Estimation and use of risk aversion coefficient for traditional dryland agriculture in western Sudan

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ABSTRACT


This paper explores the importance of including risk when modeling subsistence-oriented agriculture in a developing nation. The specific objective is to measure the degree of risk aversion for typical farmers in the smallholder traditional agriculture of the Sudan. The procedure followed is to impute the farmer's risk aversion coefficient through a mathematical risk programming technique. Imputed farmers' risk aversion coefficients were used to validate the model specified and identify, for further analysis, a single risk optimal farm plan for each of the different farm situations studied.

INTRODUCTION

The study area is the Nuba Mountains, South Kordofan Province in western Sudan. This province is well-endowed with cultivable land. The traditional agriculture of this region is better described as subsistence agriculture combined with a limited amount of cash cropping. Shifting cultivation ('harig'), which involves only the use of hand tools and no fertilization, is commonly practiced. There are two important elements of

1 Harig cultivation is the cultivation of fallow land after the bush is burned.
TABLE 1

Matrix of correlation coefficients

<table>
<thead>
<tr>
<th>Activity</th>
<th>SORGUM</th>
<th>SESAME</th>
<th>SSCOWP</th>
<th>SSESAM</th>
<th>SORCOP</th>
</tr>
</thead>
<tbody>
<tr>
<td>SORGUM (sorghum)</td>
<td>1.0</td>
<td>-0.29</td>
<td>-0.81</td>
<td>0.89</td>
<td>0.79</td>
</tr>
<tr>
<td>SESAME</td>
<td>1.0</td>
<td>-0.17</td>
<td>0.06</td>
<td>-0.21</td>
<td></td>
</tr>
<tr>
<td>SSCOWP (sorghum, sesame, and cowpeas)</td>
<td>1.0</td>
<td>0.71</td>
<td>0.97</td>
<td></td>
<td></td>
</tr>
<tr>
<td>SSESAM (sorghum and sesame)</td>
<td>1.0</td>
<td>0.61</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>SORCOP (sorghum and cowpeas)</td>
<td>1.0</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Coefficient of variation</td>
<td>30.4</td>
<td>41.9</td>
<td>20.5</td>
<td>19.8</td>
<td>24.9</td>
</tr>
</tbody>
</table>

risk in the traditional agriculture, variation in physical output and fluctuation in output prices. These two elements of risk can be combined in an activity gross returns variable. This variable is uncertain to many traditional farmers in the Sudan (Table 1).

REPRESENTATIVE FARMS AND DATA

Representative farms were developed to model the four different farming conditions in the non-mechanized rainfed traditional agriculture of the province. Three sizes of farms represented the pure traditional farming systems: small, 1.68 ha; median 4.20 ha; and large 7.56 ha. A 15.12-ha farm represented the modernized traditional system. Modernized traditional farms participate in a partial mechanization program sponsored by the Nuba Mountains Agricultural Corporation. This is a national government scheme to enhance cotton production and enable small farmers to manage larger areas of food crops than when dependent only on hand cultivation.

Enumerators collected primary data on a bi-weekly basis, for one year, on resource use and costs, cultural practices, yields and commodity prices associated with production alternatives undertaken by farmers. An initial resource base survey was conducted to determine farmers’ resource availabilities, historical enterprise patterns and family consumption levels. The modal levels of these attributes were used for each of the four representative farms.

THE RISK MODEL

Only farm-level risk was considered in this study. Crop yield and price data, to quantify income variability, were based on five years’ data (1981–85) reported by Agricultural Research Corporation, Kadugli Research Station, Sudan Ministry of Agriculture. Yield variability from experiment stations
typically understates that which would be expected from farmers. The use
of annual average price for a region also understates the expected variabil-
ity of price for individual farmers. Thus, the level of income variability
associated with cropping enterprises in this study understates total variabil-
ity, but relationships among enterprises remain unaffected.

The study used a recent adaptation of Hazell's MOTAD model to evaluate
selected risk management strategies under alternative levels of risk aver-
sion. The empirical MOTAD model in matrix form is stated as:

\[
\text{MAX}_{AX} C'X - \lambda K Rd ( = \lambda \sigma )
\]

such that

\[
AX \leq B
\]

\[
DX + Id \geq 0
\]

\[
X, d, \lambda \geq 0
\]

where \( X, A, B \) and \( C \) represent activity levels, resource requirements,
resource availabilities and expectations of gross returns from activities,
respectively; \( D \) is a deviation matrix, the elements of which represent the
difference between observed and expected gross returns of risky activities
included in the model for the 5 years covered in the analysis. The vector \( d \)
represents yearly whole farm absolute negative deviations (from expected
gross returns), summed over all activities of the farm. The elements of \( d \)
\( \{d_1, \ldots, d_5\} \) are summed over all years by \( R \) a row vector of ones to give a
measure of total negative deviation. This sum is multiplied by a constant
\( K \) to transform it into an estimate of the standard deviation of farm
income. Risk for this model is developed under the assumption of
non-stochastic input costs. Gross returns from individual activities were
assumed to follow a normal distribution, thus generating multivariate
normally distributed returns for the household farm plan. Linear regression
was used to adjust gross revenue series for trends similar to Smith (1972).
Gross revenues were regressed on time and regression residuals for each
year, representing unexplained variations, are the entries in the deviation
matrix for years 1 to 5.

Special treatments were given to the livestock production and household
consumption activities (Table 2). Livestock production is a secondary
enterprise in the production domain of the Nuban farmers. Thus, the
livestock enterprise was fixed at the modal level found on the farms by

\[
K = \frac{2}{r \sqrt{r \pi / 2 (r - 1)}}, \text{ where } r \text{ is number of years} \ (r = 5, \ K = 0.56061). \text{ The factors}
\]

outside the square root radical convert total deviation to mean absolute deviation (MAD) and
the square converts the MAD to an estimate of the standard deviation (Simmons and
Pomareda, 1975).
TABLE 2
Partial tableau of the motad model for the traditional small farm

<table>
<thead>
<tr>
<th>Crop production activities (per hectare)</th>
<th>Livestock activity (TLU per farm)</th>
<th>Borrowing activities (per LS)</th>
<th>Household consumption activity</th>
<th>Capital transfer activities</th>
<th>Constraints</th>
</tr>
</thead>
<tbody>
<tr>
<td>SSCOWP</td>
<td>SSESAM</td>
<td>SORCOP</td>
<td>JUBRAK</td>
<td>Salive</td>
<td>PRCRED1</td>
</tr>
<tr>
<td>99.33</td>
<td>79.62</td>
<td>137.31</td>
<td>135.52</td>
<td>-0.98</td>
<td>-0.93</td>
</tr>
</tbody>
</table>

Resources
Land
Operating capital period 1
Operating capital period 2
Operating capital period 3
Housegarden (JUBRAK)
Livestock sales (Salive)
Home consumption (HCONS)
Informal borrowing (BORIN)
End of year cash balance (CENDY)
Farmer's own fund maintenance (FOFM)

SSCOWP, intercropping of sorghum, sesame and cowpeas; SSESAM, intercropping of sorghum and sesame; SORCOP, intercropping of sorghum and cowpeas; JUBRAK, (Jubraka) mixed cropping of maize, sorghum, groundnuts and vegetables.
Ls, Sudanese pound = US$0.40, official exchange rate for 1984.
TLU, Tropical Livestock Unit = 250 kg meat.
PRCRED, informal credit at 98% interest rate.
holding constant the livestock activity \((\text{sALIVE})\). The livestock activity uses labor and contributes cash to the resource base during the periods in which livestock are typically sold. The lack of data on variability of income associated with livestock resulted in the assumption of no income variance arising from the livestock activity. The household consumption activity was expressed in monetary terms because of inadequate information to develop a caloric intake requirement. An explicit consumption activity in conjunction with a consumption constraint requires a given seasonal pattern of yearly consumption expenditure by the farm family. For example, annual household consumption for the small farm was required to be Ls 880.2 \((73.35 + 220.05 + 586.80)\), the average annual expenditure for household consumption by the small traditional households (Table 2).

The model constrains the solution to require the farmer to at least restore his initial operating capital stock each year. This requires all obligations, including informal borrowing, be retired on an annual basis and that the farm unit have the same or greater operating capital for the next agricultural season.

The farm decision model maximizes returns to fixed resources, subject to the disutility associated with negative risk deviations of gross returns. The behavioral assumption underlying the objective function of the model is that traditional producers in a semi-subsistence agriculture are largely utility rather than profit maximizers. The primary goal of the subsistence farmer is to assure at least a minimum level of household consumption. To capture this traditional goal of the farmer, the objective function is formulated in a way that maximizes the farmer’s expected utility subject to a fixed consumption requirement, resource constraints and attitudes toward risk. More specifically, the model maximizes the farmer’s certainty equivalent in the form of expected income discounted by a risk premium. The expected utility function of the farmer was assumed to be linear in expected gross returns from cropping and negative deviations from these expectations. The farmers were assumed to display risk-averse behavior that maximized expected income and standard deviation \([E, \sigma]\) utility. Their risk aversion attitude is measured by \(\lambda\), the tradeoff between expected income and risk. Within this framework, a crop mix (or farm plan) has the desired effect for the farmer if it reduces the standard deviation of the resulting income.

**RISK AVERTION COEFFICIENT \((\lambda)\)**

The risk aversion coefficient is an average risk attitude measure which orders individual risk averters according to their willingness to accept risk. It is an average measure since it may not clearly distinguish between risk aversion in the small and in the large (Robison and Barry, 1986). That is, it
tells that a class of decision makers is less (more) risk averse than another class. A λ was established for the study farm risk models for two reasons. First, it was needed to validate the models by comparing the estimated λ values with those found in previous studies. The second and most important reason for establishing the λ was to identify for further analysis a single risk-optimal farm plan for each farm situation. Ideally, the method of identifying the risk-optimal farm plan for a sampled farmer would be to elicit his or her utility function and determine the tangency point between his function's expected utility curve and the relevant risk-efficient frontier, e.g. E–D in Fig. 1. An alternative method for identifying the risk optimal farm plan is to present the farmer with all plans contained in the risk-efficient frontier and let the farmer choose the preferred plan. However, both criteria seemed irrelevant for farmers in rural Sudan with high rates of illiteracy and ignorance of income probabilities and their associated distributions. Consequently, the procedure used in this study was considered the most appropriate for the λ used for each type of farm.

RESULTS AND DISCUSSION

Since the actual consumption level of the household was unknown, it was fixed at the maximum feasible level allowed by the risk-neutral model for each size of farm (the highest possible consumption level). This maximum level was a fraction (or whole) of the annual consumption expenditures reported by farm families. After maximum consumption was deter-

![Fig. 1. Hypothetical optimal E-D farm plan.](image-url)
determined, the risk aversion coefficient \( \lambda \) was increased parametrically from zero to derive a set (frontier) of risk-efficient LP optimal farm plans. Farm plans (MOTAD solutions) were printed at both change of basis and intervals of 0.5 \( \lambda \) within a range of 0 to 5. This range embraced all \( \lambda \) values encountered in earlier reviewed studies. The farmer's risk aversion coefficient \( \tilde{\lambda} \) was then identified for each farm situation. The value of \( \tilde{\lambda} \) was estimated using a single measure of goodness of fit. The measure is the mean absolute deviation (MAD) of the solution values (cropping activity basis) from their counterparts' land area allotted to the actual mix of cropping activities grown by the sampled farmers during the year of the survey. The value of \( \lambda \) at the minimum MAD was chosen as the \( \lambda \) coefficient for each farm situation. In case of a tie, (MAD\(^2\)) was used instead of MAD (Table 3). The single farm plan, identified by \( \tilde{\lambda} \) is risk optimal while other plans, though risk-efficient are not risk-optimal.

Although \( \lambda \) was parameterized from 0 to 5, the choice of \( \tilde{\lambda} \) was limited to the range 0.67 to 2.68 (Thell, 1971). Hazell and Norton (1986, p. 91) pointed out that, when income is normally distributed, a \( \lambda \) value of 1.65 corresponds to a one-tail, 5% confidence test on selected hypotheses. Following the same argument, \( \lambda \) values of 0.67 and 2.68 corresponds to 25% (highest) and 0.5% (lowest) confidence limits on extreme income fractiles for a normal distributed farm income (Thell, 1971). A 25% income fractile is the value of income which will be exceeded 75% of the time. Similarly, the 0.5% income fractile will be exceeded 99.5% of the time. To explain the above argument in a risk context, assume there are two farmers, the first with a \( \lambda \) of 0.67 and the second with a \( \lambda \) of 2.68. The first farmer will not accept a risky farm plan unless he is 75% confident that the plan will equal or exceed its expected income. The latter farmer requires a 99.5% assurance before he will accept the same risky farm plan. The farmer with a \( \lambda \) of 0.67 is said to be less risk-averse than a farmer with a \( \lambda \) of 2.68. These figures illustrate why values greater than 2.68 were ignored in the present study.

**IMPUTED FARMER'S RISK AVersion COEFFICIENT (\( \tilde{\lambda} \))**

The empirical MOTAD model of the study imputed \( \tilde{\lambda} \) values of 1.93, 1.50 and 2.54 for small, median and large traditional farmers and 2.50 for the modernized traditional farmers (Table 4). These \( \tilde{\lambda} \) values fall toward the maximum value of the theoretical range specified in the last section. These high values of \( \lambda \) agree with the economic reasoning that small farmers are highly averse to risk. Purely traditional farmers (median farm) indicated a willingness to accept a risky farm plan only when they were at least 93% confident that the plan would generate an income greater or equal to its
TABLE 3
Selected crop production plans for the traditional small farm with alternative levels of risk aversion coefficients

<table>
<thead>
<tr>
<th>Item</th>
<th>Risk aversion coefficient [λ]</th>
<th>Actual farm plan</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>0.0</td>
<td>1.70</td>
</tr>
<tr>
<td>Farm income (Ls)</td>
<td>230.0</td>
<td>218.73</td>
</tr>
<tr>
<td>Total negative deviation (Ls)</td>
<td>78.71</td>
<td>68.26</td>
</tr>
<tr>
<td>Standard deviation (Ls)</td>
<td>44.13</td>
<td>38.27</td>
</tr>
<tr>
<td>Risk premium (Ls)</td>
<td>0.0</td>
<td>65.06</td>
</tr>
<tr>
<td>Coefficient of variation (Ls)</td>
<td>19.19</td>
<td>17.50</td>
</tr>
<tr>
<td>Cropping pattern (ha)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>sscowp (sorghum, sesame, and cowpeas)</td>
<td>0.0</td>
<td>0.0</td>
</tr>
<tr>
<td>sesam (sorghum and sesame)</td>
<td>0.0</td>
<td>0.20</td>
</tr>
<tr>
<td>sorcop (sorghum and cowpeas)</td>
<td>1.30</td>
<td>1.10</td>
</tr>
<tr>
<td>jubra (Jubraka or mixed cropping)</td>
<td>0.38</td>
<td>0.38</td>
</tr>
<tr>
<td>MAD</td>
<td>0.52</td>
<td>0.43</td>
</tr>
<tr>
<td>MAD²</td>
<td>0.475</td>
<td>0.363</td>
</tr>
</tbody>
</table>

Farm income = expected income from cropping net of costs of hired labor and interest on borrowed funds.
MAD = mean absolute deviation of the solution values from actual cropping pattern values:

\[ \text{MAD} = \sum_{i} |A_i - A_i^m|/n \quad i = 1, 2, \ldots, n \]

where

- \( A_i \) actual hectares of cropping activity \( i \) planted during the base period of 1984/85
- \( A_i^m \) model solution values for \( i \)
- \( \text{MAD}^2 \) sum of squared deviations.

expected income. Likewise, the modernized traditional farmers required a higher confidence level of 99% before acceptance of the risky farm plans. Previous work imputed average \( \lambda \)s of 1.5 for Mexican irrigated agriculture (Hazell et al., 1983), 3.28 for small farmers in north Brazil (Goodwin et al., 1980), 2.0 for groundnut producers (Nieuwoudt et al., 1976) and only 0.25 for Indiana cornbelt farmers in the United States (Brink and McCarl, 1978). Considering the above-mentioned \( \lambda \)s from previous studies, the range of 1.5 to 2.54 found in this work is seen as a reasonable validation of the farm models developed for traditional agriculture in Sudan.

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3 A one-tail hypothesis test was applied to the estimated \( \lambda \) values. The forms of farm income probabilities are \( \Pr (Y_i < Y_0) = 0.07 \) for pure traditional farmers and 0.01 for modernized traditional farmers; \( Y_i \) is the expected income from the risky farm plan and \( Y_0 \) is the required income of a risk-averse farmer.
TABLE 4
Comparison between risk-neutral and risk-optimal farm plans of the smallholder agriculture in Sudan

<table>
<thead>
<tr>
<th>Item</th>
<th>Pure traditional farms</th>
<th></th>
<th>Modernized farm</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>small</td>
<td>median</td>
<td>large</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Risk aversion coefficient</td>
<td>0.0</td>
<td>1.93</td>
<td>0.0</td>
</tr>
<tr>
<td>Farm income (Ls)</td>
<td>230.00</td>
<td>217.04</td>
<td>625.22</td>
</tr>
<tr>
<td>Total negative deviation (Ls)</td>
<td>78.71</td>
<td>66.7</td>
<td>203.19</td>
</tr>
<tr>
<td>Standard deviation (Ls)</td>
<td>44.13</td>
<td>37.39</td>
<td>113.92</td>
</tr>
<tr>
<td>Risk premium (Ls)</td>
<td>0.0</td>
<td>72.16</td>
<td>0.0</td>
</tr>
<tr>
<td>Coefficient of variation (Ls)</td>
<td>19.19</td>
<td>17.23</td>
<td>18.22</td>
</tr>
<tr>
<td>Cropping pattern (ha)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>SSCOWP (sorghum, sesame, and cowpeas)</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
</tr>
<tr>
<td>SSESAM (sorghum and sesame)</td>
<td>0.0</td>
<td>0.23</td>
<td>0.0</td>
</tr>
<tr>
<td>SORCOP (sorghum and cowpeas)</td>
<td>1.30</td>
<td>1.07</td>
<td>1.20</td>
</tr>
<tr>
<td>SORGUM (sorghum)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>SESAME</td>
<td>1.22</td>
<td>1.16</td>
<td>4.46</td>
</tr>
<tr>
<td>COTTON</td>
<td>2.36</td>
<td>1.22</td>
<td>2.15</td>
</tr>
<tr>
<td>COWPEA</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>JUBRAK (Jubraka, mixed cropping)</td>
<td>0.38</td>
<td>0.38</td>
<td>0.62</td>
</tr>
<tr>
<td>Percent of income sacrificed</td>
<td>0.0</td>
<td>5.3</td>
<td>0.0</td>
</tr>
</tbody>
</table>

N, risk-neutral.
O, risk-optimal.
RISK-AVERSE BEHAVIOR OF THE SAMPLED FARMERS

Assuming rationality, farmers will grow more of a crop if it has high expected returns, low variance (or absolute negative deviation), and low positive covariance with the returns of all other activities. By contrast, a cropping activity may prove attractive if its returns are negatively correlated with other enterprises in the farm plan even though it is risky in terms of its own negative deviation (or variance). Thus, a tradeoff between expected returns and risk is inevitable. Farmers faced with such choice will rank their enterprises according to expected returns and their willingness to accept the associated risk. This tradeoff between farm returns and farm level risk (total negative deviation) is illustrated in Fig. 2 for the different farm situations studied. The producer attains a higher risk-efficient frontier as the size of the farm he operates increases, given the present level of technology. This means that for a given risk level expected income increases as the farm size increases in the smallholder dryland agriculture in Sudan.

Table 4 presents selected farm plans from the expected income-negative deviation (E–D) frontier derived for each farm situation. These production plans contrast risk-neutral and risk-optimal farm plans and the incomes from these two situations. If risk had not been incorporated in the farm models, farm income would be overestimated. This behavioral relationship is consistent with risk theory. Risk averse farmers typically prefer farm plans that provide a satisfactory level of security, even if this means

![Fig. 2. Risk efficiency frontiers for the four farm sizes under conditions of present technology and resources availability.](image-url)
sacrificing some income (Dillon and Scandizzo, 1978; Binswanger, 1980). The sacrifices in farm income ranged from 5% to 9% for the sampled farmers. These reductions in farm income are due to the fact that risk optimal farm plans at the estimated \( \lambda \) coefficients involved fewer hectares of the high-income but risky enterprises. Risk-neutral \((\lambda = 0)\) farmers typically specialized in one or two enterprises, given that the size of the household garden (Jubraka) was fixed. Risk was highest in the risk-neutral plans in terms of negative deviation, standard deviation and coefficient of variation for all farm situations. When farmers were assumed to be risk averters \((\lambda > 0)\) they hedged against risk by either diversifying into a greater number of enterprises or avoiding a cropping activity with large negative deviation in favor of those with lower negative deviations. The risk-averse behavior is shown by changes in enterprise mix as the risk aversion coefficient was increased from zero to respective farmer’s \( \lambda \). Cropping activities experiencing declining hectares as \( \lambda \) increased either had large variances and/or large positive covariances with all other activities. Under the present level of technology the intercropping activity, sorcolp was the most unfavorable one in terms of expected returns–risk tradeoff among the cropping alternatives available to the traditional small farmers. sorcolp hectares declined from 1.30 ha under risk-neutral condition \((\lambda = 0)\) to 1.07 ha at \( \lambda = 1.93 \). sesame was the riskiest enterprise among the crops grown by traditional median and modernized farmers. For instance, sesame hectares declined from 2.36 \((\lambda = 0)\) to 1.22 ha at \( \lambda = 1.50 \) for traditional median farm, and from 14.88 \((\lambda = 0)\) to 4.98 ha at \( \lambda = 2.50 \) for the modernized farm. Non-inclusion of cotton in the present enterprise mix of modernized farms is attributable to both low expected returns and high variability of gross returns from the cotton enterprise. On the other hand, the apparent increase of cowpea by modernized farmers from zero hectares at the risk-neutral solution to 0.95 hectares at \( \lambda = 2.50 \) is a direct reflection of this crop’s low variability of income and/or its negative covariance with the returns of other crops.

CONCLUSIONS

This paper has demonstrated the importance of incorporating risk when modeling traditional agriculture in developing countries. It revealed that farm plans will include too many high risk–high income enterprises and that farm incomes will be overestimated if risk is not included in subsistence farm models. The portion of farm income sacrificed reached up to 9% in this study. The risk aversion coefficient \( \lambda \) imputed in the study
ranged from 1.50 to 2.54 for the smallholder traditional farmers in western Sudan. These risk aversion coefficients can be used to validate future models or directly used as risk parameters for programming models to be developed for the traditional agriculture in Sudan. Finally, it is worth noting that the risk aversion coefficients developed by this programming method are sensible to errors in model specification.

REFERENCES


