Heterogeneity is a striking feature of agriculture. This is obviously true of farm sizes, ranging from the Cuban ‘Agrocombinat’ (over 100,000 ha and 1,000 workers) to the small plot of the Senegalese woman, growing paddy for her family, of less than 0.5 ha. The technical heterogeneity is not less remarkable. Even without considering the variety of production, from pork and poultry to grain, the same commodity, rice for instance, at the same competitive international price, may be produced almost without capital in Africa or almost without labour in Texas.

Now, if there exists something like an optimal firm size, competition should select it as the only feasible one. If there exist different input/output ratios, one of them should imply a lower cost than the others, and should emerge as the only feasible technique after the competitive adjustment of prices to costs. Since this kind of equalising process does not occur, something must be very peculiar in the agricultural production function.

It is the purpose of this paper to seek plausible reasons for this situation, and consider their practical consequences from a policy point of view. First, the absence of optimal size and the existence of an optimal structure will be demonstrated in a static framework. Then, the dynamic implication of this situation will be examined. Finally, consequences on structural policies will be drawn.

FARM SIZES AND STRUCTURES IN STATIC

The absence of optimal farm size
In the classroom, the size of a production unit is unambiguously defined by the quantity of the unique output. A great advantage of this definition is that the size is then completely independent of prices. The optimal size is more difficult to assess, because it depends upon the criterion chosen for optimality. In that respect, economists are accustomed to minimise

*I am greatly indebted to Denis Bergmann for helpful comments on a previous version of this paper. He does not share my views, however.
unit costs, not so much on the ground of some metaphysical creed in the
virtue of cheap production, but simply because competition will
automatically select the associated techniques in a liberal organisation of
the society, whatever the goals of individual producers.

Even so, with several inputs, the optimal size will in general remain
price dependent, because the unit cost is the sum of input quantities
weighted by their prices. However, in this context, the optimal size is not
always defined. Let us consider a production function, \( q = f(y) \), where \( q \)
is the quantity of output, and \( y \) is a column vector of input quantities. If
\( f(y) \) is homogenous and of degree 1, i.e., \( f(\lambda y) = \lambda f(y) \), whatever the real
valued scalar, \( \lambda \), then it is easy to show that there is no value of \( q \)
iminising the unit cost \( xy/q \), where \( x \) is any row vector of input prices.
Thus, in this case, any size of firm is feasible in a competitive economy,
whatever the price system.

In the more realistic case of a multiproduct firm the above properties
remain; it is impossible to define the size of a farm out of price
considerations, because the size is then a weighted sum of outputs or of
inputs. Two farms can eventually be ranked differently by two different
systems of weights. But if the production function (now expressed as
\( f(q,y) = 0 \), where \( q \) is a vector of output and \( y \) input quantities) is
homogenous and of degree 1, then it is impossible to find any vector \( q \)
iminising \( xy/pq \), whatever the price vector \( (p, x) \).

The considerations outlined above are restricted because the absence
of optimal size depends upon one special criterion of optimality, and also,
upon the idea that actual production functions can be linearly homoge­
neous. But at the same time, they are fairly general, because the criterion
in question imposes itself very naturally as the only feasible one in a
competitive situation, and which is more, because the degree of
homogeneity is an intrinsic property of the production function
absolutely independent of prices. For these reasons, in a competitive
economy, unless the production functions are homogeneous and of
degree 1, all firms disappear, except those which, by chance or skill,
can stay in the vicinity of the optimal size. For instance, in car
manufacturing industries, economies of scale quickly pushed out
individual producers, without leaving them any chance of recovery.
Clearly, the situation is quite different in the case of agriculture. Even
artificial regulations, such as preventing small farms having access to
government subsidies, do not discourage small and part-time farming.

The fact that several farm sizes can coexist for a long time within a
common economic environment is an indirect proof of the linear
homogeneity of the production function in agriculture. The direct proof is
more difficult to bring about, because it needs a particular analytical
specification of the production function. The pitfalls\(^1\) of this kind of
exercise are numerous. The most serious studies show that increasing
returns to scale and indivisibilities are not, strictly speaking, absent, but
they are counterbalanced by decreasing returns in other fields, and
statistically negligible. We shall refer to other authors (for instance,
Boussard 1976) for details, and turn here our attention towards the consequences of this situation.

Without economies of scale, there are no incentives to the homogenisation of farm sizes. Thus, the heterogeneity of farm sizes derive straightforwardly from the specificity of the agricultural production function. What about farm structures?

THE EXISTENCE OF OPTIMAL STRUCTURES

If there exist no optimal farm sizes, there exist optimal farm structures. However, for this statement to be valid, it is necessary to define precisely the term 'structure', the meaning of which is really too vague in ordinary language. Let us call structure the ratio between the various quantities of fixed factors: If \( z = \{z_1 \ldots z_k \ldots z_K\} \) is the vector of the \( K \) available quantities of fixed factors, the structure is a vector \( s \) of dimension \( K - 1 \), the current element of which is \( z_K / z_K \), the \( K^{th} \) factor being conventionally taken as reference. Thus, land (or a certain quality of land) being the reference factor, the elements of the structure are the number of fixed permanent workers of such or such qualification per ha of that type of land. If it is possible to define structure in that way, then the existence of optimal structures is a direct consequence of the elementary theory of production.

Consider a farmer maximising his income, \( F = pq - xy \), under a production function constraint: \( f(q, y, z) \leq 0 \), where the column vector \( z \) with \( K \) elements denotes the quantities of fixed factors, and \( f \) is a function increasing with each element of \( q \), and decreasing with each elements of \( y \) and \( z \). \( f \) is homogeneous of degree one, and the constraint is convex. Then, there is no optimal value for \( z \): by the convexity of \( f \) if any triplet \( (q, y, z) \) is feasible, the triplet \( (\theta q, \theta y, \theta z) \), where \( \theta \) is any positive scalar, is also feasible, so that \( F \) is unbounded for infinite values of \( \theta \).

But if one element of \( z \), say \( z_K \), is fixed, at the level \( z_K^* \), \( F \) is actually bounded. Since the feasible set is convex, this means that there exists a finite unique maximum for \( F \) with respect to \( q, y, \) and the \( K - 1 \) other elements of \( k \). Let us denote the solution by \( \hat{q}, \hat{y}, \hat{z} \). The vector \( \hat{s} \), with \( K - 1 \) elements, given by \( \hat{s}_k = \hat{z}_k / z_K \) is called the optimal structure.

An optimal structure is linked with optimal production plans; in effect, the examination of the solutions of the maximizing problem above shows that all farms for which \( k = \lambda(\hat{s}, 1) \) will produce the same outputs, and need the same inputs in the same proportions, because they are making use of the same techniques. They will be homothetic. At the same time, it is obvious that, in reality, agricultural firms are seldom homothetic. It is therefore necessary to explain why, despite the existence of optimal structures, farms are still technically heterogeneous.

This is a consequence of a second peculiarity of optimal structures, their dependence on prices. Actually, the solution of the maximisation problem stated above depends upon the price system, so that \( \hat{s} \) is
price-dependent. This is the reason why one speaks of several optimal structures. Each variation of the prices of outputs or of variable inputs will imply a corresponding change in the optimal structure.

Again, there is a strong difference between structures and sizes: the absence of optimal size was a consequence of the specification of the production function $f$, which, it must be recalled, is perfectly free of any price consideration. On the contrary, $s$ cannot be defined before the price system is known. Therefore, it is not surprising that two firms, in two different price contexts, for instance, in Senegal and in Texas, have two different structures. However, observation shows that even firms placed within the same price environment may differ in structure. How can this happen? For answering this question, dynamic considerations must enter the analysis.

THE DYNAMIC HETEROGENEITY OF AGRICULTURE

When considering a dynamic version of the static model which has been sketched above, it is necessary to distinguish between the situation of the individual producer and the behaviour of the industry as a whole.

The individual producer in dynamics and the turnpike theorem

The basic phenomenon here is that structures, which were fixed in the short term, are now variable, at least to a certain extent, because fixed factors can be produced or purchased. The idea of buying a certain quantity of fixed factor may seem self-contradictory. The contradiction vanishes if one recalls that a factor is not fixed once and for all. It is fixed when its marginal value product falls between its acquisition price and its salvage value (Johnson 1959). When saving is abundant, its opportunity cost becomes lower, so that it may be profitable to buy new units of previously fixed inputs. In that way, it is possible to modify the vector $z$, and to consider it as endogenous.

Obviously, such modifications of $z$ are not random, but directed toward the necessity of narrowing the gap between the actual and the optimal structure. Since the available resources in saving or in own produced capital items are limited, it will not always be possible to reach at once the optimal structure. Nevertheless, after a few years, repeated increments of the quantities of the most productive factors should enable any producer to stay on the optimal expansion path defined by the optimal structure. This is the basic meaning of the famous 'turnpike theorem'.

It is beyond the scope of this paper precisely to state it (or rather them, because there is a variety of different formulations).² Let us only say that, under fairly general conditions, the individual producer, if his planning horizon is long enough, will be dynamically led to an optimal structure which is independent of his own utility function $U$, and is determined by the production function alone.

The fact that the optimal structure is independent of $U$ is important for
our discussion, because, whatever the tastes of a farmer, provided that he is interested in something which is an increasing function of output, he should come to the optimal structure. Such a result contradicts the arguments of Tchajanov and others, who relate the heterogeneity of farms with differences in objective functions. By contrast, the optimal structure is not independent of prices, because the production function \( f \) incorporates now the possibility of purchasing inputs by selling outputs. There is therefore a deep difference between the static production function, which could be considered as purely technical, and the dynamic production function. In addition, the prices in question are not observed, but expected: thus, two farms in the same situation may have two different optimal structures only because the first farmer is pessimistic, and the second optimistic. Now, variations in prices are frequent, for inputs as well as for outputs, and are not purely random, but market driven. The consequences of this fact must be drawn.

The interactions between optimal structures and markets
Thus, reaching the 'turnpike' for a given system of prices, means producing a certain set of commodities, and requiring a certain set of inputs, all in the same proportions. Nothing guarantees that the market is ready to absorb these commodities and provide these inputs. For instance, the optimal structure may require that 50 per cent of the cash receipt of farms be made from grains, and, at the same time, consumers are ready to spend only 25 per cent of their food budget on this kind of commodity. In such a situation, if the market is re-equilibriated by a change in prices, this will also, in general, change the optimal structure, so that the situation after adjustment can be no better than before. Even more, since the reaction of farm production systems may take several years, there is a possibility that the reaction of the market be far larger than that which should in principle be necessary to reach an equilibrium: for instance, the price of one specific commodity can fall far under the level for which the optimal structure meets consumers' wants. In that case, firms will be misled, because they will have to direct their adjustments towards a structure which cannot warrant market equilibrium.

An additional complication arises because farmers are not immortal; at each generation, newcomers have to buy again all existing assets to continue to produce. Since they start from scratch, they are free to choose the current optimal structure at prevailing prices. But since these prices are bound to change, the structure of the newly acquired farms quickly become out of the optimal expansion path. This element of perturbation is essential, as we shall see now, to understand if not how heterogeneity perpetuates itself, at least how it can be generated from an initially homogeneous farm population.

It would be difficult, in this paper, to develop a formal model embodying the preceding considerations. Rather, let us examine the results of a simplified, computable version of such a model. Assume a
sample of $N$ farms and a Cobb Douglas production function, with two inputs, $K$ and $L$, in quantities $k_i$ and $l_i$ for farm $i$. $q_{it}$ being the quantity of the one output produced by farm $i$, at time $t$, $q_{it} = k_{it}^{\alpha}l_{it}^{1-\alpha}$. $K$ is a variable, non-durable factor, supplied at fixed price $p$. $L$ is a durable factor, which can be purchased or sold in quantity $i_{it}$ ($i_{it} > 0$ if $L$ is purchased, and $i_{it} < 0$ if it is sold). The price of $L$ is $p_{it}$ if $L$ is purchased, and $\theta p_{it}$ if it is sold: thus, $\theta$ is an index of the fixity of $L$, with this factor perfectly liquid if $\theta = 1$, and perfectly fixed if $\theta = 0$. $p_i$ is determined in such a way that: $\Sigma i_{it} = 0$. The output is sold at price $p$ subject to a demand curve specified by: $p_{qt} = a(\Sigma q_{it})^\beta$ where $\beta < 0$ is the elasticity of output demand with respect to price, and $a > 0$, a scale factor.

Farmers’ incomes are given by: $m_{it} = p_{qt}q_{it} - p_kk_{it}$.

A fraction $c$ of this income is consumed, so that at the beginning of each year, a farmer is endowed with a quantity of money, $e_{it} = p_{qt-1}q_{it-1} - c m_{it}$ and a quantity of $L$ given by: $l_{it} = l_{it-1} + i_{it}k_{it}$ and $i_{it}$ are subject to a liquidity constraint:

\[
P_k k_{it} + P_1 i_{it} = e_{it}, \text{ if } i_{it} > 0, \text{ or } \]
\[
P_k k_{it} + \theta p_1 i_{it} = e_{it}, \text{ if } i_{it} < 0.\]

Finally, each year, a number $n$ of farmers are removed from the sample. Their assets are sold (which increases the supply for $L$) to the same number of newcomers, each of them being endowed with an exogenous fixed quantity of money $e^*$. Thus, the total quantity of $L$ is fixed, but its distribution among farms can vary. The set of equations just presented, and the assumption according to which the income $m_{it}$ is maximised, determine each year the set of endogenous variables ($q_{it}$, $k_{it}$, $l_{it}$, $p_{qt}$, and $p_{it}$) from the situation of the preceeding year, and the parameters $\alpha$, $\beta$, $\theta$, $N$, $n$ and $e^*$.

In fact, these equations represent a dynamic general equilibrium model, endowed with a Walrassian ‘tatonement’ process. Thus, the successive solutions should converge toward a steady state from any feasible starting point. The steady state itself can be readily computed, by the two conditions that the total quantity of money flowing out the system through consumption should equate the quantity of money flowing into it, through the $e^*$s, and that all farms should have the same optimal ratio $k/l$. Actually they do so, but the convergence is not necessarily quick: it is well known that the Walrassian tatonement is a poor algorithm for the search of general equilibrium solutions. In the meanwhile, the sample remains heterogeneous if it was so at the origin, and, even more, becomes heterogeneous if it was not, as shown on Figures 1a to 1f.

The corresponding results were obtained with $N = 50$, $n = 2$, $\alpha = \theta = C = 0.5$, $\beta = -0.5$, $e^* = 100$, and all farms identical in period 0, with $l_i = 50$, and $k_i = 5$, but similar results were derived from other values of these parameters. Heterogeneity within the sample is measured at each time by the coefficients of variation of the relevant variables, $y_{it}/l_{it}$ for techniques (this variable is an index of ‘intensity’), and $y_{it}$ for sizes. It
Changing environment and structural heterogeneity

FIGURE 1 Results of the simulation of a sample of 50 farms over 50 years

is significant for both variables, (Figures 1a and 1b), although greater for sizes than for techniques. This is not surprising, for at equilibrium, techniques should be homogeneous, but not necessarily sizes. Anyway, equilibrium, in that case, is very far reached, and, because of that, heterogeneity introduces itself into the sample, despite the absence of
any random event but the starting point, and despite the fact that this starting point is itself completely homogeneous.

This process of heterogenisation is driven by oscillations of prices and quantities, as pictured on Figures 1c to 1e. They swing up and down in a cobweb style, although the origin of the cycle is quite different from what it is in the traditional cobweb model: instead of being produced by errors in expectations, the cycles are triggered by the liquidity constraint, which is tightened when prices are low, and loosened when they are high. For that reason, cycles are less regular than those of the cobweb, and eventually, their periods are longer.4

Whatever their sources, these cycles introduce, even at period 1, an element of difference between the newcomers and the other farms, because the former can invest in the optimal proportions, whereas the latter cannot, as they are tied by the liquidity constraint and the difficulty of selling their fixed inputs. But since the optimal proportions vary with prices (Figure 1f), each new farm is different from the others. Thus, once the process of differentiation is triggered, it cannot end before the equilibrium is reached, very far away from the fortuitous initial disequilibrium.

In a real-life situation, the behaviour of the system would be complicated by three additional considerations:

(i) The existence of technological change: since the equilibrium is far reached, technology can vary exogenously during the adjustment process, with the consequence that the convergence can be delayed indefinitely.

(ii) The existence of several outputs, and of more than two inputs: it is dubious that their introduction could change significantly the behaviour of the system, but it can considerably complicate the time path of the variables, thus increasing the degree of heterogeneity.

(iii) The existence of a risk averse behaviour of farmers: although it is difficult to assess exactly the consequences of explicitly introducing risk in this matter, I would hypothesise that it would lead to a smoothening of the curves pictured in Figure 1. In fact, risk will lead to more cautious investments and, perhaps, induce farmers to hoard at least a fraction of their liquidities. In that case, they would react less fiercely in response to market incentives. The adjustments would take a longer time. It is likely (but not proved), that this would result in less dramatic accidents than those which are observable with the model without risk. This is a reason for heterogeneity being reduced. At the same time, by widening the wedge between the price at which an asset can surely be bought and that at which it can be sold, uncertainty increases the degree of fixity of any factor; in that way, heterogeneity is increased. Which of these two contradictory tendencies supersedes the other is a matter of discussion, and of further empirical as well as theoretical research.

Anyway, no complete study of the mathematical properties of the model described above has been attempted, as far as I know, although some references could be the point of departure of such an investigation.5
A wide, non-conventional field of research is thus open, and will probably be the object of developments in the next few years.

CONCLUSION: POLICY IMPLICATIONS

Thus, farm size heterogeneity is a consequence of the absence of economies of scale. Farm structure heterogeneity is a consequence of the interactions between a dynamic process of adjustment toward optimal price dependent structures, and of market constraints which perturb this adjustment, through pseudo-random historical events. In both cases, it is a consequence of deeply rooted natural mechanisms, which are not easily modified by policy measures. This may be a reason for the failure of 'structural policies' in agriculture.

A first rationale for such policies is that increasing their size would make small farmers in a position to increase their incomes, by benefiting from economies of scale. This is two inconsistencies in one statement. First, the existence of economies of scale is problematic, as we have seen. Second, small farmers’ income is not the difference between the value of outputs and that of inputs. With a production function homogeneous and of degree 1, this difference is simply zero. Actually, a farmer’s income is the cost of own supplied inputs, evaluated at implicit prices if they are fixed, and at market prices if they are variable. In that context, if additional factors are given to farmers as a gift, it is possible to increase their income by an amount exactly equal to the value of the gift, computed at reference price, after suitable adjustment to convert capital stocks into income flows. This is perfectly tautological. But subsidising them, or giving them credit facilities for acquiring such or such input is not likely to produce any increase of income in excess of the opportunity value of the subsidy. In fact, since opportunity costs for fixed inputs are always less than the market price, the value of the subsidy for the farmer is always less than the nominal cost for the government, so that the latter would make a more efficient use of its money by distributing it without conditions.

A second rationale for structural policies is sounder. Since, because of delay in adjustments, the market is likely to keep a large number of farms very far from the optimal structure, and, even more, from the optimal structure which should balance supply and demand, it may be wise to try to correct this inefficient situation. Thus, the structural adjustment could be speeded up or slowed down in order to avoid forecasted detrimental disequilibria. Actually, this should be the role of the government agencies. Unfortunately, this is not the way they usually operate. There are a number of reasons. Setting up a true contracyclical policy requires reliable long-term forecasts which are not available in the present state of the science. It would also require that governments be free of short-term pressures, which is not true.

Moreover, these policies would be extremely costly. Actually, by Euler’s theorem (and contrary to common creed), it costs just as much for a government to buy or sell production factors in excess or in short supply as
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to buy or sell commodities on unbalanced output markets. In fact, there are reasons for the opinion that these interventions on factors are even more expensive than interventions on output markets; since the demand for a particular factor is relatively elastic (because of substitution), whereas the demand for output is rigid (because of the properties of Engel's curves), it is likely that the total sum of government money required for a given effect is greater on the factor side than on the output side (this is not always true, however, and the statement deserves a number of qualifications, which could be the object of further research). In any case, the cost of structural policies is large. It is the basic reason for the failure of the PIK Program in the US (where excess land was hired by the government in order to be 'frozen' and removed from production), which did not resist the cut in budgetary expenditure of the Reagan administration. Another example is the farm retirement programme in France (IVD), where old farmers are offered a pension in exchange for their commitment to leave, in the avowed purpose of decreasing the man/land ratio; this programme is still in action, but to keep its cost within reasonable limits, pensions are so meagre that it is dubious if they had any significant influence on actual retirement decisions (Klatzmann 1981). Other similar examples could easily be found.

What remains, then, of the ambitious structural policies which should furnish governments with the possibility of influencing production decisions and income distribution at low cost? I am afraid the answer is almost nothing, except the faculty for a number of local notabilities to claim for their skill in pushing administrative cases through the bureaucratic labyrinth. Actually, all the beneficial effects of structural policies can be achieved by a sound output price policy. This is why, in general, the so called 'structural policies' are extremely efficient in wasting government money. They have at least the advantage of providing safe positions to a large number of civil servants who, otherwise, would have to seek more productive employment – a difficult task in the present situation of the world economy.

NOTES

1See, for instance, Zellner et al. (1966).
2The best article on the question is probably McKenzie (1976). The first version of the theorem was published by Dorfman, Samuelson and Solow (1958), inspired themselves by Von Neumann. On agricultural applications, see Boussard (1971).
3See, for instance Ginsburg and Walbroeck (1981).
4This is the kind of result which has been published by Day in a large number of references (for instance, Day 1982, or Day and Tinney 1969). I am greatly indebted toward this author, most of the theoretical ideas exposed in this paper having been drawn from his work.

REFERENCES


