UPDATING THE ESTIMATION OF THE SUPPLY OF STORAGE MODEL

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Introduction

Since Working's seminal articles appeared in 1948 and 1949, the supply of storage has been investigated extensively. Most commonly, a single regression equation has been estimated involving stocks, usually measured as the stocks-to-use ratio, and the inter-temporal spread between a distant futures price and a nearby futures price or cash price, adjusted for storage cost. This study updates the estimation of the supply of storage model to reflect recent developments in the theoretical and empirical literature.

Recent theory (see Khoury and Martel (1989) for an early example) suggests that price variability should influence the level of stocks. However, no study has been located that incorporated price variability into an estimation of the supply of storage using observed data.

Recent empirical work has explored how to measure convenience yield (see Brennan (1991), Milonas and Thomadakis (1997a and 1997b) and Heaney (2002), among others). In this analysis, we use a proxy measure proposed by Heaney. Building upon Longstaff (1995), Heaney argues that convenience yield is the value of the option to sell stocks before the end of the storage period should a high price occur. He shows that the value of this option is related to the difference between the variability of the cash price and the variability of the futures price at the end of the storage period.

Last, using contemporaneous data for variables in a regression equation reduces measurement error by aligning the price quoted at a given time with the information available to the market at that time. The advent of option trading makes it possible to obtain contemporaneous market-determined measures for price variability.

The rest of the paper is structured as follows. The next section contains a review of the literature related to the supply of storage. Then, a supply of storage model is developed, measurement issues and data are discussed, estimation issues are addressed, and the results are discussed. The last section contains a summary, conclusions, and suggestions for further research.

Literature Review

The literature review is grouped into two types of studies of the supply of storage theory. The first category consists of studies that empirically estimate the supply of storage theory. The other category includes studies that use option pricing theory to measure convenience yield.

Empirical Studies of the Supply of Storage

In his seminal paper, Working (1948, 1949) posits that an inter-temporal price spread, i.e., the difference between a nearby and a distant price for the same commodity, is a return to storing the commodity over the time interval. Thus, both negative inter-temporal spreads (i.e., nearby price exceeds distant price) and positive inter-temporal spread are a market determined return to storage. Working uses Kaldor's (1939) idea of convenience yield to explain the holding of stocks when inter-temporal spreads are negative. Kaldor argues that convenience yield is the benefit that accrues to a stock holder from being able to continue producing during a time of scarcity and from avoiding the cost of ordering frequent deliveries and/or waiting for deliveries. Working argues that this convenience yield is greatest when stocks are small and smallest (even zero) when stocks are large. In essence, Working argues that convenience yield offsets the loss from the expected decline in price forecast by the inter-temporal spread.

Telser (1958) develops a theory of stockholding in the presence of futures markets. Demand and supply functions for storage in a two-period model are posited. Convenience yield is used to explain the holding of stocks when the inter-temporal spread between nearby and distant futures contract is negative. As predicted by the theory, the inter-temporal spread for cotton and wheat is inversely related to the size of stocks over the 1926 -1954 period.

Brennan (1958) develops theoretical demand and supply functions for storage in the context of a two-period model with uncertainty for a profit maximizing storage firm. Marginal storage cost is identified as the marginal outlay on physical storage plus a marginal risk premium minus marginal convenience yield. For several agricultural commodities, Brennan plots the relationship between end-of-month stocks and net marginal storage cost, which is measured as the inter-temporal price spread minus marginal outlays for physical storage. A negative relationship is found.

Weymar (1966) develops an inter-temporal pricing model which reveals that the inter-temporal spread between cash and future prices is a function of expected inventory behavior, not current inventory as Working posits. Weymar argues that Working's supply of storage model is likely to hold when the expected future inventory pattern can be approximated by current inventory level. He expects this condition to hold for agricultural commodities because their limited harvest period means that inventory levels usually decline continuously between harvest periods.

Gray and Peck (1981) analyze the pricing performance of the Chicago Board of Trade (CBOT) wheat futures during delivery. The analysis was prompted by a Commodity Futures Trading Commission (CFTC) order that terminated trading in the CBOT 1979 March wheat futures contract. Their analysis does not support the CFTC's conclusion that a distortion existed. The inter-temporal spreads involving the 1979 March contract are similar to the historical relationship between these spreads and U.S. stocks of soft red wheat and in particular to soft wheat stocks at Chicago. However, unlike Working, they find that the March spreads are no longer related to U.S. wheat stocks. They attribute this finding to changes in the U.S. wheat market.

Using data from the U.S. wheat market from the 1970s, Sharples and Holland (1981) find that publicly-held stocks displace, at least in part, privately held stocks. Specifically, they find that a one bushel increase in wheat stocks held in the publicly-subsidized Farmer Owned Reserve increases total U.S. wheat stocks by 0.86 bushels.

Thompson (1986) estimates supply of storage equations using New York and London futures prices between 1964 and 1982 for cocoa and between 1973 and 1982 for coffee. A relationship is found between world stocks of cocoa carried between crop years and the price spread involving the September (old crop) and December (new crop) contracts. However, no relationship is found between various measures of spreads and world stocks for coffee. Although the relationship is highly variable, the best fit for a coffee price of storage curve is obtained using U.S. stocks.

Fama and French (1987) test both Kaldor-Working's theory of storage and Keynes' theory of risk premium. They use data for 21 commodities, including metals, agricultural, and wood products. To test the theory of storage, they regress the cash-futures basis against the nominal interest rate and monthly seasonal dummies. They find consistent evidence that the basis varies one-for-one with the nominal interest rate and that seasonals exist in the basis for many of the seasonally produced agricultural commodities. Both results support the theory of storage. To test for a risk premium, they regress the difference between the futures price at time t for maturity T and the cash price realized at time T against the cash-futures basis at time t. As a group, the evidence for a risk premium is mixed. The authors conclude that they find more evidence in support of the theory of storage than the risk premium theory.

Brennan (1991) posits several theoretical models, each with a different specification of convenience yield. Maximum Likelihood estimates of the models are reported for precious and commercial metals over several sample periods from January 1966 though December 1984. The estimated value of convenience yield differs significantly from zero for most of the metals and sample periods for only one of the four models. The estimates of convenience yield derived from this model are negatively related to the level of stocks, consistent with Kaldor's and Working's characterization of convenience yield.

Consistent with the price of storage theory, Heaney (1998) finds that a single cointegrating vector exists among a constant term, interest rate, three month lead futures price at the London Metals Exchange (LME), cash LME lead price, and the total stocks held in LME-approved warehouses. Physical storage cost is assumed to be a fixed proportion of the spot price, and thus is part of the constant term. Stocks are used to proxy for variables related to the level of stocks, with the two most likely being convenience yield and risk premium. The data involves quarterly observations from March 1970 through June 1995.

Sorensen (2002) develops a pricing model that includes the seasonality of prices found in the term structure of futures prices. The model is estimated using weekly futures data for corn, soybeans, and wheat traded at the Chicago Board of Trade between January 1972 and July 1997. Consistent with Kaldor and Working, an inverse relationship is found between convenience yield and the ratio of U.S. stocks to production.

Convenience Yield as an Option Value

Heinkel, Howe, and Hughes (1990) note that convenience yield can be recast as an option value available only to holders of stocks. The option value is derived from the ability to sell the cash commodity for a higher price should it materialize while the stock is being held. They construct a three-date theoretical model in which demand was uncertain. Storage agents are assumed to be risk neutral and sign a contract at time 0 to sell any stock remaining at time 2 for the futures price quoted for time 2 at time 0. As with the traditional view of convenience yield, the model reveals that the level of stocks is negatively related to the option value measure of convenience yield. It also reveals that the option value measure of convenience yield is positively related to the marginal cost of production and negatively related to the serial correlation in spot prices. The higher the marginal cost of production, the less likely current production will occur to meet unexpected demand. Thus, the higher the option value to sell at intermediate time 1. The more negative the serial correlation among spot prices, the more likely that low (high) futures prices at time 0 are associated with a high (low) cash price at time 1. Thus, the option value of holding stocks at time 0 is higher (lower).

Bresnahan and Spiller (1986) note that Keynes (1930) proposes two explanations for negative inter-temporal spreads. One is the commonly-investigated risk premium theory. The second is the "liquid stocks" theory. The latter argues that the positive probability of a stock-out, i.e. no stock, situation can cause the cash price to exceed the futures prices. In such a situation, the cash price must be high enough to postpone demand until the arrival of new supplies. Bresnahan and Spiller show that, if uncertainty about supply exists, the probability of a stock-out occurring is always positive.

Milonas and Thomadakis (1997a and 1997b) construct a three-date storage model in which a storage decision is made at the intermediate date between the beginning and end of the crop cycle. They find that the decision to store or sell at the intermediate date had a payoff structure similar to a call option. This call option, which is a measure of convenience value, has value if a stock out is a possibility at the intermediate date. Their model implies that the value of the call option is positively related to the variability of cash price, and inversely related to the size of stocks, the time left until the end of the crop cycle, and the correlation between the intermediate period cash and futures prices. The model is tested using data from the copper, corn, soybean, and wheat markets for the period 1966 through 1995. Fisher's option valuation model is used to derive the call option estimate of convenience yield. Support is found for each of the hypothesized relationships.

Heaney (2002) estimates the call option value of convenience yield by adopting a valuation technique proposed by Longstaff (1995). Longstaff uses option pricing theory to estimate the upper bound on the value of liquidity in financial markets when restrictions exist on selling an asset. The upper bound equals the present value of the cash flow that could have been obtained if, during the time the asset was illiquid, a trader with perfect foresight could have sold the asset at what was known to be its highest price. Longstaff shows that this value equals the value of a call option with a strike price equal to the price of the asset when the restriction on selling the asset existed.

Heaney adopts Longstaff's technique to compute the value of profitable trading opportunities associated with holding a cash position instead of holding a futures position in an asset. The strike price of this call option is the futures price. Value of the call option is a nonlinear function of the price volatility of the underlying cash asset, price volatility of the futures contract, and the time to maturity of the futures contract. Heaney computes the call option value of convenience yield using data from cash and futures contracts traded for copper, lead, and zinc at the London Metals Exchange. He then compares the observed futures prices with theoretical futures price derived from the cost of carry model. Inclusion of the estimated convenience yield in the calculation significantly reduces the difference between the observed and theoretically derived futures prices.

Litzenberger and Rabinowitz (1995) posit that the current value of oil in a reserve can be conceptualized as the value of a call option written at a strike price equal to the extraction cost of the marginal producer. They show that the value of oil in reserve also equals the sum of the discounted difference between the futures price and the extraction cost, plus the value of the option to forego production in the future period. Both their two-period and multi-period models reveal that the existence of the call value on future production will cause the discounted futures price to be less than the current cash price at all times in the oil market. Furthermore, the futures price will be less then the current cash price if the uncertainty about future price is sufficiently large. Their model implies that, when riskiness increases, oil production is non-increasing and inter-temporal oil price spreads are non-decreasing. These implications are consistent with empirical tests conducted using data on U.S. oil production, U.S. oil reserves, and west Texas intermediate futures and options prices over the period from December 1986 through December 1991.

Richter and Sorensen (2002) posit a model that assumes that commodities exhibit seasonality patterns in both cash price level and volatility. Price dynamics are modeled using stochastic differential equations that are heterogeneous in time and are affine asset pricing models. Their model is estimated using a quasi maximum likelihood approach and a panel data of soybean futures and options prices from the Chicago Board of Trade for October 1984 through March 1999. Seasonal patterns exist in both volatilities and convenience yields. Consistent with the price of storage theory, a negative relationship exists between stocks and convenience yield. However, in contrast to the studies discussed above, no significant correlation exists between convenience yield and volatility. This finding is inconsistent with the argument that convenience yield can be modeled as a timing option.

Fackler and Livingston (2002) examine the option value of storage from a different perspective. They argue that in most situations the grain storage and marketing decisions of farmers are irreversible because high transaction costs prohibit the replenishment of grain once it is sold. This irreversibility creates an option value similar to that found in other irreversible economic decisions, such as wilderness preservation and private investments with large sunk costs. When an investment is irreversible, the optimal decision rule is to invest if the investment's net present value exceeds the sunk investment cost plus the American option value of waiting. A model of dynamic stockholding is developed for a risk neutral farmer. The marketing problem is found to have a number of commonalities with the optimal stopping problem of determining when to exercise an American option. The optimal sales rule reduces to the following condition based on current price: sell everything when the current price is high; otherwise sell nothing. Numerical computation is used to calculate the cutoff between high and low prices for soybean storage in central Illinois over the period from November 1975 through October 1997. The results reveal that including the value of the American option in the marketing strategy substantially increases storage returns.

Supply of Storage

This section presents the conventional price of storage model, as well a simplified version of recent price of storage models that incorporate risk. Next, the critical issue of measuring the variables is discussed. Included in this discussion is a recently proposed technique for generating a proxy measure of convenience yield.

Supply of Storage Models

The most commonly estimated price of storage equation is:

$$y_t = \beta_0 + \beta_1 x_{1,t} + \varepsilon_t \tag{1}$$

where y_t = stock to use ratio at time *t*,

$x_{1,t}$ = storage cost adjusted price spread at time *t*,

, ε_t = random error term, and

β_0, β_1 are coefficients.

Stock-to-use ratio is used instead of stock level because, everything else constant, the level of stocks carried by storage agents is expected to increase as the size of the market increases. Size of the market has conventionally been measured by quantity of consumption. The storage cost adjusted price spread is most often measured as an inter-temporal price spread involving a distant futures price and either a nearby futures price or a cash price, adjusted for the cost of storage over this time period.

More recent models incorporate risk. The model that follows is a simplified version of Khoury and Martel's (1989) price of storage model. Their model is a two-period model with a risk averse representative storage firm. The firm owns quantity Q of a commodity at time 0, the first period. It chooses between selling all, part, or none of Q at time 0 and storing the remainder for sale at the cash price that prevails at time 1. A futures market is assumed to exist, thus providing information that the firm can use to predict the spot price at time 1. Unlike the model presented in this paper, Khoury and Martel assume that the firm hedges the stocks it does not sell at time 0.

Assume the storage firm has a constant (local) relative risk coefficient, γ. Thus, its utility function can be written as:

$$U(R_1) = (\frac{1}{\gamma})(1 - e^{-\gamma R}) \qquad (2)$$

This representative storage firm seeks to maximize its expected utility from the revenue it expects to generate from its storage and marketing strategy by the end of time 1 as of time 0. Its revenue maximization problem can thus be stated as:

$$Max_X E[U(R_{0,1})] = Max_X E[(\frac{1}{\gamma})(1 - e^{-\gamma R_{0,1}})] \qquad (3)$$

where, $R_{0.1} = (Q_0 - X_0) S_0 \exp(1 + r) + X_0 (S_{0,1} - C)$ (4)

Q_0	=	quantity of commodity owned at time 0,
X_0	=	quantity of commodity stored at time 0,
r	=	risk free interest rate prevailing at time 0,
S_0	=	spot price of the commodity at time 0,
$S_{0.1}$	=	spot price of the commodity at time 1 expected at time 0, and

C = physical storage costs per unit during the storage period.

If $R_{0,1}$ is distributed normally as $N(\mu_{R_{0,1}}, \sigma^2_{R_{0,1}})$, equation (3) can be rewritten as:

$$\Phi(X) = \mu_{R_{0,1}} - (\frac{\gamma}{2})\sigma^2_{R_{0,1}} \tag{5}$$

where, $\mu_{R_{0,1}} = E((Q_0 - X_0)S_0 \exp(1+r) + X_0(S_{0,1}) - C)$ (6)

and $\sigma^2_{R_{0,1}} = X^2 \sigma^2_{S_{0,1}}$ (7)

Substituting equations (6) and (7) into equation (5) and taking the first order derivative with respect to stocks X_0 yields the following relationship:

$$\frac{d\Phi(X)}{dX_0} = -S_0 \exp(1+r) + S_{0,1} - C - (\frac{\gamma}{2})(2X\sigma^2_{E_{S0,1}}) \tag{8}$$

Rearranging the terms in equation (8), the optimal level of stocks, X_0*, is:

$$X_0{}^* = \frac{S_{0,1} - S_0 \exp(1+r) - C}{\gamma\sigma^2_{S_{0,1}}} \tag{9}$$

If the futures market provides an unbiased estimate of the future spot price, i.e., $F_{0,1} = E(S_{0,1})$ and the futures-cash basis at contract expiration is zero, equation (9) can be rewritten as:

$$X_0{}^* = \frac{F_{0,1} - S_0 \exp(1+r) - C}{\gamma\sigma^2_{F_{0,1}}} \tag{10}$$

Equation (10) reveals that the representative storage firm's optimal quantity of stocks is positively associated with the storage cost adjusted spread between the cash and futures price (i.e., the numerator), and inversely related to both the firm's degree of risk aversion and the current variability of the futures price for the contract for delivery at the end of the inventory holding period.

Variable Measurement

Measurement of risk aversion is difficult. Furthermore, a time series of risk aversion measures is needed for storage firms. No such data set exist. Thus, risk aversion is not included in this estimation of the price of storage curve.

The storage cost adjusted spread depends on a distant futures price, nearby futures price or cash price, and storage cost. Storage cost conventionally equals the sum of physical storage costs and interest, minus convenience yield. To minimize measurement error, it is desirable that each of these variables, along with stocks-to-use ratio and price variability be measured contemporaneously. In this context, contemporaneous means that each variable is measured as the value of the variable that the market is using to determine price. Contemporaneous thus aligns the information set with the price determined using the information set. In other words, variables are not measured at different times in terms of the dynamics of market price.

The advent of options trading makes it possible to extract market determined measures not only of the level of prices and inter-temporal price spreads but also the variability of prices. Specifically, implied volatility estimates can be derived from the options price. Implied volatility and the inter-temporal spread can be measured contemporaneously.

Equation (10) implies that the optimal quantity of stocks is less than zero if the expected return from storage is less than zero. However, as noted earlier, convenience yield has been proposed as a return to holding the cash commodity that offsets an expected loss from storing an asset when returns from storage is expected to be less than zero. While the existence of convenience yield is highly controversial, this study will examine whether an option pricing measure of convenience yield improves the empirical estimation of the supply of storage equation. Specifically, this study will utilize the method proposed by Heaney (2002).

Consider an arbitrage model in which an arbitrager buys and holds a cash asset while selling a futures contract whenever the expected net return to storing the asset is positive, i.e., futures minus cash spread exceeds the cost of storing the asset. On the other hand, if expected net return storage is negative, the arbitrager buys a futures contract and sells the asset in the cash market. The standard arbitrage model assumes that all positions are held until futures contracts mature. However, this assumption must be relaxed when investigating convenience yield because convenience yield can be greater than zero only when the inventory holder has the right to use or sell the asset at any time.

Heaney modifies the standard arbitrage model to account for convenience yield. Drawing on Longstaff's (1995) model for estimating the value of marketability (liquidity) of securities, Heaney notes that convenience yield attains maximum value to a trader if the trader has perfect foresight about the market that allows him to sell the asset at the highest price that will occur between the current time and the end of the storage period. Once this trader sells the asset at its highest price, he/she will invest the proceeds at then risk-free rate, and then buy the asset on the cash market at the lower price on the futures contract maturity date. .

A mathematic representation of the maximum price over the storage period from time t to time T can be expressed as follows,

$$Max(S) = \max_{t \le \tau \le T}\{\exp[r(T-\tau)]S_\tau\} \tag{11}$$

where, t = beginning of storage period

T = end of the storage period

S_τ = maximum cash price observed at time τ, where $t \prec \tau \prec T$

The convenience yield value of holding the cash commodity can be approximated as the value of an option to sell the commodity if price rises sufficiently to generate an arbitrage profit when the commodity is brought back at the end of the storage period. The value of this option, designated as $V(S_t,T)$, is:

$$V(S_t,T) = \exp[-r(T-t)]E(Max(S)) - \exp[-r(T-t)]E(S_T) \tag{12}$$

Heaney proposes that the value of this option (i.e., convenience yield) can be proxied through the following calculations:

$$cy_{tT} = v_{tT}(S_t,T) - v_{tT}(F_{tT},T) \tag{13}$$

where cy_{tT} = convenience yield of holding stock at time t, with latest sale at time T

$$v_{tT}(S_t,T) = \ln\{[2+\frac{\sigma_S{}^2(T-t)}{2}]N[\frac{\sqrt{\sigma_S{}^2(T-t)}}{2}] + \sqrt{\frac{\sigma_S{}^2(T-t)}{2\pi}}\exp[-\frac{\sigma_S{}^2(T-t)}{8}]\} \tag{14}$$

$$v_{tT}(F_{t,T},T) = \ln\{[2+\frac{\sigma_F{}^2(T-t)}{2}]N[\frac{\sqrt{\sigma_F{}^2(T-t)}}{2}] + \sqrt{\frac{\sigma_F{}^2(T-t)}{2\pi}}\exp[-\frac{\sigma_F{}^2(T-t)}{8}]\} \tag{15}$$

$\sigma_S^{\ 2}$ = variance of cash prices

$\sigma_S^{\ 2}$ = variance of futures prices

$N(\bullet)$ $N(\bullet)$ = cumulative normative distribution.

Equation 14 provides an estimated value based on the variability of the cash market. Equation 15 provides an estimated value based on the variability of the futures contract at the end of the storage period. Because convenience yield is the option value of potentially selling the commodity before the end of the storage period, the difference between these two values will be related to the convenience yield. In essence, the greater the variability of the cash market relative to the futures market, the greater is the value of having the potential option to sell before the futures contract matures.

Simultaneous Equation System

A causal relationship exists between convenience yield and the storage cost adjusted spread. As convenience yield increases, the storage cost adjusted spread becomes more negative, everything else held constant. Furthermore, the optimal level of stocks is related to the storage cost adjusted spread, among other factors. Thus, convenience yield, storage cost adjusted spread, and stocks are determined simultaneously. Hence, the following two-equation simultaneous system is proposed:

$$y_{1,t} = \alpha_0 + \alpha_1 x_{1,t} + \alpha_2 x_{2,t} + \alpha_3 x_{3,t} + \varepsilon_t \qquad (16)$$

$$x_{1,t} = \beta_0 + \beta_1 x_{4,t} + \upsilon_t \qquad (17)$$

where $y_{1,t}$ = stock-to-use ratio at time t,

$x_{1,t}$ = storage cost adjusted price spread,

$x_{2,t}$ = price volatility of futures contract for delivery at the end of storage period,

$x_{3,t}$ = price volatility of futures contract squared,

$x_{4,t}$ = Heaney's (2002) proxy measure of convenience yield.

This simultaneous equation system incorporates more information about the price of storage relationship, including information about price volatility and the non-observable convenience yield. Volatility is measured using a quadratic term in order to capture possible high-order nonlinear impacts of volatility on the stock-to-use ratio. In summary, this simultaneous equation system offers the potential to provide a richer understanding of the supply of storage theory.

Data

The supply of storage equation is estimated using data from the U.S. soybean market. The soybean market is selected because, among major U.S. crops, it never had farm program acreage set asides. Public stocks also have been limited in size and duration. Lastly, soybean options are among the most heavily traded commodity options markets.

The analytical period begins with stocks carried out of the 1988/89 crop year and ends with stocks carried out of the 2003/2004 crop year. While option trading on soybeans began during the 1984/1985 crop year, substantial public stocks of soybeans existed during the 1985/86 and 1986/87 crop years. Studies have documented that public stocks can displace privately held stocks and thus affect the price of storage equation (for example, see Sharples and Holland (1981)). To avoid this issue, this study uses data for 1987/88 though 2003/04 crop years.

Data used in this study are futures prices, options on futures prices, ending stocks and consumption for the current crop year, physical storage costs, and U.S. 6-month Treasury-Bill rates. Each variable is measured as of the release of the U.S Department of Agriculture (USDA) *World Agricultural Supply and Demand Estimates* (*WASDE*). The contemporaneous nature of this data set is a unique feature of this study.

WASDE is released each month throughout the year. It contains the latest USDA forecasts of U.S. and world supply and use balance sheets for the major grains, soybeans and soybeans products, and cotton, as well as U.S. sugar and livestock products for the current crop year. Beginning with May, it contains forecasts for the upcoming U.S. crop year.

The *WASDE* reports used in this study are the ones issued in February, April, and June. These months are selected because they are non-delivery months and thus avoid potential pricing problems that occur during delivery month. .Because ending stocks are analyzed, the futures prices are for the nearby contract and for the November contract. The nearby contract is March for February, May for April, and July for June. The November contract is considered the first new crop contract. Thus, the storage intervals of February-November, April-November, and June-November bridge the old and new crop years.

Prices and option premiums are the settlement values for the first non-limit trading day after the release of WASDE. This collection rule allows the market time to incorporate any new supply and demand data contained in the WASDE release.

The futures and option prices are from a data base maintained by the AgMAS project located at the University of Illinois at Champaign-Urbana. .The six month Treasury Bill rates are collected from the U.S. Federal Reserve Bank of St. Louis. Physical storage costs are collected from the U.S. Department of Agriculture, Commodity Credit Corporation. Implied volatility is calculated using Black's option pricing model for soybean option premiums and futures prices for the November contract.

Estimation Issues

Because stocks-to-use ratio, storage cost adjusted inter-temporal spread, and convenience yield are determined simultaneously, correlations might exist between the error terms of the two equations. A standard econometric procedure for addressing this estimation problem is three-stage least squares (3SLS). 3SLS is a system method that estimates all of the coefficients of the model, forms weights, and then re-estimates the model using the estimated weighting matrix. Because heteroskedasticity and autocorrelaton have been identified as potential statistical issues when using futures price data, heteroskadasticity and autocorrelation consistent (HAC) covariance estimation procedures are used in conjunction with the 3SLS estimation method.

Standard hypothesis-tests and statistical inferences are based on strong parametric assumptions. A critical assumption in classical multiple regression analysis is that the variable have a normal distribution. However, this assumption is generally not reasonable when using data from a small sample size, leading to the potential for distorted estimation results and statistical inferences. Bootstrap is a statistical technique commonly used to improve the power of statistical tests in the presence of small sample problems.

Bootstrap methods include both a nonparametric and a parametric mode. Nonparametric bootstrap, the original bootstrap, re-samples the values of variables by drawing from the empirical distribution with replacement. Parametric bootstrap re-samples residuals. Unlike parametric bootstrap, nonparametric bootstrap does not depend on a particular class of distributions. Both procedures assume that the sample's distribution is a good estimate of the population distribution.

This study uses the nonparametric bootstrap because it more effectively addresses heteroskadasticity than parametric bootstrap (Wu, 1986). The nonparametric bootstrap is implemented as follows: (1) draw a random sample (with replacement) from the empirical distribution of the original sample with a size equal to the size of the empirical sample; (2) calculate the statistic of interest; and (3) apply a Monte Carlo-style procedure by repeating steps one and two a large number of times. A sampling distribution of the statistic of interest is generated. This distribution is used to draw inferences about the population parameter. This study uses the bootstrap estimation method in Matlab. A total of 5000 simulations are run.

Estimation of Heaney's (2002) proxy for convenience yield requires only three variables, underlying commodity cash price volatility, futures price volatility, and the futures contract time to maturity. For this study, price volatility of the nearby futures contract (March, May, and July) are used instead of cash price volatility. Volatility of the November contract is used as the measure of futures price volatility. An historical volatility is calculated using the daily returns for the 20-trading-days immediately preceding the *WASDE* report release dates for February, April, and June. Heaney used these same estimation parameters.

Empirical Results

The storage cost adjusted price spread, or net storage return, is constructed as: ln(November futures price) – ln(nearby futures price + interest cost + physical storage cost over the storage window). The value of this variable is plotted against the stocks-to-use ratio for the February, April, and June observation dates in Figure 1. Examination of Figure 1 reveals that the relationship between these two variables takes the form of a natural logarithm. Thus, the stocks-to-use variable is measured as the ln of the stocks-to-use ratio. Previous studies also have mentioned this nonlinear relationship (see Gray and Peck, 1981, for example).

The estimation of the bootstrap equation (1) and the bootstrap three-stage least squares estimate of equations (16) and (17) are presented in Table 1. The estimation is conducted using Eviews 5.0 version.

Consistent with previous empirical studies and theory, the storage cost adjusted, new crop-old crop spread is statistically significant and has a positive relationship with carryout stocks-to-use. This relationship is found in both the single variable supply of storage modes and the multiple variable supply of storage model. Statistical significance occurs at the 99 percent level of statistical confidence in all six possible cases.

R^2 for the single equation supply of storage model is lower than the R^2 of the first equation of the updated supply of storage equation although little difference existed between the two R^2s for April. Thus, the evidence is mixed in terms ability of the multiple variable, multiple equation supply of storage model to increase the empirical explanation of the year-to-year variation in the soybean carryout stocks-to-use ratio.

The coefficients on both the linear and squared volatility terms are significant at the 99 percent level of statistical confidence except for the squared term in the April regression, which is significant at the 95% level of statistical confidence. Implied volatility is negatively related to carryout stocks-to-use while the squared term of the implied volatility is positively related to carryout stocks-to-use. This finding suggests that price volatility has a nonlinear relationship with carryout stocks-to-use. To explore this nonlinear relationship, a fitted stocks-to-use ratio is estimated for each observation month using the estimated parameters, mean value of the observed spreads, and observed values of implied volatility. The fitted ln values of the stocks-to-use ratio are converted to stocks-to-use ratios by using the exponential function. Results of this analysis reveal that, as volatility increases, the stocks-to-use ratio declines until a minimum level of 10% to 12% (see Figure 2). This non-linear relationship is not consistent with the theory developed in this paper and needs to be further explored.

Turning to the second equation of the simultaneous equations system, a statistically significant negative relationship is found between Heaney's proxy for convenience yield and the storage cost adjusted inter-temporal spread. Specifically, each one percent increase in Heaney's proxy for convenience yield results in the storage cost adjusted spread becoming more inverted by 2.8 to 3.55 percentage points. This relationship is consistent with Working's argument that convenience yield and an inter-temporal spread for a storable commodity are inversely related.

R^2 for the convenience yield equations are 0.58 and 0.59. A visual picture of this regression analysis is presented in Figure 3.3, which contains a scattergraph of the data used to estimate this relationship for each of the three observation periods.

Summary and Conclusions

This study updates the estimation of the supply of storage model to reflect recent developments in the theoretical and empirical literature. This is the first empirical study to incorporate both a measure of price variability, specifically implied volatility, and a proxy measure of convenience yield, specifically a measure proposed by Heaney (2002) based on work by Longstaff (1995). Heaney's proxy measure is based on the notion that convenience yield is the value of an option to sell stocks before the end of the storage period should a high price occur.

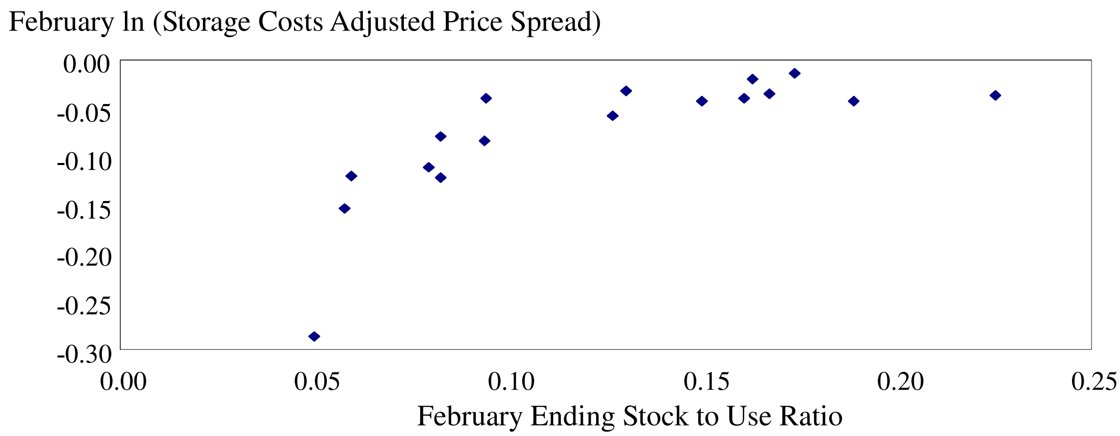
A simultaneous two-equation system model is estimated, consisting of a supply of storage equation and a price spread convenience yield equation. The mode is estimated for the U.S. soybean market using data from the 1987/1988 through the 2003/2004 crop years. All variables are measured contemporaneously to the release of the U.S.

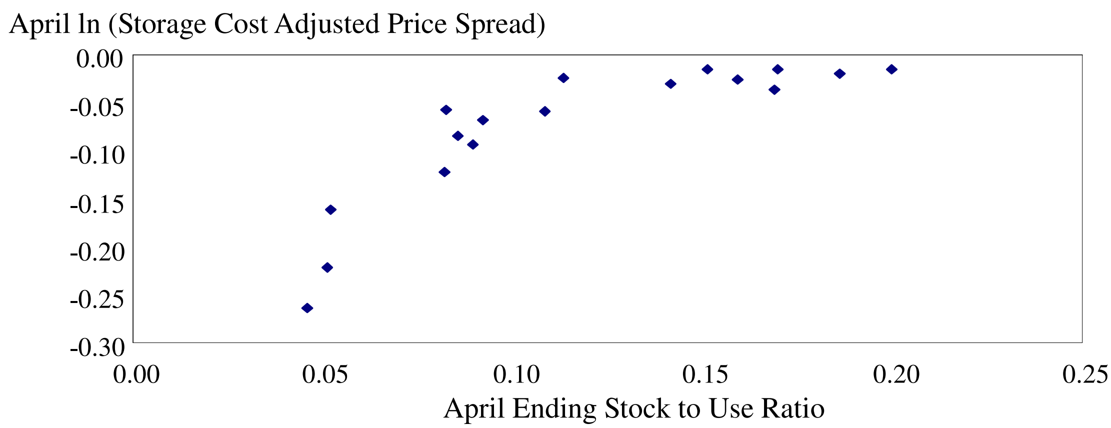
Department of Agriculture's *World Agriculture Supply and Demand Estimates* in February, April, and June.

A positive relationship is found between stocks-to-use and storage cost adjusted price spread, which is commonly accepted in the literature. While the theoretical literature proposes an inverse relationship between stocks-to-use and price risk, this study finds that the relationship is curvilinear. Once implied price volatility exceeds 25% stocks-to-use ratio begins to increase. An inverse relationship is found between the storage cost adjusted price spread and the proxy measure of convenience yield. This finding is consistent with Working's argument that convenience yield is a return to storage that can offset, at least partially, some of the loss expected from storing when the storage cost adjusted inter-temporal price spread is negative.

In summary, this study provides richer understanding of the supply of storage theory and convenience yield theory for the U.S. soybean market. It would useful to determine if these same results can be replicated in other commodity markets. Future research could also further examine the nonlinear relationship between price variability and stocks-to-use, including the development of a theoretical model to support such a relationship. Last, the relationship between the storage spread and Heaney's proxy measure, while significant, generates an explanatory power that is between 55% and 60%. Thus, additional work is needed on the measurement of convenience yield and its relationship to the storage cost adjusted inter-temporal spread.

Figure 1: Relationship between Futures Price Spread and Ending Stock-to-Use Ratio, U.S., February, April, and June *World Agriculture Supply and Demand Estimates* Release Dates, 1988-2004.





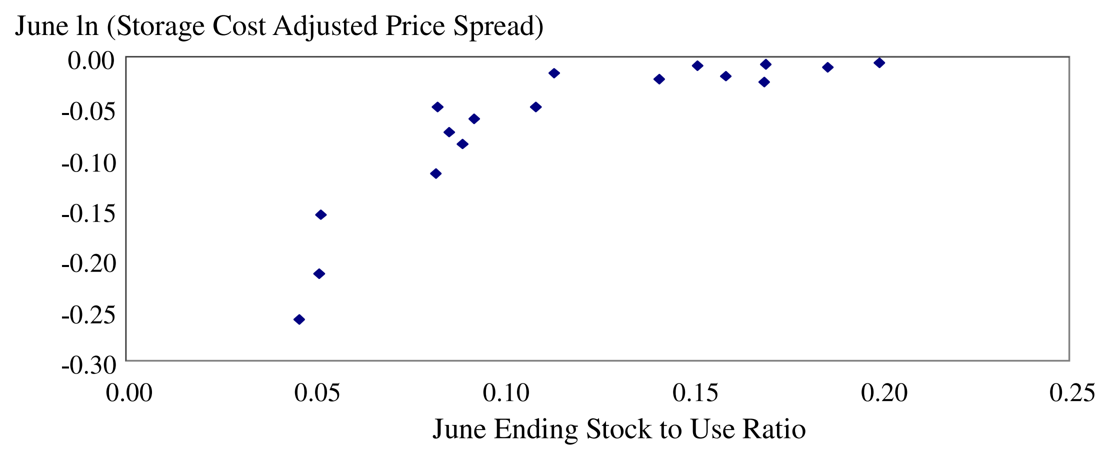


Figure 2: Plot of Fitted Stock-to-Use Ratio against Implied Volatility, U.S., February, April, and June *World Agriculture Supply and Demand Estimates* Release Dates, 1988-2004.

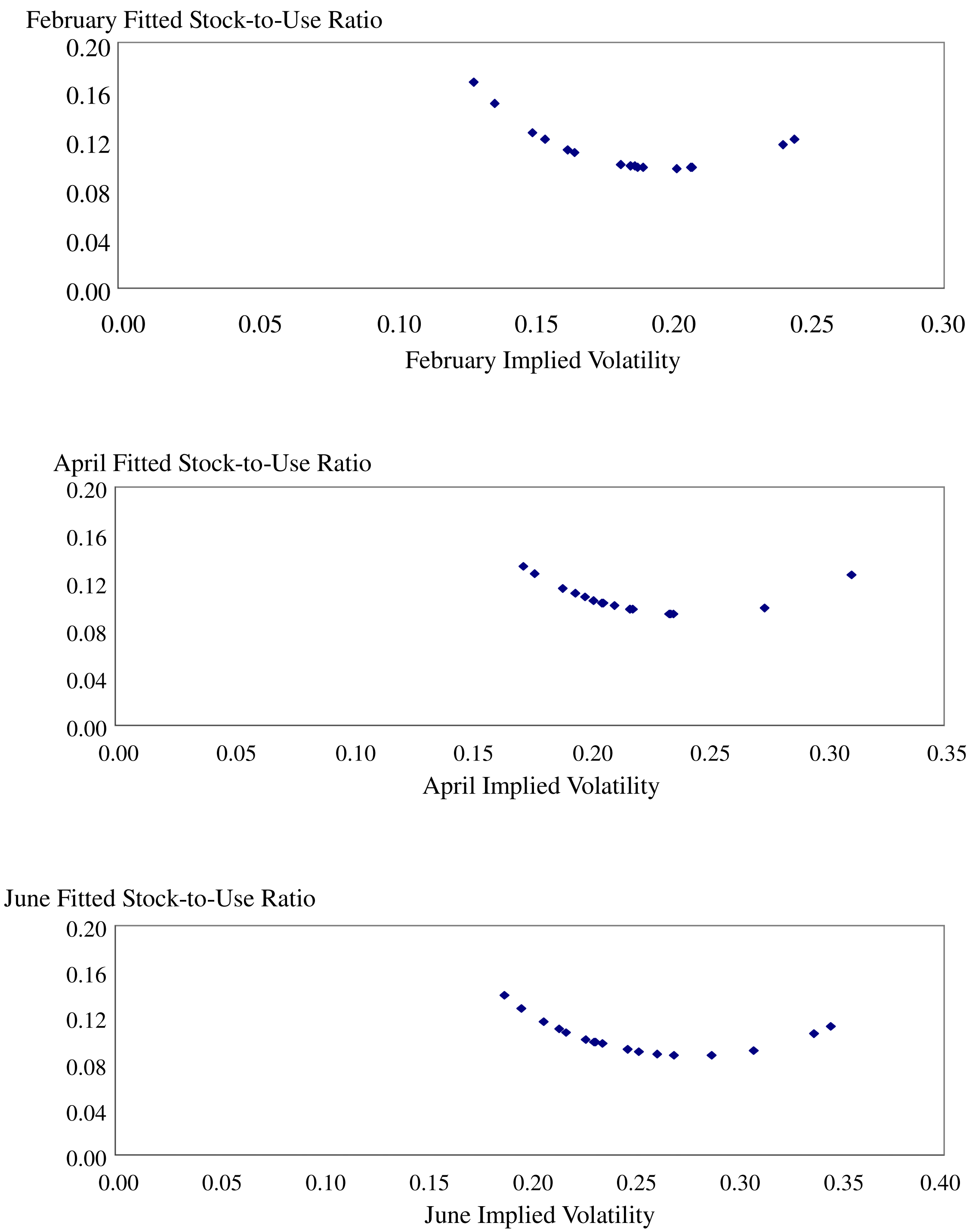


Figure 3: Plot of ln (Storage Cost Adjusted Price Spread) against Heaney's (2002) Proxy of Convenience Yield for February, April, and June, 1988-2004.

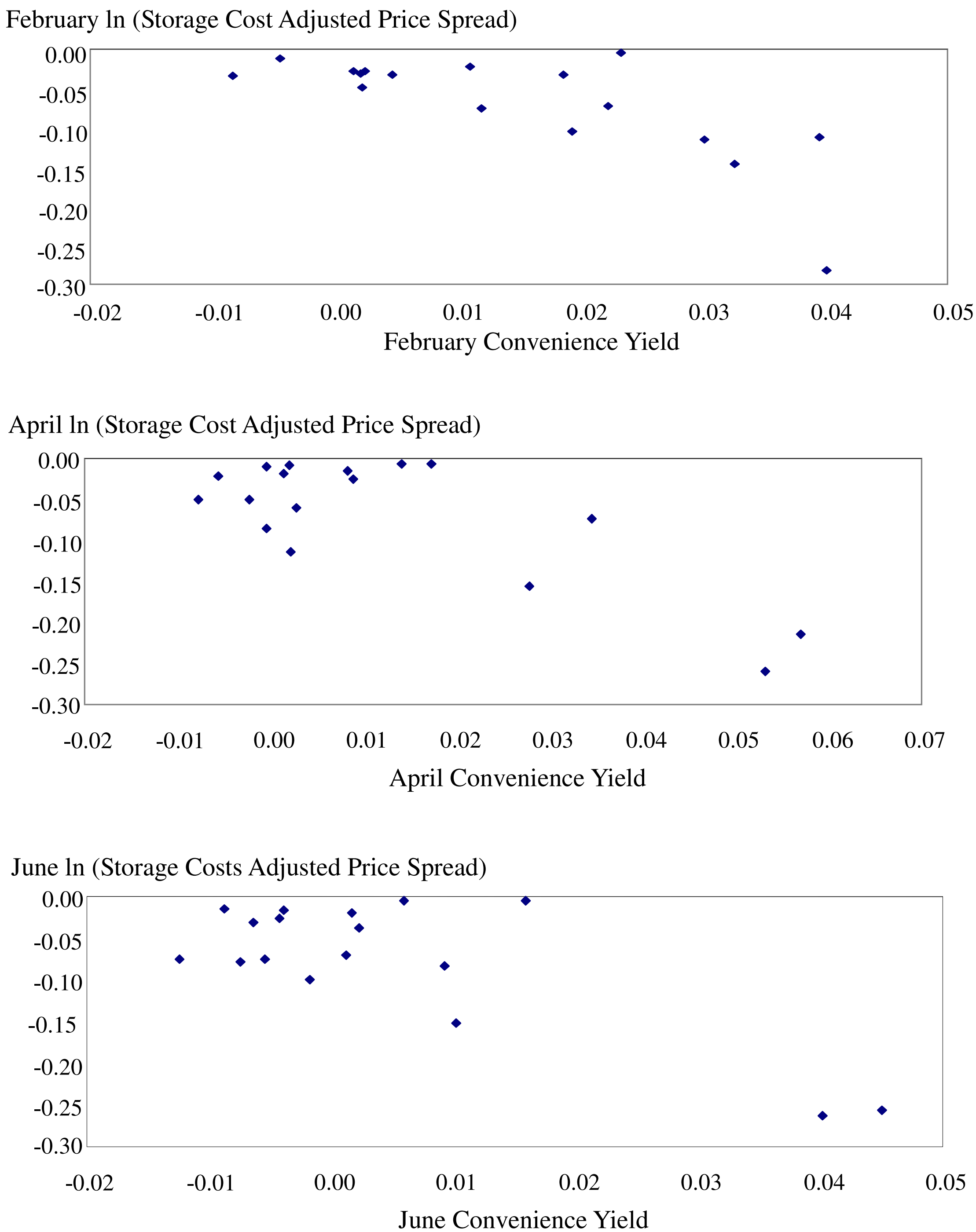


Table 1: Supply of Storage for Soybeans Estimated using Bootstrap Method, U.S., February, April, and June *WASDE* Release Dates, 1988-2004.

Model	*World Agriculture Supply and Demand Estimates* Release Date		
	February	April	June
Panel A: Traditional Supply of Storage Model (HAC)			
Intercept	-1.76**	-1.85**	-1.89**
	(0.08)	(0.06)	(0.074)
Ln (Spread)	6.48**	5.91**	5.42**
	(0.94)	(0.64)	(0.73)
R^2	0.74	0.84	0.76
Panel B: Updated Supply of Storage Equation (3 Stage Least Squares)			
Equation 1			
Intercept	2.51*	2.29	2.30*
	(1.18)	(2.54)	(1.17)
Ln (Spread)	7.47**	5.88**	3.96**
	(1.03)	(0.98)	(0.84)
Implied Volatility	-44.11**	-35.40*	-32.08**
	(12.68)	(18.15)	(8.86)
Volatility Squared	112.52**	73.49*	57.78**
	(34.94)	(40.16)	(16.70)
R^2	0.82	0.85	0.79
Equation 2			
Intercept	-0.02	-0.03*	-0.06**
	(0.011)	(0.0115)	(0.0112)
Convenience Yield	-3.30**	-2.84**	-3.55**
	(0.65)	(0.544)	(0.65)
R^2	0.58	0.58	0.59

Notes: (a) Each variable is measured on the month's release date of the U.S. Department of Agriculture's *World Agriculture Supply and Demand Estimates.* (b) Estimated coefficients and standard errors are presented. (c) ** and * denote significance at the 1% and 5% test levels, respectively. (d) A one-tailed test is used for all variables except the intercept. (e) Dependent variable in Panel A's equation and in equation 1 of Panel B is [ln(stock-use ratio)]. The spread is measured as [ln{futures price spread adjusted for storage cost}]. Convenience yield is measured using a procedure proposed by Haney.

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