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# Soil Degradation, Policy Intervention and Sustainable Agricultural Growth

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## Abstract

Sustainable agricultural growth in developing countries is jeopardized by soil degradation consequent upon intensive cultivation and use of increasing doses of chemical inputs. To pave the way to sustainable agricultural growth we develop a model that incorporates organic fertilizer into the production technology as an input having a double role of enhancement of soil fertility and conservation of the ecosystem. The results show that public intervention can make equilibrium agricultural growth sustainable by maintaining conservation of soil fertility in a setting with non-convex resource regeneration. The equilibrium is found to be dynamically stable. On the basis of our comparative static results we argue for soil preserving and productivity-enhancing technological innovations and suggest a combined tax and direct payment scheme to encourage the use of soil conserving inputs.

**Keywords:** soil depletion, organic fertilizers, replenishment, sustainable growth, incentive payments

**JEL:** Q01, Q16, Q24

## 1 Introduction

Excessive extraction of ground water and declining soil fertility consequent upon continuous and intensive cultivation using chemical inputs in inappropriate doses are causing serious threats to the sustainability of agricultural growth in developing countries. The WORLD DEVELOPMENT REPORT 2008 observes that modern varieties have been widely adopted in the cultivation of rice, wheat, maize and sorghum in South Asia, East Asia and Pacific countries. The green revolution in Asia doubled cereal production between 1970 and 1995. But intensification has brought environmental problems of its own. In intensive cropping systems the excessive and inappropriate use of agrochemicals pollutes waterways, poisons people and upsets ecosystems. The on-site and off-site effects of intensive agriculture have caused soil

degradation (salinization, nutrient depletion, loss of soil organic matter, ground water depletion, agrochemical pollution and loss of local biodiversity). TILMAN et al. (2002) estimate that since 1945, approximately 17% of vegetated land has undergone human-induced soil degradation and loss of productivity, often through poor fertilizer and water management, soil erosion, and shortened fallow periods. Continuous cropping and inadequate replenishment of nutrients – removed in harvested materials or lost through erosion, leaching, or gaseous emissions – deplete fertility and cause soil organic matter levels to decline. It is also mentioned that crop rotation, reduced tillage, cover crops, fallow periods, manuring and balanced fertilizer application can help maintain and restore soil fertility.

Land is the most important ingredient of agricultural production and the level of soil fertility depends on the rate of depletion, natural regeneration and artificial replenishment of soil nutrients. Regeneration depends on various soil treatment measures like crop rotation, fallow system, application of green manures and organic fertilisers. The depletion rate is determined by intensity and technique of farming. This is the human-nature interaction in natural resource use that we study in this paper. The soil fertility level regenerates at a natural rate that may depend upon the current soil fertility level (KRAUTKRAEMER, 1994; BARRETT, 1991). BARRETT (1991) notes that in traditional agriculture soil fertility is maintained by returning cropland to fallow. But under the pressure of rising human population fallow periods have grown shorter. Crop rotation may be another mode of soil conservation. For example, a typical crop rotation system in West Bengal, India, was jute-paddy-potato which has been replaced by mono-cropping of high-yielding (HY) paddy throughout the year. Another method of replenishment of soil nutrients was the application of cattle dung, plant leaves and compost manures which are no longer in use in adequate doses. In the Indian state of Punjab extensive use of nitrogen fertilizers and pesticides is found to have increased concentration of nitrates and pesticides residues in water and food above tolerance limits. The subsidies on water and fertilizer are found to be more wasteful in input use and they discourage a shift to alternative cropping patterns (WORLD DEVELOPMENT REPORT, 2008). TIETENBERG (2005) mentions similar problems in other countries and describes how the persistence of residues of pesticides in the environment kills many useful species and contaminates water supplies. Subsidies encourage excessive extraction of ground water making water salty in the aquifer and land less productive. The subsidies and government support measures have encouraged overexploitation of resources in many cases. The private property owners do not find it profitable to adopt suitable conservation measures; instead they prefer cheap fertilisers to replace the lost nutrients of the soil (TIETENBERG, 2005). This is consistent with the empirical findings of SASMAL (2006) that productivity of land in West Bengal, where HY paddy has been cultivated intensively for a long time, has declined by 6-16.5% over a period of 13 years despite

25-50% increase of fertiliser use during this period. As a response to declining productivity in agriculture the government has undertaken a project to prepare a soil nutrient (deficiency) map. While arguing for crop diversification in favour of less water intensive crops to preserve ground water, SASMAL (2006) shows with a time series analysis based on Indian data that ground water extraction and fertiliser use in India have been significantly influenced by subsidised input prices and price support for food grains. The Indian experience provides a lot of insight for this paper. As nitrogenous fertilisers (N) are subsidised more than potassium (K) and phosphate (P) fertilisers, nitrogen is used in excessive doses adversely affecting soil profile and causing deficiency in P, K and micronutrients. The same picture is found in the studies of SIDHU (2002) and BHULLAR and SIDHU (2006) for the state of Punjab in India. These studies indicate that perverse incentives may cause distortions in resource use and input applications. So, the misallocation of resources is not just a question of market failure, it is rather one of government failure too. This paper contributes to the analysis how government policies can be reoriented towards conservation of soil fertility but still in line with growth of agricultural production.

DASGUPTA and MÄLER (2009) draw attention to the externality problems associated with the use of natural resources. They also point to the problem of under-priced natural resources and the importance of shadow prices in the proper valuation of ecological services and natural assets. A suitably chosen set of taxes and subsidies may be helpful in this context to reach the social optimum. Furthermore, poverty and non-convex ecosystem structure are of great relevance in the analysis of resource management. Given a non-linear and non-convex regeneration function of the ecosystem, the market fails to deliver an efficient resource allocation. However, non-market institutions (governmental policy measures) also fail in many cases (DASGUPTA and MÄLER, 2009). Poverty remains to be one important cause of resource degradation. The poor typically depend far more heavily on natural assets than the rich and deepening poverty seems to go hand-in-hand with resource dependence and degradation (SASMAL, 2003 and 2009; BARRETT, 2006). The poor are more concerned about their immediate consumption and least bothered about sustained soil fertility and future return from land. In a micro-level study in the Indian context SASMAL (1992) finds that cultivation intensity of HY varieties is higher among the small farmers and it is explained by government support measures. Poverty and support measures of the government are important factors in the management of soil fertility. Non-convex resource systems imply that the resource base potentially collapses if its use crosses a threshold. BARRETT (2006) notes that renewable resource dynamics are typically non-linear, non-convex and generally logistic-shaped. He suggests taking appropriate management interventions (e.g. short fallows, application of green manures or organic fertilisers) during the early stages of degradation.

The problem of soil management has been addressed in different ways. GOETZ (1997) has suggested crop-diversification and optimal choice of soil depth to minimize soil erosion. FEINERMAN and KOMEN (2005) have analysed the effects of organic versus chemical fertilizers on soil quality. WEIKARD and HEIN (2011) study a Sahelian rangeland and suggest to reduce grazing pressure. ZILBERMAN (2006) suggests price, policy and technological reforms for the conservation of resources. If subsidization of inputs and price support for the crops result in over-exploitation of resources, withdrawal of subsidy, taxation and permits may support resource conservation. But in a poor agrarian system it may not be a desirable solution in the absence of any effective alternative policy option. Therefore, it is necessary to encourage farmers to use resource conserving inputs and farming practices. Public support for longer fallow periods, suitable crop rotation or use of resource-friendly inputs like organic or green manures may be the alternative policy options. Technological change is essentially the substitution of natural capital by man-made capital and puts emphasis on innovation for getting along with lower rates of natural resource use (SOLOW, 1992).

HARRINGTON, KHANNA and ZILBERMAN (2005) have introduced conservation capital into the production technology of an endogenous growth model where conservation capital plays the twin role of enhancing the efficiency of production capital as well as helping the abatement of pollution. ZILBERMAN, LIPPER and MCCARTHY (2006) propose Payments for Environmental Services (PES) programmes. These programmes may include shifting of land from more resource intensive to less resource intensive crops and encourage the use of resource conserving inputs and techniques. In the state of Punjab in India incentive payments to the farmers have been recommended by an Expert Committee to encourage changes in cropping pattern in favour of resource conservation (BHULLAR and SIDHU, 2006).

This paper argues for reorientation of public policies in favour of conservation of natural resources pertaining to growth in agriculture and proposes incentive payments to encourage the use of resource and environment conserving inputs and farming practices. The state of soil fertility could be described in a variety of ways – soil depth, soil moisture, organic matter, soil nitrogen or an index that combines relevant soil characteristics. The natural rate of regeneration of soil fertility level can depend upon the current soil fertility level. Soil fertility affects soil regeneration through the deposition of residue organic matter whose quantity can depend on the level of soil fertility. Consequently, the natural rate of soil regeneration is low when the soil fertility level is low, increases over some range of fertility levels, reaches a peak and then declines. More formally, the natural regeneration of the resource takes the shape of logistic growth function. Soil fertility on fallow land or in an undisturbed landscape is in equilibrium at the maximum level (KRAUTKRAEMER, 1994; ELIASSON and TURNOVSKY, 2004). Given the characteristics of non-linearity and non-convexity in

the natural regeneration function, there will be multiple equilibria and threshold effects indicating that if the resource stock declines below a critical level, there will be irreversible damage to the resource base. If the resource stock remains above that threshold, a stable equilibrium can be reached at a high level of stock (MAY, 1977; DASGUPTA and MÄLER, 2003; BARRETT, 2006). In fact, the interaction between the human decision of “harvesting” soil fertility and the natural rate of regeneration of the resource will trace out the dynamics of the resource stock. The human decision of optimal depletion of soil fertility may not be compatible with the level permissible in nature. The problem is that the depletion level is determined by socio-economic factors and market forces whereas natural regeneration follows its own law. Naturally, market failure due to externalities and perverse government interventions leading to excessive depletion of the resource generates a mismatch between the two. If the use of chemical fertilizers is taken as a measure to intensify farming and if the rate of depletion of soil fertility is directly related to that, then the dynamics of soil fertility will be related to the use of fertilizers. Here we determine the optimal path of intensification (exemplified as the use of fertilizers) in a dynamic perspective because it will trace out the optimal paths for depletion and regeneration of soil. The regeneration and depletion rates at different points of time are linked together. Essentially this gives a dynamic optimization problem because the optimal path for the use of the input will finally determine the dynamics of the resource stock. The government may subsidize soil-conserving inputs or provide incentive payments for adopting suitable measures for replenishment of soil fertility. This paper is not a study of costs and benefits of alternative policies and techniques for soil management. The conventional policy intervention of taxation of polluting inputs and subsidization of resource preserving inputs is there to maintain balance in resource stock. But the main focus of this study is how the policy instruments can be used to increase the resilience of the soil system or even to avoid multiple equilibria in a non-linear and non-convex resource system in order to stabilize agricultural growth at a high level of soil fertility.

The paper is arranged as follows: section 2 develops a theoretical model of unsustainable agricultural growth due to externalities and market failure leading to excess depletion of soil fertility in a decentralised framework. Section 3 constructs a theoretical model to demonstrate how effective government intervention can make agricultural growth sustainable by encouraging the use of fertility augmenting inputs and to maintain the stock of soil fertility in a non-convex resource system. Section 4 derives comparative static results and stability conditions. A summary of results and policy implications are presented in section 5.

## 2 Soil Degradation and Unsustainable Agricultural Growth

Let  $Q$  be agricultural output,  $N$  the stock of natural fertility of soil and  $z$  some variable input, say, chemical fertilizer, that captures the intensity of farming. We consider the following production function for a representative farmer in a decentralised agrarian system:

$$Q = Q(N, z). \quad (1)$$

We assume that  $Q_N > 0$ ,  $Q_{NN} < 0$ ,  $Q_z > 0$ ,  $Q_{zz} < 0$  and also that  $Q = 0$  if  $N = 0$ ,  $Q > 0$  if  $z = 0$  such that soil fertility is an essential input for agricultural production.

Soil fertility is a renewable resource with a continuous natural regeneration. Following ELIASSON and TURNOVSKY (2004) and KRAUTKRAEMER (1994) the dynamics of the resource depends on natural regeneration and depletion rates. At any point of time, the net rate of change of the resource is given by

$$\dot{N} = G(N) - L(z) \quad (2)$$

where  $G(N)$  describes the gross regeneration rate of the resource and  $L$  is the rate of depletion determined by the intensity of farming denoted by  $z$ . The natural rate of soil regeneration can be low when soil fertility level is low, increase over some range of fertility levels, reach a peak and then decline. We assume a maximum level of soil fertility, denoted by  $N_{\max}$ , and that the growth of the resource  $G(N)$  is governed by the logistic function

$$G(N) = \rho N^\alpha \left( 1 - \frac{N}{N_{\max}} \right),$$

where  $\rho$  is the intrinsic rate of growth of soil fertility,  $N_{\max}$  is the upper bound on soil fertility and  $\alpha$  is a constant. We assume  $\alpha > 1$  which implies depensation at low levels of stock, i.e. average growth  $G(N)/N$  is increasing in the stock of soil fertility; see Figure 1.

In an undisturbed landscape where harvest of the resource is zero (i.e.  $L = 0$ ), soil fertility  $N$  converges to its maximum sustainable level  $N_{\max}$ . Given this resource structure, the depletion of the resource is determined by the economic decisions. Human-nature interaction will trace out the path for the resource stock.

Some soil conservation inputs like green manure or organic fertilizer etc. may be used or soil treatment measures like fallow system may be adopted by individual farmers.

As a base case we assume that farmers do not take any such measure for soil treatment because this is not profitable for them.

For simplicity, we may assume linearity of  $L$  in  $z$  i.e.,  $L = hz$  where  $h$  is “harvest” of soil fertility per unit of use of  $z$ . Here,  $h > 0$  is constant. Intensive agriculture, as measured by  $z$ , causes environmental degradation by creating pollution and loss of biodiversity. But farmers do not take this external cost into their consideration while making decisions on the quantities of  $z$  to be used in production. The utility function of the representative household is

$$U = U(Y, E)$$

where  $Y$  is income and  $E$  is environmental quality. Income is defined as

$$Y = qQ - pz \quad (3)$$

where  $q$  is the price of agricultural produce and  $p$  is the price of inputs. Pollution is an external cost and is, therefore, not directly reflected in the income function. Since environmental quality is conceived as a public good and an individual cannot influence it, it does not affect individual choices. The objective of the household effectively becomes maximisation of income. In a decentralised system output and input prices are given although there may be subsidies on inputs and support measures for output – these we address in the next section.

The representative farmer disregards social costs of using  $z$  and maximizes discounted total income from agricultural production over an infinite planning horizon by choosing an optimal path for  $z$ . Assuming risk neutrality, the optimisation problem of the farmer can be expressed as

$$\text{Max}_z \int_0^{\infty} Y \cdot e^{-rt} dt \quad (4)$$

$$\text{s.t.} \quad \dot{N} = G(N) - L(z)$$

$$\text{and} \quad N(0) = N_0, N(t) \text{ free as } t \rightarrow \infty.$$

Here  $r$  is the consumption discount rate. Problem (4) is a dynamic optimisation problem which can be solved by optimal control theory. The regeneration of soil fertility depends on the deposition of residue of organic matter whose quantity depends on the level of soil fertility and the rate of its depletion over time. The rate of depletion, as noted before, depends on the intensity of farming  $z$ . So, the individually optimal path of  $z$  in a dynamic perspective will give  $L(z)$  and  $\dot{N}$ . Therefore, we need



to solve problem (4) and trace out the optimal path for  $z$ . For the mathematical underpinnings of the solution we refer the reader to DOREMAN (1969), KAMIEN and SCHWARTZ (1981) and CHIANG (1992).

Following the Maximum Principle of the optimal control theory, the current value Hamiltonian of the above problem can be written as

$$H = q \cdot Q(N, z) - pz + \lambda_N (G(N) - L(z)).$$

In this model,  $N$  is stock variable,  $z$  is control variable and  $\lambda_N$  is the co-state variable and is interpreted as shadow price of the state variable  $N$ .

We obtain the first order necessary conditions:

$$\frac{\partial H}{\partial z} = 0 = qQ_z - p - \lambda_N L_z \quad (5)$$

$$-\frac{\partial H}{\partial N} = \dot{\lambda}_N - r\lambda_N = -qQ_N - \lambda_N G_N \quad (6)$$

$$\dot{N} = G(N) - L(z). \quad (7)$$

The transversality conditions are

$$N(0) = N_0; \lim_{t \rightarrow \infty} \lambda_N = 0.$$

The specifications of the production function, soil fertility regeneration and harvest functions jointly satisfy the conditions of strict concavity of  $H$  in  $N$  and  $z$  (see Appendix I). So, the Mangasarian sufficiency condition (CHIANG, 1992) is satisfied. Furthermore, the theorems of STEINBERG and STALFORD (1973) and GALE and NIKAIDO (1965) guarantee the existence of the globally unique solution to this optimal control problem.

Equation (5) provides a condition for optimal use of chemical fertilizer at each point in time. The optimal value of the control variable  $z^*$  is globally and uniquely determined in terms of the state and co-state variables and parameters. Now, the necessary and sufficient conditions for a solution to this optimal control problem can be expressed by the differential equations (6) and (7) along with the transversality conditions. The resulting system of equations will give the optimal paths for  $N$  and  $\lambda_N$ . Since  $z$  and  $Q$  are also linked in the system, their optimal paths are also determined from these equations.

We are now interested to see whether optimal growth, i.e. whether the solution to (4) will yield a sustainable growth path. Consider  $\dot{N}=0$  i.e. resource is maintained at constant level. Then we have from (7),  $G(N)=L(z)$ . If efficient level of  $z$  exceeds ecologically sustainable level, there will be degradation of  $N$  i.e.,  $\dot{N} < 0$ . Agricultural growth through intensification means that  $\dot{z} > 0$ . Then  $L(z)$  would be growing over time and a constant level of soil fertility  $N$  cannot be maintained. In addition to that,  $z$  may be used at a level exceeding the sustainable limit due to externalities and government support measures. From (6), we can determine optimum value of  $z$  at each point of time in terms of the state and co-state variables and set of parameters i.e.,

$$z^* = z^*(N, \lambda_N, r, q, p).$$

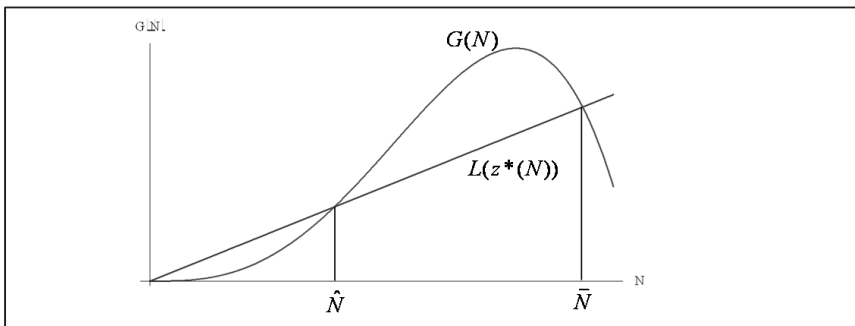
In the presence of externalities a subsidy on the input reduces  $p$  and price support increases  $q$  and as a result income maximising value of the input  $z^*$  may be higher than its sustainable value. In that case, we have that  $L(z) > G(N)$  and  $\dot{N} < 0$ . Now, we can write

$$L = L(z^*(N, \lambda_N, r, q, p)).$$

Clearly the depletion rate depends on the resource stock, shadow price of the resource and the input-output rice ratio along with other parameters. Given the parameters and ratio of input-output prices, depletion rate  $L$  can be expressed as a positive and linear function of  $N$ .

In a non-linear and non-convex resource system, there will be multiple equilibria and a threshold level of soil fertility which will determine where agricultural growth is sustainable or not. This is illustrated in Figure 1.

**Figure 1. For low stocks losses exceed regeneration and the system collapses**



Source: authors' own drawing

If soil fertility is sufficiently low such that  $N < \hat{N}$ , then optimal soil depletion exceeds regeneration such that  $\dot{N} < 0$ . Here, not only the resource base will degrade, it will collapse and stabilise at zero. If  $N > \hat{N}$ , then  $\dot{N} > 0$  and the stock of soil fertility stabilises at a high level of soil fertility  $\bar{N}$ .  $\hat{N}$  is the threshold level of soil fertility representing an unstable equilibrium. Therefore, the depletion of soil fertility must not reduce the fertility level below  $\hat{N}$  if resilience is to be maintained and a level of soil fertility  $N > 0$  is to be sustained. Here “resilience” refers to the capacity of the ecosystem to return to the stable equilibrium  $\bar{N}$  after a distortion affecting  $N$ .

### 3 Policy Intervention, Soil Conservation and Sustainable Agricultural Growth

In this section we introduce soil conserving inputs like green manure or organic fertilisers which farmers do not use (or only use to a limited extent) simply because they are not profitable. We consider the use of such inputs if the government provides some incentive payment to the farmers. The payment is justified on the ground that it enhances soil fertility and at the same it protects the biodiversity and generates ecological services to the society. This input is similar to conservation capital introduced in HARRINGTON, KHANNA and ZILBERMAN (2005). Let us now define the production function as in section 2 with some modification :

$$Q = Q(N, z, m) \quad (8)$$

where  $m$  is soil conservation input having dual role of enhancement of soil fertility and generation of environmental services. For convenience we assume that  $m$  is green manure. It is also assumed that  $Q_m > 0$ ,  $Q_{mm} < 0$  and there is some degree of substitution  $z$  and  $m$ . The specifications about  $N$  and  $z$  are same as before.

The resource dynamics changes to

$$\dot{N} = G(N) - L(z) + \psi(m), \quad (9)$$

where, as before  $G$  is gross regeneration rate of soil fertility,  $L$  is depletion rate of soil nutrients and  $\psi$  is artificial regeneration rate of soil fertility from  $m$ . It is assumed that  $\psi_m > 0$ ,  $\psi_{mm} < 0$ .

The direct marginal return from  $m$  may not be profitable for the farmers. But social marginal benefits of  $m$  are greater than its private marginal returns; so, the use of  $m$  may be subsidized. Since  $m$  has positive externalities and it generates environmental services, the government introduces a subsidy  $\zeta(m)$  as a policy instrument encouraging

the use of  $m$ . The incentive function for using  $m$  is assumed to be concave,  $\zeta_m > 0$  and  $\zeta_{mm} < 0$ .

Now, agricultural income can be written as

$$Y = qQ - pz - vm + \zeta(m) - \tau(z) \tag{10}$$

where  $v$  is the price of green manure  $m$  and  $\tau(z)$  is an environmental (Pigovian) tax that reflects the environmental cost of using  $z$ . We assume  $\tau_z > 0$  and  $\tau_{zz} > 0$ .

The utility function is as before a function of income and environmental quality  $U = U(Y, E)$ . Again an individual cannot influence  $E$  and it is provided exogenously as a public good.

Since the use of  $z$  and  $m$  has externalities, public intervention imposes a tax on  $z$  and introduces incentive payments for the use of  $m$  in order to improve social welfare. Here the objective of the private agent is to maximize the discounted total income over the infinite planning horizon subject to the given constraints as

$$Max_{z,m} \int_0^{\infty} Y \cdot e^{-rt} dt \tag{11}$$

s.t.  $\dot{N} = G(N) - L(z) + \psi(m)$  and  $N(0) = N_0, N(t)$  free as  $t \rightarrow \infty$ .

This dynamic optimization problem can be solved by using optimal control theory as in section 2. The current value Hamiltonian of the problem in (11) is

$$H = q \cdot Q(N, z, m) - pz - vm + \zeta(m) - \tau(z) + \lambda_N (G(N) - L(z) + \psi(m)).$$

The necessary and sufficient conditions for solution to this optimal control problem can now be expressed in terms of the following differential equations along with transversality conditions :

$$\frac{\partial H}{\partial z} = 0 = qQ_z - p - \tau_z - \lambda_N L_z \tag{12}$$

$$\frac{\partial H}{\partial m} = 0 = qQ_m - v + \zeta_m + \lambda_N \psi_m \tag{13}$$

$$-\frac{\partial H}{\partial N} = \dot{\lambda}_N - r\lambda_N = -qQ_N - \lambda_N G_N \tag{14}$$

$$\dot{N} = G(N) - L(z) + \psi(m) \quad (15)$$

$$\text{and } \lim_{t \rightarrow \infty} \lambda_N = 0;$$

$$N(0) = N_0, N(t) \text{ free as } t \rightarrow \infty$$

The Mangasarian sufficiency condition is fulfilled by strict concavity of  $H$  in state and control variables ( $N$ ,  $z$  and  $m$ ) jointly (see Appendix II). The conditions (12) and (13) have important economic implications. The marginal gain of using  $z$  is equal to its (after tax) price plus cost of not preserving the resource for future use at each point of time. In case of  $m$ , the marginal benefit includes the value of marginal product of  $m$ , future gain in income due to conservation of soil fertility, marginal incentive payment from the government. The costs comprise its price  $v$ . Equation (14) shows the rate at which shadow price of stock  $N$  changes over time. As we do not impose a balanced-budget condition, the total of incentive payments for  $m$  is usually not equal to the tax revenues from  $z$ . We assume that the deficit, if any, is financed from outside the agricultural sector which can be justified by the environmental benefits from the use of  $m$ .

Like in section 2, using the theorems of STEINBERG and STALFORD (1973) and GALE and NIKAIDO (1965) the optimal values of the control variables can be globally and uniquely determined in terms of state variables, co-state variables and the set of parameters as

$$z^* = z^*(N, \lambda_N; \chi)$$

$$m^* = m^*(N, \lambda_N; \chi)$$

where  $\chi = \{q, p, v, \zeta, \tau, r, N_0\}$  is the set of parameters.

Equation (12)-(15) will give the optimal paths for  $N, \lambda_N, z$  and  $m$ .

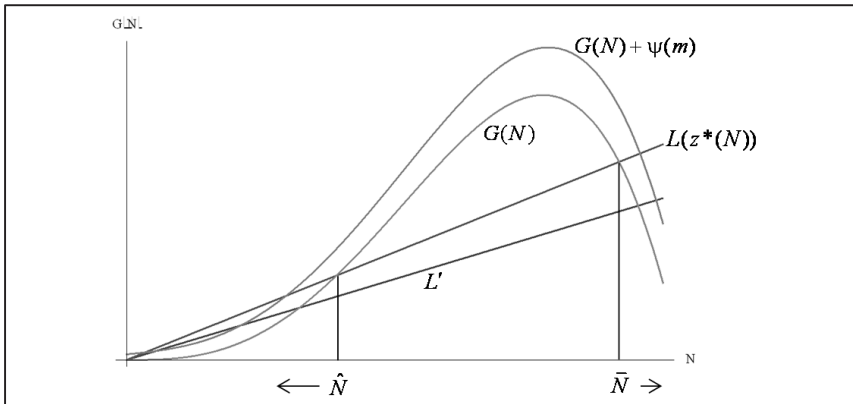
Significant implications follow from the first order conditions. Equations (12) and (13) determine the optimal values of  $z$  and  $m$  and suggest that use of inputs is socially optimal for appropriate choices of  $\zeta$  and  $\tau$ . In section 2, there was no public intervention. As a result, social cost of using  $z$  was ignored by the private agent with the result that there was over use of the input. On the other hand, the resource-preserving input  $m$  was not used because private benefit was less than the price of  $m$  and there was no provision for subsidy to encourage the use of the input. In the current setting, a subsidy encourages the use of  $m$  whereas the tax reduces the use of  $z$ . As a result of public intervention in the form of taxation and subsidies, there will be substitution between  $z$  and  $m$ . The use of  $m$  will reduce the use of  $z$ . The use of  $m$  not

only helps regeneration of soil fertility it also acts as an input in production. Therefore, it will partly replace the use of  $z$  and offset the effect on soil depletion. Besides,  $m$  will have its direct effect on regeneration of soil fertility.

On a sustainable growth path it is required that soil depletion is balanced by its regeneration and the resource is maintained at a constant level. Here,  $z$  and  $m$  are to grow in such a way that  $\dot{N} = 0$ . Here again we have three equilibrium points where  $\dot{N} = 0$  i.e.,  $G(N) + \psi(m) = L(z)$ .

The dynamic implication of this result is that in a non-convex resource system, we will get three such equilibrium (as shown in Fig. 1 in section 2). Out of these three equilibria, one is unstable at  $N = \hat{N}$  and two others are stable at  $N = 0$  and  $N = \bar{N}$ . Hence, optimal policy will determine taxes and subsidies such that equilibrium is reached at  $N = \bar{N}$ . With the use of  $m$  that reduces the use of  $z$ , the depletion curve moves downward and the regeneration curve shifts upward. This is illustrated by the curves  $L'$  and  $G(N) + \psi(m)$  in Figure 2. In that case, the threshold point moves down increasing the resilience of the system. If the effects of the policy measures on soil regenerations and loss reduction are sufficiently strong, the intersection of the two curves can even be avoided making the system fully resilient.

**Figure 2. Due to taxes and subsidies the system is resilient in a larger range**



Source: authors' own drawing

Thus, if  $\dot{N} = 0$  and soil fertility level is maintained at the stable equilibrium  $\bar{N}$ , agricultural growth is sustainable. However, as  $N$  remains constant, it will not be a balanced growth even if  $z$  and  $m$  grow at the same rate. In balanced growth all the related variables grow at the same rate. In the present case, balanced growth requires that

$$g_Q = g_N = g_z = g_m$$

But here,  $g_N = 0$ .

Balanced growth is feasible if the production function exhibits constant returns to scale and the sum of the production elasticities of the inputs is equal to unity i.e.  $\sigma_N + \sigma_z + \sigma_m = 1$  (see HARRINGTON et al., 2005). Here, if we consider a CRS production function and  $N$  is measured in efficiency terms (efficiency follows from exogenous technological progress), we can get sustainable balanced growth. Let us take the production function in Cobb-Douglas form as

$$Q = (TN)^\alpha z^\beta (m)^{1-\alpha-\beta} \quad (16)$$

where  $T$  is a (dynamic) efficiency parameter capturing technological progress.

Taking log in (16) and differentiating w.r.t. time we get

$$\frac{\dot{Q}}{Q} = \alpha \frac{\dot{T}}{T} + \alpha \frac{\dot{N}}{N} + \beta \frac{\dot{z}}{z} + (1-\alpha-\beta) \frac{\dot{m}}{m} \quad (17)$$

$$g_Q = \alpha g_T + \alpha g_N + \beta g_z + (1-\alpha-\beta) g_m \quad (18)$$

using  $g_N = 0$  on a sustainable growth path. So, if there is (exogenous) technological progress at the same rate as  $z$  and  $m$ , we obtain a sustainable balanced growth path. But if technological progress does not take place,  $g_T = 0$ , and then

$$g_Q = \beta g_z + (1-\alpha-\beta) g_m$$

Here even if  $g_z = g_m$ ,

$$g_Q = (1-\alpha) g_z$$

$$\text{i.e., } \frac{g_Q}{g_z} = (1-\alpha) < 1$$

which means, we obtain a growth path with diminishing returns.

#### 4 Comparative Statics and Stability of Equilibrium

After the values of  $z$  and  $m$  are globally and uniquely determined from (12)-(13) we derive the following comparative static results (see Appendix III).

$$\frac{\delta z^*}{\delta N} = 0$$

$$\frac{\partial z^*}{\partial p} = [q \cdot Q_{mm} + \zeta_{mm} + \lambda_N \Psi_{mm}] < 0$$

$$\frac{\partial z^*}{\partial v} = 0, \quad \frac{\partial z^*}{\partial q} = [-Q_z \cdot (q \cdot Q_{mm} + \zeta_{mm} + \lambda_N \Psi_{mm})] > 0$$

$$\frac{\partial m^*}{\partial N} = 0$$

$$\frac{\partial m^*}{\partial v} = [q \cdot Q_{zz} - \tau_{zz} - \lambda_N L_{zz}] < 0$$

$$\frac{\partial m^*}{\partial p} = 0$$

$$\frac{\delta m^*}{\delta q} = [-Q_z \cdot (q \cdot Q_{zz} - \tau_{zz} - \lambda_N L_{zz})] > 0.$$

Our comparative static results have very clear policy implications. The taxation of soil degrading inputs will discourage their use. On the other hand, subsidies can help the use of resource friendly inputs. So, government intervention in the form of taxes and subsidies can help soil conservation and sustainability of agricultural growth. It is also revealed that increase in output price increases the optimal use of  $z$  and  $m$  at each point of time because income rises as a result of it.

##### *Stability of the Steady-State Equilibrium*

In the long-run the state and co-state variables converge to stationary values where  $\dot{N} = 0, \dot{\lambda}_N = 0$ .

In this dynamic optimization problem the state, co-state and control variables are interlinked and optimal paths for their equilibrium values are simultaneously determined with the stationary values of  $N$  and  $\lambda_N$  in the long run.

We check the stability of the equilibrium path by characteristic roots of the Jacobian matrix for the system of the differential equations:



$$\dot{N} = G(N) - L(z) + \psi(m)$$

$$\dot{\lambda}_N = r\lambda_N - qQ_N - \lambda_N G_N$$

We form the Jacobian matrix and evaluate it at  $N = \bar{N}$  with

$\dot{N} = 0, \dot{\lambda}_N = 0$  as

$$J_E = \begin{bmatrix} \frac{\partial \dot{N}}{\partial N} & \frac{\partial \dot{N}}{\partial \lambda_N} \\ \frac{\partial \dot{\lambda}_N}{\partial N} & \frac{\partial \dot{\lambda}_N}{\partial \lambda_N} \end{bmatrix} \quad (\dot{N} = 0, \dot{\lambda}_N = 0).$$

According to CHIANG (1992), the qualitative information about the characteristic roots  $r_1$  and  $r_2$  are needed to confirm a saddle point by the result

$$r_1 r_2 = |J_E|.$$

If  $|J_E|$  is negative, the roots have opposite signs and the equilibrium is locally a stable point.

To show the stable or saddle branches in a phase-diagram the signs of the two isoclines  $\dot{N} = 0$  and  $\dot{\lambda}_N = 0$  can be specified by applying the implicit function rule as in CHIANG (1984) and GOETZ (1997).

In the long run,  $\dot{N} = G(N) - L(z) + \psi(m)$

$$\dot{\lambda}_N = r\lambda_N - qQ_N - \lambda_N G_N$$

And,  $\frac{\partial \dot{N}}{\partial N} = G_N > 0, \frac{\partial \dot{N}}{\partial \lambda_N} = 0$

$$\frac{\partial \dot{\lambda}_N}{\partial N} = -q \cdot Q_{NN} - \lambda_N G_{NN} > 0$$

$$\frac{\partial \dot{\lambda}_N}{\partial \lambda_N} = r - G_N > < 0.$$

The sufficient condition for  $\frac{\partial \dot{\lambda}_N}{\partial \lambda_N} < 0$  is  $G_N > r$

Therefore,  $|J_E| < 0$ . It ensures that the steady state equilibrium is locally stable.

The results in sections 3 and 4, demonstrate how government intervention in the form of taxation, subsidy and technological progress can ensure a dynamically stable sustainable balanced growth in agriculture. However, the model is not closed as we do not require a balanced-budget constraint. Here, technological progress takes place exogenously and its cost is covered by the non-agricultural sector which enjoys the positive externalities of resource management in agriculture. The tax revenue from polluting inputs used in agriculture will usually not equal the subsidy paid for the use of resource and environment-friendly inputs. The deficit may be covered from government's general tax revenues because the ecological benefits of using environment preserving inputs are enjoyed by the whole society. However, a closed model with a budget balancing condition may be considered in further extension of this work.

## 5 Summary Results and Policy Implications

This paper addresses the problem of sustainable agricultural growth and soil degradation consequent upon intensive cultivation and externalities in production. It demonstrates with the help of a theoretical model that if the harvest rate of soil nutrients exceeds its natural regeneration rate, the stock of soil fertility will decline in the absence of any soil treatment measure. So, there will be decay of the resource base making agricultural growth unsustainable in the long-run. Since individual agents in a decentralized system do not take into account the social cost of environmental degradation, resource-damaging inputs will be overused.

This paper argues for public intervention in resource use and suggests that agricultural research for technological innovations may facilitate sustainable agricultural growth. It is suggested that the government can adopt direct incentive payment schemes to encourage the use of resource and environment friendly inputs. Such payments can be justified to the extent that they reduce externality problems and that use of such inputs generates environmental and ecological services to the whole society. If sufficient funds cannot be raised from the agricultural sector, it may be collected from other sectors because the whole society benefits from improved ecological conditions and resource conservation. In fact, agriculture and resource management is financed by the non-agricultural sector in many of the cases.

This paper constructs a growth framework that incorporates an input like green manure or organic fertilizer into the production technology having its twin role of enhancing soil fertility and generating ecological services. It has introduced an incentive payment as subsidy to encourage the use of such a resource and environment friendly input in

production. Agricultural growth becomes sustainable in a steady state path and the equilibrium is dynamically stable under the condition of technological progress. The comparative static results suggest that imposing a tax discourages the use of polluting inputs whereas subsidies encourage the use of resource friendly inputs.

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## Acknowledgement

The authors gratefully acknowledge valuable comments from Professor Sugata Marjit and two anonymous reviewers.

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### Appendix I

Differentiation of (6) to (9) w.r.t.  $z$ , and  $N$  gives

$$\begin{bmatrix} q \cdot Q_{zz} - \lambda_N L_{zz} & q \cdot Q_{zN} \\ q \cdot Q_{Nz} & q \cdot Q_{NN} \end{bmatrix}$$

$|D_1| < 0, |D_2| > 0$  assuming  $Q_{zN} = 0$ .

### Appendix II

Differentiation of (12) to (14) w.r.t.  $z$ ,  $m$  and  $N$

$$\begin{bmatrix} q \cdot Q_{zz} - \tau_{zz} - \lambda_N L_{zz} & q \cdot Q_{zm} & q \cdot Q_{zN} \\ q \cdot Q_{mz} & q \cdot Q_{mm} + \varsigma_{mm} + \lambda_N \Psi_{mm} & Q_{mN} \\ q \cdot Q_{Nz} & q \cdot Q_{Nm} & q \cdot Q_{NN} + \lambda_N G_{NN} \end{bmatrix}$$

So,  $|D_1| < 0, |D_2| > 0, |D_3| < 0$  assuming  $Q_{zm} = Q_{Nm} = Q_{Nz} = 0$

### Appendix III

Total differentiation of (12) to (13) gives

$$A + B \begin{bmatrix} \frac{\partial z^*}{\partial N} & \frac{\partial z^*}{\partial \lambda_N} & \frac{\partial z^*}{\partial p} & \frac{\partial z^*}{\partial v} & \frac{\partial z^*}{\partial q} \\ \frac{\partial m^*}{\partial N} & \frac{\partial m^*}{\partial \lambda_N} & \frac{\partial m^*}{\partial p} & \frac{\partial m^*}{\partial v} & \frac{\partial m^*}{\partial q} \end{bmatrix} = 0$$

$$B = \begin{bmatrix} q \cdot Q_{zz} - \tau_{zz} - \lambda_N L_{zz} & q \cdot Q_{zm} \\ q \cdot Q_{mz} & q \cdot Q_{mm} + \varsigma_{mm} + \lambda_N \Psi_{mm} \end{bmatrix}$$

$$|B| = [(q \cdot Q_{zz} - \tau_{zz} - \lambda_N L_{zz})(q \cdot Q_{mm} + \varsigma_{mm} + \lambda_N \Psi_{mm})]$$

$|B| > 0$  (assuming  $Q_{zm} = Q_{mz} = 0$ )

$$A = \begin{bmatrix} q \cdot Q_{zN} & -L_z & -1 & 0 & Q_z \\ q \cdot Q_{mN} & \psi_m & 0 & -1 & Q_m \end{bmatrix}$$