Environmental Restoration of Invaded Ecosystems: How Much Versus How Often?

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Selected Paper Prepared for Presentation at the AAEA Meetings, July 2005 and Society for Risk Analysis Meetings, December 2005

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Abstract

This paper derives the optimal level of restorative efforts required to restore environments degraded by invasive species invasion. Specific attention is focused on a case when the restoration faces the risk of failure through relapse of the restored environment. The level of restored environment may also play a role in its future improvement or susceptibility to failure. The tradeoff between the optimal level of environmental quality and number of restorative efforts required to attain that given environmental quality is analyzed.

Keywords: Environmental restoration, Resilience, Restoration failure, Invasive Species
**Introduction**

Invasive species are a noticeable source of biodiversity degradation (Glowka et al. 94). Lately, invasive species have become a subject of widespread concern due to enormous economic and environmental damages they inflict upon society (Pimentel et al. 1999, 2000). While a number of options exist to prevent the advent of invasive species, none of them are foolproof. Once the species have invaded a given eco-system, steps could be taken to either control them in part or eradicate them. However, it is rarely economically or physically viable to eradicate them. Yet, in most cases the invaded environment could be restored to a certain extent in order that society may continue to derive economic and environmental services from it.

Recent studies on the economics of invasive species management include those by Shogren (2000), Knowler and Barbier (2000), Olson and Roy (2002), Eiswerth and Van Kooten (2002), Perrings (2003), etc. While these studies focus mostly on the optimal combination of prevention and control options, one possible option is to take restoration measures to bring the invaded eco-system close to its pre-invaded state through eradication of the pest and recovery of the eroded environment. This process of restoration may be more relevant for recovery of habitats such as grasslands, lakes, wetlands, etc., which provide environmental services to the society in their un-invaded states.

This paper looks at the important issue of the extent of optimal restoration of an invaded environment that provides economic amenities to society, but is faced with the threat of repeated invasions\(^1\). Possibility of re-invasion, which might lead to failure of the restoration project, is considered. While it is possible for a restoration project to fail
independently of any invasion, this possibility is ruled out here to concentrate on the issue of failures caused by re-invasions only. This also allows us to focus on the important issue of extent of restoration when the risk of failure may be related to the level of restoration.

Current work on restoring invaded ecosystems has been mostly confined to the field of restoration ecology. Yet, there are significant issues of economic importance that come into play while deciding the extent of restoration. Total restoration may neither be feasible nor desirable for most invaded ecosystems due to the high costs associated with achieving and maintaining them. Further, restored eco-systems face the risk of falling back into degraded states from repeated invasions. Therefore, restoration efforts that do not incorporate this possibility of failure are bound to lead to inefficient outcomes. Most restoration efforts, after the initial investment, require substantial subsequent efforts to constantly monitor and fight the invasives for sustained periods of time. This is an essential feature of restorative efforts that are specifically targeted against invasions. The restored environment may face continuous threats from invasion even as restoration efforts are underway.

Experimental work on restoration ecology has revealed that degraded eco-systems may be resilient to restoration efforts owing to changes in landscape connectivity and changes in native species pools from invasion by exotics (Suding et al. 2004). Suding et al. point out that understanding the nature of feedback between the abiotic and biotic factors in the degraded state of the environment is crucial for the success of restoration projects. These feedbacks often lead to degraded states that are resilient to restorative efforts. By simply restoring the abiotic factors and omitting the biotic factors, the pristine
level of environmental quality may not be achieved. In such cases, either due to loss of dominant species, or significant changes in the environment, there is a regime change which is non-linear in nature and which might exhibit hysteresis (Mayer and Reitkerk, 2004 and references therein). Suding et al. cite several reasons for this resilience effect seen in degraded environments. These include the ‘species effects’, ‘trophic interactions’ and ‘landscape connectivity. The species effect occurs when the new species has changed the characteristics of the invaded environment in its favor (and against the native species) eventually carving out a niche for itself. Thus, by doing so it makes the degraded environment much more resilient to restoration efforts. One case is that of grasses in Hawaii woodlands that make the environment fire prone by altering the nitrogen cycle, thus eliminating the native shrubs (Suding et al. 2004). Similarly, habitat destruction can cause a reduction in the native seed-source, thus making the restoration of the native species much more difficult. Finally, trophic interactions between the herbivores in the Savannas have led to the replacement of the woody grasses with the inedible invasive shrubs. In fact, most systems that exhibit strong or moderate abiotic regimes, such as arid grasslands, rangelands, wetlands, etc. are prone to regime shifts that lead to resilience in restoration (Didham et al. 2004).

A crucial economic issue is then over the extent of restorative efforts to be undertaken per period when the environment exhibits resilience and the risk of failure of restoration projects is real. For instance, invasive species that lead to frequent fires may be countered by planting other species that compete with them and are fire resistant. However, in case of a fire break-out species of both kinds would get eliminated, thereby negating all the previous efforts of restoration. Another related issue is over the level of
restorative efforts when risks are stock-dependent. In the above case, the more the species of fire-resistant kind are planted; the lower would be the risk of failure of restorative efforts. Further, higher stock of fire-resistant species may exhibit stock-dependent resilience, i.e., once a threshold level of fire-resistant species has been reached, there may be a sharp decline in the level of other restorative efforts required to preserve the level of restored environment.

Restoration and resilience improving measures under risk have been found to be at the center of issues that concern invaded ecosystems in the ecology literature. However, these issues also make the economic analysis fairly complicated, as the non-linear attributes of the ecological processes must be included in a traditional cost-benefit approach. Currently there are no known applications of restoration risks on invasives species in the economics literature.

In this paper, a model of environmental restoration is designed that incorporates the risk and resilience effects associated with environmental restoration. The issue of how much restoration effort to undertake is then looked at in an inter-temporal cost-benefit analysis setting. When risk of failure may be stock dependent, the question of how much restoration versus how often becomes relevant, as the costs of continual but lower restoration must be weighed against the costs of less frequent but larger restorative efforts leading to a higher environmental quality. This also determines under what circumstances a more resilient state is desirable given the higher costs associated with its attainment. Numerical simulations reinforce these analyses. The literature on the management of ecosystems faced with non-linear dynamics and regime changes provides guidance over choice of consumption paths and control efforts that might prevent costly
reversals from one state to another (Maler et al. 2003). This paper extends that analysis and applies to the case of invaded ecosystems and their restoration when multiple equilibriums are possible. Significantly, valuable insights are derived over the optimal level of restoration of invaded environments faced with multiple threats of invasion, the risk of which might be stock-dependent. The model also provides insights into the tradeoff between the levels of restoration and the time required to attain it.

The paper first starts with a deterministic model, where restoration efforts are not faced with the threat of failure, in order to understand the role of resilience associated with environmental restoration. The analysis delves into the existence of multiple equilibriums with respect to environmental restoration. Next, risk of failure is introduced into the model. Finally, the tradeoff between the level of restoration and the frequency of failure of restoration is taken up in the above setting.

**Basic Model**

Consider a degraded environment that could provide recreational and environmental benefits upon restoration. The stock of environmental quality \( q \) evolves as:

\[
\dot{q} = \alpha d + \eta \frac{q^a}{q^a + b} - \delta q
\]

Environmental quality stock \( q \) improves from restoration efforts net of any natural rate of decay, \( \delta \). The amount of environmental quality lost to decay increases as the level of environmental quality improves. This assumption is made in order to make unlimited improvements in environmental quality difficult. Furthermore, it also captures the fragile balance that certain complex ecosystems like wetlands and lakes exhibit and which are prone to degradation from slightest perturbations caused by human and natural
forces. For instance, the restoration of Florida everglades requires amongst other things introduction of native species and restoration of water flow to prevent decay of vegetation, etc. It is likely that as the quality of restoration is improved through an increase in stock of native species, a larger quantity of the same would get lost from the stagnating vegetation if water flow is not improved.

There may be multiple options available for restoration; but in order to simplify things, we assume that it is possible to combine these options together into a single restoration variable ($l$). $\alpha$ is a parameter that transforms restoration efforts into stock of environmental quality. The second term on the right hand side in equation (1) captures a sharp upward jump exhibited in environmental quality once a threshold level is crossed. This term captures the resilience aspect of degraded ecosystems. Conventionally, resilience has been defined in two ways in the ecology literature. First one, termed as the ‘engineering resilience’ defines it as the speed of bouncing back of any perturbed system (Pimm 1984). The other one, termed the ‘ecological resilience’, is about the amount of stress that the system can tolerate before flipping from its original state to another stable but degraded state (Holling 1995, Carpenter and Cottingham 1997). In this paper we follow the ‘ecological resilience’ definition to model the impact of restoration. Parameters $\eta$, $a$ and $b$ define the rate and magnitude of this effect. This functional form is associated with the process of hysteresis in the ecology literature and is characterized by a sharp jump (but not irreversible) in the states of the ecosystem that make it costlier to revert back to. In this paper, restoration induced jump in environmental quality is defined in a positive sense, as beyond a certain threshold of environmental restoration the environment shifts into a better state and is more responsive to restoration efforts.
Alternatively, this formulation mandates that a willful restorative perturbation in environmental quality would not lead a system out of its degraded state unless some threshold is crossed.

Note that the restorative efforts do not necessarily have to add in environmental stock from outside. In most cases restorative efforts are simply about removing the cause of trouble. In most cases, even the degraded environments may have a capacity to grow back to their full potential, but are overshadowed by the negative forces such as pests that cause its degradation through a complex interaction with natural forces such as fire, droughts, floods, diseases etc. One particular example is the case of Buffel grass invasion in Queensland, Australia on the native species such as the Brigalow and Gridgee. Buffel grass pastures increase the risk of fire amongst these native species, and the more fire-infested the surrounding gets, the higher is the density of Buffel grass over time. Thus, in a positive feedback relationship with fire and the native species, Buffel grass has been able to replace most of these native species over time (Butler and Fairfax, 2003). Other examples of models involving resilience in grasslands can be found in Perrings and Walker (1997, 2004).

Benefits \( m(q) \) are derived per period from environmental quality. The cost of restoration \( c(l) \) is convex in restorative efforts, thus making unlimited restoration prohibitive. Let \( \mu \) be the shadow price of environmental quality and \( r \) the social discount rate. Society maximizes benefits from environmental quality net of restoration costs as:

\[
\text{(2)} \quad \text{Max} \int_{0}^{\infty} \{m(q) - c(l)\} e^{-rt} \, dt
\]
subject to the constraints posed on environmental quality evolution as given by equation (1). The current value Hamiltonian \((cvh)\) is written as:

\[
(cvh) = m(q) - c(l) + \mu (\alpha l + \eta \frac{q^a}{q^a + b} - \delta q)
\]

First order condition with respect to restorative efforts implies that per unit cost of restoration must be equated to the shadow value of that marginal unit of restoration.

\[
\frac{c'(l)}{\alpha} = \mu
\]

Co-state variable \(\mu\) evolves as:

\[
\dot{\mu} = -m'(q) + (r + \delta)\mu - \mu \frac{\eta ab q^{a-1}}{(q^a + b)^2}
\]

From (4) and (5), the time path of restorative efforts could be derived as:

\[
\dot{t} = \frac{-m'(q)\alpha}{c^a(l)} + \frac{c'(l)}{c^a(l)}(r + \delta - \frac{\eta ab q^{a-1}}{(q^a + b)^2})
\]

In a steady state, restorative efforts and environmental quality are held constant. From (1) and (6) we get:

\[
\alpha l + \frac{\eta q^a}{q^a + b} = \delta q
\]

\[
\frac{c'(l)}{c^a(l)} = \frac{c'(l)}{c^a(l)}(r + \delta - \frac{\eta ab q^{a-1}}{(q^a + b)^2})
\]

Equations (7) and (8) define a relationship between environmental quality and restorative efforts, which could be solved to derive their steady state values. The isoclines for which the levels of restorative effort and environmental quality are constant are represented in figure 1 below\(^vi\).

INSERT FIGURE 1 HERE
Note that there exist three possible equilibriums $L, U$ and $R$, the low, middle and the high environmental qualities respectively. Of the three, the low and the high equilibriums are the stable ones with the middle one being unstable. The resilience effect is depicted by a jump in environmental quality once environmental quality crosses the threshold given by the crest in the $\dot{q} = 0$ curve. The state below this threshold is the degraded state. Also notice that $R$ is the resilient equilibrium as environmental quality can be reduced significantly without letting the system flip back to the low quality steady state. The threshold, below which environmental quality falls into the ‘degraded’ state, is given by region on the right hand side of the unstable equilibrium. The state left to this threshold is the high-quality state or the resilient state. The ‘high equilibrium’, which is the resilient state, may not be possible to reach from a degraded state in all cases. If benefits from environmental restoration are lower than the costs incurred, or if the discount rate is high, or if the resilience effect is not very significant, the high equilibrium may not be desirable. Similarly, the low equilibrium may not be desirable under certain circumstances. Figure 2 depicts a case where benefits of restoration exceed their costs, thus leading to high equilibrium as the only possibility.

INSERT FIGURE 2 HERE

The effect from varying discount rate is depicted in figure 3 below. As the discount rate increases, only equilibrium that is possible is the low environmental quality one, whereas, with low discounting high quality is the only possible equilibrium. Consequently, time preferences play an important role in deciding the level of environmental restoration.

INSERT FIGURE 3 HERE
**Restoration with Relapse**

One issue that restoration projects are faced with is the relapse of restored ecosystems into their original degraded states. This could be caused by a number of factors such as renewed infestations which could be seasonal, climate-induced or man-made. Further, once the system flips back into the degraded state, restoration has to start all over again as environmental quality built up in the past is gone. This possibility is explored next. The methodology used for modeling the risk of invasion in this system is that of a Poisson process, based upon the work of Clarke and Reed (1994), Reed (1988) and Reed and Heras (1994). Further, Knowler and Barbier (2005) have applied this approach to model invasive species management under uncertainty.

In each time period, assume that the restored environment faces an instantaneous probability of failure denoted as $p$. The timing of failure is defined as a random variable, and for tractability assume that this variable has a Poisson distribution. Conditional on failure having not yet occurred, the probability of failure in any interval $dt$ is $pdt$ where:

$$\int_{0}^{t} pdt = \lambda(t)$$

If $T$ is a stochastic variable that represents the time of failure, the cumulative probability density function for failure is $F(t) = \Pr(T < t)$ and $F(t) = 1 - e^{-\lambda(t)}$, given the Poisson specification. The survivor function is $S(t) = \Pr(T > t) = 1 - F(t)$ which is defined as the probability of project survival up to time period $t$.

The manager is faced with the challenge of incorporating such possibilities into her optimization framework. The manager’s task is to maximize her long term value $V(q_0)$, starting with an initial level of quality $q_0$ as:
subject to (1), where \( p \) is the constant hazard rate of invasion characterized by a Poisson process. In equation (10), the term \( pV(q_0))e^{-\lambda(t)\cdot rt} \), when integrated up to infinity, captures the expected value from the system flipping back into the original state at time \( t \) and the manager having to start all over again. \( V(q_0) \) represents the value function from starting all over again from the initial level of environmental quality \( q_0 \). Equation (10) in its extended form can be re-written as:

\[
V(q_0) = \max \left[ \int_0^\infty (m(q) - c(l))e^{-\lambda(t)\cdot rt} dt + \int_0^\infty (pV(q_0))e^{-\lambda(t)\cdot rt} dt \right],
\]

given that \( \lambda(t) = pt \), the above relation can be further simplified as:

\[
V(q_0) = \max \left[ \int_0^\infty (m(q) - c(l))e^{-\lambda(t)\cdot rt} dt + pV(q_0) \frac{1}{r^2 + p} \right]
\]

Equation (12) reduces to:

\[
V(q_0) = \frac{r + p}{r} \max \left[ \int_0^\infty (m(q) - c(l))e^{-pt\cdot rt} dt \right]
\]

Setting up the current value Hamiltonian for the above problem, we get:

\[
(m(q) - c(l))e^{-\lambda(t)} \frac{r + p}{r} + \xi (\alpha d + \frac{\eta q^2}{q^2 + b} - \delta q),
\]

where \( \xi \) is the shadow price of environmental quality

The first order condition with respect to restorative effort yields:

\[
\xi = \frac{1}{\alpha} c'(l)e^{-\lambda} \frac{r + p}{r}
\]
Let \( \xi \beta = \beta \), be the adjusted shadow price of environmental quality. The rate of evolution of the shadow price is determined by the no-arbitrage condition as:

\[
\dot{\xi} = -m'(q) e^{-\xi} \frac{r + p}{r} + \xi (-\eta \frac{abq^{a-1}}{(q^a + b)^2} + \delta + r)
\]

Therefore, the rate of change of the adjusted shadow price \( \beta \) is given by:

\[
\dot{\beta} = -m'(q) \frac{r + p}{r} + \beta (-\eta \frac{abq^{a-1}}{(q^a + b)^2} + \delta + r + p)
\]

In steady state, \( \dot{\beta} = 0 \), implying:

\[
\beta = \frac{m'(q) \frac{r + p}{r}}{(-\eta \frac{abq^{a-1}}{(q^a + b)^2} + \delta + r + p)}
\]

Substituting for \( \beta \) from (15) above, we get the steady state relationship between restoration efforts and environmental quality as:

\[
c'(l) = \frac{m'(q) \alpha}{(-\eta \frac{abq^{a-1}}{(q^a + b)^2} + \delta + r + p)}
\]

Notice that in the no-risk case derived before, the steady state evaluation of equation (6) would yield:

\[
c'(l) = \frac{m'(q) \alpha}{(-\eta \frac{abq^{a-1}}{(q^a + b)^2} + \delta + r)}
\]

Equation (20) is similar to equation (19) except for the extra term \( p \) in the denominator of equation (19). When restoration efforts are faced with an ever present constant exogenous risk of invasion, this risk acts as an additional discounting term.
Consequently, the above analysis reveals that the steady state restorative efforts would be lower in the case when there is a risk of relapse as compared to no-risk case.

Notice that in the above equations (19 & 20); the increment in resilience from a change in stock serves as an adjustment to the discount rate which is further augmented by the natural rate of decay of environmental quality. From the way this resilience effect has been specified in the model, some interesting implications can be deduced for the optimal restoration path. The level of environmental quality shows a sharp jump upwards once a certain threshold level has been reached. Due to this reason, as long as environmental quality is lower than this threshold, the resilience effect will not be significant. Therefore, the discounting effect brought by a change in resilience due to enhanced environmental stock, kicks in only beyond that threshold level of stock. As a consequence, the change in the optimal steady state level of restoration effort and environmental quality from some external disturbance in parameters would be significantly higher if the steady state is closer to this threshold. In lay terms, the incentives for restoration efforts are higher; the closer the system is to the flipping threshold. Next we consider the case when risk of relapse may be stock dependent.

**Stock Dependent Risk**

In the stock-independent risk case, the relationship between the hazard rate and environmental quality is specified as:

(21) \( \hat{\lambda} = p(q) \)

The value function can be specified as before:
which can be further expanded for a starting level of environmental quality as:

\[
V(q_0) = \max \left[ \int_0^\infty (m(q) - c(l))e^{-\lambda - r} \, dt + \max_{\int_0^\infty} (p(q)V(q_0))e^{-\lambda - r} \, dt \right]
\]

Rewriting above we get:

\[
V(q_0) = \max \left[ \int_0^\infty (m(q) - c(l))e^{-\lambda - r} \, dt \right]_{(1 - \int_0^\infty (p(q))e^{-\lambda - r})}
\]

subject to the equations of motion for the hazard function as given by (21) and the environmental stock as given by (1). It is not very straightforward to analytically perform dynamic optimization on the above problem as it is not in a standard format; therefore, we take recourse to numerical simulationsvii.

Figure 4 shows the time paths of environmental quality from a slight increase in the hysteresis effect of \( \eta = .06 \) from the base case (see the Appendix for base case values) and with a lower discount rate of \( r = .03 \). We can see that the higher steady state equilibrium becomes attainable as the starting level of environmental quality increases \( (q_0 = 4.8) \).

INSERT FIGURE 4 HERE

Next we compare the time paths of restorative efforts under constant and environmental quality-dependent risk of failure with the no risk case. In the constant risk case, the hazard rate is assumed to be 0.1, where as in the environmental quality dependent risk case the hazard function is defined as:
\[(29) \quad p = p_0(1 - \theta q(t)), \quad \text{where} \quad p_0 = .01, \quad \text{and} \quad \theta = 1 & q(t) < 10\]

In the above equation \( p_0 \) is the constant component of the hazard rate. Clarke and Reed (1994) refer to \( p_0 \) as the policy-independent component of the hazard rate, as it is an exogenous parameter. Whereas, \((1 - \theta q(t))\) is the policy-dependent component that is endogenous to environmental quality in which \( \theta \) is the parameter that determines the effect of stock of environmental quality on hazard rate. Figure 5 shows the time paths of restorative efforts and environmental quality under the three cases for a starting level of environmental quality of 0.8. Notice that the highest level of environmental quality is attained when there is no risk of failure. Under a constant risk of failure, environmental quality attained is the lowest, whereas the endogenous-risk case has a higher environmental quality than the constant-risk case. The effect of risk is primarily to discount the future benefits from environmental quality. However, when the risk is stock dependent, environmental quality is increased to capitalize on its risk reducing impacts.

**INSERT FIGURE 5 HERE**

**How much versus how Often**

When restoration projects are faced with the risk of collapsing back into a degraded state, the question of how much effort to put in becomes important. If the ecosystem keeps collapsing into the degraded state time and again, it may take a long time before the desirable level of environmental stock is attained. Therefore, it may happen that systems that require a low level of restorative effort but are faced with high risks of reversal may take a longer time to reach their steady states as compared to systems that may require a higher level of restorative effort but are faced with a lower level of risk. Note that the risk of project failure has been optimally accounted for in the above models. However,
the above models do not say anything about the number of times the project would fail before a steady state is reached. The time taken to reach the steady state when there are no setbacks to the restoration project would differ from the case when setbacks exist. The actual time taken to reach a desirable level of environmental restoration would depend upon the number of times relapses happen before the goal is achieved. This concept is explored further in the setting of the model described above.

Let $t^*$ be the time it takes for the ecosystem to reach the steady state level of environmental quality without collapsing, when there is a constant risk of reversal back to the initial degraded level\textsuperscript{viii}. In presence of a constant risk of reversal, the expected time $E(t^*)$ taken to reach the steady state would be given by:

$$E(t^*) = \int_{0}^{t^*} \left[ s + E(t^*) \right] pe^{-ps} \, ds$$

(30)

Notice that in the above formulation, once the system reverts back into the degraded state it has to start all over again, and therefore, would take the same amount of expected time thereafter. $s$ is the time at which the restoration project fails, thus sending the system back to its initial level. Also, $s$ ranges from 0 to $t^*$, and the probability of failure is exponentially distributed with hazard rate $p$. Moreover, the system faces risk of reversal even after the steady state has been reached, but, by the optimal nature of that steady state it would mean that restorative efforts and environmental stock maximize expected benefits at that level of risk. Integrating the above term we get:

$$E(t^*) = \left[ -spe^{-ps} - \frac{e^{-ps}}{p} + E(t^*)(-e^{-ps}) \right]^{t^*}_{0}$$

(31)

Solving the above equation one derives the expected time as:
Figure 6 below plots the contours for $E(t^*)$ for a range of values of the hazard rate $p$ and $t^*$. Notice that the expected time taken is much higher when either $t^*$ or $p$ are higher.

The case of stock-dependent risks is slightly complicated. The time taken to reach the steady state without any interruptions is a function of the discount rate, the marginal benefits and costs of restoration, rate of decay of environmental stock and the resilience parameter. For example, a higher rate of discount would require a lower environmental stock and thus would take lesser time to reach as compared to a case when the benefits from environmental stock are higher or the costs of restoration are lower. Whereas, a lower rate of discount would make the resilient state more desirable thus requiring more time to traverse. Similarly, a lower level of $p_0$ (the constant component of the hazard rate) would make a higher environmental quality attractive. This is shown in figure 7 below, where maximum possible level of environmental quality falls with an increase in $p_0$. However, it can be numerically verified that the expected time to steady state $E(t^*)$ is actually lower for the case when $p_0$ is 0.1 (420 time units) as compared to the case when $p_0$ is 0.5 (37279 time units). That is, even though the desirable level of environmental quality increases with a reduction in risks, the expected time taken to reach it falls. Using a time horizon of 250, the time to reach the steady state without relapses, ($t^*$), is 211 units for $p_0 = .1$ and 162 units for $p_0 = .5$. This is because the steady state level of environmental quality falls as $p_0$ rises. Notice that even as ($t^*$)

\begin{equation}
E(t^*) = \frac{e^{pt^*} - 1}{p} - t^*
\end{equation}
decreases as risk increases, the expected time to steady state, \( E(t^*) \) increases. This is so as an increase in \( p_0 \) also increases the number of relapses, thus increasing the total expected time.

**INSERT FIGURE 7 HERE**

If the hazard rate falls sharply with a marginal increase in the environmental stock, it would reduce the expected number of relapses over the same period, as the expected duration for a single relapse increases. This would have an effect of reducing the expected time to reach the steady state. However, the negative effect of environmental quality on the hazard rate would also make it beneficial to strive for a higher environmental quality as the hazard rate comprises one of the elements of adjusted discount rate as derived in equation (14) above. This is shown in figure 8 below where an increase in \( \vartheta \), parameter which determines the influence of environmental quality stock on hazard rate, leads to an increase in the maximum possible environmental stock. It can also be numerically verified that the expected time to steady state, \( E(t^*) \), decreases as \( \vartheta \) increases even though the time taken to reach the steady state without a relapse, \( (t^*) \), is higher for higher levels of \( \vartheta \).

**INSERT FIGURE 8 HERE**

The net impact that a stock of environmental quality could have on project success through its influence on hazard rate is slightly complicated. On one hand, it offers the incentive to attain higher stock of environmental quality, as a higher quality yields direct utility and also reduces the risk of relapse. On the other hand, a higher goal for environmental stock also means that a higher restoration effort is required to reach there. If costs are convex in restoration efforts, restoration efforts may need to be
stretched over a longer period of time. Consequently, the more time that is required for reaching the steady state, the higher would be the expected number of relapses. Therefore, a tradeoff between how much quality to strive for and how many failures in order to reach it is highlighted in the case of stock dependent risks of restoration.

The issue of expected time to steady state is important to policy makers as one important goal of restoration projects is to bring the system back to a level at which it could be exploited for direct economic uses. In the case when consumption of environmental quality leads to a reduction in its stock, additional restorative efforts will need to be taken in order to maintain the optimal steady state level.

Conclusion

In this paper, the role of restoration measures in improving environmental quality was looked at through the application of the concept of resilience. Optimal restoration efforts were derived when environmental quality impacted the risk of failure of restoration projects. It was shown that the environmental and economic parameters determined the desirability of the level of resilience, and a highly resilient environment might not always be either feasible or desirable.

The tradeoff between the extent of restoration and the number of failures was derived. It was shown that the expected time to reach the desirable state in the event of multiple relapses is a function of both the hazard rate ($p$) and the time taken to reach the steady state without a relapse ($t^*$). This relationship between $p$ and $t^*$ is convex, implying that the expected time to steady state under the possibility of relapses could be same for high risks of collapse but lower $t^*$ and low risks of collapse but a higher $t^*$. Note that $t^*$ could
be low due to the influence of several factors such as the discount rate, benefits and costs of restoration, etc. It also turned out that no straightforward derivation of expected time to steady state is possible when risks are stock dependent.

There exist several other challenges to restoration projects. Some even oppose the idea of human interferences in degraded environments. Holling and Meffe (1996) in an influential paper argue in favor of natural disturbances that help build the resilience of a system rather than human interventions that shield it against them. Conflicting opinions exist towards the choice of restoration tools, with some even claiming that exotic species themselves may play beneficial role in restoration of the environment as human interferences lead to further disturbances (Antonio and Meyerson 2002). However, when restorative options are available and their advantages are clear, it may be worthwhile to apply them, especially when the benefits from their restoration span economic and environmental goods. In case of environments invaded by alien species, the need for restoration is an urgent one, as invasive species pose serious threats of extinction of valuable native ecosystems. It must also be kept in mind that restoration projects need to incorporate longer time horizons and utilize the resilience effects offered by higher levels of environmental quality in order to be able to ward off current and future threats of invasion.
References


Appendix

Table 1: Parameters used for Simulation

<table>
<thead>
<tr>
<th>Parameters</th>
<th>$a$</th>
<th>$b$</th>
<th>$\gamma$</th>
<th>$\delta$</th>
<th>$n$</th>
<th>$\eta$</th>
<th>$\alpha$</th>
<th>$r$</th>
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</thead>
<tbody>
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<td>10000</td>
<td>2</td>
<td>.01</td>
<td>4</td>
<td>.05</td>
<td>.02</td>
<td>.05</td>
</tr>
</tbody>
</table>

Table 2: Functional forms of Cost and Benefit Functions

\[
m(q) = q^7
\]
\[
c(l) = l^n
\]
Figure 1: Steady State Levels of Environmental Quality and Restoration

\[
\begin{align*}
q &= 0 \\
\dot{q} &= 0 \\
\dot{l} &= 0 \\
\end{align*}
\]
Figure 2: A Case of Resilient Steady State (high stock benefits)
Figure 3: Restorative Effort Isoclines for various Discount Rates
Figure 4: Time paths of restorative Efforts from two Starting Levels of Environmental Quality When Hysteresis Effect is Substantial

Note: The direction of evolution of environmental quality from their starting levels is illustrated through the arrows. The transition path has been solved through discrete dynamic optimization using GAMS.
Figure 5: Restoration and Quality Levels under no-risk, constant-risk and quality stock-dependent-risk
Figure 6: Contours of Expected Time to Steady State in Presence of Risk
Figure 7: Environmental Quality Levels under Varying Levels of Policy-Independent Component of the Hazard Rate (\( p_o \))
Figure 8: Environmental Quality Levels under $\varphi(\mu)$
i The level of amenities depends upon the level of environmental quality, which in turn would be affected by the extent of restoration.

ii For example, invasive plant species may survive through the next season through their seeds, which may be hard to eliminate.

iii It also allows us to explore steady state properties of the system. When the decay parameter is absent from equation (1), it is possible to make unlimited improvements in environmental quality. However, situations where the costs of restoration are significant and the time horizon not too long, the net benefits from the environment would converge, thus making an expected cost-benefit analysis feasible.

iv For instance, Keohane et al. (2000) incorporate both natural decay and recovery of the environment while modeling the optimal restoration problem.

v These benefits are ecological benefits that do not deplete from public consumption. Ecosystems such as grasslands, forests and fisheries are also subjected to direct harvests that lead to a reduction in the environmental stock. This has not been modeled here as the primary goal of restoration may not be immediate consumption in most cases.

vi The shapes of the cost and benefit curves are assumed to be non-linear, the relevant parameters of which are shown in the Appendix.

vii In order to reduce the problem into a standard framework one may define two more state variables as $z_1$ and $z_2$, whose rate of change is defined as:

$$
\dot{z}_1 = (m(q) - c(l))e^{-\lambda - rt} \quad \&
\dot{z}_2 = p(q)e^{-\lambda - rt}
$$

Now the above problem in equation (23) reduces to:

$$
V(q_0) = \text{Maximize} \frac{z_1}{1 - z_2}, \text{ subject to }
$$

$$
\dot{q} = ad + \eta \frac{q^\alpha}{q^\alpha + b} - \delta q, \quad \dot{\lambda} = p(q), \quad \dot{z}_1 = (m(q) - c(l))e^{-\lambda - rt} \text{ and } \dot{z}_2 = p(q)e^{-\lambda - rt}
$$

The current value Hamiltonian of the above problem that would maximize $V(q_0)$ is defined as:

$$
\frac{z_1}{1 - z_2} + \gamma_1 (ad + \eta \frac{q^\alpha}{q^\alpha + b} - \delta q) + \gamma_2 p(q) + \gamma_3 (m(q) - c(l))e^{-\lambda - rt} + \gamma_4 (m(q) - c(l))e^{-\lambda - rt}
$$

The first order conditions along with the equations of motion for the co-state variable would yield a time path for the restorative efforts and the environmental quality. Though, this approach is presented here for academic interest, the numerical simulation approach is much simpler to solve through GAMS.

viii Analytically, in most steady state problems it may take an infinite amount of time for the system to reach the steady state. But, for practical purposes, $t^*$ can be calculated using dynamic optimization software such as GAMS.