaning weight. He finds negative coefficient of herd size, which means the annual cost per cow will decrease with the herd expansion. But there is no explanation if the relationship between herd size and cost is linear or curvature. So he does not explain if optimal herd size exists by his data.

Ian McNinch (2000) does a study for the cow-calf production in Saskatchewan. By plotting the relationship between average cost and cow wintered, he finds the downward slope of the average cost, which means the cost is decreased with the increase of output. However, by regression, he finds the optimal size of cow production does exist. Sara D. Short (2001) analyzed the variable cost in different regions in the U.S. He also finds that the larger acreage size of operations in the West and Southern Plains of the U.S can support more cows and take advantage of economies of scale because spreading the fixed investment over more units of production.

Though research on economies of size (scale) in beef-cow production is sound, some problems still exist. One of them is that the parameters in cost system always assume exogenous not endogenous. However, in the real world, especially when farmers observed are varied at different periods, the parameters may depend on different farmers in sample. To incorporate parameter endogeneity in the cost system estimation is one of the objectives in this paper.

3. Data description and model construction

3.1. Data description

3.1.1 Frequency of Observations

The data are for cow-calf farmer inputs and outputs from Alberta Agriculture. The data are unbalanced with T=8 years and N=$\sum_{k=1}^{K} N_k$ =196 farmers. The total number of observation is $n = \sum_{k=1}^{K} N_k * k$ =333. The observation frequencies are as follows:

Table1 Frequency Analysis

K Observation frequency (years)	N_k number of farmers	percentage
1	118	60%
2	47	24%
3	13	7%
4	10	5%
5	6	3%
6	2	1%
Total (N)	196	

The number of farmers with only single observations accounts for 60% of the total number of farmers. The number of most observed years is 6. This means that the data may not be adequate in explaining time varying effects.

3.1.2 Deflation of cost by price indices

To decrease the measurement error from the data and get the real value of all the variables, farm input price indices are applied. The price indices used for deflation are western Canada animal production input index, feed price index, hired labor price index, legume and grass input price index and building and fencing price index. These price indices come from Statistics Canada CANSIM II TABLE 3280014 and CANSIM II TABLE 3280001. The total variable cost is deflated by the western Canada animal production input index, the feed cost is deflated by feed input price index, the hired labor cost is deflated by hired labor input price index, the pasture cost is deflated by the legume and grass input price index, other variable cost is calculated by the deflated total variable cost minus deflated feed cost, hired labor cost, pasture cost. Because the building and fencing price changed little during the observed period, it had almost no effect on the capital costs, which are mainly attributable to building depreciation. As a result, the capital cost (fixed cost) is not deflated.

Table 2 Price Indices

Price indices	West Canada animal production		Feed		Hired labor		Legume and grass		Building and fencing	
Time	Annual	Change	Annual	Change	Annual	Change	Annual	Change	Annual	Change
1995	110.60		122.32		105.89		149.26		114.17	
1996	112.22	0.01	148.13	0.21	110.10	0.04	154.89	0.04	116.51	0.02
1997	122.90	0.10	140.69	-0.05	115.44	0.05	194.98	0.26	120.37	0.03
1998	117.66	-0.04	124.12	-0.12	115.91	0.00	270.46	0.39	116.93	-0.03
1999	120.30	0.02	112.90	-0.09	111.00	-0.04	305.10	0.13	123.20	0.05
2000	133.60	0.11	110.30	-0.02	117.10	0.05	296.90	-0.03	118.60	-0.04
2001	141.00	0.06	123.40	0.12	123.80	0.06	281.80	-0.05	118.50	0.00
2002	136.60	-0.03	144.40	0.17	126.00	0.02	278.50	-0.01	121.60	0.03
Aver.		0.03		0.03		0.03		0.10		0.01

Source: Statistics Canada CANSIM II TABLE 3280014 and CANSIM II TABLE 3280001

3.1.3 Average cost in sample

The average costs are calculated from the original dataset. The average feed and bedding cost is an aggregation of wintered feeding and bedding cost divided by weaned calf pounds; The average pasture cost is pasture expenditure divided by weaned calf weight; The average labor cost is an aggregation of paid labor and unpaid labor cost divided by weaned calf pounds; Average other cost is an aggregation of other variable costs and capital costs divided by weaned calf weight.

We get the average cost **per pound weaned calf** before deflating as following:

Table 3 Average Cost per pound Weaned Calf before Deflated

Year	Average feeding and bedding cost	Average pasture cost	Average labor cost	Average other cost	Average production cost
1995	0.57	0.26	0.28	0.58	1.69
1996	0.74	0.26	0.41	0.57	1.98
1997	0.65	0.25	0.29	0.56	1.76
1998	0.61	0.24	0.26	0.51	1.62
1999	0.39	0.22	0.17	0.39	1.17
2000	0.41	0.30	0.17	0.33	1.21
2001	0.52	0.30	0.13	0.33	1.28
2002	0.65	0.38	0.15	0.40	1.58

The average cost **per pound weaned calf** after deflating is:

Table 4 Average Cost per pound Weaned Calf after Deflated

year	Average feeding and bedding cost	Average pasture cost	Average hired labor cost	Average other cost	Average production cost
1995	0.46	0.18	0.26	0.64	1.55
1996	0.50	0.17	0.38	0.75	1.79
1997	0.46	0.13	0.25	0.64	1.48
1998	0.49	0.09	0.22	0.61	1.41
1999	0.34	0.07	0.15	0.43	1.00
2000	0.37	0.10	0.15	0.32	0.94
2001	0.43	0.10	0.10	0.31	0.94
2002	0.45	0.14	0.12	0.49	1.20

To illustrate the cost component changes over time more clearly, the following two graphs (figure2 and figure 3) are provided:

Figure 2 Average Cost per Pound Weaned Calf before Deflation

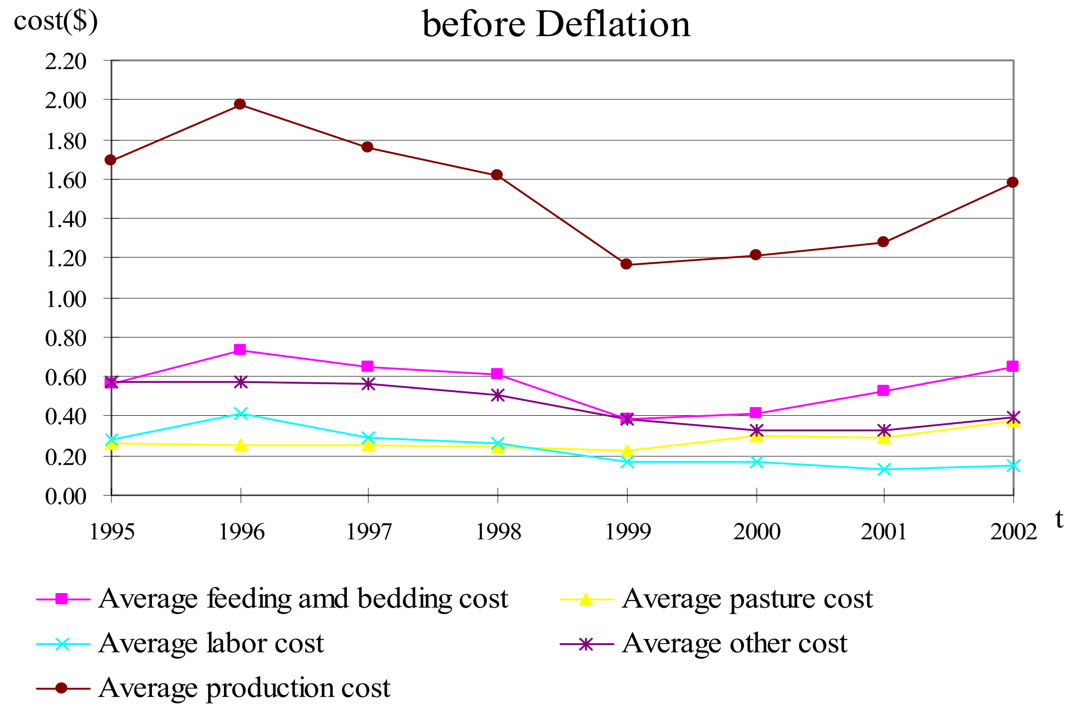
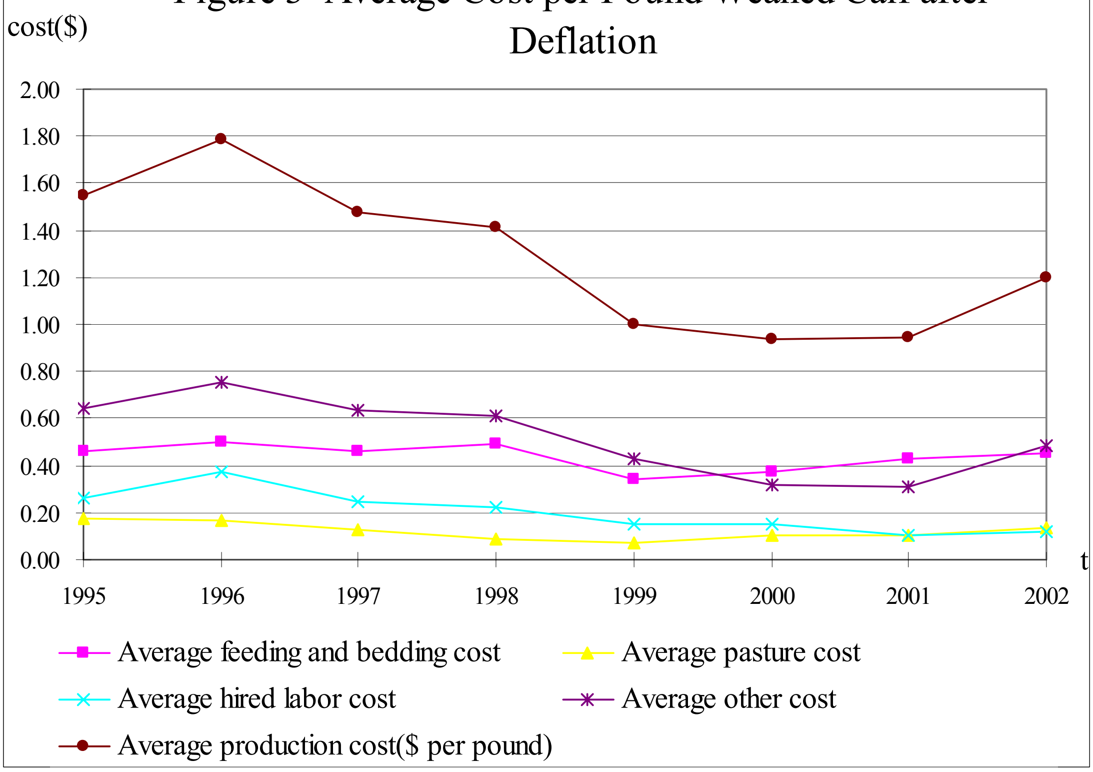


Figure 3 Average Cost per Pound Weaned Calf after Deflation



From this picture, we can see that the average production cost of per pound weaned calf was fluctuating over time. From 1995 to 1996, the sample average cost of production increased to the highest level among the 8 years. A decrease then occurred between 1996 and 1999. From 1999 to 2001, the average production cost remained at the lowest level during the eight years. After 2001, the average total cost of production was increasing again. From the components of average production cost, we can find that the average wintered feed and bedding cost showed similar trend as the average total cost of production; the average labor cost was increased from 1995 to 1996 and then decreased gradually; the average other cost was continuously decreasing from 1996 to 2001 and increased in 2002; the average pasture cost was also fluctuating over time. From the average cost figures, there is not a significant difference between deflated and not deflated value. However, the variances of the cost may differ between the two cases. To get a good estimation result and decrease measurement error from data, the deflated data set is used for the econometric model.

3.1.4 Relation between average production cost and output

The average production cost per pound weaned calf and weaned calf pounds (output) are as follows:

Table 5 Weaned Calf Output and Average Production Cost before Deflation

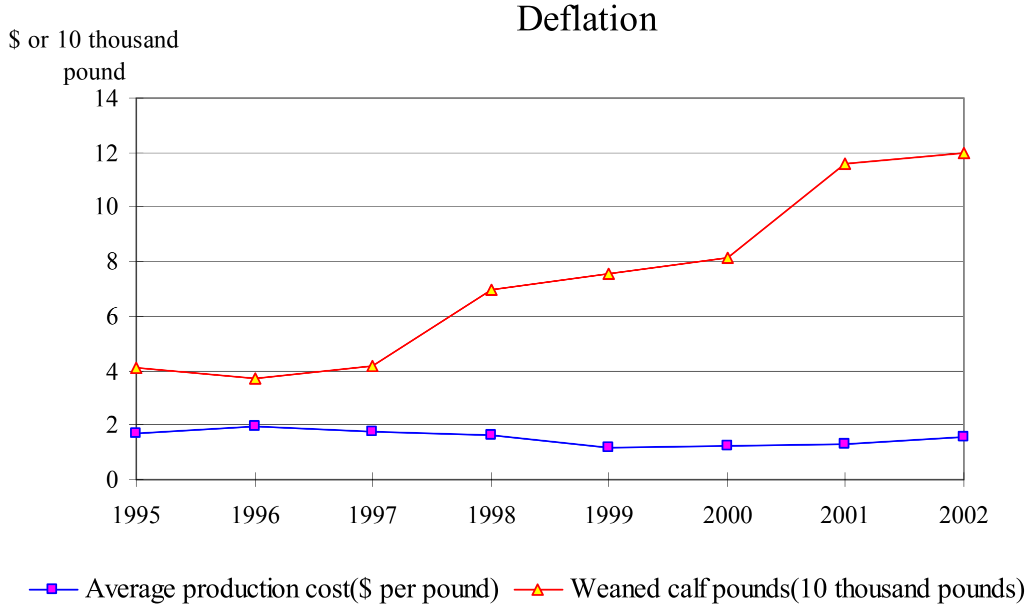
Year	Average production cost($ per pound)	Growth rate	Weaned calf pounds (10 thousand)	Growth rate
1995	1.69		4.12	
1996	1.98	0.17	3.73	-0.09
1997	1.76	-0.11	4.15	0.11
1998	1.62	-0.08	6.94	0.67
1999	1.17	-0.28	7.58	0.09
2000	1.21	0.04	8.17	0.08
2001	1.28	0.05	11.61	0.42
2002	1.58	0.24	12.01	0.03

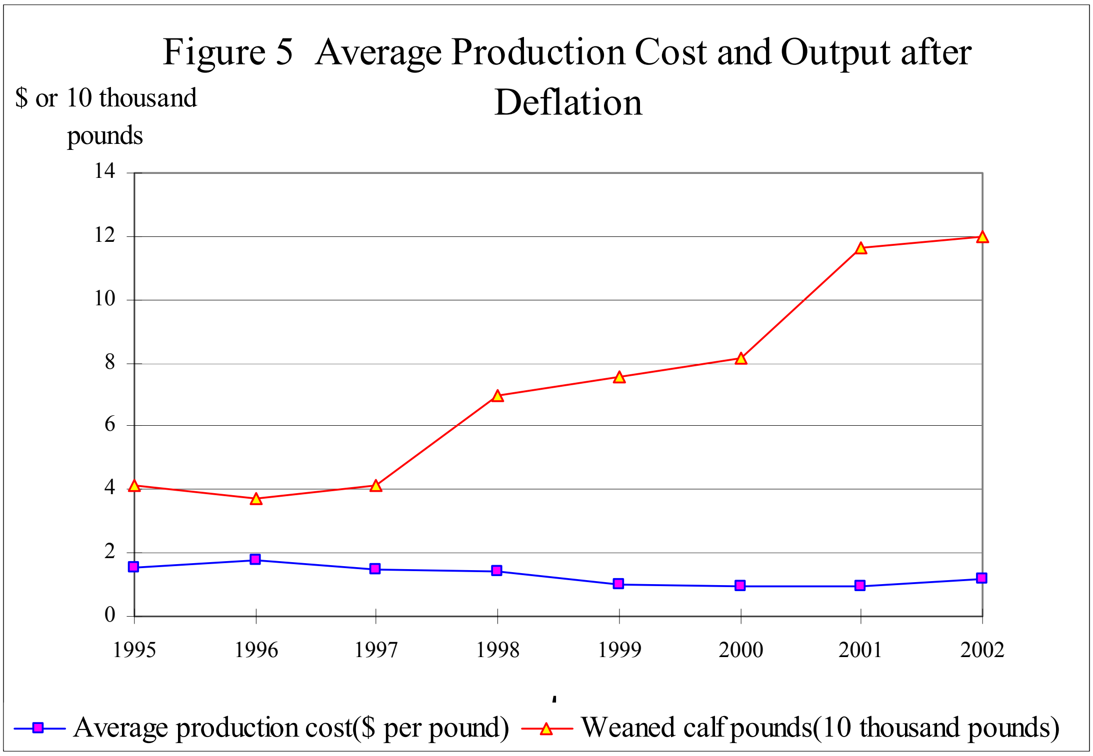
Table 6 Weaned Calf Output and Average Production Cost after Deflation

Year	Average production cost($ per pound)	Growth rate	Wean calf lbs(10 thousand)	Growth rate
1995	1.55		4.118407	
1996	1.79	0.16	3.729604	-0.09
1997	1.48	-0.17	4.153656	0.11
1998	1.41	-0.05	6.94191	0.67
1999	1.00	-0.29	7.579018	0.09
2000	0.94	-0.06	8.169994	0.08
2001	0.94	0.00	11.61182	0.42
2002	1.20	0.27	12.00763	0.03

The tendency of average production cost per pound weaned calf can be shown in the following graph.

Figure 4 Average Production Cost and Output before Deflation





Note: In above graphs, we put the average production cost per pound weaned calf and the weaned calf pounds together to study the relationship between the two.

The two graphs show no significant difference between deflated and undeflated values. From 1995 to 1996, the weaned calf output was decreased while the average production cost was increased, which meant diseconomy of size existed. From 1996 to 1999, the weaned calf output was increasing continuously. However, during the same period, the average production cost was decreasing, which means economies of size existed. From 1999 to 2001, the weaned calf output was increasing faster than average cost increase (or decrease in deflation case), which again showed economies of size. However, after 2001, the average production cost rised faster than the weaned calf output, showing the growth turns from economy of size to diseconomy of size. Because the average production cost is increasing faster and faster with output increase, the growth derived from economies of size becomes less and less and finally negative. One point worth noticing is that the economies of size described here is just based on the simple average of production cost among sample farmers. It is possible for bias existing in averaging process. Therefore, a model fitting for different sample farmers would be necessary to evaluate the economies of size more precisely.

3.2. Model Construction

3.2.1 Variable Definition

C is the total cost for each sample farmer, which corresponds to the deflated total cost in the original data set.

Wi is the price of input i. Specifically, w_1 is the input price for wintered feed and bedding, which is calculated by the deflated wintered feed and bedding expenditure divided by wintered feed and bedding input ($ per ton). This assumes that wintered feed and bedding inputs are separable to the labor input or other inputs; w_2 is input price of pasture, which is calculated by deflated pasture expense divided by the pasture input ($ per AUM); w_3 is labor input which is calculated by the aggregation of deflated paid labor and unpaid labor expenditures divided by paid labor and unpaid labor input($ per hour);other input price is calculated by total deflated other cost(including variable cost and fixed cost assuming long run cost minimum) divided by the wintered cow number($ per head).

Y is the output (lbs) represented by the weaned calf weight, which is calculated by weaned calf weight multiplied by weaned calf head.

To check if different regions have different costs in cow-calf production, regional dummy variables are included. Specifically, Di is dummy variables for different type of grassland. D_1 =1, Fescue grassland; 0 otherwise. D_2 =1, Moist Mixed Grassland; 0, otherwise. D_3 =1, Aspen Parkland; 0, otherwise. D_4 =1, Mixed Grassland; 0, otherwise. D_5 =1, Boreal Transition; 0, otherwise. D_6=1, Peace Lowland; 0, otherwise.

t is time trend, indexed by 1, 2,..8.

3.2.2 Empirical model

3.2.2.1 Translog cost system

The total cost function based on cost minimization is

$$TC = f(W, Y, T) \qquad 1$$

It is well known that lnTC (W, Y, T) provides a second-order approximation to an unknown cost function at an arbitrary point (Christensen, Jorgenson, and Lau, 1973). Applying second order Taylor expansion and considering time trend as well as land conditions[1], we get the following translog cost function:

$$\begin{aligned} LnTC = {} & a_0 + \sum_{i=1}^{4} a_i * LnW_i + \delta_1 * LnY + \frac{1}{2}\sum_{i=1}^{4}\sum_{j=1}^{4}\beta_{ij} * LnW_i * LnW_j + \frac{1}{2}\delta_2 * (LnY)^2 \\ & + \sum_{I=1}^{4}\gamma_i * LnW_i * LnY + \tau_1 * t + \frac{1}{2}\tau_2 * t^2 + \sum_{i=1}^{4}\psi_i * t * LnW_i + \omega * t * LnY + \sum_{i=1}^{6}\theta_i * D_i \end{aligned} \qquad 2$$

Applying Shepherd's lemma and symmetry, we can get the following cost share functions:

$$S_i = a_i + \sum_{j=1}^{4}\beta_{ij} * LnW_j + \gamma_i * LnY + \psi_i * t \qquad i = 1,2,3,4 \qquad 3$$

According to the economic theory, non-negativity, symmetry, linear homogeneity,

[1] see Baltagi, Griffin(1988) for technical change measurement.

monotonicity, continuity and concavity need to be satisfied in the cost system above (see appendix for details about these assumptions).

In our specified model, non-negativity, symmetry and linear homogeneity are imposed. The assumptions of monotonicity and concavity will be tested after the model estimation. We represent the linear homogeneity and symmetry conditions as:

$$a_4 = 1 - a_1 - a_2 - a_3 \qquad i = 1,2,3,4$$
$$\beta_{i4} = -\beta_{i1} - \beta_{i2} - \beta_{i3}$$

$$\sum_{i=1}^{4} \gamma_i = 0 \, ; \sum_{i=1}^{4} \psi_i = 0 \, ; \beta_{ij} = \beta_{ji}$$

Through the symmetry and linear homogeneity conditions, we can normalize the cost system by some input price, which can decrease the number of parameters to estimate and increase degree of freedom. Normalizing the cost function and share equations by price of other inputs, we get:

$$Ln\frac{TC}{W_4} = a_0 + \sum_{i=1}^{4} a_i * Ln\frac{W_i}{W_4} + \delta_1 * LnY + \frac{1}{2}\sum_{i=1}^{4}\sum_{j=1}^{4} \beta_{ij} * Ln\frac{W_i}{W_4} * Ln\frac{W_j}{W_4} + \frac{1}{2}\delta_2 * (LnY)^2$$
$$+ \sum_{I=1}^{4} \gamma_i * Ln\frac{W_i}{W_4} * LnY + \tau_1 * t + \frac{1}{2}\tau_2 * t^2 + \sum_{i=1}^{4} \psi_i * t * Ln\frac{W_i}{W_4} + \omega * t * LnY + \sum_{i=1}^{6} \theta_i * D_i \qquad 4$$

$$S_1 = a_1 + \beta_{11} * Ln\frac{W_1}{W_4} + \beta_{12} * Ln\frac{W_2}{W_4} + \beta_{14} * Ln\frac{W_3}{W_4} + \gamma_1 * LnY + \psi_1 * t \qquad 5$$

$$S_2 = a_2 + \beta_{12} * Ln\frac{W_1}{W_4} + \beta_{22} * Ln\frac{W_2}{W_4} + \beta_{24} * Ln\frac{W_3}{W_4} + \gamma_2 * LnY + \psi_2 * t \qquad 6$$

$$S_3 = a_3 + \beta_{13} * Ln\frac{W_1}{W_4} + \beta_{23} * Ln\frac{W_2}{W_4} + \beta_{34} * Ln\frac{W_3}{W_4} + \gamma_3 * LnY + \psi_3 * t \qquad 7$$

Because of adding-up constraint on the demand equations, the last cost share equation is eliminated to avoid singularity of variance-covariance matrix. The parameters of the last cost share equation are estimated by the combinations of the parameters from other cost share equations. The econometric model for equation 4 to 7 can be written compactly as:

$$y_{(ik)t} = X_{(ik)t}\beta_{(ik)} + u_{(ik)t}$$
$$k = 1,...K; \qquad i = 1,....N_k; \qquad t = 1,.....,k. \qquad 8$$

Where $u_{(ik)t} \sim IIN(0_{ik}, \Omega_{ik})$

Due to the unbalanced panel data, the farmers are observed in at least 1 year and at most K years. N_k represents the number of farmers that are observed in exactly k years. t indexes the observation number. So total number of farmers in the panel is $N = \sum_{k=1}^{K} N_k$ and the total number of observations is $n = \sum_{k=1}^{K} N_k * k$. ik indexes the i'th farmer who is in those observed in k years. $\beta_{(ik)}$ is the coefficient vector of plant ik, in which some elements may be random and depend on different observed farmers(see appendix for random parameter model construction).

3.2.2.2 Translog cost frontier

When incorporating inefficiency error term into the cost function, it becomes stochastic cost frontier model. The new model is as following[2]:

$$Ln\frac{TC}{W_4}=a_0+\sum_{i=1}^{4}a_i*Ln\frac{W_i}{W_4}+\delta_1*LnY+\frac{1}{2}\sum_{i=1}^{4}\sum_{j=1}^{4}\beta_{ij}*Ln\frac{W_i}{W_4}*Ln\frac{W_j}{W_4}+\frac{1}{2}\delta_2*(LnY)^2$$
$$+\sum_{I=1}^{4}\gamma_i*Ln\frac{W_i}{W_4}*LnY+\tau_1*t+\frac{1}{2}\tau_2*t^2+\sum_{i=1}^{4}\psi_i*t*Ln\frac{W_i}{W_4}+\omega*t*LnY+\sum_{i=1}^{6}\theta_i*D_i \qquad 9$$
$$+v+u$$

To write it compactly, we have:

$$y_{(ik)t}=X_{(ik)t}\beta_{(ik)}+u_{(ik)t}+v_{(ik)t}$$
$$k=1,..K; \qquad i=1,....N_k; \qquad t=1,.....,k. \qquad 10$$

Where $u_{(ik)t}$ may distributed with half normal, truncated normal, exponential or gamma distribution. $v_{(ik)t}\sim N(0,\sigma_v^2)$.

Other restrictions for the cost frontier (model 10) are the same as the cost system without the inefficiency term (model 8).

3.3 Estimation procedure

The maximum likelihood estimations are used in both model 8 and model 10.

In model 8, the joint log-likelihood function of farmer (ik), i.e. $y_{(ik)}$ conditional on $x_{(ik)}$ is :

$$L_{(ik)}=-\frac{Gk}{2}Ln(2\pi)-\frac{1}{2}Ln|\Omega_{ik}|-\frac{1}{2}[y_{(ik)}-X_{(ik)}\beta]'\Omega_{ik}^{-1}[y_{(ik)}-X_{(ik)}\beta]$$

We write the log-likelihood function of all observation of y conditional on all observations of x as:

---

[2] Note: for the frontier cost function model, as argued by Greene(1980), the share equations can not be included.

$$L=\sum_{k=1}^{K}\sum_{i=1}^{N_k}L_{(ik)}=-\frac{Gn}{2}Ln(2\pi)-\frac{1}{2}\sum_{k=1}^{K}\sum_{i=1}^{N_k}Ln|\Omega_{ik}|-\frac{1}{2}\sum_{k=1}^{K}\sum_{i=1}^{N_k}[y_{(ik)}-X_{(ik)}\beta]^{'}\Omega_{ik}^{-1}[y_{(ik)}-X_{(ik)}\beta]$$

In model 10, the maximum likelihoods of farmer (ik) for different specifications u are as following (Greene, 2002)

Half normal:

$$L_{(ik)}=-\log(2\pi)-\log\sigma-\frac{1}{2}(\varepsilon/\sigma)^2+\log\Phi[-d\varepsilon_i\lambda/\sigma]$$

Truncated normal:

$$L_{(ik)}=-\log(2\pi)-\log\sigma-\frac{1}{2}[d(\varepsilon_i+\mu)/\sigma]^2+\log\Phi[(1/\sigma)(d(\mu/\lambda-\varepsilon_i\lambda)]-\log\Phi(\mu/\sigma_u)$$

Exponential:

$$L_{(ik)}=-\log(\theta)-\frac{1}{2}\theta^2\sigma_v^2-\theta d\varepsilon_i-\log\Phi[-(d\varepsilon_i/\sigma_v+\theta\sigma_v)]$$

The definitions of these variables can be found in Greene’s limdep manual (Greene,2002). For simplicity, we don’t present the aggregate likelihood here.

4. Results from cost system estimation (model 8)

4.1 Model selection tests

Table 7 Fit Statistics

Information criteria		Fixed effect	Random coefficient[3]
	-2 Log Likelihood	-3124.3	-3266.8
Akaike’s Information Criteria	AIC (smaller is better)	-3052.3	-3174.8
Corrected form of Akaike’s Information Criteria	AICC (smaller is better)	-3050.2	-3171.4
Bayesian Information Criteria	BIC (smaller is better)	-2915.2	-3024

We can use the Hausman test to check if there is fixed effect in the model and select

3 This random coefficient model assumes parameters representing second-order terms in the cost function are constant. Actually, if we let those parameters to be random, the likelihood function cannot converge.

either fixed effect or random effect panel data model. However, one problem as suggested by Hsiao (2003) is that the alternative hypotheses are not clear in the Hausman test. For example, if we reject the null hypothesis (random effect model), the alternatives can be fixed effect or random parameter model or some other model. Therefore, Hsiao suggests using AIC, BIC or other methods to select among different type of panel data models. The AIC procedure (Akaike, 1974) is used to evaluate how well the candidate model approximates the true model by assessing the difference between the expectations of the independent variables under the true model and the candidate model using the Kullback-Leibler (K-L) distance4. The BIC procedure (Schwarz, 1978) also uses Kullback-Leibler (K-L) distance, but has different criteria. From the above results, we can find that random parameter model is preferred in that it can better fit the dataset.

4.2 Estimated random coefficient model

Table 8 Random Coefficient Model Estimation

Variable name	Label	Estimate[5]	Standard Error	t value	p-value>|t|[6]
Constant	a_0	3.1688	1.4129	2.24	0.0253
P1 (price of feeding and bedding)	a_1	0.1039	0.1026	1.01	0.3123
P2 (price of Pasture)	a_2	0.2269	0.03874	5.86	<.0001

[4] The Kullback-Leibler distance is a natural distance function from a "true" probability distribution, p, to a "target" probability distribution, q.

[5] The estimators of P1, P2, P3, P4 and output are expectations of the random parameters. The solution for random parameters is omitted here because it is too large, but it is available from author upon request.

[6] The p-values here are for asymptotical t test.

P3 (price of labor)	a_3	0.6823	0.06785	10.06	<.0001
P4 (other cost per cow)	a_4	-0.0132	0.07368	13.75	<.0001
Output (weaned calf weight)	δ_1	0.1509	0.2844	0.53	0.5962
P1*P1	β_{11}	0.06403	0.01393	4.6	<.0001
P1*P2	β_{12}	-0.00965	0.005025	-1.92	0.0554
P1*P3	β_{13}	0.01582	0.009893	1.6	0.1104
P1*P4	β_{14}	-0.0702	0.007914	-8.87	<.0001
P2*P2	β_{22}	0.07865	0.004522	17.39	<.0001
P2*P3	β_{23}	-0.02602	0.005382	-4.83	<.0001
P2*P4	β_{24}	-0.04298	0.00382	-11.25	<.0001
P3*P3	β_{33}	0.04326	0.01272	3.4	0.0007
P3*P4	β_{34}	-0.03306	0.006907	-4.79	<.0001
P4*P4	β_{44}	0.1462	0.007195	20.32	<.0001
Square of output (weaned calf weight)	δ_2	0.05045	0.02917	1.73	0.0843
P1*y	γ_1	0.02874	0.009483	3.03	0.0026
P2*y	γ_2	0.003301	0.003316	1	0.3199
P3*y	γ_3	-0.03814	0.005453	-6.99	<.0001
P4*y	γ_4	0.006104	0.00649	0.94	0.3474
t	τ_1	-0.2603	0.1003	-2.59	0.0097
t square	τ_2	0.01581	0.00597	2.65	0.0083
P1*t	ψ_1	0.008808	0.002848	3.09	0.0021
P2*t	ψ_2	0.00243	0.001022	2.38	0.0178

P3*t	ψ_3	-0.00832	0.001715	-4.85	<.0001
P4*t	ψ_4	-0.00292	0.002001	-1.46	0.1456
y*t	ω	0.01467	0.009964	1.47	0.1415
Fescue Grassland	θ_1	-0.1491	0.06656	-2.24	0.0255
Moist Mixed Grassland	θ_2	-0.07146	0.07588	-0.94	0.3467
Aspen Parkland	θ_3	-0.06407	0.05428	-1.18	0.2384
Mixed Grassland	θ_4	0.009074	0.06716	0.14	0.8926
Boreal Transition	θ_5	-0.01983	0.05261	-0.38	0.7064
Peace Lowland	θ_6	0.2954	0.2481	1.19	0.2343

Null Model Likelihood Ratio Test

DF	Chi-Square	Pr > ChiSq
19	1986.98	<.0001

Note: All variables are in log form expect time and regional dummies

We can see the p-values for logged pasture, labor and other input prices are less than 0.05, which means that they have significant effect on the total production cost. Though the logged price of feeding and bedding is not significant individually, it's square term and cross product with logged other input price are statistically significant. Most cross products among logged input prices are highly significant, which shows that the relations among input factors in production are very strong. The p values of time trend variable t and t square are 0.0097 and 0.0083, less than 0.05, which means that production costs are non-linearly related with the time variable The p values for different land types are not significant except the p value for Fescue Grassland, which means there is almost no significant difference at production costs for different land types.

4.3 Estimation of Economies of Size

Economies of Size (ES) is calculated by the following formula:

$$ES = \frac{\partial LnC(W,Y)}{\partial LnY} = \delta_1 + \delta_2 LnY + \sum_{i=1}^{4} \gamma_i * LnW_i + \omega * t$$

If ES>1, diseconomies of size exist; if ES<1, economies of size exist. It should be noted that the measure of economies of size here is a random variable because the parameter δ_1 is random.

By the results from random parameter model, we have the formula of predicted ES as follows:

$$ES_k = 0.1509 + 0.05045 LnY + 0.02874 * LN(W_1 / W_4)$$
$$+ 0.003301 * LN(W_2 / W_4) - 0.03814 * LN(W_3 / W_4) + 0.01467 * t + \delta_{1k}$$

where k is kth farmer. δ_{1k} is the random parameter with respect to kth farmer.

It should be noticed that the derivative of predicted ES with respect to lny is positive, which means that increase of output will make ES bigger and bigger and the production goes toward diseconomies of size. Also, the derivative of ES with respect to time t is positive, which means with time passing by, diseconomies of size will present. The prices have different effects on ES. Feed and bedding price, pasture price have positive effects on ES while labor price has negative effect on ES. It is probably because that increase of labor price raises the productivity of labor, which in turn, leads to economies of size. However, this kind of effect is decreasing because of the concavity of ES on wage rate. The random parameters correspond to the individual farmers, which mean that different farmers have different economies of size. The descriptive analysis of predicted ES is as following:

Table 9 Descriptive Analyses of Predicted ES

ES	Value
Mean	0.84
Standard Error	0.01
Minimum	0.65
Maximum	1.00
Sum	165.13
Count	196

From the above results, we know that the monotonicity restriction: non-decreasing in y has been satisfied. By the annual average logged output, prices, we calculate predicted ES as following:

Table 10 Annual ES

Year	Mean	Std Dev	Minimum	Maximum
1995	0.75	0.04	0.69	0.82
1996	0.76	0.05	0.61	0.85
1997	0.79	0.03	0.72	0.83
1998	0.83	0.04	0.68	0.90
1999	0.85	0.03	0.76	0.92
2000	0.87	0.03	0.79	0.94
2001	0.90	0.03	0.84	0.96
2002	0.92	0.04	0.79	1.00
Average	0.85	0.07	0.61	1.00

The average ES from 1995 to 2002 is 0.85, which means that the cow-calf production in Alberta is still within economies of size. But as we have analyzed above, the range of economies of size become less and less from 1995 to 2002. In other words, economies of size have been almost fully exploited.

4.4 Tests for economic assumptions

The hypotheses of non-decreasing in w and concavity in w are tested. To check property of non-decreasing in w, we need to examine the predicted cost shares.

Table 11 Predicted Cost Shares

Variable	N	Mean	Std Dev	Minimum	Maximum
Share1	333	0.35	0.06	0.12	0.59
Share2	333	0.10	0.03	-0.04	0.19
Share3	333	0.15	0.04	0.04	0.30
Share4	333	0.40	0.06	0.02	0.55

The predicted shares are all positive, which means that the monotonicty restriction: non-decreasing in w has been satisfied by the model.

Diewert and Wales (1987) show that the Hessian of the TL cost function will be negative semidefinite, providing C(p, y, t)>0, if and only if the matrix G given below is negative semidefinite. The ijth element of G is defined as:

$$g_{ij} = \beta_{ij} - S_i \delta_{ij} + S_i S_j \qquad i, j = 1,.......n$$

with $\delta_{ij} = 1$ if $i = j$ and 0 otherwise. S_i is the share of ith input. In our model, because panel data is used and random parameters are involved, we modify the requirement as:

$$g_{(kt)ij} = \beta_{ij} + \beta_{(k)ij} - S_{(kt)i}\delta_{ij} + S_{(kt)i}S_{(ktt)j} \qquad i, j = 1,.......n \quad k = 1,.....K; t = 1,.....T$$

All the farmers in the sample are evaluated based on their input shares during different years. The results show that determinants of the second order, third order minors and the whole G matrix for different farmers are very small. The predicted G matrix and its determinants are as follows:

Table 12 Predicted G Matrix and Its Determinants

Variable	N	Mean	Std Dev	Minimum	Maximum
g11	196	-0.15	0.04	-0.21	0.01
g12	196	0.02	0.02	-0.04	0.08
g13	196	0.07	0.03	0.00	0.16
g14	196	0.06	0.03	-0.05	0.12
g22	196	-0.01	0.04	-0.13	0.10
g23	196	-0.01	0.02	-0.06	0.06
g24	196	0.00	0.02	-0.05	0.07
g33	196	-0.08	0.04	-0.18	0.01
g34	196	0.03	0.03	-0.03	0.16
g44	196	-0.08	0.03	-0.15	0.04
h1	196	-0.15	0.04	-0.21	0.01
h2	196	0.00	0.01	-0.01	0.01
h3	196	0.00	0.00	0.00	0.00
h4	196	0.00	0.00	0.00	0.00

where hi is the determinant of ith order minor of the predicted G matrix. Based on the above results, the negative semi definiteness of the G matrix and therefore, the negative semi definiteness of the translog cost function is approximately satisfied.

4.4 Estimation of Allen-Uzawa partial elasticity of substitution

The Allen-Uzawa partial elasticity of substitution is calculated by:

$$\sigma_{ii} = \frac{C*C_{ii}}{C_i*C_i} = \frac{\beta_{ii} + S_i(S_i - 1)}{S_i^2} \quad ; \quad \sigma_{ij} = \frac{C*C_{ij}}{C_i*C_j} = 1 + \frac{\beta_{ij}}{S_i*S_j}$$

Table13 Average Allen-Uzawa Partial Elasticity of Substitution (n=333)

Input	Feeding and bedding	Pasture	Labor	Other

Feeding and bedding	-1.40 (0.75)	0.61 (0.33)	1.40 (0.26)	0.42 (0.27)
Pasture		8.27 (60.52)	-1.62 (1.80)	-0.62 (2.13)
Labor			-3.68 (1.02)	0.18 (1.41)
Other				0.15 (12.19)

Note: standard error in parentheses; Allen-Uzawa partial elasticity of substitution is symmetric between two inputs

Two of the average Allen-Uzawa own elasticities of substitution are negative and two are positive, which may not be reliable measure. The standard deviations for pasture and labor own elasticities of substitution are very large. By checking the predictions of elasticities of different farmers, we find that elasticities at several observation points are very large[7]. If getting rid of those abnormal prediction points by restricted the own price elasticities of pasture and other inputs within 10, we get:

Table 14 Adjusted Average Allen-Uzawa Partial Elasticity of Substitution (n=288)

	Feeding and bedding	Pasture	Labor	Other
Feeding and bedding	-1.44 (0.78)	0.67 (0.20)	1.42 (0.27)	0.41 (0.23)
Pasture		0.06 (2.67)	-1.26 (1.28)	-0.22 (0.65)
Labor			-3.74 (1.03)	0.24 (0.52)
Other				-0.51 (0.34)

It seems there is no big change in the elastisticites except the own elastisticites of

[7] Specifically, id=2137,period=5;id=2450,period=4; id=3074,period=4;id=3074,period=7; id=2525,period=4;id=2910,period=5, etc. At these data points, the prediction of own price elasticities for pasture or other inputs are very big, which is not reliable.

pasture and other inputs. The deviation for the two own price elasticities become far smaller than in the previous table, which shows that the predictions are more reliable. The sign of own price elasticity of other input becomes negative, which is reasonable. However, the own price elasticity of pasture is still positive, which means that the pasture input is somewhat inferior.

When checking the cross elasticities of substitution among inputs, we find that substitution and complementary relationship exist. The feeding and bedding input is a substitute for pasture, labor and other inputs. The pasture input and labor input are complements, the same as the pasture input and other inputs. The labor input and other input here are substitutes.

4.5 Estimation of own and cross price elasticities

The own and cross price elasticities are calculated by following formula:

$$\varepsilon_{ii} = \frac{\partial x_i}{\partial w_i}\frac{w_i}{x_i} = \sigma_{ii} S_i \quad ; \quad \varepsilon_{ij} = \frac{\partial x_i}{\partial w_j}\frac{w_j}{x_i} = \sigma_{ij} S_j \, ; \quad \varepsilon_{ji} = \frac{\partial x_j}{\partial w_i}\frac{w_i}{x_j} = \sigma_{ij} S_i \quad (i \neq j)$$

We should note here that the cross price elasticity is not symmetric between two inputs. The results are shown in tables 15 and 16.

Table 15　Average Own and Cross Price Elasticities (n=333)

	Feeding and bedding	Pasture	Labor	Other
Feeding and bedding	-0.43 (0.08)	0.07 (0.04)	0.20 (0.08)	0.17 (0.10)
Pasture	0.23 (0.14)	0.12 (0.72)	-0.19 (0.24)	-0.16 (0.42)
Labor	0.49 (0.16)	-0.11 (0.10)	-0.50 (0.11)	0.12 (0.16)
Other	0.16 (0.15)	-0.02 (0.12)	0.05 (0.11)	-0.19 (0.30)

Table 16 Adjusted Average Own and Cross Price Elasticities (n=288)

	Feeding and bedding	Pasture	Labor	Other
Feeding and bedding	-0.44 (0.08)	0.08 (0.04)	0.20 (0.08)	0.16 (0.10)
Pasture	0.25 (0.12)	-0.07 (0.23)	-0.13 (0.11)	-0.05 (0.18)
Labor	0.48 (0.16)	-0.11 (0.10)	-0.49 (0.11)	0.12 (0.16)
Other	0.16 (0.08)	-0.01 (0.06)	0.05 (0.07)	-0.21 (0.07)

Again, the standard deviation of own price elastisities of pasture and other inputs are far smaller under adjusted table 16[8] than that under original prediction. Also the own price elasticity of pasture becomes negative, which is more reasonable prediction. The own price elasticities are all negative. The signs of cross price elasticities under adjusted table are the same as the original table. The feeding and bedding input is substitute to pasture, labor and other inputs. The pasture input is complements to labor input and other inputs. The labor input and other inputs are substitutes. These cross price elasticities are very small, which means the substitution or complement is not easy among inputs. We should note that the levels of substitution or complement among inputs evaluated at cross price elasticities and that evaluated under Allen-Uzawa cross elastisities of substitution are different because the calculation and averaging processes of them are different. However, the signs of substitution and complement among inputs are almost the same under both cases.

4.6 Estimation of Technical Change and Total Factor Productivity

[8] Here we use the same adjustment as that of Allen-Uzawa elasticity of substitution.

Technical change and total factor productivity are calculated by following formula:

$$\hat{T}_k = \frac{\partial LnC_k}{\partial t} = \tau_1 + \tau_2 * t + \sum_{i=1}^{4} \psi_i * LnW_{ik} + \omega * LnY_k$$

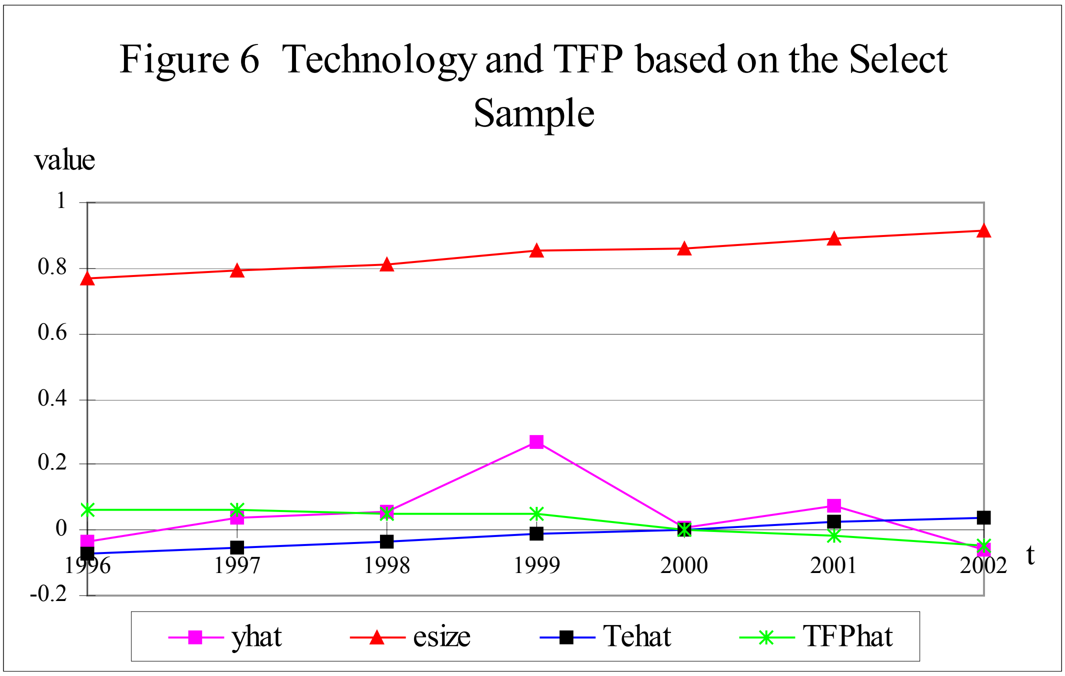
$$\hat{TFP}_K = -\hat{T}_K + (1 - ES_k)\hat{Y}_K \qquad k=1,2,\ldots..K$$

Where k is the kth farmer. $\hat{TFP}$ is total factor productivity; $\hat{T}$ is the rate of technical progress; $\hat{Y}$ is the rate of output increase. $\hat{T} < 0$, Technical progress; $\hat{T} > 0$, technical recession. ES is economies of size. Because the farmers with only one year observations are dropped from the analysis, the value of average ES based on the farmers with more than one year observations is a little different from previous predicted ES. The growth of output based on the farmers with more than one year observations is also different from the previous descriptive analysis, which based on the whole sample farmers. Taking the average value based on different years, we get the following tables.

Table 17 Average Output Increase, ES, Technical Progress rate and Total Factor Productivity

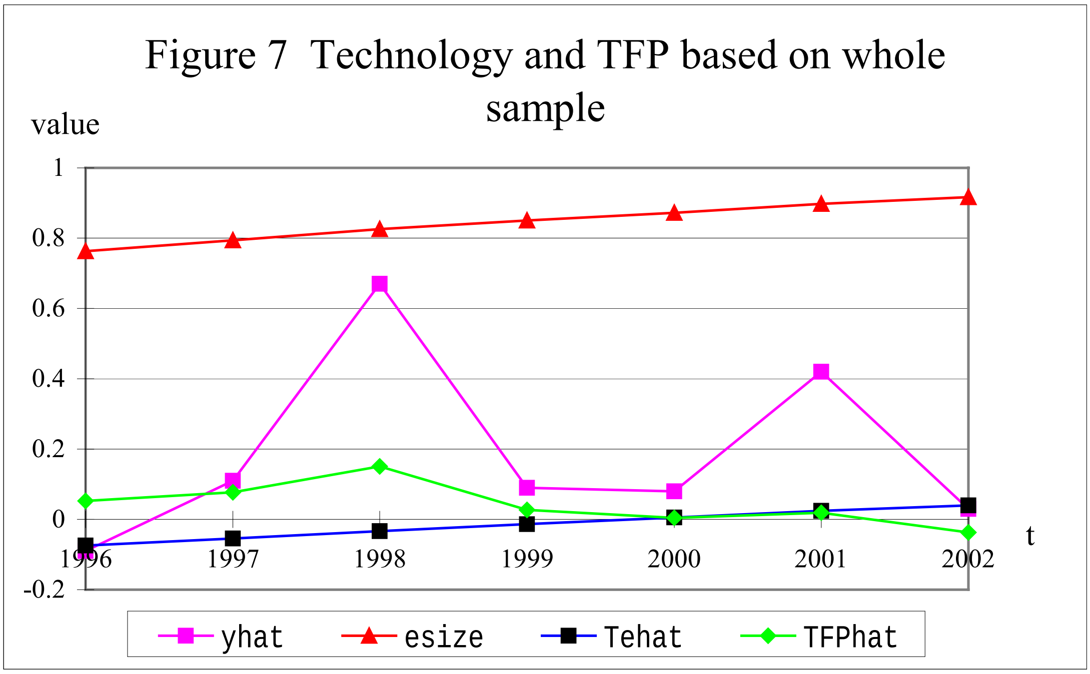
Year	Yhat	ES	$\hat{T}$	$\hat{TFP}$
1996	-0.04	0.77	-0.07	0.06
1997	0.04	0.79	-0.06	0.06
1998	0.06	0.81	-0.04	0.05
1999	0.27	0.86	-0.01	0.05
2000	0.01	0.86	0.00	0.00
2001	0.08	0.89	0.02	-0.02
2002	-0.06	0.91	0.04	-0.05
average	0.05	0.84	-0.02	0.02

Note: the output increase (Y hat) is calculated by LnY_t-LnY_{t-1} .



Note: Tehat is predicted technical change rate; TFPhat is the predicted total factor productivity.

From the above picture, we can analyze the joint effect of output, ES and technical change on total factor productivity. Obviously, the measure of ES is rising, meaning exploitable economies of size decrease over time. The measure of technical change becomes positive after 2000, suggesting technical recession. The output is fluctuating over time, but during 1997 to 2001, output is increasing. All these factors contribute to the change of Total Factor productivity. As showing by the table 17, TFP is gradually decreasing with time. However, on average, the value of TFP is positive and the technical change is negative, suggesting productivity growth and technical progress. Because we use farmers with more than one year observations, the results may exist some error. However, when we take all the sample farmers into account, we have the following graph:



Comparing the figure 6 and figure 7 above, we can find that although the fluctuation of output change is bigger, the other measurements for ES, technology are almost the same. The predicted TFP is still on the path of decreasing.

5. Results from cost frontier estimation (model 10)

5.1 Model selection

Based on the result from different specification of u, the truncated normal is not suitable for the dataset[9]. The exponential and half normal are suited for the dataset. For simplicity, the half normal distribution is specified in the cost frontier model. Also, linear homogeneity is tested based on the cost frontier model. The test result suggests that linear homogeneity is satisfied (Wald test = 9.90, Sig. level = .19406). So the final model used is restricted cost frontier model based on half normal specification of

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9 The cost frontier model under truncated normal specification doesn't converge by maximum likelihood estimation.

u.

5.2 Estimated Cost Frontier Model

Table 18 cost frontier estimation

Variable name	Label	Estimate	Standard Error	t value	p-value>|t|[10]
Constant	a_0	9.2684	4.1788	2.22	0.0266
P1 (price of feeding and bedding)	a_1	0.7341	1.2674	0.58	0.5624
P2 (price of Pasture)	a_2	0.5374	1.2805	0.42	0.6747
P3 (price of labor)	a_3	1.7302	1.5410	1.12	0.2615
Output (weaned calf weight)	δ_1	-0.3844	0.6494	-0.59	0.5539
P1*P1	β_{11}	0.2355	0.2324	1.01	0.3110
P1*P2	β_{12}	0.1250	0.1758	0.71	0.4769
P1*P3	β_{13}	-0.2838	0.2844	-1.00	0.3183
P2*P2	β_{22}	0.3785	0.1556	2.43	0.0150
P2*P3	β_{23}	-0.3077	0.2203	-1.40	0.1624
P3*P3	β_{33}	0.7241	0.4609	1.57	0.1161
Square of output (weaned calf weight)	δ_2	0.0863	0.0562	1.54	0.1246
P1*y	γ_1	-0.0643	0.0923	-0.70	0.4861
P2*y	γ_2	0.0091	0.0894	0.10	0.9193
P3*y	γ_3	-0.0311	0.1105	-0.28	0.7781
t	τ_1	-0.3942	0.2139	-1.84	0.0653

[10] The p-values here are for asymptotical t test.

t square	τ_2	0.0103	0.0104	0.99	0.3210
P1*t	ψ_1	-0.0142	0.0321	-0.44	0.6579
P2*t	ψ_2	0.0061	0.0286	0.21	0.8319
P3*t	ψ_3	-0.0173	0.0452	-0.38	0.7024
y*t	ω	0.0255	0.0156	1.63	0.1023
Fescue Grassland	θ_1	-0.1411	0.1092	-1.29	0.1962
Moist Mixed Grassland	θ_2	-0.0519	0.1080	-0.48	0.6310
Aspen Parkland	θ_3	-0.0660	0.0718	-0.92	0.3581
Mixed Grassland	θ_4	-0.1103	0.0848	-1.30	0.1932
Boreal Transition	θ_5	-0.0319	0.0689	-0.46	0.6432
Lambda		1.0094	0.2495	4.05	0.0001
Sigma(u)		0.2115	0.0418	5.05	0.0000

The t test of lambda suggests that cost inefficiency exists in the model. From Table18, we can see that the parameters for prices and output are all not significantly different from zero. Among the 13 parameters for cross product terms, only one is significant. These results suggest that collinearity may exist in the model. Further, the results don't change much even deleting the regional dummies. Some other problems include the estimates of price elasticities and G matrix (for testing concavity). These estimates are as follows:

Table 19 estimations of price elasticities, G matrix

	Mean	Std.Dev.	Skewness	Kurtosis	Minimum	Maximum	NumCases
S11	0.1275	0.3150	6.2618	60.3326	-0.0295	3.7655	333
S12	0.5087	0.2333	3.8983	31.4681	0.1987	2.7812	333
S13	-0.7814	0.4668	-4.5926	39.0819	-5.5924	-0.2711	333
S14	0.1451	0.1082	-4.3687	38.2778	-0.9543	0.3207	333

S22	3.9915	3.5605	5.8468	57.6087	0.6433	44.7929	333
S23	-3.8299	2.9071	-5.7724	56.7071	-37.0182	-1.0191	333
S24	-2.1348	1.8628	-5.6503	54.7501	-23.1878	-0.1255	333
S33	5.1071	2.6955	1.3579	5.3259	0.8857	17.2539	333
S34	-0.6947	0.5171	-1.3714	5.5851	-3.0155	0.1322	333
S44	0.5452	0.9064	15.6551	269.1020	0.2730	16.2129	333
G11	-0.2650	0.2266	-1.0064	4.7932	-1.3788	0.1831	333
G12	0.0914	0.0155	-0.5375	3.0658	0.0369	0.1227	333
G13	-0.3327	0.0196	-0.5601	3.3492	-0.4096	-0.2884	333
G14	-0.2078	0.0297	0.7984	3.7065	-0.2729	-0.0971	333
G22	0.2669	0.0581	-1.3369	5.3909	0.0247	0.3702	333
G23	-0.3219	0.0084	-1.2728	4.5610	-0.3528	-0.3094	333
G24	-0.2356	0.0219	-1.3855	6.8055	-0.3569	-0.1971	333
G33	0.5504	0.0955	-2.1838	11.9364	-0.1009	0.6828	333
G34	-0.1907	0.0285	-0.8534	3.8587	-0.3119	-0.1339	333
G44	-0.1580	0.1804	-0.3157	3.4452	-0.8225	0.3809	333
K1	-0.2650	0.2266	-1.0064	4.7932	-1.3788	0.1831	333
K2	-0.0836	0.0688	-1.3628	5.8001	-0.4492	0.0316	333
K3	-0.0682	0.0467	-1.4200	6.3044	-0.3253	0.0021	333
K4	0.0158	0.0073	-0.5826	2.8882	-0.0052	0.0296	333

Here Sii is the own price elasticity of input i; Sij is the cross price elasticity of input i and j; Gij is the ijth element of G matrix; Ki is the determinant of i'th minor of G matrix. One big problem is that the own price elasticities are all positive. This implies that all of the inputs are inferior goods, which may not be reasonable. The value of Ki can be used to check the concavity of Hessian matrix (Diewert and Wales.1987). If concavity holds, K_2 should be bigger than zero. However, K2 is less than zero, which violates concavity. The model with exponential specification of u is also estimated and checked for these estimations. The same problems exist.

Given these problems, the translog cost frontier model doesn't seem applicable for the cost function estimation although it is very flexible. To analyze the effect of efficiency change on productivity, the non-parametric approach without model specification can be used. There are two advantages in non-parametric analysis of efficiency. One is that we don’t need to specify the function form; another is that not as many observations are required. Further, a comparison can be made between the TFP estimates from the cost function system and TFP estimates from non-parametric analysis.

6. Results from non-parametric estimation

The approach used for non-parametric analysis of TFP is the Malmquist productivity index. In the previous review of productivity measurement, the Malmquist productivity index has been introduced. The data are clustered by farmers with different number of observations (see Table 1). Of the total sample, 60% percent are farmers with one observation and 24% are farmers with two observations. Because of the unbalanced panel data nature, some farmers don’t have continuous observations. To evaluate the mean TFP, we use farmers with continuous observations. Also, because the farmers with more than four observations are really few, we only consider farmers with 2, 3 and 4 period observations. The results for the Malmquist productivity index are summarized as follows:

Table 20 The Malmquist TFP index

Mean	2 period	3 period	4 period	Weighted mean (weight is observation number)
Observation number	36	6	8	
Technical efficiency change	0.903	1.038	1.043	0.9416
Technology change	1.114	1.005	0.961	1.07644
Pure technical efficiency change	0.884	1.02	1.08	0.93168

Scale efficiency	1.022	1.018	0.966	1.01256
TFP change	1.006	1.043	1.002	1.0098

The Malmquist TFP index under Variable Return to Scale can be decomposed into technology change, technical efficiency change (including pure technical change and scale efficiency). We have explained the decomposing procedure in the review of TFP measurement. Comparing Tables 20 and Table 17, we can find that the non-parametric measure and parametric measure of TFP and Technical Change are different. Specifically, the mean of Malmquist productivity is 1.0098, which suggests that the total factor productivity is increasing 1% for each observed year, while the TFP from econometric estimation of cost system is 2% for each observed year. The technical change from decomposition of Malmquist TFP index is 1.07644, which means that the technology is improved around 8% per year, while the technical change from econometric estimation of cost system is only 2% per year. There are several reasons for these differences. First and the most important reason is the samples used in cost system and in Malmquist TFP index are different. This is due to the unbalanced panel data nature. In Malmquist TFP index, we need the continuous observations to calculate the TFP index and therefore lost the information from the farmers with only one observation or with not continuous estimations. Second, in econometric estimation of cost system, we have considered the contribution of economies of size in TFP growth but in Malmquist TFP index, the contribution of economies of size is not covered in TFP growth. Third, the calculation methods are different, which may produce differences between the estimated TFP from cost system and that from Malmquist TFP index.

Though the values are different, the signs of TFP and Technical Change from cost system estimation and that from Malmquist TFP index are the same. Under Malmquist index, the efficiency is decreasing, as shown by the value of technical efficiency being 0.9416, less than 1. This suggests there is some problem in the farmers' management ability or some technical application. Also, we must notice that this measure is just partially correct because of the sample limit.

7. Conclusions and Some Future Work

This paper adopts two approaches for estimation of total factor productivity and other economic indices. One is econometric approach and another is non-econometric approach. The data is unbalanced panel data, which come from 196 farmers for cow-calf production in Alberta from 1995 to 2002. Under the econometric approach, a translog cost system and a translog cost frontier are estimated. In the translog cost system estimation, a random parameter cost system is preferred based on the AIC and BIC model selection tests. All basic economic assumptions are either imposed or tested in the random parameter cost system. Specifically, linear homogeneity and symmetry are imposed. Concavity and monotonicity are tested after model estimation. The results show that monotonicity and concavity are satisfied. Through the estimated random parameter model, we get the predicted Economies of Size(ES) and find that output and prices of feed and bedding, pasture and other input have negative effects on the ES. One interest thing is that the labor price has positive effect on ES, which may be due to the stimulus of wage improvement on productivity. However, this kind of stimulus is decreasing because the concavity of ES on wage rate. The annual ES is decreasing as reflected by the negative coefficient of time variable t, meaning that the cow-calf production in Alberta has less and less size economy. The average TFP is positive and average technical change is negative (technical advancement), which means that growth of cow calf production is still on a good pattern. However, the technical progress rate also becomes less and less important in production growth. As shown at table 17, the technology and ES are decreasing simultaneously. Therefore, the crucial problem facing cow-calf production in Alberta is how to improve technology and maintain economies of size. If the technology is highly advanced, there’s limit space for technical progress and we need to focus on maintaining economies of size. One possibility is to change the shape of long run average cost. This may be realized by the improvement of technical efficiency and market efficiency.

The estimations of Allen-Uzawa partial elasticity of substitution and price elasticities show that the substitution and complement relationship exist simultaneously among the feeding and bedding input, pasture input, labor input and other input. The adjusted predictions for these elasticities seem more reliable. As shown by the cross price elasticities among inputs, the substituting or complement effect are with absolute values less than one, which means that it is not easy to substitute or complement among these inputs.

Because in cost system we can not get the measure of efficiency, a translog cost frontier is also estimated. Though the statistical test for efficiency suggests that the inefficiency does exist in the model, the parameter estimation is not good. Certain colinearity exists in the model. Furthermore, the price elasticities are all positive under the cost frontier estimation, meaning an irrational input allocation. The concavity constraint is also violated. All these problems cannot be solved under different specifications of inefficiency error term. Therefore, the translog cost frontier is not suitable for the efficiency estimation.

Now that the translog cost function is already flexible function, it may be hard to find some more flexible function forms. To address efficiency component, we go to non-parametric analysis. Under the non-parametric approach, Malmquist TFP index are applied. The sample farmers are selected from the original data set. For more accuracy, the farmers with only one observation or with no continuous observations are not covered in the sample. After Malmquist TFP index estimation, we find that the signs of average TFP and average technical change are the same as those under econometric estimations, meaning that beef-cow production is still at a good growth pattern. However, the efficiency measure is 0.9416, meaning inefficiency exists in beef cow production.

There are still some problems left to be answered. One of them is that some exogenous variables are not covered in the study because of lack of data. While these variables may have great impact on cow-calf production. For example, BSE impact may affect the cow-calf production in Alberta to a certain extent and may incur some cost in cow-calf production. However, the input and output data for Alberta beef-cow farmers in 2003 and 2004 are not available yet. Upon these data is available, a structure break test can be applied to check if there are significant effects of BSE on the beef-cow production in Alberta.

Appendix

A. Assumptions of cost function.

(1) Non-negativity

C (w,y)>0 for w>0 and y>0. This has been implied by the translog function form.

(2). Non-decreasing in w.

$$\frac{\partial C(W,Y)}{\partial W_i} \geq 0.$$

Through data analysis, we find that w>0 and y>0. Therefore, we have: $S_i = \frac{\partial LnC(W,Y)}{\partial LnW_i} = \frac{\partial C(W,Y)}{\partial W_i} * \frac{W_i}{C} \geq 0 \Leftrightarrow \frac{\partial C(W,Y)}{\partial W_i} \geq 0$.

This test will be done based on the predicted cost share.

(3). Non-decreasing in y.

$\frac{\partial C(W,Y)}{\partial Y} \geq 0$. Because w>0 and y>0, this means

$$\frac{\partial LnC(W,Y)}{\partial LnY} = \delta_1 + \delta_2 LnY + \sum_{i=1}^{4} \gamma_i * LnW_i + \omega * t \geq 0$$

Which implies economies of size must be non-negative. This requirement will be tested based on predicted economies of size.

(4). Second differentiable. This implies Symmetry. $\beta_{ij} = \beta_{ji}$

(5). Concave and continuous in w.

This implies the following matrix H is negative semi-definite.

H= $\begin{pmatrix} C_{11} & C_{12} & C_{13} & C_{14} \\ C_{12} & \cdots & \cdots & C_{24} \\ \cdots & \cdots & \cdots & \cdots \\ C_{14} & \cdots & \cdots & C_{44} \end{pmatrix}$ Where C_{ij} is the second derivative of cost function with respect to Wi and Wj. This requirement will be tested based on the predicted G-matrix suggested by Diewert and Wales (1987, p. 48).

(6). Homogeneous of degree one in prices. This implies:

$$\sum_{i=1}^{4} a_i = 1 ; \sum_{i=1}^{4} \beta_{ij} = \sum_{j=1}^{4} \beta_{ij} = \sum_{i=1}^{4}\sum_{j=1}^{4} \beta_{ij} = 0 ; \sum_{i=1}^{4} \gamma_i = 0 ; \sum_{i=1}^{4} \psi_i = 0$$

(7)Adding-up. $\sum_{i=1}^{4} a_i = 1 ; \sum_{i=1}^{4} \beta_{ij} = 0 ; \sum_{i=1}^{4} \gamma_i = 0 ; \sum_{i=1}^{4} \psi_i = 0$

B. Random parameter model and estimation

The model 8 can be transformed into the following model assuming some parameters are random:

$$y_{(ik)t} = X_{(ik)t}\beta + \eta_{(ik)t}$$
$$\eta_{(ik)t} = X_{(ik)t}\rho_{(ik)t} + u_{(ik)t}$$

$X_{(ik)t}$, $u_{(ik)t}$ and $\rho_{(ik)t}$ are all stochastically independent.

$$u_{(ik)t} \sim IIN(0_{GI}, \sum{}^{u}), \rho_{(ik)t} \sim IIN(0_{MI}, \sum{}^{\rho})$$

Where G is number of equations. M is number of independent variables.

$$\sum{}^{u} = \begin{pmatrix} \sigma^{u}_{11} & \cdots & \sigma^{u}_{1G} \\ \vdots & & \vdots \\ \sigma^{u}_{G1} & \cdots & \sigma^{u}_{GG} \end{pmatrix} ; \sum{}^{\rho} = \begin{pmatrix} \sigma^{\rho}_{11} & \cdots & \sigma^{\rho}_{1M} \\ \vdots & & \vdots \\ \sigma^{\rho}_{M1} & \cdots & \sigma^{\rho}_{MM} \end{pmatrix}$$

So $\eta_{ik} \mid X_{(ik)}$ are stochastically independent. $\eta_{ik} \mid X_{(ik)} \sim N(0_{GK,I}, \Omega_{IK})$

Ω_{ik} is GK*GK matrix.

$$\Omega_{ik} = X_{ik} \sum{}^{\rho} X'_{ik} + I_k \otimes \sum{}^{u} .$$

The joint log-likelihood function of farmer (ik), i.e. $y_{(ik)}$ conditional on $x_{(ik)}$ is :

$$L_{(ik)} = -\frac{Gk}{2} Ln(2\pi) - \frac{1}{2} Ln|\Omega_{ik}| - \frac{1}{2}[y_{(ik)} - X_{(ik)}\beta]' \Omega_{ik}^{-1} [y_{(ik)} - X_{(ik)}\beta]$$

We write the log-likelihood function of all observation on y conditional on all observations of x as:

$$L = \sum_{k=1}^{K}\sum_{i=1}^{N_k} L_{(ik)} = -\frac{Gn}{2} Ln(2\pi) - \frac{1}{2}\sum_{k=1}^{K}\sum_{i=1}^{N_k} Ln|\Omega_{ik}| - \frac{1}{2}\sum_{k=1}^{K}\sum_{i=1}^{N_k}[y_{(ik)} - X_{(ik)}\beta]' \Omega_{ik}^{-1} [y_{(ik)} - X_{(ik)}\beta]$$

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