Abstract: This paper investigates the strategic behavior between countries that have purchasing power on the world market for a certain good. Tariffs and quotas are not equivalent protection instruments in this oligopsonistic market. Policy active importers would be better off by colluding and setting their trade instrument cooperatively. In a non-cooperative setting, if production decisions occur before consumption decisions, the ex-ante optimal policy is not time consistent because the ex-post elasticity of the residual foreign export supply curve is lower than the ex-ante elasticity. However, we show that the importers’ inability to irrevocably commit to their trade instrument may be welfare superior to the precommitment solution. The negative welfare implication of non-cooperative behavior may be balanced off by the welfare effect of the ex-post elasticity. A numerical example is proposed to provide insights on the theoretical results.

Keywords: Precommitment, time consistency, optimal tariff and quota, oligopsony.

JEL Classification: F13, Q17, D4.

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1 - Introduction

Much has been written on the theory of optimal tariffs (see Corden (1994) for a detailed survey). However, as pointed out by Grant and Quiggin (1997), when there is more than one country with world market power for a good, it raises strategic issues that have not been addressed formally, but are obviously related to the problems of oligopoly theory.

The contribution of this paper to the literature is twofold. First, it investigates the strategic behavior between countries that have purchasing power on the world market. This strategic game between policy active importers has been introduced first by Bergstrom (1982) and later by Karp and Newbery (1991,1992). We formalize the non-equivalence of tariffs and quotas given the structure of the world market and the non-cooperative behavior among importers. Importers set their trade instrument given the optimal instrument set by the other importers. When the strategy space is restricted to the use of a tariff, the Nash equilibrium entails lower tariffs for each country than in the situation when they collude and act as a single monopoly importer. If the strategy space is restricted to the use of a quota, the non-cooperative solution implies that a too large quantity is imported in each country, thus driving the world price above the optimal level. Each country would be better off by colluding and importing a smaller quantity.

The main contribution of this paper analyses time consistency issues. As pointed out by Staiger (1995), time consistency problems and rules versus discretion issues have occupied a major place in the macroeconomics and public finance literature, but less so in

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1 These papers analyze the strategic behavior between importers of a depletable resource. There is a significant difference between the optimal tariff for an ordinary good compared to an exhaustible resource such as oil. Oil is available in a fixed amount and, if costless to extract, its supply will be inelastic. However, in a trade context, exports are not inelastic, i.e. there is a role for demand.
the international trade field. With a sufficient degree of discretion, an optimal trade policy is bound to lack credibility because it is almost surely time inconsistent. Most of the time consistency issues addressed in the economic literature emphasize the inferiority of the no commitment solution.

We assume there is a lag between production and consumption, and that all countries can change their policy between the two stages. For example, this setting applies to agricultural markets with spring planting and fall harvest\(^2\). In the case of tariffs, the *ex-post* (given production decisions) tariff will be higher than the *ex-ante* (before production decisions are made) tariff because the residual export supply curve elasticity faced by each country is lower *ex-post*. With perfect foresight, foreign producers will fully anticipate the time consistent tariff and decrease their production accordingly. Therefore, the lower *ex-post* elasticity of the residual foreign export supply curve may be welfare increasing for the policy active importers compared to the *ex-ante* situation because of its off-setting welfare effect with respect to the trade instrument competition. The same argument also applies to the strategic quota game.

This paper is organized as follows. First, it provides a review of literature relevant to the proposed problem to be studied. The theoretical model is set out in the second section to address optimal trade policy and time consistency issues. Next, we develop a numerical example to illustrate our various results. The last section provides concluding remarks and suggests some extensions.

\(^2\) For example, the same issue would arise if there was a lag between capital/investment decisions and labor decisions.
2 - Review of Literature

According to the well-known theory of Johnson (1954), the optimal tariff for a large country equals the reciprocal of the foreign export elasticity of supply. Lapan (1988) points out that, in the case where production decisions are made before consumption and trade decisions, and that the government can readjust its tariff between the two stages, the standard optimal tariff will not be time consistent. From an \textit{ex-post} perspective, \textit{i.e.} once production decisions are made, the foreign export supply elasticity is lower than the \textit{ex-ante} elasticity. Therefore, policy makers have an incentive to set \textit{ex-post} tariffs at a higher level than they would if they could precommit to the \textit{ex-ante} tariff.

The foreign and domestic producers, knowing that the \textit{ex-ante} tariff is not time consistent will adjust their production accordingly (\textit{i.e.} foreign production will be lower than if the large country could precommit to the \textit{ex-ante} optimal tariff) and both countries will be worse off. The importance of the timing assumption is immediate once it is recalled that a tariff can always be decomposed into a production subsidy and a consumption tax on the importable good.

Maskin and Newbery (1990) model the behavior of a large importer of oil unable to commit to future tariffs. Time consistency models of exhaustible resources point out that scarcity can be artificially induced by the exercise of market power. Suppliers not only have to make their extraction (and hence their production) decision according to current prices, but also by comparing anticipated future prices, which will depend on future levels of a tariff. In their two period model, if the importer places sufficient weight on
second period consumption of oil, and can revise costlessly the tariff set in the first period, the welfare associated with the dynamically consistent tariff may be less than the free-trade welfare level.

Karp and Newbery (1991, 1992) build a continuous time model where oligopsonistic importers choose a time path of tariffs to maximize their domestic welfare. They show that the open loop strategy is not time consistent. They rely on numerical methods to illustrate the welfare inferiority of the closed loop solution compared to the open loop solution. However, they simplify the model so that there is only one large importer. In that case, there is disadvantageous market power for the importer. In their 1991 paper, they illustrate the differences between the time consistent tariff and the importers' welfare for different sequential games between policy active importers and competitive exporters.

Karp and Perloff (1995) consider the impacts of government commitment on output subsidies and investment subsidies. This paper differs from theirs because they develop a model of oligopolistic competition à la Brander-Spencer to help domestic firms gain a strategic advantage in trade. Output policies based on static models are not altered for a dynamic model. However, investment decision depends on future as well as current government policies. If precommitment is not feasible, investment policy is of a limited strategic use.\(^3\)

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\(^3\) Other interesting issues of time consistency in international trade have been addressed in the literature. Staiger and Tabellini (1987, 1989) considered the credibility issue arising from the use of tariffs as a redistributive tool. Tornell (1991) and Wright (1995) provide explanations for the empirical observation that future tariff removal is time inconsistent if protection was granted to provide incentives for the firms to reduce their costs. Brainard (1994) stresses the time inconsistency problem arising when policymakers are unable to precommit to a certain policy in a declining industry. All of these papers address redistributive or second-best issues.
3 – Model

Consider a partial equilibrium model. Suppose there are \( N \) importing nations with purchasing power on the world market for a certain good. Their inverse domestic demand is denoted by \( p_i(d_i) \) where \( d_i \) and \( p_i \) are the domestic demand and domestic price respectively. Denote the world price and foreign exports from the rest of the world by \( \bar{p} \) and \( \bar{X} \) respectively. The foreign export supply is defined by:

\[
\bar{X} = \bar{Q}(\bar{p}^e) - \bar{D}(\bar{p})
\]

where \( \bar{Q}(\bar{p}^e) \) and \( \bar{D}(\bar{p}) \) are the foreign supply and demand respectively. The superscript \( e \) is used to denote producers’ price expectations when production decisions are made. This notation is introduced to model the time consistency issue later in the paper. Foreign supply depends on the producers’ price expectations and hence on their expectation of the trade policies, whereas foreign demand depends on the realized world price. This structure gives rise to a foreign export supply curve: \( \bar{X}(\bar{p}, \bar{p}^e) \). In the case where production and consumption decisions are carried out simultaneously, \( \bar{p}^e \equiv \bar{p} \), and thus the foreign export supply curve is:

\[
\phi(\bar{p}) \equiv \bar{Q}(\bar{p}) - \bar{D}(\bar{p})
\]

Denote by \( \phi' \) the slope of the \textit{ex-ante} foreign export supply curve so that \( \phi' = \bar{Q}' - \bar{D}' \).

3.1 Tariff competition

If \( \tau_i \) is the \textit{ad-valorem} tariff imposed by country \( i \) on imports, we have the following arbitrage condition between the domestic and world price: \( p_i = \bar{p}(1 + \tau_i) \). If \( q_i \) is the quantity produced in country \( i \), imports are defined by \( m_i(p_i^e, p_i) = d_i(p_i) - q_i(p_i^e) \), where again the superscript \( e \) denotes producers’ expectation. From an \textit{ex-ante} perspective, \( p_i^e \equiv p_i \), and the slope of the import demand is:

\[
m_i'(p_i) = d_i' - q_i'.
\]
market equilibrium implies \( \phi(p) = \sum_j m_j(p(1+\tau_j)) \). For further reference, totally differentiate this equilibrium condition to obtain:

\[
\frac{\partial \phi}{\partial \tau_i} = \frac{m'_j p}{\phi - \sum_j m_j'(1+\tau_j)} < 0
\]  (1)

Sufficient conditions for (1) to be negative are to have positively sloped foreign and domestic supply and negatively sloped foreign and domestic demand. The welfare of foreign exporters is increasing with the world price and so decreasing in every tariff. The objective function of the government in country \( i \) is to maximize domestic welfare defined as the sum of consumer surplus\(^4\), producer surplus and tariff revenue. Domestic welfare of country \( i \) is:

\[
W_i = \int_0^{d_i} p_i(y_i)dy_i - p_i d_i + p_i q_i - \int_0^{q_i} c'(z_i)dz_i + \tau_i p m_i
\]  (2)

such that \( d_i = q_i + m_i \), and \( p_i(q_i + m_i) = \bar{p}(1+\tau_i) \). Rewrite (2) as:

\[
W_i = \int_0^{q_i+m_i} p_i(y_i)dy_i - c(q_i) - \bar{p} m_i
\]  (3)

To solve for the optimal ex-ante tariff, totally differentiate equation (3). After some simple manipulations, you get:

\[
dW_i = [p_i(d_i) - c'(q_i)] dq_i + [p_i(d_i) - \bar{p}] dm_i - m d\bar{p}
\]  (4)

The expression in the first bracket on the right hand-side of (4) is equal to zero absent government domestic policy since we assume perfect competition in production. The world price is determined according to the ex-ante residual foreign export supply:

\(^4\)Consumer surplus is an exact measure of consumer welfare if demand is derived from a quasi-linear utility function.
\[
\phi(\bar{p}) - \sum_{j \neq i} m_j (\bar{p}(1 + \tau_i)) - m_i = 0
\]  

(5)

Differentiating the behavioral equation in (5), given the other countries’ tariff choice, yields:

\[
\left[ \phi' - \sum_{j \neq i} m'_j (1 + \tau_j) \right] d\bar{p} - dm_i = 0
\]  

(6)

To get the optimal tariff, set equation (4) to zero and substitute (6) in (4) for \( dm_i \). Finally, divide both sides of (4) by \( d\tau_i \) to get:

\[
\frac{\partial W_i}{\partial \tau_i} = \left( \tau_i \bar{p} \left[ \phi' - \sum_{j \neq i} m'_j (1 + \tau_j) \right] - m_i \right) \frac{\partial \bar{p}}{\partial \tau_i} = 0
\]  

(7)

Assuming that the welfare function in (3) is strictly concave in its own tariff everywhere, the second order condition for a maximum is satisfied. Equation (7) implicitly yields the reaction function of country \( i \) as a function of its belief about other countries’ tariff, \( \tau_i^p = R(\tau_1, \ldots, \tau_{i-1}, \tau_{i+1}, \ldots, \tau_N) \). The Nash equilibrium will be the set of tariffs \( (\tau_1^p, \ldots, \tau_i^p, \ldots, \tau_N^p) \) such that \( \tau_i^p = R(\tau_1^p, \ldots, \tau_N^p) \) and \( \tau_j^p = R(\tau_1^p, \ldots, \tau_N^p) \), \( \forall i, j \). After some manipulations, the \textit{ex-ante} (or precommitment) tariff in elasticity form is:

\[
\tau_i^p = \frac{\alpha_i}{\xi^p + \sum_{j=1}^{N} \alpha_j \eta_j^p}
\]  

(8)

where \( \xi^p \equiv \frac{\phi' \bar{p}}{\phi} \) is the foreign export supply elasticity with precommitment, \( \alpha_i \) is country \( i \)'s share of world imports, and \( \eta_j^p = -m'_j p_j / m_j \) is the import demand elasticity of country \( j \). Therefore, the denominator in (8) represents the elasticity of the residual
foreign export supply curve. If all countries are symmetric, their share of world imports is equal to $1/N$. As $N \rightarrow \infty$, the optimal policy becomes free trade.

Consider the case where the $N$ countries maximize their joint welfare. Intuitively, countries will internalize the adverse effect on world price given by the change of other countries’ imports with respect to their own tariff. The joint welfare of the importers is:

$$ W = \sum_{j=1}^{N} \int_{0}^{q_{j}+m_{j}} p_{j}(y_{j})dy_{j} - \sum_{j=1}^{N} c(q_{j}) - \sum_{j=1}^{N} \bar{p}m_{j} $$

(9)

Differentiating (9) with respect to $\bar{p}$, $q_{i}$ and $m_{i}$ yields after some manipulations:

$$ dW = \left[p_{i}(d_{i}) - c'(q_{i})\right]dq_{i} + \left[p_{i}(d_{i}) - \bar{p}\right]dm_{i} - \sum_{j} m_{j}d\bar{p} $$

(10)

Since the expression $\left[p_{i}(d_{i}) - c'(q_{i})\right]$ equals zero, (10) can be rewritten as:

$$ dW = \tau_{i}\bar{p}dm_{i} - \sum_{j} m_{j}d\bar{p} $$

(11)

Differentiating the behavioral equation (5) yields $\phi'd\bar{p} = dm_{i}$. Use this to substitute in (11) for $dm_{i}$, set (11) to zero to find a maximum, and divide both sides by $d\tau_{i}$ to yield:

$$ \frac{\partial W}{\partial \tau_{i}} = \left[\tau_{i}\bar{p}\phi' - \sum_{j} m_{j}\right]\frac{\partial \bar{p}}{\partial \tau_{i}} = 0 $$

(12)

Solving (12) yields $\tau^{*}_{i} = 1/\xi_{p} \forall j$, the optimal collusive tariff for importer $i$. It is proportional to the inverse of the foreign export supply elasticity, which is the standard result in the optimal tariff literature. Evaluate (12) at $\tau_{i} = \tau'_{i}$ to get:

$$ \frac{\partial W}{\partial \tau_{i}}\bigg|_{\tau_{i} = \tau'_{i}} = \sum_{j \neq i} \left[\tau_{i}\bar{p}m'_{j}(1 + \tau_{j}) - m_{j}\right]\frac{\partial \bar{p}}{\partial \tau_{i}} > 0 $$

(13)

\[\text{Obviously, the choice of instrument does not matter for the collusive and full information case. It can be readily shown that quotas and tariffs are equivalent instruments under collusion. Therefore, differentiating}\]
Equation (13) provides the following ranking between the precommitment tariff and collusive tariff: $\tau_i^p < \tau_i^c$. Countries would be better off acting cooperatively; thus maximizing their joint welfare. In the non-cooperative setting, when a country decreases its tariff, it fails to consider the reduction in other countries’ welfare that is caused by the ensuing increase in the world price.

3.2 Quota competition

We will now restrict the strategy space of the $N$ importers to a quota. To do so, set $\tau_j = 0 \quad \forall j$ and use imports as the choice variable. The domestic price is determined by:

$$m_i = d_i(p_i) - q_i(p_i^c)\quad \text{In the \textit{ex-ante} situation, when production decisions are carried out simultaneously with consumption, } p_i = p_i^c.$$  

The domestic price is then $p_i(m_i)$. If domestic imports are the choice variable, the \textit{ex-ante} residual foreign export supply faced by country $i$ is defined by:

$$\phi(\bar{p}) - \sum_{j \neq i} m_j - m_i = 0$$

(14)

Auctioning the quota licenses raises government revenue $(p_i - \bar{p})m_i$. Therefore, country $i$’s welfare is still given by (3). Differentiating the residual export supply curve in (14) given the other countries’ quota gives:

$$\phi’d\bar{p} - dm_i = 0$$

(15)

Using (15) to substitute for $d\bar{p}$ in (4), we get:

$$\frac{\partial W_i}{\partial m_i} = (p_i - \bar{p}) - \frac{m_i}{\phi’} = 0$$

(16)

equation (5), we treat imports $m_j, j \neq i$, as given.
We assume the welfare function to be concave everywhere in its own imports. Equation (16) yields the reaction function of country $i$. The intersection of every countries’ reaction function gives the Nash equilibrium precommitment quota $m_i^p$. Define $\theta_i^p$ as the tariff equivalent measure of the difference between the domestic price and the world price given $m_i^p$. From (16), the precommitment tariff equivalent in elasticity form is:

$$\frac{p_i - \bar{p}}{\bar{p}} = \theta_i^p = \frac{m_i}{\bar{p}\phi} = \frac{\alpha_i}{\xi^p}$$  \hspace{1cm} (17)

As we mentioned before, in the case of collusion, the optimal policy is independent of the instrument used. The optimal collusive quota is $m_i^*$ and the collusion tariff equivalent of the import quota in elasticity form is: $\theta_i^* = \tau_i^* = 1/\xi^p$. It can readily be shown that the collusion quota is lower than the *ex-ante* Nash equilibrium quota. Under the strategic quota game, when a country increases its quantity imported, it fails to consider the reduction in other countries’ welfare that is caused by the ensuing increase in the world price.

**Proposition 1**: Assuming symmetry among the policy active importers, the *ex-ante* Nash equilibrium quota will induce a higher price differential between the domestic price and the world price than the optimal *ex-ante* tariff. Moreover, the importer’s welfare associated with the *ex-ante* quota will be higher than the welfare associated with the precommitment tariff.

**Proof**: Evaluating (7) at $\tau_i = \theta_i^p$ yields:

$$\frac{\partial W_i}{\partial \tau_i} \bigg|_{\tau_i = \theta_i^p} = -\theta_i^p \bar{p} \left[ \sum_{j \neq i} m_j^p (1 + \tau_j) \frac{\partial \bar{p}}{\partial \tau_i} \right] < 0$$  \hspace{1cm} (18)
From (1) and because the import demand is negatively sloped, the expression in (18) provides the following ranking: \( \tau_i^p < \theta_i^p \), given symmetry among policy active importers. Because tariffs and quotas are equivalent under collusive behavior, we have the following rankings: \( \tau_i^p < \theta_i^p < \tau_i^* = \theta_i^* \). Therefore, it must be that the \textit{ex-ante} quota brings a higher welfare level than the \textit{ex-ante} tariff. This is so because domestic welfare for each country under both Nash equilibria declines as the instrument set under collusion produces the global optimum for both sets of strategy. QED.

The intuition behind proposition 1 is simple because it is tantamount to the standard oligopoly theory. By using a tariff as their trade instrument, each country faces a more elastic residual foreign export supply at the tariff set by the other countries than in the monopsony case. Moreover, the residual foreign export supply curve will be even more elastic than under the strategic quota game. When countries use a quota, the Nash equilibrium will induce a higher welfare than tariffs because countries reducing their imports by one (differential) unit cause an increase in the world price of \( \tilde{p}'(\tilde{X}) \). Tariffs do not have an equivalent effect on the world price, since imports of other countries also vary following a change in one importer's tariff.

The results in proposition 1 contrast with the bilateral monopoly case (two-good, two-country retaliation world). The use of a quota in our model does not eliminate trade as in the Rodriguez’s model (1974). In the quota-retaliation framework, each country cannot enforce a favorable terms of trade shift. Although both wish to achieve the same level of trade restriction on a certain good, they have different preferred levels of trade in the
other good. In our model the foreign exporters are passive. Therefore, competing importers are able to induce a terms of trade shift, but this is not optimal because they fail to take into account (or do not care) about the consequence of their trade policy on the other policy active importers’ welfare. The same basic story applies to the bilateral monopoly with tariffs as strategic variables. In that situation, it is possible to find an equilibrium where both countries are worse off than under free trade. This is ruled out in our setting since every country gains by imposing a tariff, and importer $i$ gains when importer $j$ imposes a tariff.

4 - Time consistency of trade policies

This section derives the time consistent trade instrument when there is a lag between production and consumption decisions. The timing of events is of great importance. We follow the hypothesis made in Lapan (1988). First, each country announces its tariff given its own belief about the tariff choice of other countries. Then, production decisions are made according to price expectations of domestic and foreign producers. Before consumption decisions (and trade decisions) are made, each government can costlessly revise the level of its trade instrument set at the beginning of the game. Finally, consumption decisions are made and trade between countries is carried out.

Because there is a lag between production and consumption and all countries can change their tariff after production decisions are made, the time consistent tariff is higher than the precommitment tariff. This is so because the $ex$-$post$ residual foreign export supply elasticity faced by country $i$ is lower than the $ex$-$ante$ elasticity. The lack of commitment
by importers may be collectively beneficial since the *ex-post* tariff is bounded below by the *ex-ante* tariff. With the perfect foresight assumption, foreign producers fully anticipate the tariff change after production decisions are made. Since the fully anticipated time consistent tariff is large, the lower world price causes a contraction in production. Therefore, the lack of commitment can increase domestic welfare by offsetting the policy competition welfare effect. However, if foreign supply is very elastic, thus making the residual *ex-post* export supply elasticity much lower than the *ex-ante* elasticity, this potential gain of not committing to a tariff before production decisions are made can vanish.

4.1 *Tariff competition*

Formally, the slope of the *ex-post* foreign export supply curve is: \[ \frac{\partial X}{\partial \bar{p}} = \phi' - \bar{Q}' = -\bar{D}'. \]

Clearly, given output levels, the slope of the foreign export supply is smaller *ex-post* than *ex-ante* if \( \bar{Q}' > 0 \). Restricting the strategy space to the use of a tariff, the following arbitrage condition between the world price and domestic price must hold if imports in country \( i \) are positive: \( p_i = \bar{p}(1 + \tau_i) \). From an *ex-post* perspective, *i.e.* when production decisions are made: \( \frac{\partial m_i}{\partial p_i} = m_i' + q_i' \). The welfare function of the government, once production decisions are made is still defined as in (3). The *ex-post* residual foreign export supply is:

\[\bar{X}(\bar{p}, \bar{p}') = \sum_{j \neq i} m_j \left( \bar{p}(1 + \tau_j), \bar{p}'(1 + \tau_j) \right) - m_i = 0\]  

(19)
Totally differentiate (19) given the tariff choice of other countries and producers’ expectations. Assume perfect foresight so that producers correctly anticipate the tariffs set by governments \( \text{ex-post} \), and so \( \tau_i = \tau_i^e, \bar{p} = \bar{p}^e \) and \( p_i = p_i^e \).

\[
\left[ \bar{X}_\bar{p} = \sum_{j\neq i} \frac{\partial m_j}{\partial p_j} (1 + \tau_j) \right] d\bar{p} - dm_i = 0 \tag{20}
\]

Set equation (4) to zero, substitute (20) in (4) for \( dm_i \) and divide both sides by \( d\tau_i \). After some manipulations, you get the \( \text{ex-post} \) tariff reaction function implicitly defined by:

\[
\frac{\partial W_i}{\partial \tau_i} = \left( \tau_i \bar{p} \left[ \bar{X}_\bar{p} - \sum_{j\neq i} \frac{\partial m_j}{\partial p_j} (1 + \tau_j) \right] - m_i \right) \frac{\partial \bar{p}}{\partial \tau_i} = 0 \tag{21}
\]

The tariff reaction function for country \( i \) is: \( \tau_i^e = R(\tau_1, ..., \tau_{i-1}, \tau_{i+1}, ..., \tau_N, \beta) \) where \( \beta \) is a vector of all other factors in the information set (such as observed domestic and foreign production levels). Imposing a subgame perfect equilibrium, the time consistent tariff in elasticity form is:

\[
\tau_i^e = \frac{\alpha_i}{\xi^e + \sum_{j\neq i} \alpha_j \eta_j^e}.
\]

**Proposition 2**: Assuming symmetry among the policy active importers, the time consistent tariff is higher than the \( \text{ex-ante} \) tariff. Moreover, the inability to precommit to the \( \text{ex-ante} \) tariff may result in a higher welfare \( \text{ex-post} \) for all importing countries.

**Proof**: Evaluating (21) at the precommitment solution \( \tau_i^p \), we get:

\[
\frac{\partial W_i}{\partial \tau_i} \bigg|_{\tau_i = \tau_i^p} = \tau_i^p \bar{p} \left( -\bar{Q} - \sum_{j\neq i} q_j^p (1 + \tau_j^p) \right) \frac{\partial \bar{p}}{\partial \tau_i} > 0 \tag{22}
\]
Equation (22) provides the following ranking between the ex-post and the ex-ante tariffs, $\tau_i^p < \tau_i^c$. Also, as shown in (13), we have: $\tau_i^p < \tau_i^*$. Domestic welfare must be lower under both the ex-ante and ex-post equilibrium tariffs than under the collusive tariff since the ex-ante collusive tariff yields the global optimum. Since the domestic welfare function is continuous and monotonic over the interval $[\tau_i^p, \tau_i^*]$, a sufficient condition for the time consistent tariff to be welfare superior to the ex-ante tariff is for it to fall within the previous interval. However, a Nash equilibrium resulting in an ex-post tariff set higher than the collusion ex-ante tariff has an indeterminate effect on domestic welfare compared to the precommitment tariff. QED.

The intuition behind proposition 2 is the following. The ex-ante Nash equilibrium entails tariffs set too low compared to the collusive equilibrium. This is so because all countries face a residual foreign export supply curve which is more elastic due to the tariff competition between countries. If foreign and domestic production is fixed, the residual foreign export supply curve is less elastic ex-post, so the ex-post welfare may be closer to the collusive ex-ante welfare. However, if foreign production is very elastic, then welfare can be lower ex-post since the equilibrium ex-post will be far apart from the ex-ante equilibrium.

4.2 Quota competition

The time consistent quota is derived in a similar way to the time consistent tariff. Imposing a quota on imports is equivalent to setting the domestic price. In this case, since $m_i = d_i(p_i) - q_i(p_i^*)$, the domestic price is $p_i(m_i, p_i^*)$. The residual foreign
export supply curve faced by country $i$ is $X(\bar{p}, \bar{p}^e) - \sum_{j \neq i} m_j - m_i = 0$. Differentiate the latter expression to get: $X_p(\bar{p}, \bar{p}^e) - dm_i = 0$. Perfect foresight implies $p_i^e = p_i$ and $\bar{p}^e = \bar{p}$. Setting equation (4) to zero after appropriate substitutions implicitly yields the ex-post quota reaction function $m_i^e = g(m_1, \ldots, m_{i-1}, m_{i+1}, \ldots, m_N, \beta)$. Imposing a subgame perfect equilibrium, the time consistent tariff equivalent quota in elasticity form can be written as:

$$\frac{\partial W_i}{\partial m_i} = -\frac{m_i}{X_p} + (p_i - \bar{p}) = 0 \Rightarrow \theta_i^e \equiv \frac{\partial}{\partial \bar{p}} = \frac{\alpha_i}{\xi_i^e}$$

(23)

**Proposition 3**: Assuming symmetry among the policy active importers, the time consistent quota is lower than the precommitment quota. It induces a higher price differential between the domestic price and the world price ex-post than in the ex-ante case. Moreover, the inability to precommit to the ex-ante quota may be collectively welfare improving for all importing countries.

**Proof**: Evaluating (23) at the precommitment solution $m_i^p$, we get:

$$\frac{\partial W_i}{\partial m_i} \bigg|_{m_i=m_i^p} = p_i - \bar{p} - \frac{m_i^p}{\phi' - \bar{Q}'}$$

$$= \frac{\phi'}{\phi' - \bar{Q}'} \left[ p_i - \bar{p} - \frac{m_i^p}{\phi'} \right] - \frac{(p_i - \bar{p}) \bar{Q}'}{\phi' - \bar{Q}'}$$

$$= -\frac{(p_i - \bar{p}) \bar{Q}'}{\phi' - \bar{Q}'} < 0$$

(24)

Equation (24) gives the following ranking between the ex-ante Nash equilibrium quota and the time consistent ex-post quota, $m_i^e < m_i^p$. Similarly to the case of tariffs, the
incapability to precommit to a quota can be welfare improving since we have proved the following rankings: \( m_i^p > m_i^c \) and \( m_i^p > m_i^p \); and that the \textit{ex-ante} collusive quota yields the global optimum. The world market structure in our model implies that the quantity imported in each country is too large \textit{ex-ante}. Since \textit{ex-post}, the residual foreign export supply is less elastic, each country will import a smaller quantity. QED.

**Proposition 4:** Assuming symmetric policy active importers who cannot precommit to a trade policy, the time consistent tariff is not necessarily an inferior instrument to the time consistent quota. The ranking between the two instruments is generally indeterminate.

**Proof:** The ranking of the time consistent quota and tariff involves the ranking of two second best policies. But clearly, because:

\[
\frac{\partial W_i}{\partial \tau_i} \bigg|_{\tau_i = 0} = -\theta_i^p \left( \sum_{j \neq i} \frac{\partial m_j}{\partial p_j} (1 + \tau_j) \right) \frac{\partial p}{\partial \tau_i} < 0,
\]

then \( \tau_i^c < \theta_i^c \). Thus, in the case where \( \tau_i^c > \tau_i^* \), tariffs are welfare superior to quotas because the optimal \textit{ex-post} tariff is in the interval \([\theta_i^*, \theta_i^c]\). This is more likely to happen when foreign demand elasticities are very small and/or foreign supply elasticities are very large. QED.

5 - A Numerical Example

This section tries to illustrate the welfare implications of the precommitment and time consistent trade instrument discussed in propositions 1 to 4. Suppose domestic preferences are represented by a quasi-linear utility function: \( U(w_i, x_i) = w_i + \frac{a}{b} x_i - \frac{x_i^2}{2b} \).
where \( w_i \) is a numéraire good. These preferences implies the domestic demand (\( d_i \)) for good \( x_i \) is: \( d_i = a - bp_i \), where \( a \) and \( b \) are positive constants and \( p_i \) is the domestic price.

Domestic producers of the importable good in country \( i \) have the following cost function:

\[
c(q_i) = \frac{q_i^2}{2g} - \frac{c}{g} q_i.
\]

Competitive domestic markets imply the domestic supply function is:

\[
q_i = c + gp_i,\ 
\text{where}\ g \text{ is a positive constant. Assume for simplicity the available policy is a specific tariff}\ ^6, \text{ so that } p_i = \bar{p} + t_i. \text{ The import demand function of country } i \text{ is then:}
\]

\[
m_i = a - c - (b + g)(\bar{p} + t_i).
\]

The welfare function for country \( i \) is the sum of consumer surplus, producer surplus and tariff revenue:

\[
W_i = \int_0^{m_i+q_i} \left( \frac{a - y_i}{b} \right) dy_i - c(q_i) - \bar{p}m_i = \frac{a}{b} d_i - \frac{d_i^2}{2b} - \frac{q_i^2}{2g} + \frac{c}{g} q_i - \bar{p}m_i, \tag{25}
\]

such that \( d_i = q_i + m_i \). The export supply curve of the rest of the world is:

\[
\bar{X}(\bar{p}, \bar{p}^e) = \bar{Q}(\bar{p}^e) - \bar{D}(\bar{p}) = (\delta - \alpha) + \beta \bar{p} + \gamma \bar{p}^e \ 
\text{At the ex-ante level}\ \bar{p}^e = \bar{p}, \text{ so the ex-ante foreign export supply curve is: } \phi(\bar{p}) = (\delta - \alpha) + (\beta + \gamma)\bar{p}. \text{ World equilibrium implies } \sum_{j=1}^{N} m_j = \phi(\bar{p}). \text{ Solving for } \bar{p} \text{ yields: } \bar{p} = \frac{N(a-c) - (\delta - \alpha) - (b+g)\sum_{j} t_j}{(\beta + \gamma) + N(b+g)}.\]

The first order condition of the strategic tariff game can be rewritten as \( t_i(\partial m_i / \partial t_i) = m_i(\partial \bar{p} / \partial t_i) \). Therefore, the reaction function for country \( i \) is implicitly defined by:

\[\text{In our model, proposition 1 to 4 still hold if the strategy space is restricted to specific tariffs instead of } ad\text{-valorem} \text{ tariffs. However, specific tariffs and } ad\text{-valorem} \text{ tariffs are not equivalent instruments. The use of a specific tariff is uniquely to facilitate the computation of the numerical example in this section.} \]
Imposing symmetry among the importing countries, the intersection of the reaction functions solves for the \textit{ex-ante} Nash equilibrium. To simplify the derivation of further results, rescale the following parameters: $\alpha = \alpha' N, \delta = \delta' N, \beta = \beta' N$ and $\gamma = \gamma' N$. Moreover, define $\lambda = \left( \frac{\beta' + \gamma'}{b + g} \right)$ and $A = \frac{(a-c)\lambda - (\alpha' - \delta')}{b + g}$. The precommitment optimal tariff is:

$$t_i^p = \frac{A}{\lambda(2 + \lambda) + (N-1)(1 + \lambda)^2}$$

The optimal collusive tariff is given by maximizing the sum of the $N$ countries’ welfare. This gives a set of $N$ first order conditions of the type: $t_i \sum_j \partial m_j / \partial t_i = \bar{X} (\partial \bar{p} / \partial t_i)$. Because of the symmetry between the countries, $t_i = t_j$. The solution is:

$$t_i^* = \frac{A}{\lambda(2 + \lambda)}$$

We have expressed the precommitment tariff as a deviation from the collusive tariff. Both tariffs have the same numerator and only differ in their denominator. It is readily seen that if $N > 1$, $t_i^p < t_i^*$, because $(N-1)(1 + \lambda)^2 > 0$. The latter expression illustrates the theoretical result of equation (13). The collusive tariff is higher than the non-cooperative precommitment tariff.
The time consistent tariff is found by maximizing:

\[
W_i = \int_0^{q_i^{*,m_i}} \left( \frac{a-y_i}{b} \right) dy_i - c(q_i) - \bar{p}m_i = \frac{a}{b} d_i - \frac{d_i^2}{2b} - \frac{(q_i^*)^2}{2g} + \frac{c}{g} q_i^* - \bar{p}m_i
\]

where \( q_i^* \) represents domestic production given the trade policy expectation of producers in country \( i \). From the government’s perspective, production is fixed. In this case,

\[
\bar{p} = \frac{Na + \alpha - \bar{Q} - \sum_j bt_j - \sum_j q_j^*}{Nb + \beta}
\]

and \( m_i = a - bt_i - q_i^* - b\bar{p} \). The first order condition is:

\[
t_i \left( \frac{\partial m_i}{\partial t_i} \right) = m_i \left( \frac{\partial \bar{p}}{\partial t_i} \right).
\]

Imposing perfect foresight implies, \( t_i^* = t_i^* \) and \( q_i^* = c + gp_i \). The implicit reaction function of country \( i \) is:

\[
t_i ((N-1)b + \beta) = m_i.
\]

For further reference, define the following two parameters \( \mu = \frac{b}{b+g} \in [0,1] \) and \( \mu' = \frac{\beta'}{\beta' + \gamma} \in [0,1] \). The parameters \( \mu \) and \( \mu' \) can be interpreted as the relative domestic and foreign demand responsiveness respectively. Using the symmetry across countries, the optimal time consistent tariff is:

\[
t_i^* = A \frac{1}{\lambda(2 + \lambda) + (1 + \lambda)(\lambda(N\mu' - 1) + (N - 1)\mu)}
\]

Using equations (27), (28) and (30), we can illustrate the point made in proposition 2. Because we used linear domestic and foreign demand and supply with a quadratic cost function, it should be clear that the welfare function of country \( i \) is a quadratic function.

The welfare function reaches a maximum at \( t_i^* \) and is symmetric around that point. With those properties, we show that the inability to precommit to the \textit{ex-ante} tariff increases
welfare for an importer if the following inequality holds: \( t_i^p < t_i^c < t_i^r + (t_i^r - t_i^p) \). The first inequality is ensured by proposition 2. The second inequality holds if:

\[
\Delta' = \left[ \lambda(2 + \lambda) + 2(N - 1)(1 + \lambda)^2 \right] \left[ \lambda(N\bar{\mu} - 1) + (N - 1)\mu \right] + (N - 1)\lambda(2 + \lambda)(1 + \lambda) > 0
\]

The inability to commit to the specific tariff is collectively beneficial for the policy active importers if the inequality above is satisfied. Clearly, if \( N = 1 \), \( \Delta' < 0 \) implying precommitment is desirable. For \( N > 1 \), a sufficient condition for \( \Delta' \) to be greater than zero is for \( \bar{\mu} \geq 1/N \) because then \( t_i^p < t_i^c < t_i^r \). The only remaining question is what if \( t_i^c > t_i^r \). Since \( \Delta' \) is increasing in \( N \), the likelihood that the inability to precommit is beneficial increases with \( N \). Fixing \( N = 2 \) and \( \lambda = 1 \), \( \Delta' \) becomes: \( 22\bar{\mu} + 11\mu - 5 > 0 \). Thus, for not very high relative foreign and domestic demand responsiveness, the collective inability to commit to the ex-ante specific tariff increases welfare for policy active importers.

In the case of a strategic quota game, \( m_i \) is the choice variable. Since \( d_i = q_i + m_i \), for the precommitment case, domestic market equilibrium implies: \( p_i = \frac{a - c - m_i}{b + g} \). Solving for the inverse ex-ante foreign export supply gives: \( \bar{p}(\bar{X}) = \frac{\bar{X} + (\alpha - \delta)}{\beta + \gamma} \). Maximizing domestic welfare in (25), the first order condition is: \( p_i - \bar{p} = \bar{p}'m_i \), where \( \bar{p}' \) is the derivative of the inverse foreign export supply with respect to imports in country \( i \).
If the policy active importers can precommit to their policy, the quota reaction function of country $i$ is then implicitly defined by:

$$m_i^p = \frac{a-c-m_i}{b+g} + \frac{m_i}{\beta + \gamma} = \sum_j m_j + (\alpha - \delta) \frac{\beta}{\beta + \gamma}.$$

Imposing symmetry between the countries allows to solve for the optimal Nash equilibrium quota: $m_i^p = \frac{(a-c)(\beta + \gamma) - (\alpha - \delta)(b+g)}{(N+1)(b+g) + (\beta + \gamma)}$. The tariff equivalent is given by: $\theta_i^p = p_i - \bar{p}$. With various simple manipulations, one finds:

$$\theta_i^p = \frac{A}{\lambda(2 + \lambda) + (N-1)\lambda(1 + \lambda)}$$

(31)

Note that equations (27) and (31) give the same optimal tariff when $N = 1$. This shows the equivalency result between quotas and tariffs in case of a single large country.

However, following proposition 1, if $N > 1$, $t_i^p < \theta_i^p$ since $(N-1)((1+\lambda)^2 - \lambda(1+\lambda)) = (N-1)(1+\lambda) > 0$; and quotas are welfare superior to tariffs. In the collusive case, the choice of tariff or quota is irrelevant and thus, the tariff equivalent $\theta_i^*$ is equal to $t_i^*$ in equation (28).

Finally, the time consistent quota is derived by assuming production is fixed. The price in importing countries is: $p_i = (a - m_i - q_i^e)/b$. Similarly, the foreign inverse export supply world is: $\bar{p}(\bar{X}, \bar{Q}^*) = (\alpha + \bar{X} - \bar{Q}^*)/\beta$, and the world price is determined such that $\bar{X} = \sum_j m_j$. The first order condition implicitly defines the quota reaction function for country $i$: $\beta(\alpha - m_i - q_i^e) - b(\alpha + X - Q^*) = bm_i$. Imposing perfect foresight and symmetry among the $N$ countries gives:
\[ m_i^c = \frac{\beta [(a - c)(\beta + \gamma) - (\alpha - \delta)(b + g)]}{\beta (\beta + \gamma) + (N + 1)(\gamma + \beta)(b + g) - N \gamma (b + g)}. \]

The time consistent tariff equivalent is:

\[ \theta_i^c = \frac{A}{\lambda (2 + \lambda) + (\mu N - 1)(1 + \lambda) \lambda} \quad (32) \]

Again, because we have expressed equations (31) and (32) in terms of deviation from the optimal collusive tariff equivalent, we can easily illustrate proposition 3. The importers’ collective inability to precommit to their trade policy will be welfare improving if the following inequality holds: \( \theta_i^p < \theta_i^c < \theta_i^* + (\theta_i^* - \theta_i^p) \). The first inequality in the latter expression always holds for \( \mu < 1 \). The second inequality holds if:

\[ \Delta^0 = \left[ 2N(1 + \lambda) - \lambda \right] (N\mu - 1) + (2 + \lambda)(N - 1) > 0 \quad (33) \]

If you set \( N = 1 \), then \( \Delta^0 < 0 \), and the inability to precommit to the \textit{ex-ante} quota hurts the large country [Lapan (1988)]. For \( N \neq 1 \), note that (33) is independent of the parameter \( \mu \). This means that domestic demand and supply do not play a role in the \textit{ex-post} versus \textit{ex-ante} welfare analysis in the strategic quota game. Further note that \( \Delta^0 \) is an increasing function of \( \mu \). The inability to precommit to the \textit{ex-ante} quota is more likely to be welfare improving the higher the foreign exporters relative demand responsiveness is. A sufficient condition for the inequality to hold is that: \( \mu \geq 1/N \), because then: \( \theta_i^c < \theta_i^* \).

The sign of \( \partial \Delta^0 / \partial N \) is ambiguous. The likelihood that the inability to precommit is beneficial increases with \( N \) (\( \Delta^0 \) is increasing in \( N \)) as long as \( \mu \geq \lambda / (4 + 3\lambda) \).
Assuming commitment is not feasible, (30) and (32) can be used to compare tariffs and quotas. Clearly if $\theta_i^c < t_i^*$, then quotas dominate tariffs; comparably, if $t_i^c > t_i^*$ then tariffs dominate quotas (for $(N\mu - 1) + (N - 1)\mu < 0$). Thus the ambiguity arises only for $\theta_i^c > t_i^* > t_i^c$. Using the same reasoning as earlier, tariffs will dominate quotas if and only if $\theta_i^c - t_i^* > t_i^* - t_i^c$. Hence, tariffs are superior to quotas if and only if $\Delta^\theta < 0$ where:

$$\Delta^\theta = (N\mu - 1)(1 + \lambda)(N - 1)\mu + \lambda(N\mu - 1) + (2 + \lambda)\left[\lambda(N\mu - 1) + \frac{(N - 1)\mu}{2}\right] < 0$$

Finally, in some extreme cases, an importing country can end up worse off than if it had no market power at all on the world market. In the case of tariffs, the condition is:

$$2(N - 1)\mu + \lambda N\mu(2 + \lambda) + \lambda^2(\mu - 1) < 0$$  \hspace{1cm} (34)

The inequality in (34) implies the policy active importers would collectively be better off if they had precommitted not to free trade. In the quota game, market power is collectively disadvantageous if: $\lambda N(2 + \lambda) + \lambda^2(N\mu - 1) < 0$.

6 - Concluding Remarks and Extensions

We have shown that tariffs and quotas are not equivalent protection instruments in a strategic setting where importing countries have purchasing power on the world market over a certain good. Each policy active importer would be better off by colluding and setting their trade instrument cooperatively. If production lags are present, the *ex-ante* optimal policy is not time consistent because the *ex-post* elasticity of the residual foreign export supply curve is lower than the *ex-ante* elasticity. However, we have shown that the importers’ inability to precommit to their trade instrument may be welfare superior to
the precommitment solution. The negative welfare implication of non-cooperative behavior may be balanced off by the welfare effect of the *ex-post* elasticity. Given the structure of the world market, these conclusions extend to any dynamic framework in which some inputs are committed before trade decisions are made. The political reasons why the government does not, or cannot, precommit itself to a predetermined policy is not the focus of the paper. However, it could very well be an interesting exercise to explain the precommitment incapability of the importers.

An interesting extension to our model would be to modify the time consistency game to include another stage. Suppose that the timing of economic decisions is modified as follows: First, governments of the policy active countries announce a tax/subsidy it will pay domestic producers as well as the tariff rate. Next, domestic and foreign production is made according to producer’s price expectations. Assuming precommitment is not feasible, each policy active government can revise the tariff rate or the production tax/subsidy. Finally, consumption decisions and trade are carried out. Lapan (1988) has shown that, with a single large country, the optimal production policy is a tax on domestic production of the importable. The purpose is to signal foreign producers to increase their production. If it were possible, a government would want to assure foreign producers they will not exploit the lower *ex-post* elasticity once production is made.

The first question to answer is if there exists such a production tax and/or subsidy which would increase welfare for the importers. Assuming they behave non-cooperatively, if we allow the importers the opportunity to use a production policy, it may very well be
that they will end up worse off than had they not been allowed to move before production decisions were made in the first place. In this situation, finding the cooperative solution could be interesting. It may seem odd at first to suppose importers set their production policy cooperatively, and that we later restrict their trade policy to be set non-cooperatively. However, this is not as strange as it may seem. In practice, there are some issues on which binding agreements are easily enforced, facilitating cooperation, while for other issues, the lack of any credible enforcement mechanism makes the nature of any interaction fundamentally non-cooperative [Burbidge et al. (1997)].
7 - Bibliography


